CSAS.

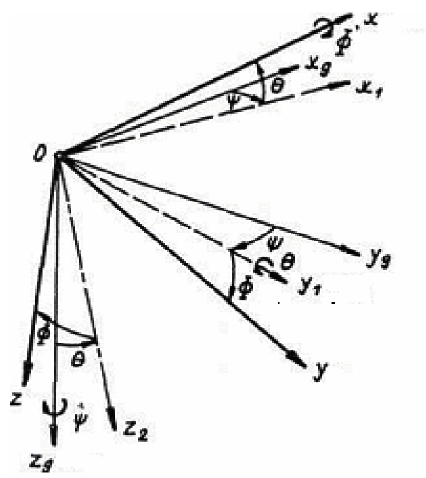
Topic: Brief introduction into flight dynamics.

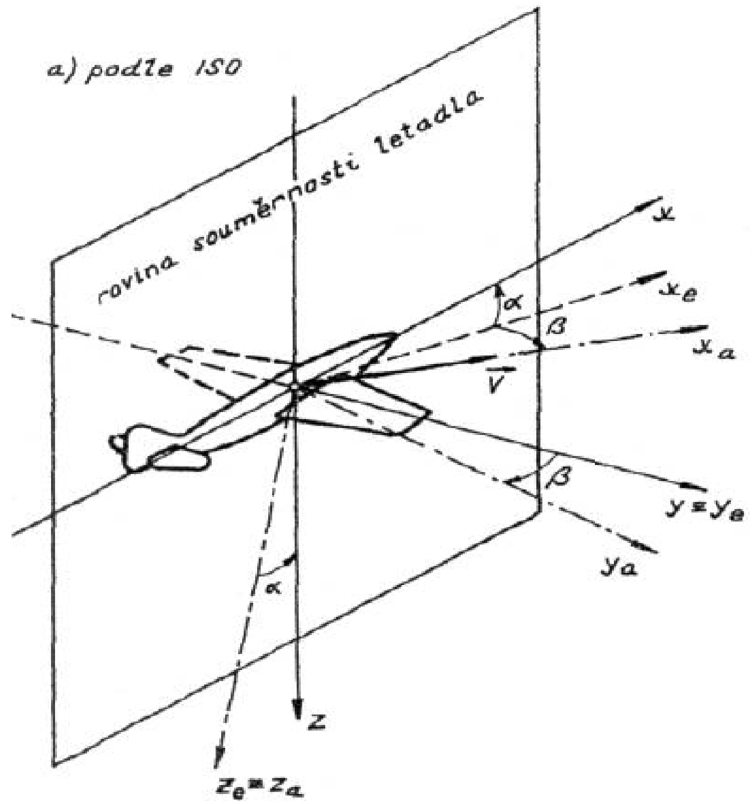
- flight dynamics equations for rigid aircraft
- linearization
- longitudinal and lateral equations
- state equations, modes, approximations

... will be accompanied / completed by case study and detailed info given by P.Hospodar

Standard aerospace coordinate systems (ISO)

- aircraft movements (pitch, …) & coordinate systems (ground, …)
- control surfaces





Flight equations. Flight variables.

