CS/ECE/ISyE 524 — Introduction to Optimization — Summer 2020

Greener Korea! - Renewable Energy Plan 2030

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1. Introduction

The objective of this project is to develop a linearlized power flow model and solve an optimization problem to determine best set of locations to construct solar and wind power plants.

The Korean government announced to shut down nuclear power plants by 2030 [1]. The government is planning to build solar and wind power plants to compensate the reduced power generation capability. Based on the solar and wind resource analysis, couple of locations are selected for additional renewable power plants. However, due to the budget limit, the Korean government is trying to determine optimal set of locations that minimizes the total generation cost. Also, it is assumed that electric load demand increases by 10 % in 2030 compared to that of 2020.

In a power grid, the real power generated from each power plants is systematically controlled to meet the total electric load (demand) of the power grid. When the mismatch between generated power and consumed power becomes large, the power grid becomes unstable which may lead to blackout. This blackout can be a significant issue in numerous facilities such as hospitals, data centers, and hazardous industries. Shutting down all nuclear power plants by 2030 is beneficial for safer and eco-friendly environment. However, this change in power grid infrastructure undermines the power generation capability of the power grid and imposes a considerable challenge on the stability of the power grid. Therefore, it is inevitable to construct more renewable power plants to maintain the stability of the power grid and avoid blackouts.

One problem with the renewable power plants such as solar and wind plants is highly location dependant. The location, where solar or wind power plants are build, should have either sufficient sunlight or wind to generate enough power. Also, building additional renewable power plants adds fixed generation costs to the grid. It is economically critical to optimize the number and location of new renewable power plants to minimize the generation cost in 2030.

For a fixed power grid, detailed power grid models are developed to calculate the power flow in the grid and are used to estimate the required power generation at each power plants. This detailed power grid models are also known as AC grid model []. Although the AC grid model can give an accurate power flow in the grid, it is computationally demanding due to highly nonlinear characterestics of the AC grid model. DC grid model is a linearized model of AC grid model which significantly reduces the computational demand to calculate the power flow in a power grid while sacrificing the accuracy of the result.

A simplifed power grid near Seoul is developed from [2] and [3] as shown in Fig. 1-1.

