I. Flight dynamics.

Equations of motion for winged aircraft. Longitudinal (LONG) and lateral (LAT) dynamics. Linearized equations. Dynamical analysis, modes, natural motions. Typical unstable zeros (non-minimum phase) in selected channels.

Linearized equations of flight dynamics

- equations of motion for rigid body aircraft
  - set of nonlinear ODE’s (forces equations, moments equations)
  - parametrized by mass characteristics of the A/C (m, Jx, Jxz, ...) as well as aerodynamics (by means of aerodynamic coefficients, clalpha ...)
  - directly useful for simulations (computer games ☺, flight trainers, flight simulators ... and ... validation of control laws)
  - not directly usable for development of control laws as the design methods rely on the linear systems and control theory
  - the nonlinearities are not “ugly”, they are “the nice ones” (compared to e.g. friction, saturations, hystheresis models, various switching dynamics etc.)
  - they are coming from the geometric transformations (goniometric functions of attitude angles – for gravity-induced forces and moments – and alpha/beta – for aerodynamic forces) and describing functions of aerodynamic coefficients (often considered as linearly dependant on flight variables, by means of aerodynamic coefficients)
  - for this reason, the linear systems and control tools are an attractive and viable option for FCS design (after local linearization or, if not sufficient, exact linearization)

Linearized equations of flight dynamics

- linearized equations of motion for rigid body aircraft

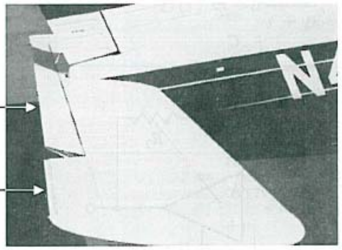
  - local linearization is considered, at trimmed conditions
  - for given flight equations a set of linearizations is delivered typically to cover important flight envelope cases (altitude, Mach number, mass and moments of inertia etc.) and/or flight regimes (steady level flight, steady climb, steady descent etc.)
  - linear control laws designed (controllers = LTI systems), augmented with a switching mechanism (gain scheduling)
  - in addition, the linear equation provide further insight into the flight dynamics analysis (e.g. stability and damping of oscilatory modes, time constants etc.)
  - technically, the aerodynamics coefficients, mass characteristics and engine/thrust parameters are filled in the state-space matrices (A,B, LONG, LAT)

  - longitudinal and lateral dynamics considered as separated (assumptions ...)
  - no “theoretical” developments needed in fact as the formulas in analytic forms were developed decades ago

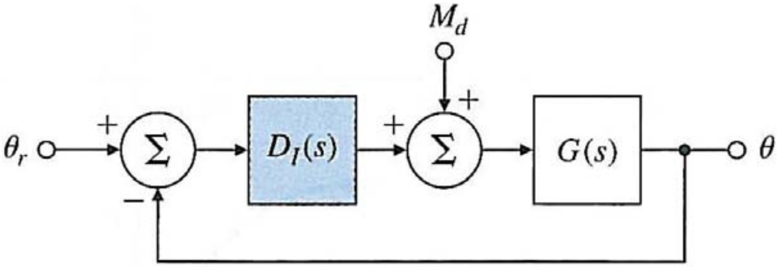
Linearized equations of flight dynamics

- trimming of aircraft – example
  - adopted from Franklin et.al., Feedback control of dynamic systems
  - LONG dynamics. inputs – elevator + trim tab
  - trim tab – set in order to take-over the steady-state deflection of elevator, e.g. at constant disturbance (modelled as constant disturbing moment), to save servos, mechanical hinge, ...)