		Značení v oblasti norem			
		ISO	GOST	Název	
LETOVÉ VELIČINY	Souřadné osy	x	x	podélná	longitudinal
		y	z	příčná	lateral
		z	-y	kolmá	normal
	Polohové úhly	θ	ϑ	podélný sklon	Pitch angle
		ϕ	γ	příčný náklon	Roll/Bank ang-le
		ψ	ψ	kurz	Yaw angle
	Úhlové rych-losti / derivace	$p=\omega_x \cong \phi^{\bullet}$	$\omega_x \cong \gamma^{\bullet}$	klonění	Roll rate
		$q=\omega_y \cong \theta^{\bullet}$	$\omega_z \cong \vartheta^{\bullet}$	klopení	Pitch rate
		$r=\omega_z \cong \Psi^{\bullet}$	$\omega_y \cong \psi^{\bullet}$	zatáčení	Yaw rate
	Úhly ofuková-ní	α	α	úhel náběhu	angle of atack
		β	β	úhel vybočení	Sideslip angle
	Lineární zrychlení	a_z	a_y	normálové	
		a_y	a_z	stranové	
SLOŽKY AERODYNA-MIC. SIL A MOMENTŮ	Síly	$X, D=q.S.C_D$	$X=q.S.c_x$	odporová síla	Drug
		$Z, L=q.S.C_L$	$Y=q.S.c_y$	vztlaková síla	Lift
		$Y=q.S.C_Y$	$Z=q.S.c_z$	stranová síla	Sideforce
	Momenty	M_x, $\bar{L} = q.S.b.C_l$	$M_x=q.S.1.m_x$	klonivý moment	Rolling mo-ment
		M_z, $N=q.S.b.C_N$	$M_y=q.S.1.m_y$	zatáčivý mo-ment	Yawing mo-ment
		M_y, $M=q.S.c.C_M$	$M_z=q.S.b_{SAT}.m_z$	klopivý moment	Pitching mo-ment
	Úhel sklonu trajektorie letu	$\gamma=\theta-\alpha$	$\theta=\vartheta-\alpha$	ve vertikál. rov.	Flight path ang.
		$\gamma_S=\psi-\beta$	$\theta_S=\psi-\beta$	v horizont. rov.	
	Výchylky kormidel	δ_e, η	δ_V	výškovka.	elevator
		δ_a, ξ	δ_K	křidélka.	aileron
		δ_r, ς	δ_S	směrovka	rudder

Flight equations. Assumptions

- rigid body
- constant weight
- inertial axes = aircraft axes
- thrust parallel to longitudinal axis
- g = const.
- equations for inertial system

Flight equations. Forces and moments. Kinematic equations.

$$F_x = X - mg\sin\theta \quad = \quad m(\dot{v}_x + \omega_y v_z - \omega_z v_y)$$

$$F_y = Y + mg\cos\theta\sin\phi \quad = \quad m(\dot{v}_y + \omega_z v_x - \omega_x v_z$$

$$F_z = Z + mg\cos\theta\cos\phi \quad = \quad m(\dot{v}_z + \omega_x v_y - \omega_y v_x)$$

$$M_x = I_x\dot{p} - (I_y - I_z)qr - I_{yz}(q^2 - r^2) - I_{zx}(\dot{r} + pq) - I_{xy}(\dot{q} - rp)$$

$$M_y = I_y\dot{q} - (I_z - I_x)rp - I_{zx}(r^2 - p^2) - I_{xy}(\dot{p} + qr) - I_{yz}(\dot{r} - pq)$$

$$M_z = I_z\dot{r} - (I_x - I_y)pq - I_{xy}(p^2 - q^2) - I_{yz}(\dot{q} + rp) - I_{zx}(\dot{p} - qr)$$

$$\omega_x = \dot{\phi} - \dot{\psi}\sin\theta$$

$$\omega_y = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi$$

$$\omega_z = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$$

Flight equations. Incremental form.

$u=u_0+\Delta u$ $v=v_0+\Delta v$ $w=w_0+\Delta w$

$p=p_0+\Delta p$ $q=q_0+\Delta q$ $r=r_0+\Delta r$

$X=X_0+\Delta X$ $Y=Y_0+\Delta Y$ $Z=Z_0+\Delta Z$

$M_x=M_{x0}+\Delta M_x$ $M_y=M_{y0}+\Delta M_y$ $M_z=M_{z0}+\Delta M_z$

$\theta = \theta_0 + \Delta\theta$

$$\Delta X - mg\cos\theta_0 . \Delta\theta = m\, d/dt\, \Delta u$$

$$\Delta Y + mg\cos\theta_0 . \Delta\phi = m\, (d/dt\, \Delta v + u_0 \Delta r)$$

$$\Delta Z - mg\sin\theta_0 . \Delta\theta = m\, (d/dt\, \Delta w - u_0 \Delta q)$$

$$\Delta M_x = I_x\, d/dt(\Delta p) - I_{zx}\, d/dt(\Delta r)$$

$$\Delta M_y = I_y\, d/dt(\Delta q)$$

$$\Delta M_z = I_z\, d/dt(\Delta r) - I_{zx}\, d/dt(\Delta p)$$

Flight equations. Linearization.

