AIRCRAFT PERFORMANCE

Czech technical university in Prague
Faculty of electrical engineering
Department of control engineering

FLIGHT CONTROL SYSTEM

Principle of flight

Horizontal flight (elementary variable)

Longitudinal static stability

Steady flight

Coordinate system

Equation of motion - 6 degree of freedom

Aerodynamic model / aerodynamic coefficient

Exercising

- Non-linear longitudinal of motion

Principle of flight

pressure lay out on foil

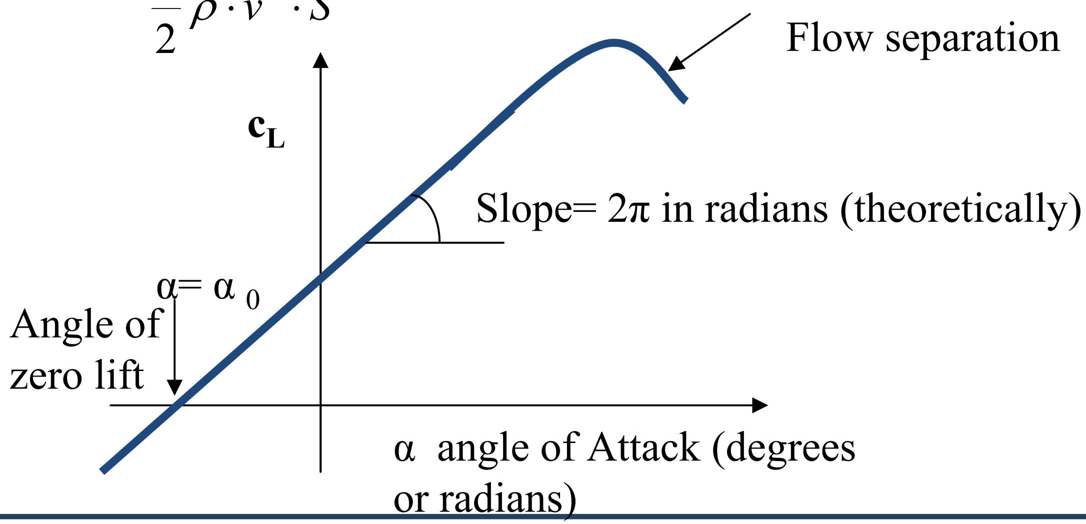
$$L' = \int_{\text{Leading Edge}}^{\text{Trailing Edge}} \left(p_{\text{lower side}} - p_{\text{upper side}} \right) dx$$

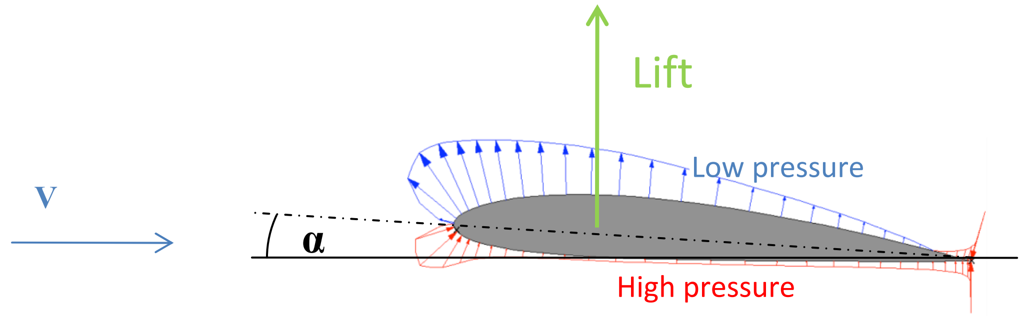
angle of attack **α**

air speed **V**

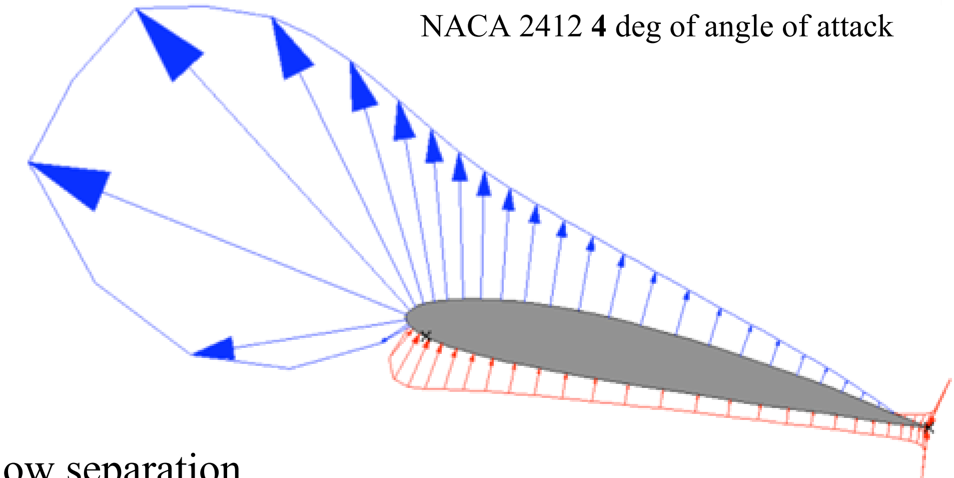
non-dimensional lift coefficient $\mathbf{c_L}$