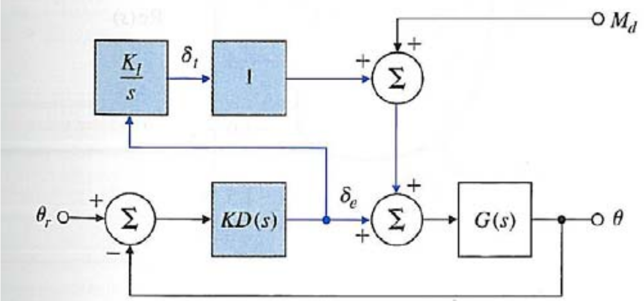
(a)



(b)



$$D_I(s) = K\,D(s)\left(1 + \frac{K_I}{s}\right)$$



(a)

Figure 5.39 Block diagram showing the trim-command loop

Linearized equations of flight dynamics

- linearized equations of LONG motion for rigid body aircraft

	a_{ij}			c_{ij}	
i\j	1	2	3	1	2
1	$-\frac{c_x^v}{A} = -\frac{1}{m}X^v$	$-\frac{c_x^\alpha}{A} = -\frac{1}{mu_0}X^\alpha$	$\frac{c_x^\theta}{A} = \frac{g}{u_0}\cos\theta_0$	$\frac{c_x^{\delta_T}}{A} = \frac{1}{mu_0}X^{\delta_T}$	$\frac{c_x^{\delta_v}}{A} = \frac{1}{mu_0}X^{\delta_v}$
2	$-\frac{c_z^v}{A} = -\frac{1}{m}Z^v$	$-\frac{c_z^\alpha}{A} = -\frac{1}{mu_0}Z^\alpha$	$\frac{c_z^\theta}{A} = \frac{g}{u_0}\sin\theta_0$		$\frac{c_z^{\delta_v}}{A} = \frac{1}{mu_0}Z^{\delta_v}$
3	$-\frac{m_y^v}{C} = -\frac{u_0}{I_y}M_y^v$	$-\frac{m_y^\alpha}{C} = -\frac{1}{I_y}M_y^\alpha$	$-\frac{B}{C}m_y^{\dot\theta} = -\frac{1}{I_y}M_y^{\dot\theta}$		$\frac{m_y^{\delta_v}}{C} = \frac{1}{I_y}M_y^{\delta_v}$
a_{30}		$-\frac{B}{C}m_y^{\dot\alpha} = -\frac{1}{I_y}M_y^{\dot\alpha}$			
	$A = \frac{mu_0}{Sq}, B = \frac{l}{2u_0}, C = \frac{I_y}{Sql_{SAT}}$				

linear ODEs

$$\dot v + a_{11}v + a_{12}\alpha + a_{13}\theta = c_{11}\delta_T$$

$$a_{21}v + \dot\alpha + a_{22}\alpha - \dot\theta + a_{23}\theta = c_{22}\delta_V$$

$$a_{31}v + a_{30}\dot\alpha + a_{32}\alpha + \ddot\theta + a_{33}\dot\theta = c_{32}\delta_V$$

state-space description

$\mathbf{x}^{\mathbf{T}} = [\mathrm{v}, \alpha, \theta, \theta'] = [\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4]$

$\mathbf{u}^{\mathbf{T}} = [\delta_T, \delta_V]$ *(thrust, elevator)*

$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

$$\mathbf{A} = \begin{bmatrix} -a_{11} & -a_{12} & -a_{13} & 0 \\ -a_{21} & -a_{22} & 0 & 1 \\ 0 & 0 & 0 & 1 \\ a_{21}a_{30} & a_{22}a_{30} - a_{32} & 0 & -(a_{33} + a_{30}) \end{bmatrix}; \mathbf{B} = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \\ 0 & 0 \\ 0 & c_{32} - c_{22}a_{30} \end{bmatrix}$$

(adopted from Pech-Věk, Systémy řízení letu, skripta ČVUT)

