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$$(X'X)^{-1}X'y = b$$

$$X' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & \cdots & & & x_n \end{bmatrix}$$

$$X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \qquad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}$$

$$X'X = \begin{bmatrix} n & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix}$$

$$X'y = \begin{bmatrix} \Sigma y \\ \Sigma xy \end{bmatrix}$$

$$(X'X)^{-1} = \frac{1}{n\Sigma x^2 - (\Sigma x)^2}\begin{bmatrix} \Sigma x^2 & -\Sigma x \\ -\Sigma x & n \end{bmatrix}$$

$$(X'X)^{-1}X'y = \frac{1}{n\Sigma x^2 - (\Sigma x)^2}\begin{bmatrix} \Sigma x^2 \Sigma y - \Sigma x \Sigma xy \\ n\Sigma xy - \Sigma x \Sigma y \end{bmatrix}$$

$$\frac{1}{\Sigma x^2 - (\Sigma x)^2}\begin{bmatrix} \bar{y}\Sigma x^2 - \bar{x}\Sigma xy \\ \Sigma xy - n\bar{x}\bar{y} \end{bmatrix}$$

$$\frac{1}{SSx}\begin{bmatrix} \bar{y}\Sigma x^2 - \bar{x}\Sigma xy \\ SPxy \end{bmatrix}$$

$$\frac{1}{SSx}\begin{bmatrix} \bar{y}\Sigma x^2 - n\bar{y}\bar{x}\bar{x} + n\bar{y}\bar{x}\bar{x} - \bar{x}\Sigma xy \\ SPxy \end{bmatrix}$$

$$\frac{1}{SSx}\begin{bmatrix} \bar{y}\,\Sigma x^2 - n\bar{x}^2 \quad + -\bar{x}(n\bar{y}\bar{x} - \Sigma xy) \\ SPxy \end{bmatrix}$$

$$\frac{1}{SSx}\begin{bmatrix} \bar{y}\,SSx \quad + -\bar{x}\,SPxy \\ SPxy \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \bar{Y} - \bar{X}\,\frac{SPXY}{SSX} \\ \frac{SPXY}{SSX} \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$b_1 = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - \bar{x})^2}$$

$$SPxy = \Sigma(x - \bar{x})(y - \bar{y})$$

$$SSx = \Sigma(x - \bar{x})^2$$

$$SP_{xy} = \Sigma(x - \bar{x})(y - \bar{y})$$

$$\Sigma xy - \Sigma\bar{x}y - \Sigma\bar{y}x + \Sigma\bar{x}\bar{y}$$

$$\Sigma xy - \bar{x}\Sigma y - \bar{y}\Sigma x + \bar{x}\bar{y}n$$

$$\Sigma xy - \frac{\Sigma x\Sigma y}{n}\left(\frac{n}{n}\right) - \frac{\Sigma x\Sigma y}{n}\left(\frac{n}{n}\right) + \bar{x}\bar{y}n$$

$$\Sigma xy - \bar{x}\bar{y}n \underbrace{- \bar{y}\bar{x}n + \bar{x}\bar{y}n}$$

$$\boxed{SP_{xy} = \Sigma xy - \bar{x}\bar{y}n}$$

$$SS_x = \Sigma(x - \bar{x})^2$$

$$\Sigma x^2 - \Sigma 2x\bar{x} + \Sigma\bar{x}^2$$

$$\Sigma x^2 - 2\bar{x}\Sigma x + n\bar{x}^2$$

$$\Sigma x^2 - 2n\bar{x}\bar{x} + n\bar{x}^2$$

$$\Sigma x^2 - n\bar{x}^2$$

$$\boxed{SS_x = \Sigma x^2 - (\Sigma x)^2}$$