$$\Delta X = \Delta X(\Delta u, \Delta w, \Delta\delta_T, \Delta\delta_V) = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_T}\Delta\delta_T + \frac{\partial X}{\partial \delta_v}\Delta\delta_v$$

$$= X^u \Delta u + X^w \Delta w + X^{\delta_T}\Delta\delta_T + X^{\delta_v}\Delta\delta_v$$

$$\Delta Y = \Delta Y(\Delta v, \Delta p, \Delta r, \Delta\delta_S)$$

$$\Delta Z = \Delta Z(\Delta u, \Delta w, \Delta w', \Delta q, \Delta\delta_T, \Delta\delta_V)$$

$$\Delta M_x = \Delta M_x(\Delta v, \Delta p, \Delta r, \Delta\delta_K, \Delta\delta_S)$$

$$\Delta M_y = \Delta M_y(\Delta u, \Delta w, \Delta w', \Delta q, \Delta\delta_T, \Delta\delta_V)$$

$$\Delta M_z = \Delta M_z(\Delta v, \Delta p, \Delta r, \Delta\delta_K, \Delta\delta_S)$$

Separation: longitudinal and lateral equations.

$$\frac{\partial \Delta Y}{\partial u} = \frac{\partial \Delta M_x}{\partial u} = \frac{\partial \Delta M_z}{\partial u} = 0$$

$$\downarrow$$

F_x: $m\Delta\dot{u} - X^u \Delta u - X^w \Delta w + mg\cos\theta_0 \Delta\theta = X^{\delta_T} \Delta\delta_T + X^{\delta_v} \Delta\delta_V$

F_z: $-Z^u \Delta u - Z^w \Delta w - Z^{\dot{w}} \Delta\dot{w} + m\Delta\dot{w} - mu_0 \Delta q - Z^q \Delta q + mg\sin\theta_0 \Delta\theta =$
$= Z^{\delta_T} \Delta\delta_T + Z^{\delta_v} \Delta\delta_V$

M_y: $-M_y^u \Delta u - M_y^w \Delta w - M_y^{\dot{w}} \Delta\dot{w} - M_y^q \Delta q + I_y \Delta\dot{q} = M_y^{\delta_T} \Delta\delta_T + M_y^{\delta_v} \Delta\delta_V$

F_y: $m\Delta\dot{v} - Y^v \Delta v - Y^p \Delta p + mu_0 \Delta r - Y^r \Delta r - mg\cos\theta_0 \Delta\phi = Y^{\delta_s} \Delta\delta_S$

M_x: $-M_x^v \Delta v + I_x \Delta\dot{p} - M_x^p \Delta p - I_{zx} \Delta\dot{r} - M_x^r \Delta r = M_x^{\delta_k} \Delta\delta_K + M_x^{\delta s} \Delta\delta_S$

M_z: $-M_z^v \Delta v - I_{zx} \Delta\dot{p} - M_z^p \Delta p + I_z \Delta\dot{r} + I_z \Delta\dot{r} - M_z^r \Delta r = M_z^{\delta_k} \Delta\delta_K + M_z^{\delta s} \Delta\delta_S$

Non-dimensional equations: LONG equations.