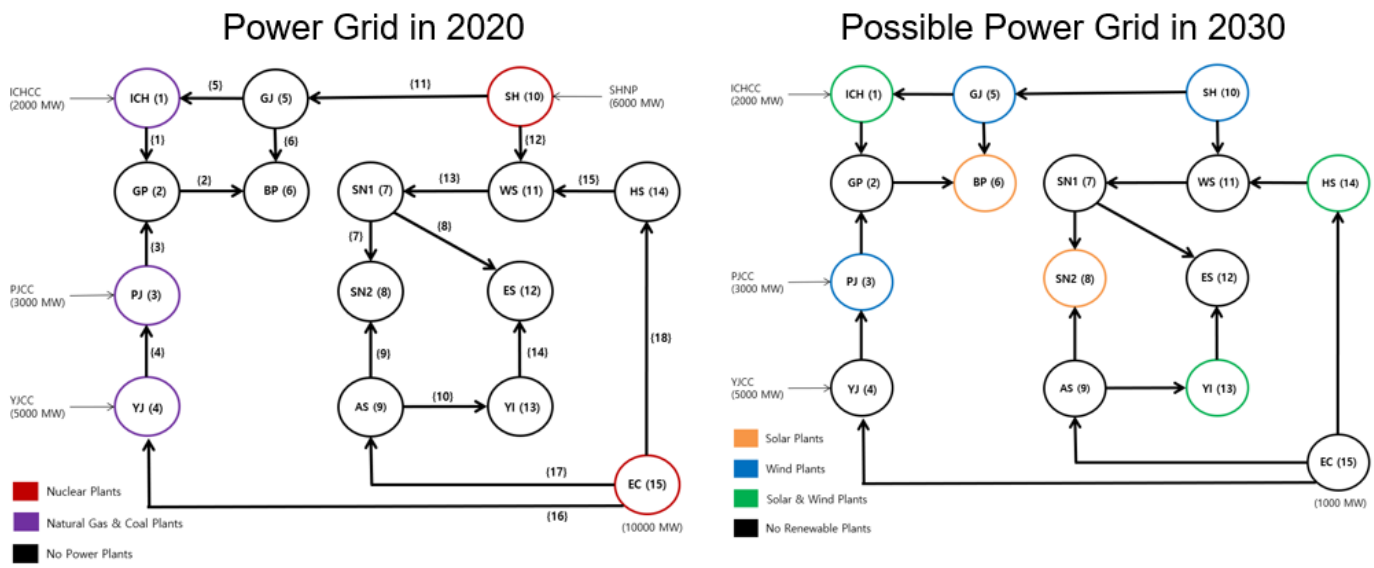

Fig. 1-1

More detailed analysis and modeling oriented materials of the power grid in Fig. 1-1 will be discussed in section 2.

Section 2 discusses the modeling of the power grid. The nonlinear power grid model is linearlized into linear (DC) power grid model under steady state operation assumption. Expression for linear (DC) power flow F in each transmission line is derived from the developed linear power grid model. The linear power grid model and linear power flow model are used to form an min-cost network flow (MCNF) problem. It should be note that MCNF is possible due to the linearlization process. This mathematical model is used to obtain the lowest generation cost per hour in year 2020. Additional decision variables z_S and z_W are added to the MCNF model to determine the optimal locations for renewable power plants in year 2030. These decision variables transforms the MCNF into a mixed integer progrem (MIP). Section 3 solves the MCNF and MIP with Julia and results are discussed in section 4. Section 5 summarizes and concluded this report.

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2. Mathematical model

2-1. Power Flow Balance Modeling in a General Power Grid [4]

A power transmission netwok or power grid can be modeled as a large circuit network with multiple power sources (generators), loads and admittances y_{kn} as shown in Fig. 2-1.