$$c_L = \frac{L}{\frac{1}{2}\rho \cdot v^2 \cdot S}$$

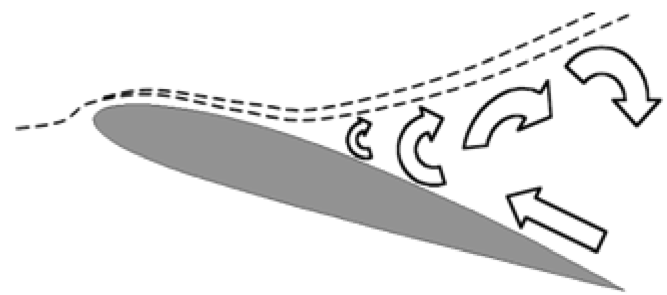




NACA 2412 **4** deg of angle of attack



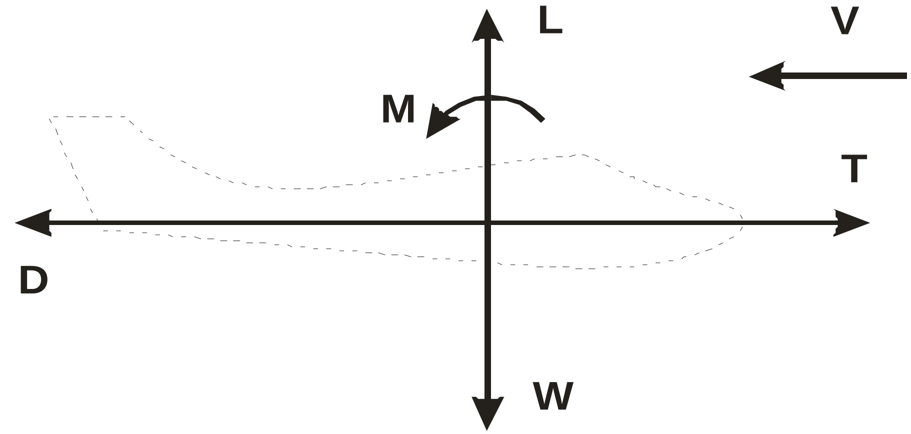
NACA 2412 **12** deg of angle of attack



NACA 2412 flow separation

Horizontal flight (logitudinal plane)

- engine thrust **T**
- aerodynamic lift **L**
- aerodynamic drag **D**
- aerodynamic pitch moment **M**
- weight of aircraft **W**
- air speed **V**



Aerodynamic forces

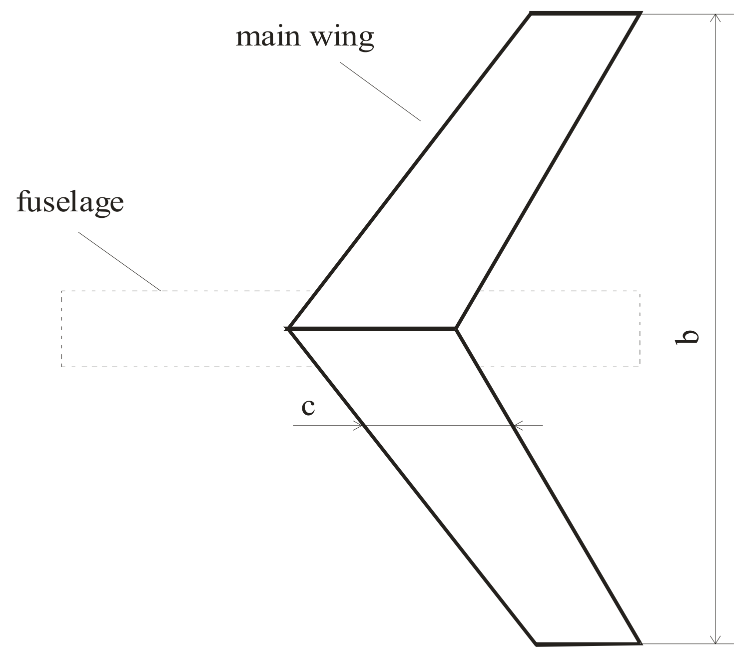
Lift equation $L = \bar{q} \cdot S \cdot c_L$

Drag equation $D = \bar{q} \cdot S \cdot c_D$

Pitch moment equation $M = \bar{q} \cdot S \cdot \bar{c} \cdot c_m$

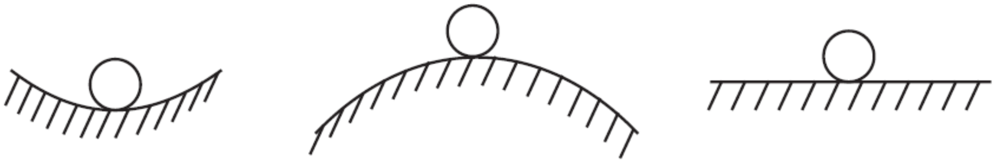
Where

- $\bar{q}$ dynamic pressure $\bar{q} = \frac{1}{2} \cdot \rho \cdot V^2$
- ρ air density
- b wing span
- $\bar{c}$ mean aerodynamic chord $\bar{c} = \int_{-b/2}^{b/2} c$
- S wing area $\bar{c} \times b$

