Assignment 3 – Question 2

Q.2)

a) Derivation of Motion tracking Equation

Here, the assumption used is the brightness consistency assumption which states that the appearance of an object does not change between consecutive frames, although it's position may change.

Let $I(x, y, t)$ represent the intensity of a pixel at position (x, y) at time t. If a pixel moves by $(\Delta x, \Delta y)$ in a small time interval Δt, the brightness consistency can be expressed as:

$$I(x, y, t) = I(x+\Delta x, y+\Delta y, t+\Delta t)$$

For small motions, we can approximate Δx & Δy using Taylor Expansion:

$$I(x+\Delta x, y+\Delta y, t+\Delta t) \approx I(x, y, t) + \frac{\partial I}{\partial x}\Delta x + \frac{\partial I}{\partial y}\Delta y + \frac{\partial I}{\partial t}\Delta t$$

Setting the left & right sides of the brightness constancy equation equal & assuming small Δt, we get;

$$0 = \frac{\partial I}{\partial x}\Delta x + \frac{\partial I}{\partial y}\Delta y + \frac{\partial I}{\partial t}\Delta t$$

Dividing through by Δt & letting $u = \frac{\Delta x}{\Delta t}$ &

$v = \frac{\Delta y}{\Delta t}$ (velocities in x & y directions), we

obtain the following optical flow Equation:

$$\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t} = 0.$$

b) Locas- kanade Algorithm with Affine Motion Model.

For the Locas- Kanade method under the assumption of affine motion; the motion model is;

$$u(x, y) = a_1 x + b_1 y + c_1$$
$$v(x, y) = a_2 x + b_2 y + c_2$$

1) Jaccobian of the Motion Model;

The Jaccobian J of this transformation with respect to the parameters $\theta = [a_1, b_1, c_1, a_2, b_2, c_2]$ is:

$$J = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix}$$

2> Formulating the Problem :

With optical flow constraint

$\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t} = 0$, you can express u & v

in terms of θ, & substitute into the equation:

$$\begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\frac{\partial I}{\partial t}$$

Substituting the expressions for u & v gives a system of equations involving θ.

3> Solving for θ:

Using least squares to solve for θ, which minimizes the error across all pixels in a window:

$$\min_{\theta} \| A\theta - b \|^2$$

Where; A is formed by stacking the Jaccobians multiplied by the image gradients; & b is the negative temporal gradient.

By solving this system, we can estimate the affine Parameters which describe the motion between two frames under assumption that this model of motion is a good fit for the actual motion observed in the image sequence.