Linearized equations of flight dynamics

- linearized equations of LAT motion for rigid body aircraft

	b_{ij}			d_{ij}	
i\j	1	2	3	1	2
1 b_{10}	$-\frac{c_y^{\beta}}{A} = -\frac{1}{mu_0}Y^{\beta}$	$-\frac{c_y^{\phi}}{A} = -\frac{g}{u_0}\cos\theta_0$ $-\frac{Bc_y^{\dot\phi}}{A} = -\frac{1}{mu_0}Y^{\phi'}$	$-\frac{B}{A}c_y^{\dot\psi} = -\frac{1}{mu_0}Y^{\dot\psi}$	$\frac{c_x^{\delta_T}}{A} = \frac{1}{mV}X^{\delta_T}$	$\frac{c_y^{\delta_S}}{A} = \frac{1}{mu_0}Y^{\delta_S}$
2	$-\frac{m_x^{\beta}}{D} = -\frac{1}{I_x}M_x^{\beta}$	$-\frac{B}{D}m_x^{\dot\phi} = -\frac{1}{I_x}M_x^{\dot\phi}$	$-\frac{B}{D}m_x^{\dot\psi} = -\frac{1}{I_x}M_x^{\dot\psi}$ $b_{20} = -\frac{E}{D} = \frac{-I_{zx}}{I_x}$	$\frac{m_x^{\delta_k}}{D} = \frac{1}{I_x}M_x^{\delta_k}$	$\frac{m_x^{\delta_s}}{D} = \frac{1}{I_x}M_x^{\delta_s}$
3	$\frac{m_z^{\beta}}{F} = -\frac{1}{I_z}M_z^{\beta}$	$\frac{B}{F}m_z^{\dot\phi} = -\frac{1}{I_z}M_z^{\dot\phi}$ $a_{30} = -\frac{E}{F} = \frac{-I_{zx}}{I_z}$	$-\frac{B}{F}m_z^{\dot\psi} = \frac{-1}{I_z}M_z^{\dot\psi}$	$\frac{m_z^{\delta_k}}{F} = \frac{1}{I_z}M_z^{\delta_k}$	$\frac{m_z^{\delta_s}}{F} = \frac{1}{I_z}M_z^{\delta_s}$
	$A = \frac{mu_0}{Sq}, B = \frac{l}{2u_0}, D = \frac{I_x}{Sql}, E = \frac{I_{xz}}{Sql}, F = \frac{I_z}{Sql}, b_{20} = -\frac{E}{D}, b_{30} = -\frac{E}{F}$				

$$\dot\beta + b_{11}\beta + b_{12}\phi - \dot\psi = d_{12}\delta_S$$

$$b_{21}\beta + \ddot\phi + b_{22}\dot\phi + b_{20}\ddot\psi + b_{23}\dot\psi = d_{21}\delta_K + d_{22}\delta_S$$

$$b_{31}\beta + b_{30}\ddot\phi + b_{32}\dot\phi + \ddot\psi + b_{33}\dot\psi = d_{31}\delta_K + d_{32}\delta_S$$

(adopted from Pech-Věk, Systémy řízení letu, skripta ČVUT)

Linearized equations of flight dynamics

- linearized equations of LAT motion for rigid body aircraft

$$\mathbf{A} = \begin{bmatrix} -b_{11} & -b_{12} & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & D & 0 & 1 \\ -\dfrac{b_{21}-b_{20}b_{31}}{(1-b_{20}b_{30})} & 0 & 0 & -\dfrac{b_{22}-b_{20}b_{32}}{(1-b_{20}b_{30})} & -\dfrac{b_{23}-b_{20}b_{33}}{1-b_{20}b_{30}} \\ -\dfrac{b_{31}-b_{30}b_{21}}{1-b_{20}b_{30}} & 0 & 0 & -\dfrac{b_{32}-b_{30}b_{22}}{1-b_{20}b_{30}} & -\dfrac{b_{33}-b_{30}b_{23}}{1-b_{20}b_{30}} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} d_{11} & d_{12} \\ 0 & 0 \\ 0 & 0 \\ \dfrac{d_{21}-b_{20}d_{31}}{1-b_{20}b_{30}} & \dfrac{d_{22}-b_{20}d_{32}}{1-b_{20}b_{30}} \\ \dfrac{d_{31}-b_{30}d_{21}}{1-b_{20}b_{30}} & \dfrac{d_{32}-b_{30}d_{22}}{1-b_{20}b_{30}} \end{bmatrix}$$