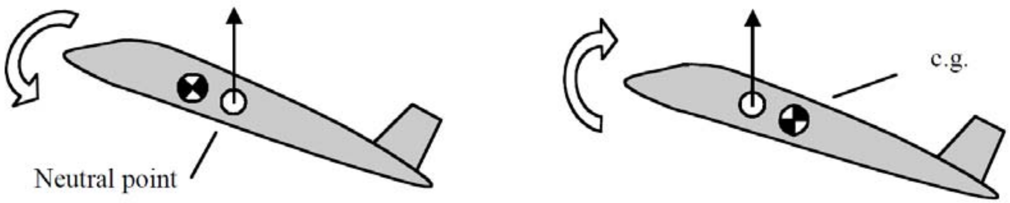


Logitudial static stability

initial tendency of a body to return to its equilibrium state after being disturbed

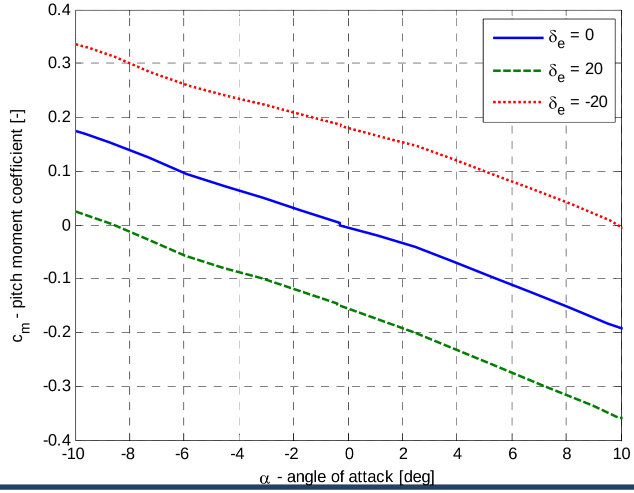


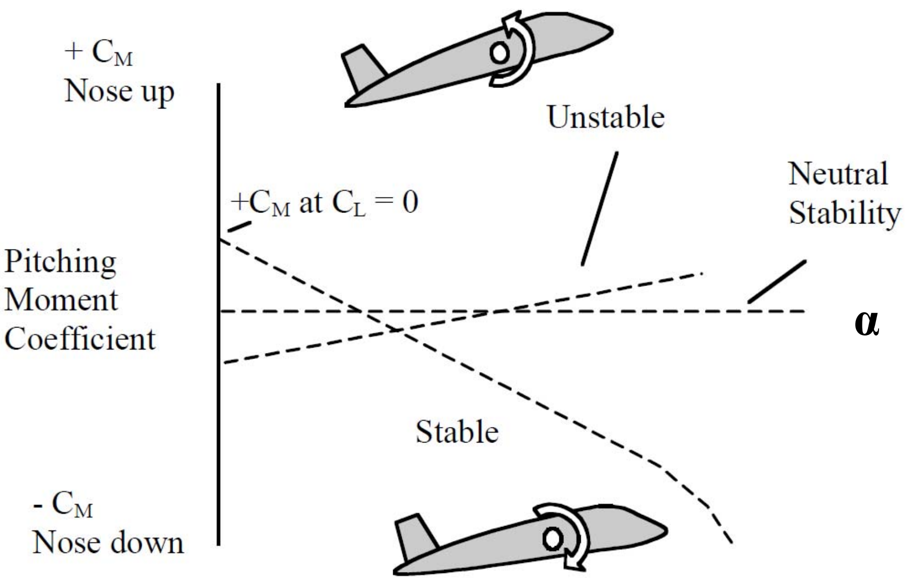
equilibrium point - moment about centre of gravity to be zero $c_m = 0$



If perturb α up, need a moment that pushes nose back down (negative)

$$\frac{\partial c_m}{\partial \alpha} < 0; \quad \frac{\partial c_m}{\partial c_L} < 0$$