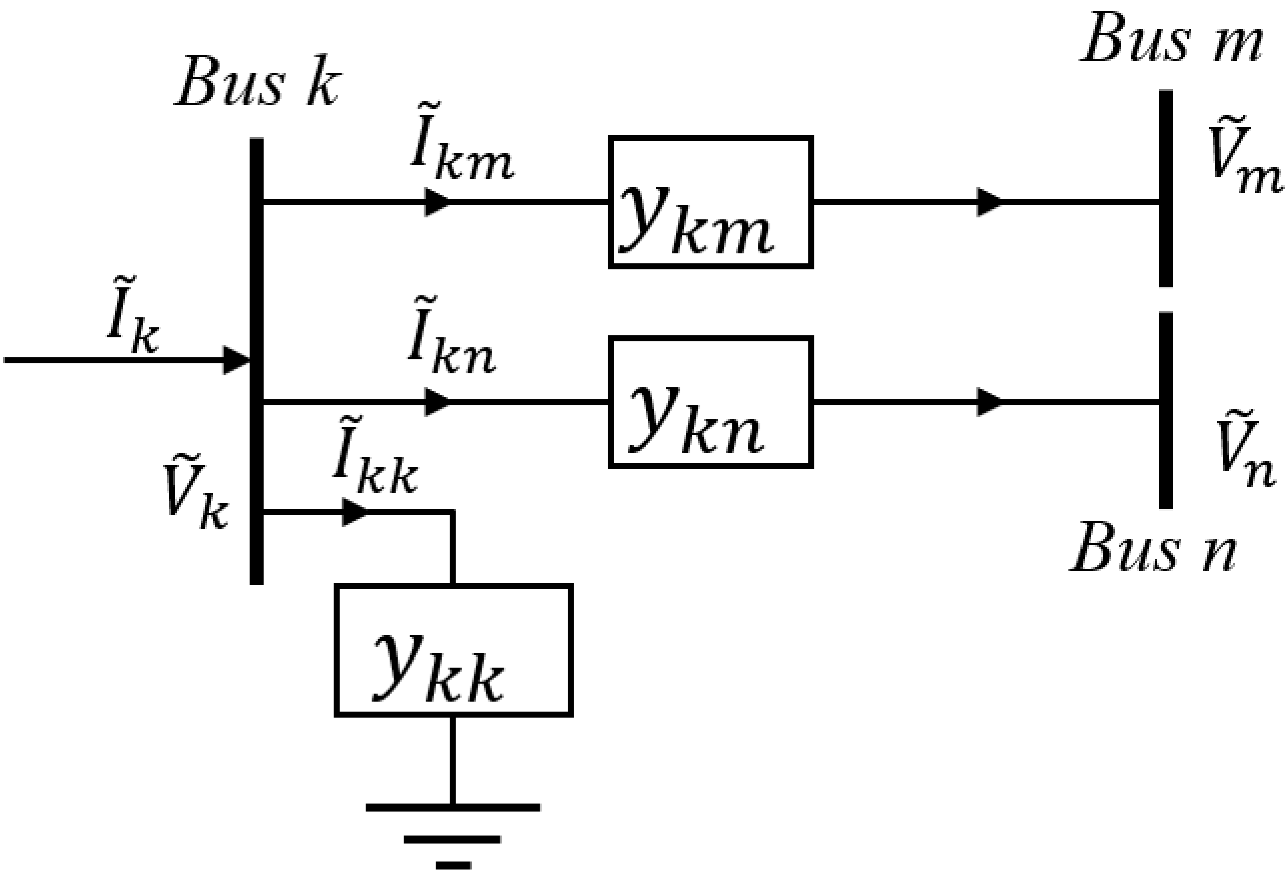

Fig. 2-1

The bus voltage of k bus $\widetilde{V}_k$ is defined as (1)

$$\widetilde{V}_k = |\widetilde{V}_k| \cdot e^{j\theta_k} \ (1)$$

where $|\widetilde{V}_k|$ is the magnitude and θ_k is the angle of the k bus voltage phasor $\widetilde{V}_k$.

The current $\widetilde{I}_k$ injected into bus k flows into adjacent buses (bus n and m in Fig. 2-1) through the transmission line while rest of the current flows to the ground via shunt admittance y_{kk}. The flow balance equation of the current at bus k is given by (2)

$$\widetilde{I}_k = \widetilde{I}_{Gk} - \widetilde{I}_{Lk}$$

$$= \widetilde{V}_k \cdot y_{kk} + \sum_{n=1}^{N} (\widetilde{V}_k - \widetilde{V}_n) \cdot y_{kn}$$

$$= \sum_{n=1}^{N} y_{kn} \cdot \widetilde{V}_k + \sum_{n \neq k}^{N} (-y_{kn}) \cdot \widetilde{V}_n \ (2)$$

where $\widetilde{I}_{Gk}$ is the current flowing from the generator to the k bus whereas $\widetilde{I}_{Lk}$ is the current flowing from the k bus to the load of the k bus.

For all bus nodes in the power grid ($k = 1, \ \cdots, \ N$), injected current $\widetilde{I}_k$ defined by (2) is reformated into a matrix form as shown in (3)

$$\begin{bmatrix} \vdots \\ \widetilde{I}_k \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & \sum_{n=1}^{N} y_{kn} & \cdots & -y_{kn} & \cdots \\ & \vdots & & \vdots & \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ \widetilde{V}_k \\ \vdots \\ \widetilde{V}_n \\ \vdots \end{bmatrix} \text{(3)}$$

where the admittance matrix of the power grid Y_{bus} is defined as (4)

$$Y_{bus} = \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & \sum_{n=1}^{N} y_{kn} & \cdots & -y_{kn} & \cdots \\ & \vdots & & \vdots & \end{bmatrix} \text{(4)}$$

When the (k, n) entry of Y_{bus} is defined as Y_{kn}, (2) can be expressed as (5)

$$\widetilde{I}_k = \widetilde{I}_{Gk} - \widetilde{I}_{Lk} = \sum_{n=1}^{N} (Y_{kn} \cdot \widetilde{V}_k) \text{ (5)}$$

where $\widetilde{I}_{Gk}$ is the current from the generator of k bus while $\widetilde{I}_{Lk}$ is the current flowing to the load of k bus.