$\mathbf{x}^{\mathrm{T}} = [\beta, \phi, \psi, \phi', \psi']$

$\mathbf{u}^{\mathrm{T}} = [\delta_{K}, \delta_{S}]$

(ailerons, rudder)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

(adopted from Pech-Věk, Systémy řízení letu, skripta ČVUT)

Dynamical modes of LONG and LAT motions

- linearized equations used
  - analysis of eigenvalues and eigenvectors of state-space matrices
  - corresponding physics insight
  - mid-sized transportation aircraft linearized models used in this presentation
  - LONG motion, $\mathbf{x}^T = [v, \alpha, \theta, \theta'] = [x_1, x_2, x_3, x_4]$ $\mathbf{u}^T = [\delta_T, \delta_V]$ *(thrust, elevator)*

```
A =
   -0.0460    0.1330   -0.2200         0
   -0.0990   -0.8950         0    1.0000
         0         0         0    1.0000
    0.0788   -3.3256         0   -2.3120

B =
    0.4000         0
         0    0.0990
         0         0
         0    3.7632
```

Dynamical modes of LONG and LAT motions

- linearized equations used
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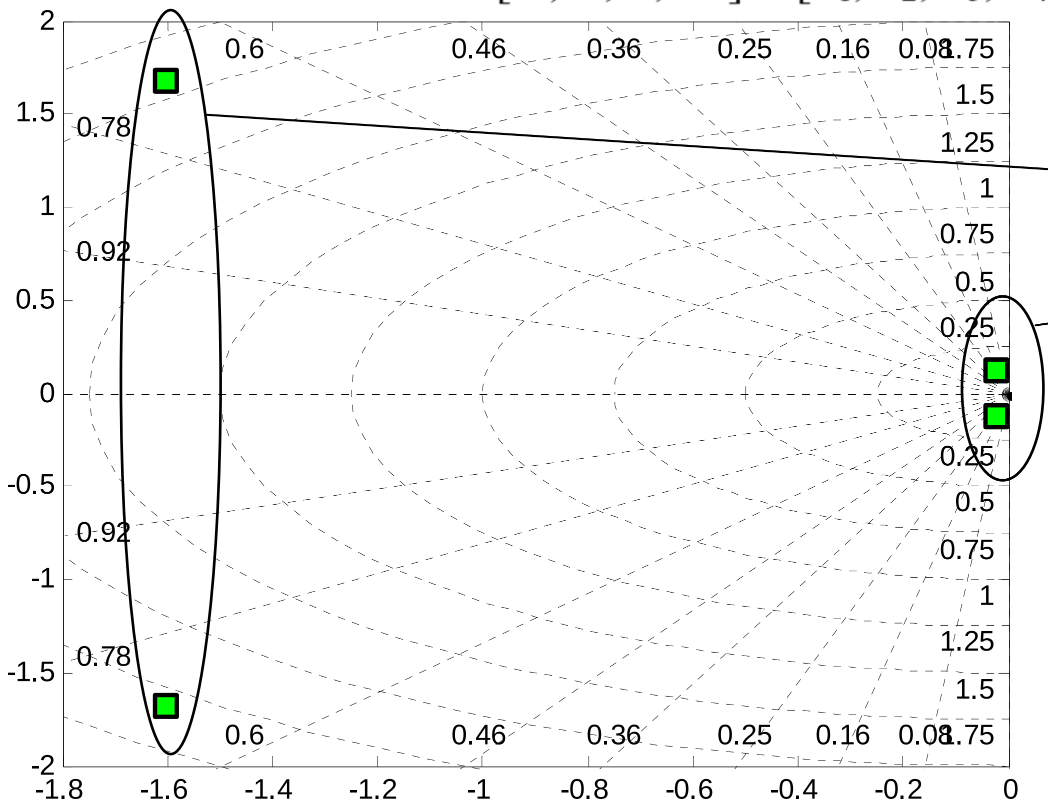
  - LONG motion, $\mathbf{x}^T = [v, \alpha, \theta, \theta']=[x_1, x_2, x_3, x_4]$



two oscilatory modes.

freq. ~ 2rad/sec
damp.rat. ~ 0.7

freq. ~ 0.2rad/sec
damp.rat. ~ 0.1

short period mode (?)

phugoid mode (?)