Steady flight - trim

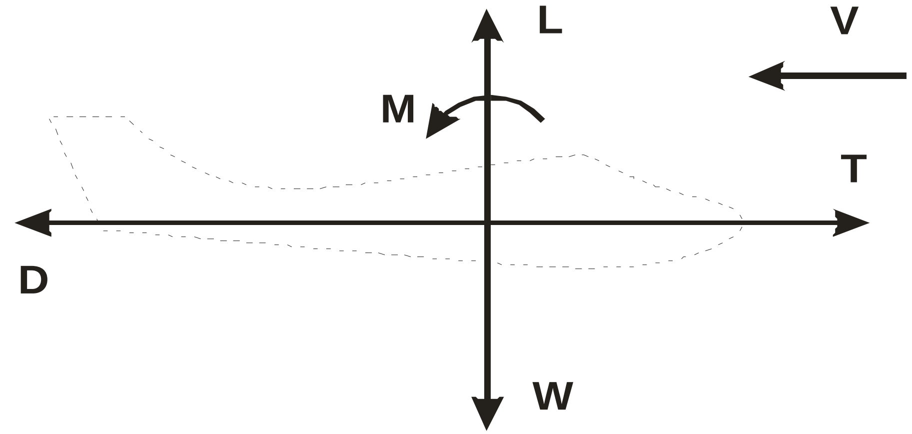
thrust of engine = drag force

$$T = D = \frac{1}{2}\rho \cdot v^2 \cdot S \cdot c_D$$

lift = weight of aircraft $W = L$

$$m.g = \frac{1}{2}\rho.v^2.S.c_L$$

zero pitch momet $\frac{1}{2}\rho.v^2.S.c_m\overline{c} = 0$



Defined values: v,m,S,ρ,g

Steady flight - trim

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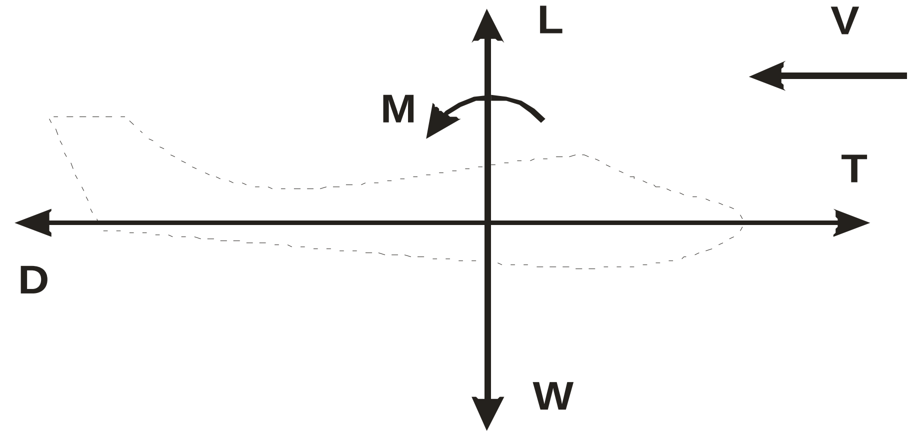

Defined values: v,m,S,ρ,g

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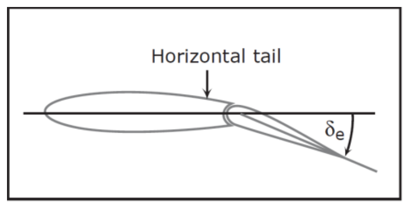
can use elevators to provide incremental lift and moments



lift coefficient $c_{L_{Trim}} = c_{L_0} + c_{L_\alpha}.\alpha + c_{L_{\delta e}}.\delta e$

$$c_{L_{Trim}} = \frac{m.g}{\frac{1}{2}\rho.v^2.S}$$

pitch moment coefficient

$$c_m = c_{m_0} + c_{m_\alpha}.\alpha + c_{m_{\delta e}}.\delta e = 0$$

two equation with two unknown

$$-c_{m_0} = c_{m_\alpha}.\alpha_{Trim} + c_{m_{\delta e}}.\delta e_{Trim}$$

$$c_{L_{Trim}} - c_{L_0} = c_{L_\alpha}.\alpha_{Trim} + c_{L_{\delta e}}.\delta e_{Trim}$$

elevator angle needed to trim

$$\delta e_{Trim} = \frac{c_{m_0}\left(c_{L_0} - c_{L_{Trim}}\right) - c_{L_\alpha} c_{m_0}}{c_{m_{\delta e}} c_{L_\alpha} - c_{m_\alpha} c_{L_{\delta e}}}$$

Aircraft coordinate system

body-fixed (aircraft system)

flight-path (wind, aerodynamic)

Transformation

Aerodynamic to body-fixed

$$\bar{\chi}_B = T^{Ba}\bar{\chi}_a = \left[T^{aB}\right]^T \bar{\chi}_a$$

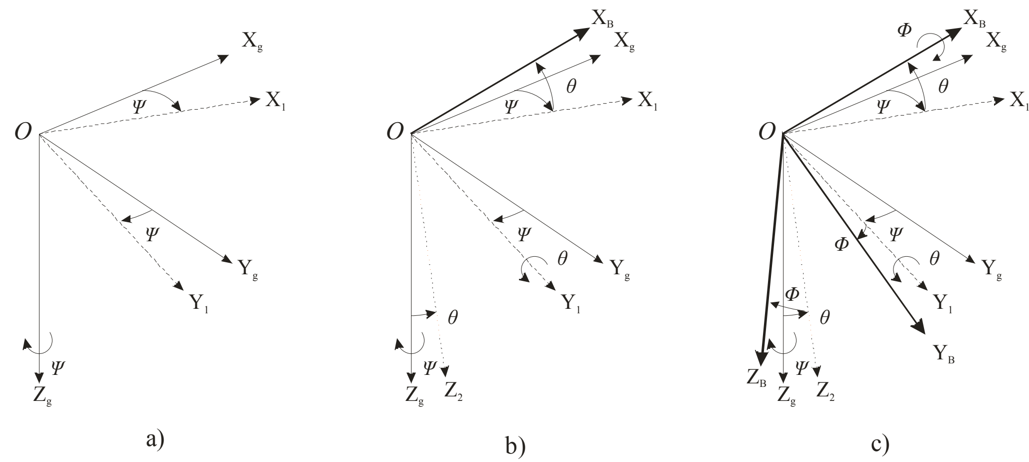
$$T^{Ba} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix}$$