$\Delta v = \Delta u / u_0$ $\qquad$ $\mathbf{y}^T = [y_1, y_2, y_3] = [\Delta v, \Delta\alpha, \Delta\theta]$

$c_x^v = \frac{u_0}{Sq} X^v$	$c_x^\alpha = \frac{1}{Sq} X^\alpha$	$c_x^\theta = \frac{mg}{Sq} \cos\theta_0$	$c_x^{\delta_T} = \frac{1}{Sq} X^{\delta_T}$	$c_x^{\delta_v} = \frac{1}{Sq} X^{\delta v}$
$c_z^v = \frac{u_0}{Sq} Z^v$	$c_z^\alpha = \frac{1}{Sq} Z^\alpha$	$c_z^\theta = \frac{mg}{Sq} \sin\theta_0$		$c_z^{\delta_v} = \frac{1}{Sq} Z^{\delta v}$
$m_y^v = \frac{u_0}{Sql_{SAT}} M_y^v$	$m_y^\alpha = \frac{1}{Sql_{SAT}} M_y^\alpha$ $m_y^{\dot\alpha} = \frac{1}{Sql_{SAT}} \frac{1}{B} M_y^{\dot\alpha}$	$m_y^{\dot\theta} = \frac{1}{Sql_{SAT}} \frac{1}{B} M_y^{\dot\theta}$		$m_y^v = \frac{1}{Sql_{SAT}} M_y^v$

S je plocha křídla a $q=1/2\ \rho V^2$ je dynamický tlak.

Non-dimensional equations: LONG equations.

$$\dot{v} + a_{11}v + a_{12}\alpha + a_{13}\theta = c_{11}\delta_T$$
$$a_{21}v + \dot{\alpha} + a_{22}\alpha - \dot{\theta} + a_{23}\theta = c_{22}\delta_V$$
$$a_{31}v + a_{30}\dot{\alpha} + a_{32}\alpha + \ddot{\theta} + a_{33}\dot{\theta} = c_{32}\delta_V$$

	a_{ij}			c_{ij}	
i\j	1	2	3	1	2
1	$-\frac{c_x^v}{A} = -\frac{1}{m}X^v$	$-\frac{c_x^\alpha}{A} = -\frac{1}{mu_0}X^\alpha$	$\frac{c_x^\theta}{A} = \frac{g}{u_0}\cos\theta_0$	$\frac{c_x^{\delta_T}}{A} = \frac{1}{mu_0}X^{\delta_T}$	$\frac{c_x^{\delta_v}}{A} = \frac{1}{mu_0}X^{\delta_v}$
2	$-\frac{c_z^v}{A} = -\frac{1}{m}Z^v$	$-\frac{c_z^\alpha}{A} = -\frac{1}{mu_0}Z^\alpha$	$\frac{c_z^\theta}{A} = \frac{g}{u_0}\sin\theta_0$		$\frac{c_z^{\delta_v}}{A} = \frac{1}{mu_0}Z^{\delta_v}$
3	$-\frac{m_y^v}{C} = -\frac{u_0}{I_y}M_y^v$	$-\frac{m_y^\alpha}{C} = -\frac{1}{I_y}M_y^\alpha$	$-\frac{B}{C}m_y^{\dot{\theta}} = -\frac{1}{I_y}M_y^{\dot{\theta}}$		$\frac{m_y^{\delta_v}}{C} = \frac{1}{I_y}M_y^{\delta_v}$
a_{30}		$-\frac{B}{C}m_y^{\dot{\alpha}} = -\frac{1}{I_y}M_y^{\dot{\alpha}}$			
	$A = \frac{mu_0}{Sq}, B = \frac{l}{2u_0}, C = \frac{I_y}{Sql_{SAT}}$				

Non-dimensional equations: LAT equations.