Dynamical modes of LONG and LAT motions

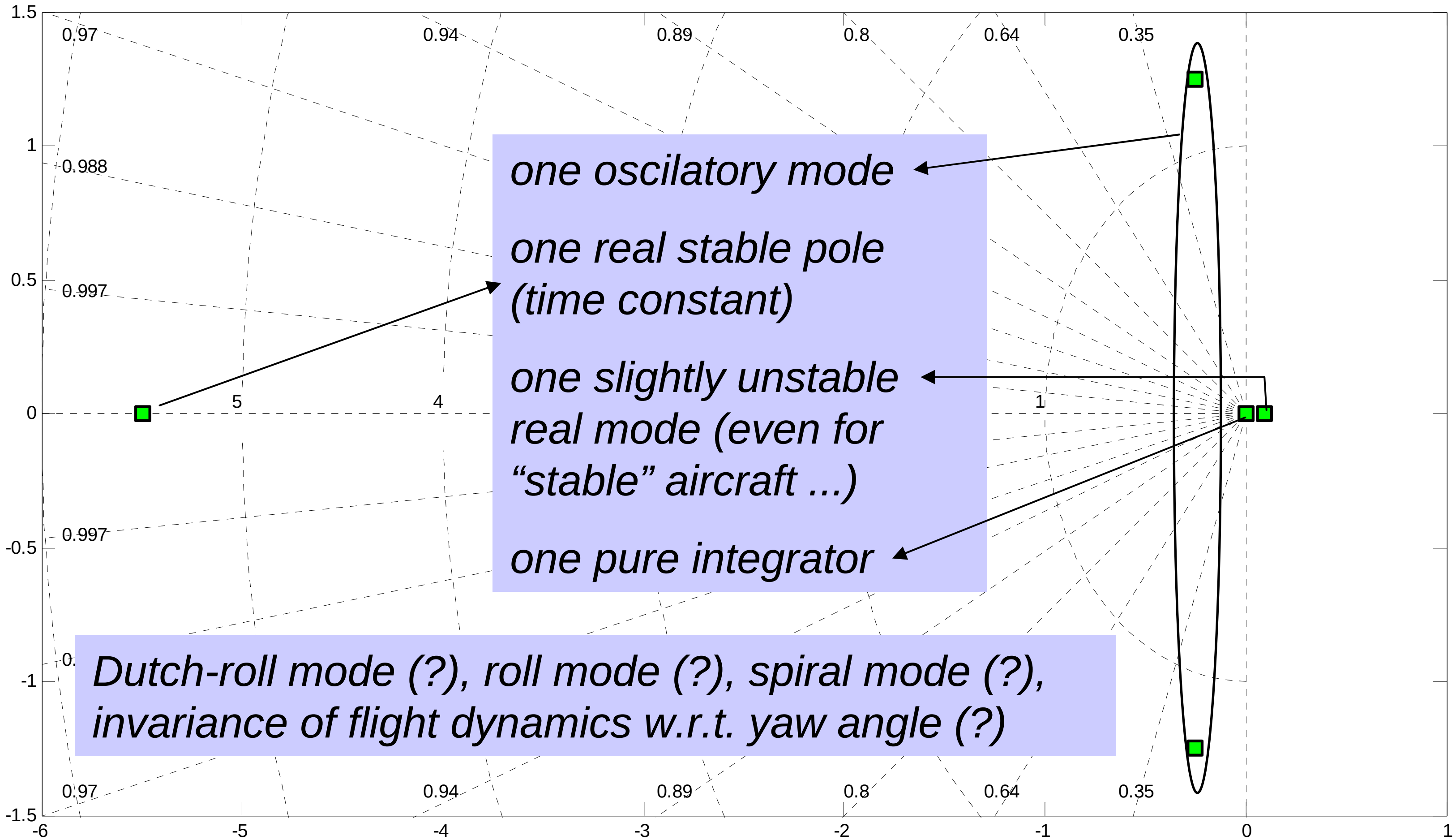
- linearized equations used
  - LAT motion, $\mathbf{x}^T = [\beta, \phi, \psi, \phi', \psi']$ $\mathbf{u}^T = [\delta_K, \delta_S]$ *(ailerons, rudder)*

```
A =
   -0.1460    0.2200         0         0    1.0000
         0         0         0    1.0000         0
         0         0         0         0    1.0000
   -1.8650         0         0   -5.0850   -2.6880
   -1.8700         0         0   -0.8590   -0.6730

B =
         0    0.0430
         0         0
         0         0
    8.5220    0.2920
    0.8370    1.7280
```