Euler angel from angular rates

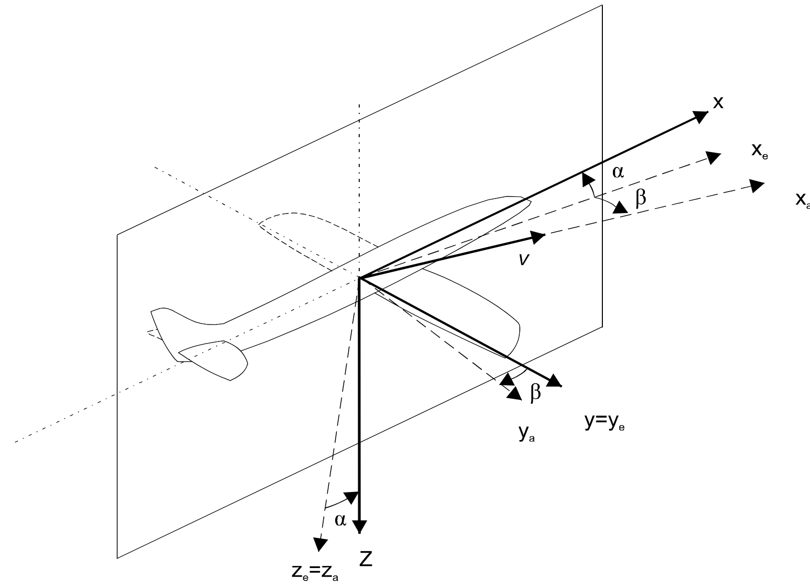
$$\dot{\phi} = p + \tan\theta\cdot(q\cdot\sin\phi + r\cdot\cos\phi)$$

$$\dot{\theta} = q\cdot\cos\phi - r\cdot\sin\phi$$

$$\dot{\psi} = \frac{q\cdot\sin\phi + r\cdot\cos\phi}{\cos\theta}$$

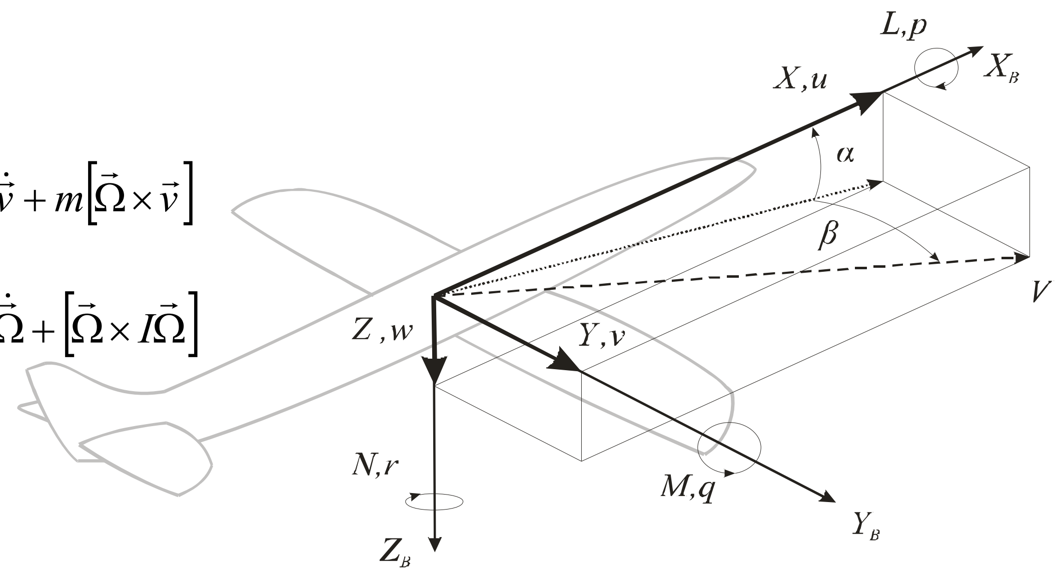


Transformation body-fixed from inertial



body-fixed and flight-path coordinate system

Equation of motion 6-DOF (Degree of freedom)



body-fixed coordiante system

Newton's second law of motion:

force equation
$$\vec{F} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} = m\dot{\vec{v}} + m\left[\vec{\Omega}\times\vec{v}\right]$$

moment equation
$$\vec{M} = \frac{d(I\Omega)}{dt} = I\frac{d\Omega}{dt} + \Omega\frac{dI}{dt} = I\dot{\vec{\Omega}} + \left[\vec{\Omega}\times I\vec{\Omega}\right]$$

$\vec{F}$	=	force vector [N]
m	=	mass [kg]
$\vec{v}$	=	linear velocity vector [m.s^{-1}]
$\vec{\Omega}$	=	angular velocity vector [rad.s^{-1}]
t	=	time
$\partial/\partial t$	=	derivative by time
$\vec{M}$	=	moment vector [N.m]
$\bar{I}$	=	moment of inertia[kg.s^{-2}]

Component form for linear velocities

$$\vec{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = V_a \cdot \begin{bmatrix} \cos\alpha\cdot\cos\beta \\ \sin\beta \\ \sin\alpha\cdot\cos\beta \end{bmatrix}$$

Aerodynamic values

$$V_a = \sqrt{\left(u^2 + v^2 + w^2\right)}$$

$$\alpha = \tan^{-1}\left(\frac{w}{V_a}\right)$$

$$\beta = \sin^{-1}\left(\frac{v}{V_a}\right)$$

V_a	=	total airspeed
α	=	angle of attack
β	=	sideslipe angle

Equation of motion 6-DOF

Newton's second law of motion:

force equation $$\vec{F}=\frac{d(mv)}{dt}=m\frac{dv}{dt}+v\frac{dm}{dt}=m\dot{\vec{v}}+m\left[\vec{\Omega}\times\vec{v}\right]$$

moment equation $$\vec{M}=\frac{d(I\Omega)}{dt}=I\frac{d\Omega}{dt}+\Omega\frac{dI}{dt}=I\dot{\vec{\Omega}}+\left[\vec{\Omega}\times I\vec{\Omega}\right]$$