The (k, n) entry of Y_{kn} is either $-y_{kn}$ or $\sum_{n=1}^{N} y_{kn}$ as shown in (4). As a result, the value of Y_{kn} can be considered as an admittance and can be decomposed into conductance G_{kn} and susceptance B_{kn} as shown in (6)

$$Y_{kn} = G_{kn} + jB_{kn}. \text{ (6)}$$

The appearant power injected to k bus S_k is given by (7)

$$S_k = S_{Gk} - S_{Lk} = P_k + jQ_k = \widetilde{V}_k \cdot \widetilde{I}_k^* \text{ (7)}$$

where S_{Gk} is the appearant power flow from the generator of k bus while S_{Lk} is the appearant power flow from the k bus to the load of k bus. The appearant power S_k can also be express with real power P_k and imaginary power Q_k where real power P_k is the power that works on the load and eventually dissipates while imaginary power Q_k is a power that circulates in the power network and cannot produce any power.

The appearant power can be expressed with the voltage of the k bus $\widetilde{V}_k$ and the injected current of the k bus $\widetilde{I}_k$ as shown by (7) where $\widetilde{I}_k^*$ is the complex conjugate of $\widetilde{I}_k$. Substituting (5) into (7) leads to (8)

$$S_k = \widetilde{V}_k \cdot (\sum_{n=1}^{N} Y_{kn} \cdot \widetilde{V}_n)^* = \widetilde{V}_k \sum_{n=1}^{N} (Y_{kn}^* \cdot \widetilde{V}_n^*) \text{ (8)}$$

The left side of (8) S_k is the appearant power injected to the k bus while the right side of (8) is the appearant power flowing out of the k bus. Therefore, (8) is a mathematical expression of appearant power flow balance at bus k. However, appearant power S_k is a complex number with both real power injected to the bus k P_k and imaginary power injected to the bus k Q_k as shown in (7). Since the real power P_k is the physical quantity that can be controlled, it is necessary to decompose (8) into real power P_k and the imaginary power Q_k. Substituting (1), (6), and (7) into (8) leads to (9)

$$S_k = \sum_{n=1}^{N} |\widetilde{V}_k| \cdot |\widetilde{V}_n| \cdot e^{j\theta_{kn}} \cdot (G_{kn} - jB_{kn})$$

$$= \sum_{n=1}^{N} |\widetilde{V}_k| \cdot |\widetilde{V}_n| \cdot (cos(\theta_{kn}) - jsin(\theta_{kn})) \cdot (G_{kn} - jB_{kn}) \ (9)$$

where $\theta_{kn} = \theta_k - \theta_n$ is the angle between θ_k and θ_n.

The real part of (9), which is the injected real power P_k is given by (10)

$$P_k = P_{Gk} - P_{Lk} = \sum_{n=1}^{N} |\widetilde{V}_k| \cdot |\widetilde{V}_n| \cdot (G_{kn} \cdot cos(\theta_{kn}) + B_{kn} \cdot sin(\theta_{kn})) \ (10)$$

where P_{Gk} is the real power flowing from the generator to the k bus and P_{Lk} is the real power flowing to the load from the k bus. It should be noted that (10) is generally referred to as real power flow blance equation at bus k.

Similarly, the imaginary part of (9), which is the injected imaginary power Q_k is given by (11)

$$Q_k = Q_{Gk} - Q_{Lk} = \sum_{n=1}^{N} |\widetilde{V}_k| \cdot |\widetilde{V}_n| \cdot (G_{kn} \cdot sin(\theta_{kn}) - B_{kn} \cdot cos(\theta_{kn})) \ (11)$$

where Q_{Gk} is the imaginary power flowing from the generator to the k bus and Q_{Lk} is the imaginary power flowing to the load from the k bus.

2-2.Linearlization of Power Flow Balance Model [4]

The real power flow balance equation at bus k (10) can represent the power flow with high accuracy. However, due to the multiplication between voltage magnitudes $|\widetilde{V}_k|, |\widetilde{V}_n|$ and trigonometic functions, (11) is a nonlinear equation. Therefore, the real power flow problem based on (11) is hard to solve and numerical methods such as newton-raphson's methods are used to solve this nonlinear equation [].