Dynamical modes of LONG and LAT motions

- linearized equations used
  - LAT motion, $\mathbf{x}^T = [\beta, \phi, \psi, \phi', \psi']$



Dutch-roll mode (?), roll mode (?), spiral mode (?), invariance of flight dynamics w.r.t. yaw angle (?)

Dynamical modes of LONG and LAT motions

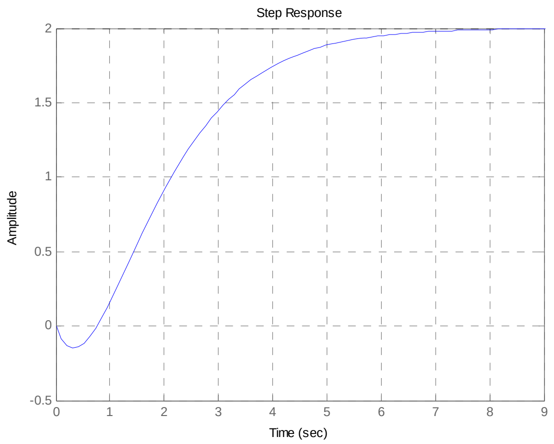
- non-minimum phase effects in LONG and LAT motions

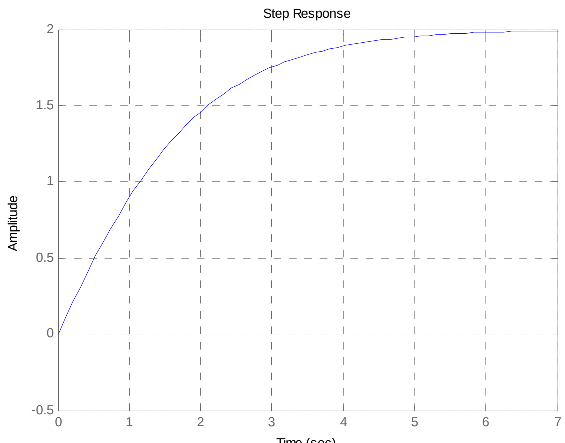
  - formally: unstable (positive-real-part) zero appears in related transfer function
  - step response: starts in the “wrong direction”, after some time goes correct ☺
  - physics insight: two effects are fighting each other after an actuator is operated: a faster and weaker effect if defeated in the end by a slower but stronger reaction
  - unlike system poles (and related dynamical modes), zeros are not “set in stone”, they are not inherent to the A/C construction
  - zeros of a channel (a TF) are determined by the arrangement of actuators and sensors

  - IMPORTANT: one can ***never*** achieve faster CL response than the unstable zeros dictate

    (compare to unstable poles – posing actually reversal CL performance requirements ...)

  - compare (a) G(s) = (2-s)/(s+1)^2 and (b) G(s) = (2+s)/(s+1)^2 step responses:





In the first picture, a 2nd mode (oscilatory) might seem to be present. No. Mind the control design consequences / differences: Is the achievable CL bandwidth restricted for system (a)? And (b)?