Equation of motion – force equation

right side

$$\vec{F} = m\dot{\vec{v}} + m\left[\vec{\Omega} \times \vec{v}\right] = m\begin{bmatrix} \dot{u} + q \cdot w - r \cdot v \\ \dot{v} + r \cdot u - p \cdot w \\ \dot{w} + p \cdot v - q \cdot u \end{bmatrix}$$

left side $\vec{F} = \vec{A} + \vec{G} + \vec{T}$

$\vec{A} = [X \quad Y \quad Z]^T$ aerodynamic force

drag force, side force, lift force*

$\vec{G} = [G_X \quad G_Y \quad G_Z]^T$ gravity force

$\vec{T} = [F_T \quad 0 \quad 0]^T$ thrust force

gravity force

$$G_x = -mg \cdot \sin\theta$$

$$G_y = mg \cdot \cos\theta \cdot \sin\phi$$

$$G_z = mg \cdot \cos\theta \cdot \cos\phi$$

total force equation

$$\vec{F} = \begin{bmatrix} X - mg \cdot \sin\theta + T_x \\ Y + mg \cdot \cos\theta \cdot \sin\phi \\ Z + mg \cdot \cos\theta \cdot \cos\phi \end{bmatrix} = m\begin{bmatrix} \dot{u} + q \cdot w - r \cdot v \\ \dot{v} + r \cdot u - p \cdot w \\ \dot{w} + p \cdot v - q \cdot u \end{bmatrix}$$

Moment equation

$$\vec{M} = \frac{d\vec{H}}{dt} + \left[\vec{\Omega} \times \vec{H}\right] = \bar{I} \cdot \dot{\vec{\Omega}} + \dot{\bar{I}} \cdot \vec{\Omega} + \left[\vec{\Omega} \times \bar{I} \cdot \vec{\Omega}\right]$$

inertia moment

$$\bar{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

right side

$$\vec{M} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \cdot \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} +$$

$$+\det\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ p & q & r \\ p \cdot I_{xx} - q \cdot I_{xy} - r \cdot I_{xz} & q \cdot I_{yy} - p \cdot I_{xy} - r \cdot I_{yz} & r \cdot I_{zz} - p \cdot I_{xz} - q \cdot I_{yz} \end{vmatrix}$$

left side $\vec{M} = [L \quad M \quad N]^T$

roll moment, pitch moment, yaw moment

total moment equation ($I_{yz} = I_{xy} = 0$)

$$\vec{M} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \dot{p} \cdot I_{xx} + q \cdot r \cdot (I_{zz} - I_{yy}) - (p \cdot q + \dot{r}) \cdot I_{xz} \\ \dot{q} \cdot I_{yy} + p \cdot r \cdot (I_{xx} - I_{zz}) + (p^2 - r^2) \cdot I_{xz} \\ \dot{r} \cdot I_{zz} + p \cdot q \cdot (I_{yy} - I_{xx}) + (q \cdot r - \dot{p}) \cdot I_{xz} \end{bmatrix}$$

* body–fixed system

Aerodynamic model

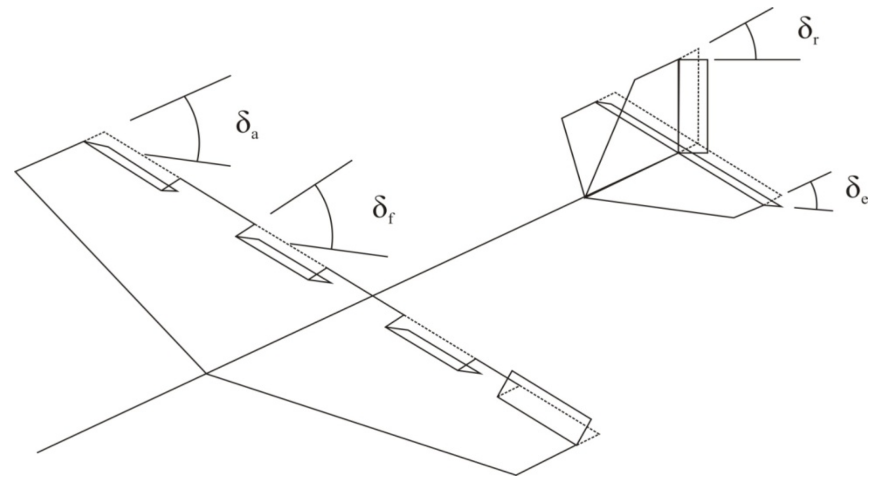
force equation

$X = \bar{q} \cdot S \cdot c_X$ drag force

$Y = \bar{q} \cdot S \cdot c_Y$ side force

$Z = \bar{q} \cdot S \cdot c_Z$ lift force

c_X, c_Y, c_Z non-dimensional force coefficient



definition of control deflection

moment equation

$L = \bar{q} \cdot S \cdot b \cdot c_l$ roll moment

$M = \bar{q} \cdot S \cdot \bar{c} \cdot c_m$ pitch moment

$N = \bar{q} \cdot S \cdot b \cdot c_n$ yaw moment

c_l, c_m, c_n non-dimensional moment coefficient

$b, \bar{c}$ wing span, mean aerodynamic chord

longitudinal coefficient

$$c_Z = c_Z\left(\alpha, \beta, \delta, M, \mathrm{Re}, \vec{\Omega}\right)$$

control coef.