Bus voltage magnitude $|\widetilde{V}_k|$ and voltage angle θ_{kn} are strictly regulated to ensure stability of the power grid. Under steady-state, the voltage magnitude $|\widetilde{V}_k|$ can be approximated to 1 p.u. where p.u. (per unit) is used to indicate that the voltage magnitude $|\widetilde{V}_k|$ is a normalized value. Also, the voltage angle θ_{kn} is regulated to stay below 30^o. When the angle θ_{kn} is small, trigonometric functions can be linearized to (12-1) and (12-2)

$$cos(\theta_{kn}) \approx 1 \ (12-1)$$
$$sin(\theta_{kn}) \approx \theta_{kn} = \theta_k - \theta_n. \ (12-2)$$

In general, the resistance in the high voltage transmission line is negligible compared to the line reactance which leads to $G_{kn} \approx 0$. Applying these three assumptions ($|\widetilde{V}_k| \approx 1, (12-2), G_{kn} \approx 0$) to real power flow balance equation (10) leads to (13)

$$P_k = \sum_{n=1}^{N} (B_{kn} \cdot (\theta_k - \theta_n)) \ (13)$$

which is a linearlized equation of (10).

The matrix form of (13) is given by (14)

$$\begin{bmatrix} \vdots \\ P_k \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & (\sum_{n=1}^{N} B_{kn}) - B_{kk} & \cdots & -B_{kn} & \cdots \\ & \vdots & & \vdots & \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ \theta_k \\ \vdots \\ \theta_n \\ \vdots \end{bmatrix} \ (14)$$

$$\overline{P} = \overline{B} \cdot \overline{\theta} \ (15)$$

where $\overline{P}$ and $\overline{\theta}$ are injected power and voltage angle at bus k respectively.

From a physical point of view, $\overline{B}$ can be obtained by (16)

$$\overline{B} = -B_{bus} \text{ (16)}$$

where entries of B_{bus} is defined by (17)

$$B_{bus,kk} = \sum_{n \neq k}^{N} b_{kn}, \ \ B_{bus,kn} = -b_{kn} \text{ (17)}$$

where b_{kn} is the susceptance of the admittance y_{kn} defined by

$$y_{kn} = g_{kn} + jb_{kn}. \text{ (18)}$$

The voltage angle θ_k needs a reference point where $\theta_k = 0$. It is possible to introduce an imaginary voltage bus with $\theta_k = 0$. However, this method makes system more complicated and adds additional bus that does not exist in reality. It is better to select an arbitrary voltage bus with generator as a slack bus with voltage angle of $\theta_{slack} = 0$. Rest of the voltage angles θ_k are determined with respect to this slack bus voltage angle $\theta_{slack} = 0$. If N bus is chosen as the slack bus, the voltage angle of N bus $\theta_N = \theta_{slack} = 0$ and is no longer needed in (15). Also, P_N is given by (19)

$$P_N = -\sum_{n=1}^{N-1} P_n \text{ (19)}$$

where lossless power grid is assumed.

As a result, the Nth row and column of $\overline{B}$ can be eliminated from (15) leading to simplifed real power flow balance equation (20)

$$\overline{P'} = \overline{B'} \cdot \overline{\theta'} \text{ (20)}$$

where $\overline{P'}$ and $\overline{\theta'}$ are matrices without Nth row and $\overline{B'}$ is without Nth row and Nth column.

2-3. Power Flow Modeling

One of the constrain in power transmission is the capacitiy limit at each trasmission line. Although (20) shows the relationship between the voltage angle θ_k and injected real power P_k at k bus, all quantities are associated with the bus and not the transmission line. The real power flow between k bus and n bus F_{kn} is given by (21)

$$F_{kn} = \frac{|\widetilde{V}_k| \cdot |\widetilde{V}_n|}{X_{kn}} \cdot sin(\theta_k - \theta_n) \text{ (21)}$$

where $G_{kn} \approx 0$ is assumed as shown in Fig. 2-2.

