

5. you are given an array containing all distinct elements.

Find the number of possible subsequences

Ex:

$N=3$ , Arr =  $[1, 2, 3]$

Ans. 7

Subsequences are:  $[1]$ ,  $[2]$ ,  $[3]$ ,  $[1, 2]$ ,  $[1, 3]$ ,  $[1, 2, 3]$   
 $[2, 3]$

Solution Approach:

What's a subsequence?

$\left[ \begin{array}{c} \_ \\ T/Nr \end{array} \right] \begin{array}{c} \_ \\ T/Nr \end{array} \begin{array}{c} \_ \\ T/Nr \end{array} \begin{array}{c} \_ \\ T/Nr \end{array} \begin{array}{c} \_ \\ T/Nr \end{array} \rightarrow$  you can decide take it (or etc.) or not take it.

\* 2 choices for each element, possible subsequences:

$\rightarrow$  no. of elements

$2^N - 1$

$\rightarrow$  when we don't take any element (that's an invalid subsequence)

$$\Rightarrow 2^3 - 1 = 7$$

of elements

Note:  $[2, 1]$  X not a subsequence as the order <sup>of elements</sup> should remain same

\*\* Instead of using pow(2, n) use a for loop to store the no. of subsequence as pow returns a floating pt. value.

6. you are given an array. Find the number of its subarrays.

Ex:

$N=3$ , Arr =  $[1, 2, 3]$  Ans = 6

Subarrays are:  $[1]$ ,  $[1, 2]$ ,  $[1, 2, 3]$ ,  $[2]$ ,  $[2, 3]$ ,  $[3]$

Note: A subarray is continuous. eg:  $[1, 3]$  X not a subarray.

It's a counting problem

\* 2 ways to see an array:

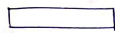
①.



No. of 1 sized array =  $N$

No. of 2 sized array =  $N-1$

No. of 3 sized array =  $N-2$



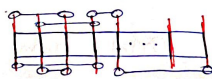
No. of  $N$  sized array = 1

$$\text{total arrays} = N + (N-1) + (N-2) + \dots + 1$$

$$= \frac{N(N+1)}{2} \quad \left\{ \begin{array}{l} \text{sum of } n \text{ natural nos} \\ \text{in } n \end{array} \right.$$

= possible subarrays.

②



for array of  $n$  elements there are  $n+1$  lines that divides them.

Choose any 2 you'll get a subarray between them.

So, in how many ways we can choose 2 lines from  $n+1$  lines

$$\boxed{{}^{n+1}C_2 = \frac{(n+1)(n)}{2}} \quad \text{combinatorics}$$

4. You are given an array. Find sum of all subarrays of the array. [Time limit: 1 sec] Constraints:  
 i.  $N \leq 100$  ii.  $N \leq 1000$  iii.  $N \leq N^5$

Example:

$N = 3$ , Arr = [1, 2, 3] Ans = 20

Subarray	[1]	[1, 2]	[1, 2, 3]	[2]	[2, 3]	[3]
Sum	1	3	6	2	5	3

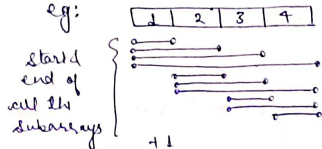
$\Rightarrow$  Total: 20

Approach 1: (using 3 for loops)

Logic?  $\rightarrow$  find start and end of all the possible subarrays

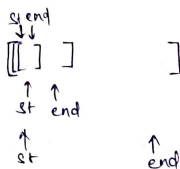
the inner loop iterates over each subarray & sum of each subarray is added to total

eg:

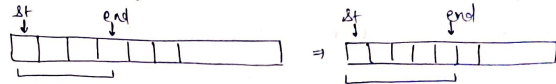


1  
3  
6  
10  
2  
5  
9  
3  
7  
4  
50

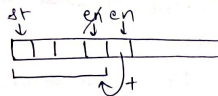
```
int total = 0;
for (int st = 0; st < n; st++) {
    for (int en = st; en < n; en++) {
        sum = 0;
        for (int i = st; i <= en; i++) {
            sum += arr[i];
        }
        total += sum;
    }
}
```



Approach 2: [Using 2 for loops]



Logic: what we are currently doing is every time the end is shifting we are looping through the entire previous subarray. Now, we will store this sum of previous subarray and add the end element to it, & use it in total



int total = 0

```
for (int st = 0; st < n; st++) {
    int sum = 0;
    for (int en = st; en < n; en++) {
        int sum += arr[en];
        total += sum;
    }
}
```

# Using the sum of previous subarray, adding just the <sup>new</sup> end value to it.

Approach 3: (Using contribution technique)  $\rightarrow$  Atomic item contribution

Contribution

Technique

Atomic Item Contribution

Extending Contribution

	1	2	3
1	1		
		2	
			3
1	1	2	
		2	3
1	1	2	3

$\uparrow \quad \uparrow \quad \uparrow$

These are basically row wise sum of the <sup>2D</sup> matrix.  
 $\downarrow$   
 If we add row wise we will get  $n^2$  cuz we saw no. of sub arrays are  $\frac{n(n+1)}{2}$

However, if we try to add it column wise, there are only  $n$  columns as 'n' elements in the array.

We can observe the no. of times an element has appeared in a subarray how many subarrays.

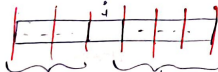
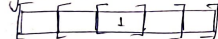
If we could just find how many times each element of array has appeared in the subarrays & sum them we will get our result.

If we could just find out <sup>each</sup> in how many subarrays <sup>each</sup> contains an element, we get the no. of times that element has appeared & multiply it with the element

1	2	3
1		
	2	
		3
1	2	
	2	3
1	2	3

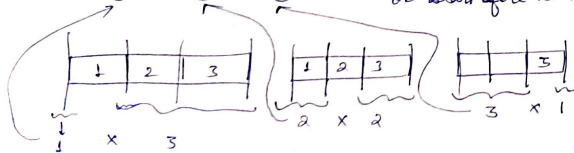
Calculate how many sub arrays contains an element:

Case by Case



A sub array can only contain  $i$ th element if it ends after it or starts before it.

$$1 \times 3 + 2 \times 2 + 3 \times 1 = 20$$



$i+1$  line  
→ arrays possible  
 $i+1 C_1 = i+1$   
total lines in array of 'n' elements  
 $= (n+1) - (i+1)$   
 $= (n-i)$   
→ arrays possible  
 $n-i C_1 = n-i$

⇒ No. of subarrays in which  $i$  comes =  $(i+1) * (n-i)$

Final code:

```
int total = 0;
for (int i = 0; i < n; i++) {
    total += arr[i] * (i+1) * (n-i);
}
```

cout << total;