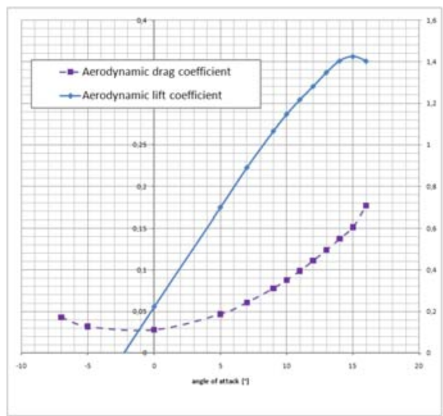
$$c_Z = \boxed{c_{Z_0} + c_{Z_\alpha}.\alpha} + \boxed{c_{Z_{\delta e}}.\delta e} + \boxed{c_{Z_q}.\frac{q.\bar{c}}{2.V}}$$

Static coef.

dynamic coef.

$$c_m = c_{m_0} + c_{m_\alpha}.\alpha + c_{m_{\delta e}}.\delta e + c_{m_q}.\frac{q.\bar{c}}{2.V}$$

$$c_X = c_{X_0} + c_{X_\alpha}.\alpha + c_{X_{\alpha^2}}.\alpha^2 + c_{X_{\delta e}}.\delta e + c_{X_q}.\frac{q.\bar{c}}{2.V}$$



lateral coefficient

$$c_Y = c_{Y_0} + c_{Y_\beta}.\beta + c_{Y_{\delta a}}.\delta a + c_{Y_{\delta r}}.\delta r + c_{Y_r}.\frac{r.b}{2.V} + c_{Y_p}.\frac{p.b}{2.V}$$

$$c_l = c_{l_0} + c_{l_\beta}.\beta + c_{l_{\delta a}}.\delta a + c_{l_{\delta r}}.\delta r + c_{l_r}.\frac{r.b}{2.V} + c_{l_p}.\frac{p.b}{2.V}$$

$$c_n = c_{n_0} + c_{n_\beta}.\beta + c_{n_{\delta a}}.\delta a + c_{n_{\delta r}}.\delta r + c_{n_r}.\frac{r.b}{2.V} + c_{n_p}.\frac{p.b}{2.V}$$

Aerodynamic coefficient

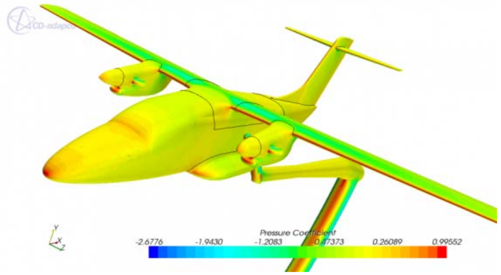
Wind tunnel

model of the aircraft in aerodynmical wind tunnel (six-component balance)

Computational methods

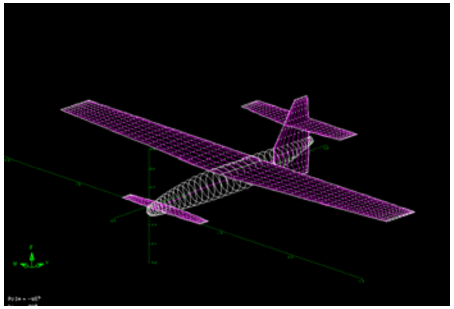
CFD (Computational fluid dynamics)

3D computer software for numerical calculations aerodynamic characteristics of aircraft



Panel methods

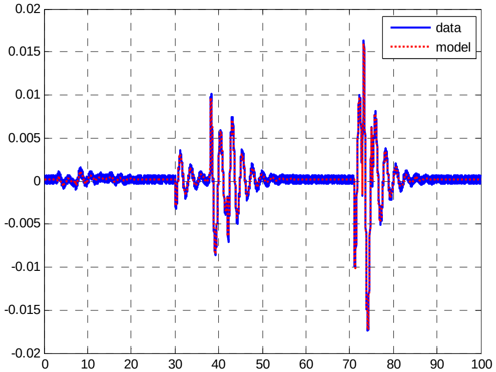
potential flow (without viscosity)



Flight test

Aircraft parameter idetification

- Least squares method
- Output error method
- Filter error method
- Frequency method



Aircraft parameter identification

Parameter	Th	s(Th)	|to|
C_x0	1.97e-004	1.73e-011	11412871.5
C_xbeta	6.41e-002	1.12e-007	572594.7
C_xp	-9.53e-002	5.36e-006	17790.0
C_xr	-1.15e-001	6.17e-007	185744.4
C_xd_ail	-1.66e-002	7.65e-007	21684.0
C_xd_rud	-6.25e-002	2.41e-007	259252.9
s = gama	2.94e-004		
R^2, %	98.5		