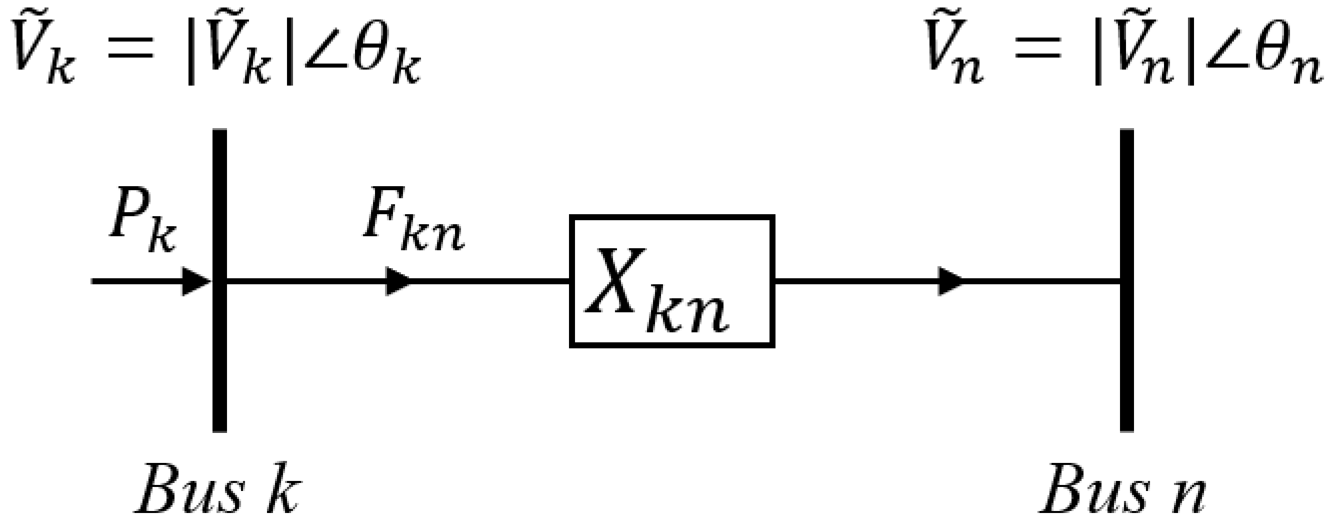

Fig. 2-2

Applying $|\widetilde{V}_k| \approx 1$ and small θ_{kn} assumptions used in 2-2. yeilds (22-1) and (22-2)

$$F_{kn} = \frac{\theta_k - \theta_n}{X_{kn}} \ (22-1)$$
$$F_{kn} = -b_{kn} \cdot (\theta_k - \theta_n) \ (22-2)$$

where $\frac{1}{X_{kn}} = -b_{kn}$.

A matrix form of (22-1) is given by (23)

$$\overline{F} = Diag(\frac{1}{X}) \cdot A^T \cdot \overline{\theta} \ (23)$$

where $Diag(\frac{1}{X})$ is a dianoal matrix given by (24)

$$Diag(\frac{1}{X}) = \begin{bmatrix} \frac{1}{X_{12}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{X_{kn}} \end{bmatrix} (24)$$

and A is the incidence matrix of the power grid network.

From (17), $\frac{1}{X_{kn}} = B_{bus,kn}$ leading to (25)

$$Diag(B_{bus}) = \begin{bmatrix} B_{bus,12} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & B_{bus,kn} \end{bmatrix} (25)$$

2-4. Generation Cost Modeling

The gernation cost/hour of power plant k is normally given by (26)

$$C(k) = \alpha_k + \beta_k \cdot P_G(k) + \gamma_k \cdot P_G(k)^2 \ (26)$$

where $C(k)$ is the generation cost per hour of the power plant k and $P_G(k)$ is the power generated from the power plant k. The cost per hour curve (26) is generally obtained imperically. The coefficient α in (26) is the constant generation cost which includes maintanance cost of the power plant. This cost is independant of power generation $P_G(k)$. On the other hand, both β_k and γ_k coefficients are associated with the cost of fuels (natural gas, coal etc) to generate power P_G. As a result, both β_k and γ_k are associated with the generated power P_G as shown in (26). Normally γ_k is very small compared to α_k or β_k and (26) can be approximated to (27)

$$C(k) = \alpha_k + \beta_k \cdot P_G(k) \ (27)$$

which makes the generation cost per hour a linear (affine) function of generated power P_G.