$\mathbf{y}^T = [\Delta\beta, \Delta\phi, \Delta\psi]$

$c_y^{\beta} = \frac{1}{Sq} Y^{\beta}$	$c_y^{\phi} = \frac{mg}{Sq}\cos\theta_0$ $c_y^{\dot\phi} = \frac{1}{Sq}\frac{2u_0}{l} Y^{\dot\phi}$	$c_y^{\dot\psi} = \frac{1}{Sq}\frac{2u_0}{l} Y^{\dot\psi}$		$c_y^{\delta_s} = \frac{1}{Sq} Y^{\delta_s}$
$m_x^{\beta} = \frac{1}{Sql} M_x^{\beta}$	$m_x^{\dot\phi} = \frac{1}{Sql}\frac{2u_0}{l} M_x^{\dot\phi}$	$m_x^{\dot\psi} = \frac{1}{Sql}\frac{2u_0}{l} M_x^{\dot\psi}$	$m_x^{\delta_k} = \frac{1}{Sql} M_x^{\delta_k}$	$m_x^{\delta_s} = \frac{1}{Sql} M_x^{\delta_s}$
$m_z^{\beta} = \frac{1}{Sql} M_z^{\beta}$	$m_z^{\dot\phi} = \frac{1}{Sql}\frac{2u_0}{l} M_z^{\dot\phi}$	$m_z^{\dot\psi} = \frac{1}{Sql}\frac{2u_0}{l} M_z^{\dot\psi}$	$m_z^{\delta_k} = \frac{1}{Sql} M_z^{\delta_k}$	$m_z^{\delta_s} = \frac{1}{Sql} M_z^{\delta_s}$

Non-dimensional equations: LAT equations.

$$\dot{\beta} + b_{11}\beta + b_{12}\phi - \dot{\psi} = d_{12}\delta_S$$

$$b_{21}\beta + \ddot{\phi} + b_{22}\dot{\phi} + b_{20}\ddot{\psi} + b_{23}\dot{\psi} = d_{21}\delta_K + d_{22}\delta_S$$

$$b_{31}\beta + b_{30}\ddot{\phi} + b_{32}\dot{\phi} + \ddot{\psi} + b_{33}\dot{\psi} = d_{31}\delta_K + d_{32}\delta_S$$

	b_{ij}			d_{ij}	
i\j	1	2	3	1	2
1 b_{10}	$-\frac{c_y^{\beta}}{A} = -\frac{1}{mu_0}Y^{\beta}$	$-\frac{c_y^{\phi}}{A} = -\frac{g}{u_0}\cos\theta_0$ $-\frac{Bc_y^{\dot{\phi}}}{A} = -\frac{1}{mu_0}Y^{\phi'}$	$-\frac{B}{A}c_y^{\dot{\psi}} = -\frac{1}{mu_0}Y^{\dot{\psi}}$	$\frac{c_x^{\delta_T}}{A} = \frac{1}{mV}X^{\delta_T}$	$\frac{c_y^{\delta_S}}{A} = \frac{1}{mu_0}Y^{\delta_S}$
2	$-\frac{m_x^{\beta}}{D} = -\frac{1}{I_x}M_x^{\beta}$	$-\frac{B}{D}m_x^{\dot{\phi}} = -\frac{1}{I_x}M_x^{\dot{\phi}}$	$-\frac{B}{D}m_x^{\dot{\psi}} = -\frac{1}{I_x}M_x^{\dot{\psi}}$ $b_{20} = -\frac{E}{D} = \frac{-I_{zx}}{I_x}$	$\frac{m_x^{\delta_k}}{D} = \frac{1}{I_x}M_x^{\delta_k}$	$\frac{m_x^{\delta_s}}{D} = \frac{1}{I_x}M_x^{\delta_s}$
3	$\frac{m_z^{\beta}}{F} = -\frac{1}{I_z}M_z^{\beta}$	$\frac{B}{F}m_z^{\dot{\phi}} = -\frac{1}{I_z}M_z^{\dot{\phi}}$ $a_{30} = -\frac{E}{F} = \frac{-I_{zx}}{I_z}$	$-\frac{B}{F}m_z^{\dot{\psi}} = \frac{-1}{I_z}M_z^{\dot{\psi}}$	$\frac{m_z^{\delta_k}}{F} = \frac{1}{I_z}M_z^{\delta_k}$	$\frac{m_z^{\delta_s}}{F} = \frac{1}{I_z}M_z^{\delta_s}$
	$A = \frac{mu_0}{Sq}, B = \frac{l}{2u_0}, D = \frac{I_x}{Sql}, E = \frac{I_{xz}}{Sql}, F = \frac{I_z}{Sql}, b_{20} = -\frac{E}{D}, b_{30} = -\frac{E}{F}$				