Summary

force equation (final form)

$$\dot{u} = r\cdot v - q\cdot w - g\cdot\sin\theta + \frac{1}{m}(X + F_T)$$

$$\dot{v} = p\cdot w - r\cdot u + g\cdot\sin\phi\cdot\cos\theta + \frac{1}{m}Y$$

$$\dot{w} = q\cdot u - p\cdot v + g\cdot\cos\phi\cdot\cos\theta + \frac{1}{m}Z$$

moment equation

$$\dot{p} = (c_1\cdot r + c_2\cdot p)\cdot q + c_3\cdot L + c_4\cdot N$$
$$\dot{q} = c_5\cdot p\cdot r - c_6\cdot(p^2 - r^2) + c_7\cdot M$$
$$\dot{r} = (c_8\cdot p - c_2\cdot r)\cdot q + c_4\cdot L + c_9\cdot N$$

where

$$\Gamma\cdot c_1 = (I_y - I_z)\cdot I_z - I_{xz}^2 \qquad \Gamma\cdot c_4 = I_{xz} \qquad c_7 = 1/I_y$$
$$\Gamma\cdot c_2 = (I_x - I_y + I_z)\cdot I_{xz} \qquad c_5 = (I_z - I_x)/I_y \qquad \Gamma\cdot c_8 = I_x\cdot(I_x - I_y) + I_{xz}^2$$
$$\Gamma\cdot c_3 = I_z \qquad c_6 = I_{xz}/I_y \qquad \Gamma\cdot c_9 = I_x$$
$$\Gamma = I_x\cdot I_z - I_{xz}^2$$

euler angle

$$\dot{\phi} = p + \tan\theta\cdot(q\cdot\sin\phi + r\cdot\cos\phi)$$
$$\dot{\theta} = q\cdot\cos\phi - r\cdot\sin\phi$$
$$\dot{\psi} = \frac{q\cdot\sin\phi + r\cdot\cos\phi}{\cos\theta}$$

Aerodynamic force and moment

$$X = \bar{q}\cdot S\cdot c_X \qquad L = \bar{q}\cdot S\cdot b\cdot c_l$$
$$Y = \bar{q}\cdot S\cdot c_Y \qquad M = \bar{q}\cdot S\cdot \bar{c}\cdot c_m \qquad \bar{q} = \frac{1}{2}\cdot\rho\cdot V^2$$
$$Z = \bar{q}\cdot S\cdot c_Z \qquad N = \bar{q}\cdot S\cdot b\cdot c_n$$

Aerodynamic values

$$V_a = \sqrt{(u^2 + v^2 + w^2)}$$
$$\alpha = \tan^{-1}\left(\frac{w}{V_a}\right)$$
$$\beta = \sin^{-1}\left(\frac{v}{V_a}\right)$$

Aerodynamic characteristic

longitudinal

$$c_Z = c_{Z_0} + c_{Z_\alpha}.\alpha + c_{Z_{\delta e}}.\delta e + c_{Z_q}.\frac{q.\bar{c}}{2.V}$$
$$c_m = c_{m_0} + c_{m_\alpha}.\alpha + c_{m_{\delta e}}.\delta e + c_{m_q}.\frac{q.\bar{c}}{2.V}$$
$$c_X = c_{X_0} + c_{X_\alpha}.\alpha + c_{X_{\alpha^2}}.\alpha^2 + c_{X_{\delta e}}.\delta e + c_{X_q}.\frac{q.\bar{c}}{2.V}$$

lateral

$$c_Y = c_{Y_0} + c_{Y_\beta}.\beta + c_{Y_{\delta a}}.\delta a + c_{Y_{\delta r}}.\delta r + c_{Y_r}.\frac{r.b}{2.V} + c_{Y_p}.\frac{p.b}{2.V}$$
$$c_l = c_{l_0} + c_{l_\beta}.\beta + c_{l_{\delta a}}.\delta a + c_{l_{\delta r}}.\delta r + c_{l_r}.\frac{r.b}{2.V} + c_{l_p}.\frac{p.b}{2.V}$$
$$c_n = c_{n_0} + c_{n_\beta}.\beta + c_{n_{\delta a}}.\delta a + c_{n_{\delta r}}.\delta r + c_{n_r}.\frac{r.b}{2.V} + c_{n_p}.\frac{p.b}{2.V}$$

Non-linear longitudinal equation of motion

zero lateral values v, p, r, ϕ (side velocity, roll rate, yaw rate, roll angle)

initial condition $m = 30kg, \quad V_a = 25m/s, \quad \rho = 1{,}225kg/m^3, \quad g = 9{,}81m.s^{-2}$

aircraft model $S = 2{,}33m^2, \quad \bar{c} = 0{,}514m, \quad I_Y = 8{,}36kg.m^2$

aerodynamic parameters

$$c_{Z_0} = -0{,}23 \quad c_{Z_\alpha} = -5{,}4 \quad c_{Z_{\delta e}} = -0{,}3 \quad c_{Z_q} = 0$$

$$c_{m_0} = -0{,}031 \quad c_{m_\alpha} = -0{,}52 \quad c_{m_{\delta e}} = -0{,}5 \quad c_{m_q} = -7{,}84$$

$$c_{X_0} = -0{,}027 \quad c_{X_\alpha} = -0{,}15 \quad c_{X_{\alpha^2}} = -0{,}016 \quad c_{X_{\delta e}} = c_{X_q} = 0$$

Create a non-linear model (separate longitudinal) **in Matlab** (m-file)

Approximatele trim aircraft for initial condition (gravity influence!)

Plot results