2-5. Analysis of Power Grid in 2020

Simplified power grid surrounding Seoul in 2020 shown in Fig. 2-3.

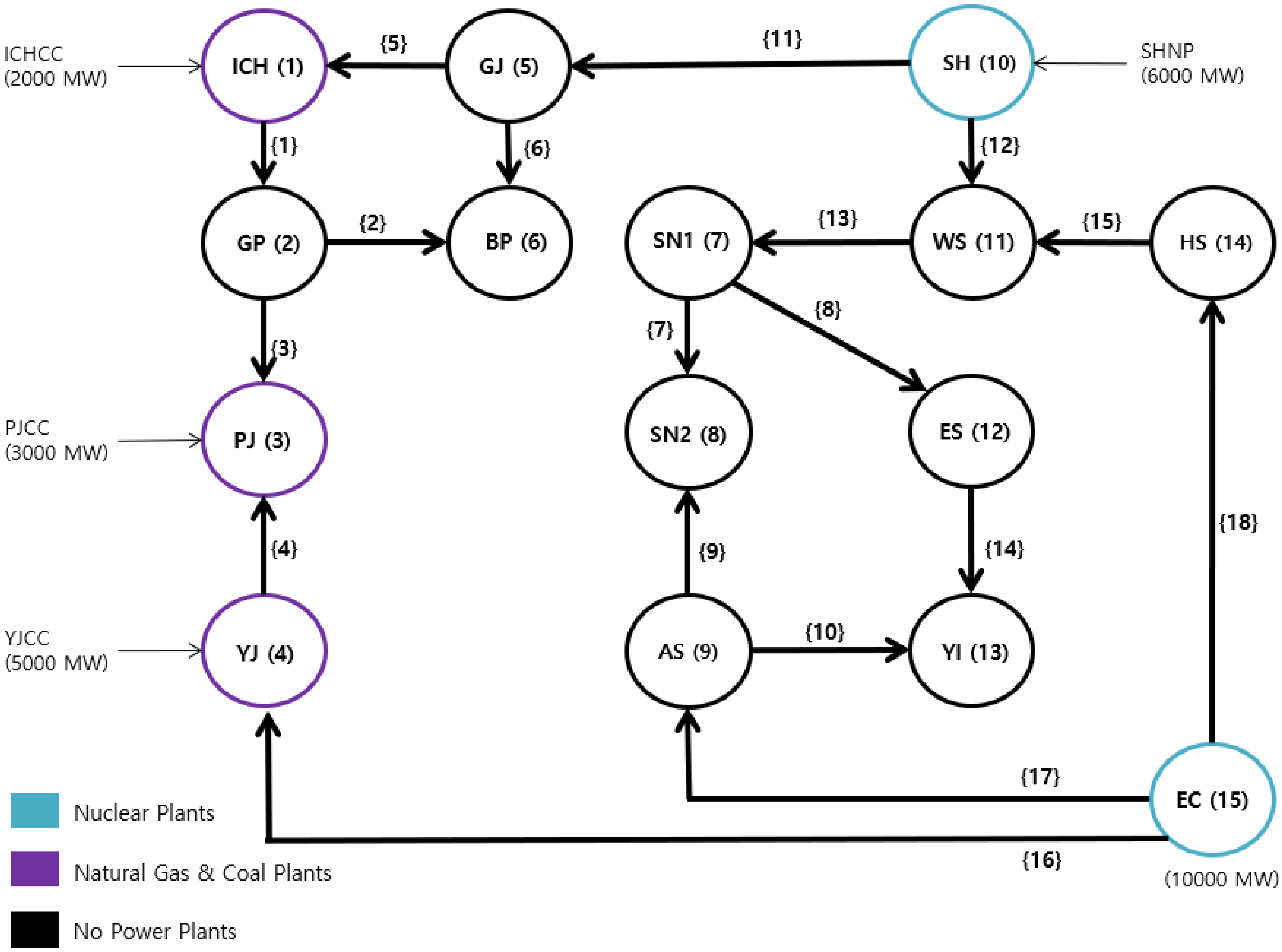

Fig. 2-3

Each node in Fig. 2-3. represents the bus in the grid while arrows represents the transmission line with directions. The direction of these transmission lines are arbitrary and power can flow in the trasmission line in either direction. If the direction of the power flow in a transmission line is the opposite of the direction of the arrow, the power flow will be negative. Thus, it is necessary to set power flow variable F as a free variable.