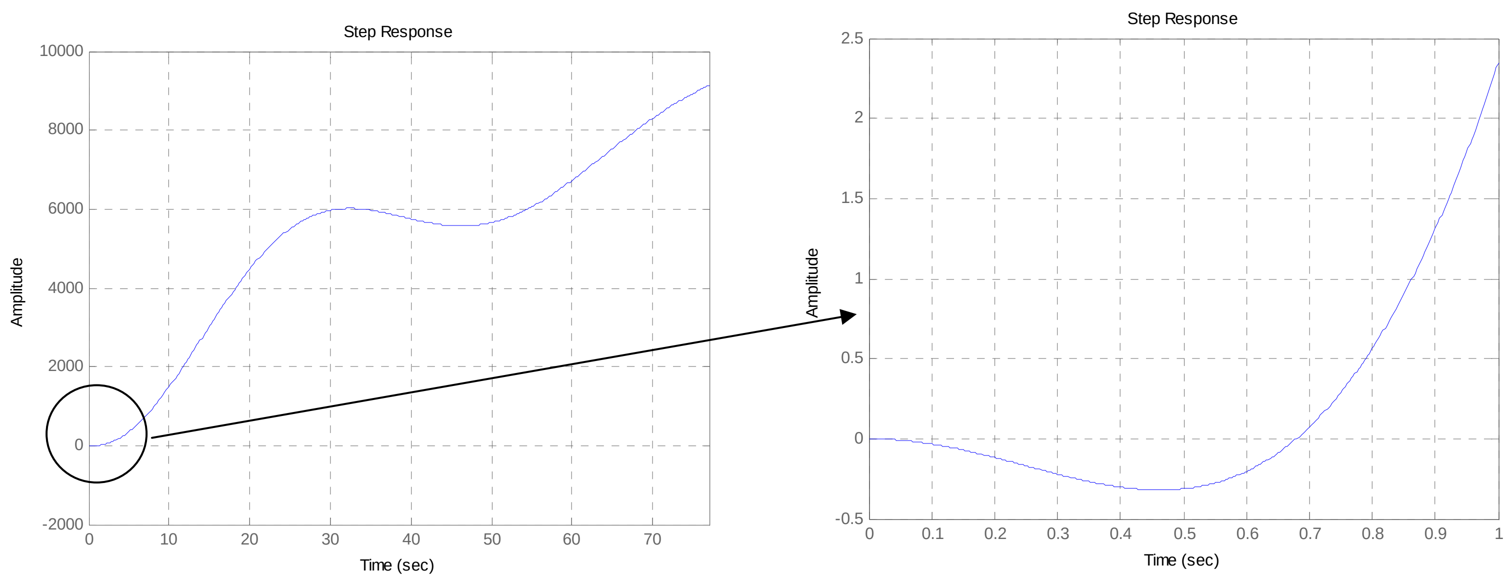
Dynamical modes of LONG and LAT motions

- non-minimum phase effects in LONG and LAT motions

similarly: response EL -> vertical speed:

```
>> sysHdot = ss([A zeros(4,1);0 -70 70 0 0],[B;0 0],eye(5),[]);
>> zpk(sysHdot(5,2))
Zero/pole/gain:
               -6.93 (s+6.82) (s-4.497) (s+0.03514)
--------------------------------------------------
s (s^2 + 0.04318s + 0.01628) (s^2 + 3.21s + 5.401)

physics-insight explanation ... ? ☺
```



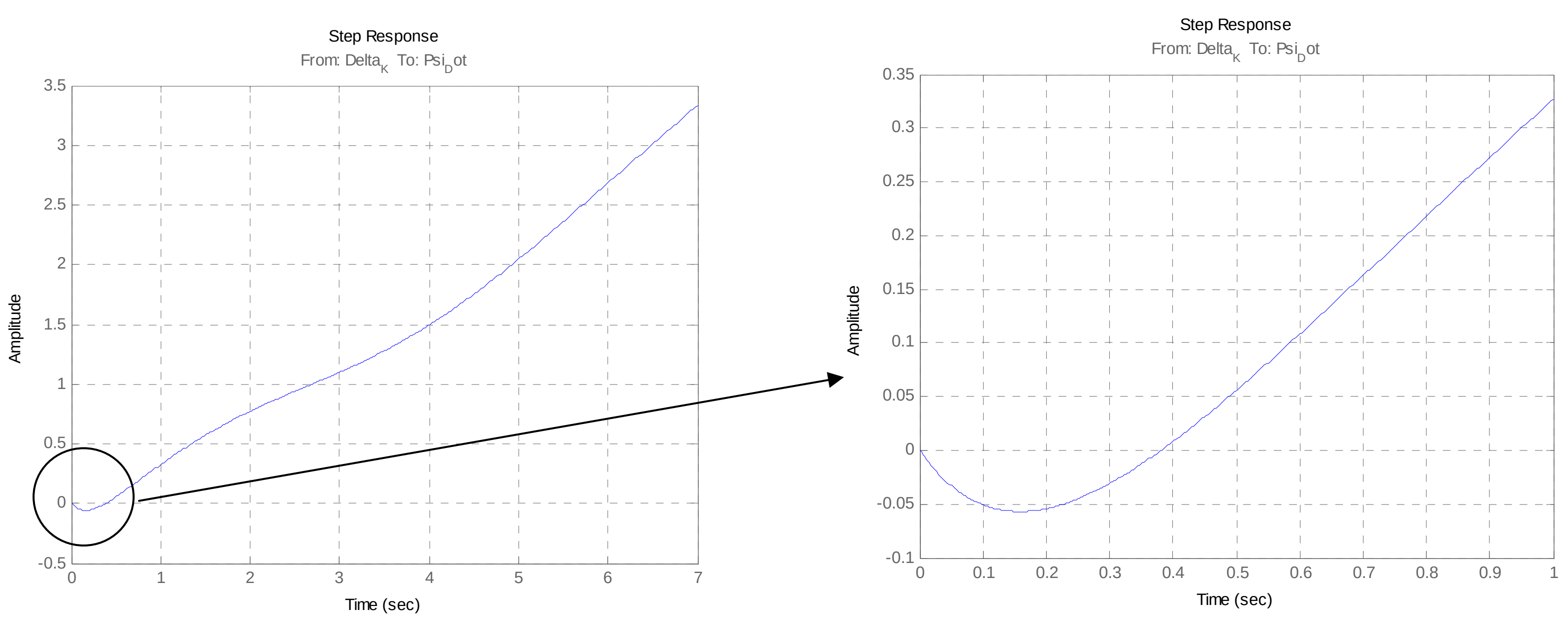
Dynamical modes of LONG and LAT motions

non-minimum phase effects in LONG and LAT motions

similarly: response AIL -> r (yaw rate):

```
>> zpk(sys(5,1))
Zero/pole/gain from input "Delta_K" to output "Psi_Dot":
   0.837 (s-3.9) (s^2 + 0.3854s + 0.9687)
---------------------------------------------
(s+5.494) (s-0.09334) (s^2 + 0.5034s + 1.618)

physics-insight explanation ... ? ☺
```



Dynamical modes of LONG and LAT motions

non-minimum phase effects for flexible A/C / missiles / rockets

- extra care should be taken
- consider flexible rocket, with pitch-rate gyro at the nose and related actuator (vectored thrust, gas rudder, whatever) at the tip. If the first-bending mode is excited, opposite \ sign is fed to the control system at the first moment, compared to the rigid-body situation. Now, if the controller is designed as fast and aggressive, w/o the bending mode excitation modelled ... disaster.
- first unsuccessful US satellite rocket launcher lost for this reason (Bryson Jr., Control systems of aircraft and spacecraft) ...
- note that if the actuator and sensor are *collocated*, no problems arise with unstable zeros ...

```
>> G_nonCol = (1-s)/(s+2)/(s^2+s+1);
>> step(G_nonCol)
```

```
>> G_Col = (1+s)/(s+2)/(s^2+s+1);
>> step(G_Col)
```