Each node has a name which is the initial of the location where each nodes are located at. For instance, ICH is an initial of Incheon city WS is an initial of West-Seoul, and PJ stands for Paju city. It is nice to keep the names of each node to indicate the actual location of each buses. However, to avoid confusion caused by unfamiliar location names (Paju, Hwaseong, etc) each buses has a designated numbers from 1 to 15. Similarly, each transmission lines are assigned with numbers to simplify the discussion and avoid confusing names such as from bus 1 to bus 2 line.

Bus 15 or EC represents the power grid of the east coast of Korea. Most of large scale power plants in Korea are located in the east coast. However, load (demand) in the east coast is low and most of the generated power flows to the west. It is generally acceptable to model the whole east coast grid into a single voltage bus with very high generation capability. Although some other form of power plants such as natural gas, coal plants or wind farm exisit in the east coast, majority of the power is generated from nuclear plant. Thus, it is assumed that bus 15 is connected to nuclear plants and colored red in Fig. 2-3. Additional nuclear plant is connected to bus 10 with generation limit of 6000 MW. These nuclear plants are the ones that will be eliminated in 2030 from the power grid. Natural gas and coal plants are connected to bus 1, 3, and 4 respectively.

Load requirments at each bus and cost per hour coefficients α, β, and γ are summarized in Table 2-1.

Bus	Load in 2020 (MW)	Max Gen (MW) in 2020	Max Gen (MW) in 2030	α	β	γ
1	1275	2000	2000	800	2	0.08
2	475.63	0	0	0	0	0
3	794.5	3000	3000	300	2	0.07
4	957.2	5000	5000	300	2	0.07
5	1306.6	0	0	0	0	0
6	726.4	0	0	0	0	0
7	794.1	0	0	0	0	0
8	858.6	0	0	0	0	0
9	1556.4	0	0	0	0	0
10	1184.2	6000	0	200	0.2	0.001
11	381.4	0	0	0	0	0
12	997.8	0	0	0	0	0
13	827.6	0	0	0	0	0
14	536.0	0	0	0	0	0
15	0	10000	1000	100	0.1	0.001

Table 2-1. Bus Information

It should be noted that maximum generation from bus 10 is reduced to 0 and that of bus 15 reduced from 10000 MW to 1000 MW in 2030. The value of γ is smaller than that of α or β which agrees with the assumption used in (27).

The line reactance and transmission capacitiy limits are available in Table 2-2.

Line	From Bus	To Bus	X_L (p.u.)	Capacity Limit (MW)
1	1	2	0.02404	1300
2	2	6	0.05809	1500
3	3	2	0.06734	2000
4	4	3	0.03751	2000
5	5	1	0.26689	2100
6	5	6	0.26689	1000
7	7	8	0.08069	2700

Line	From Bus	To Bus	X_L (p.u.)	Capacity Limit (MW)
8	7	12	0.02045	2000
9	9	8	0.04053	1500
10	9	13	0.0258	2000
11	10	5	0.06293	3300
12	10	11	0.02404	4000
13	11	7	0.05809	4000
14	13	12	0.06734	3000
15	14	11	0.03751	3000
16	15	4	0.04528	1500
17	15	9	0.3011	3000
18	15	14	0.2069	3000

Table 2-2. Transmission Line Information

2-6. Analysis of Power Grid in 2030

Although it is verified in section 3, elminating nuclear plants in bus 10 and 15 makes the MCNF infeasible. That is, it is impossible to fullfill the load requirment with a given power plants which leads to system instability and blackout. Therefore, renewable power plants such as solar and wind plants must be constructed.

Each locations have been analyzed and bus 1, 6, 8, 13, and 14 are found suitable for solar power plants while bus 1, 3, 5, and 10 are suitable for wind power plants as shown in Fig. 2-4.

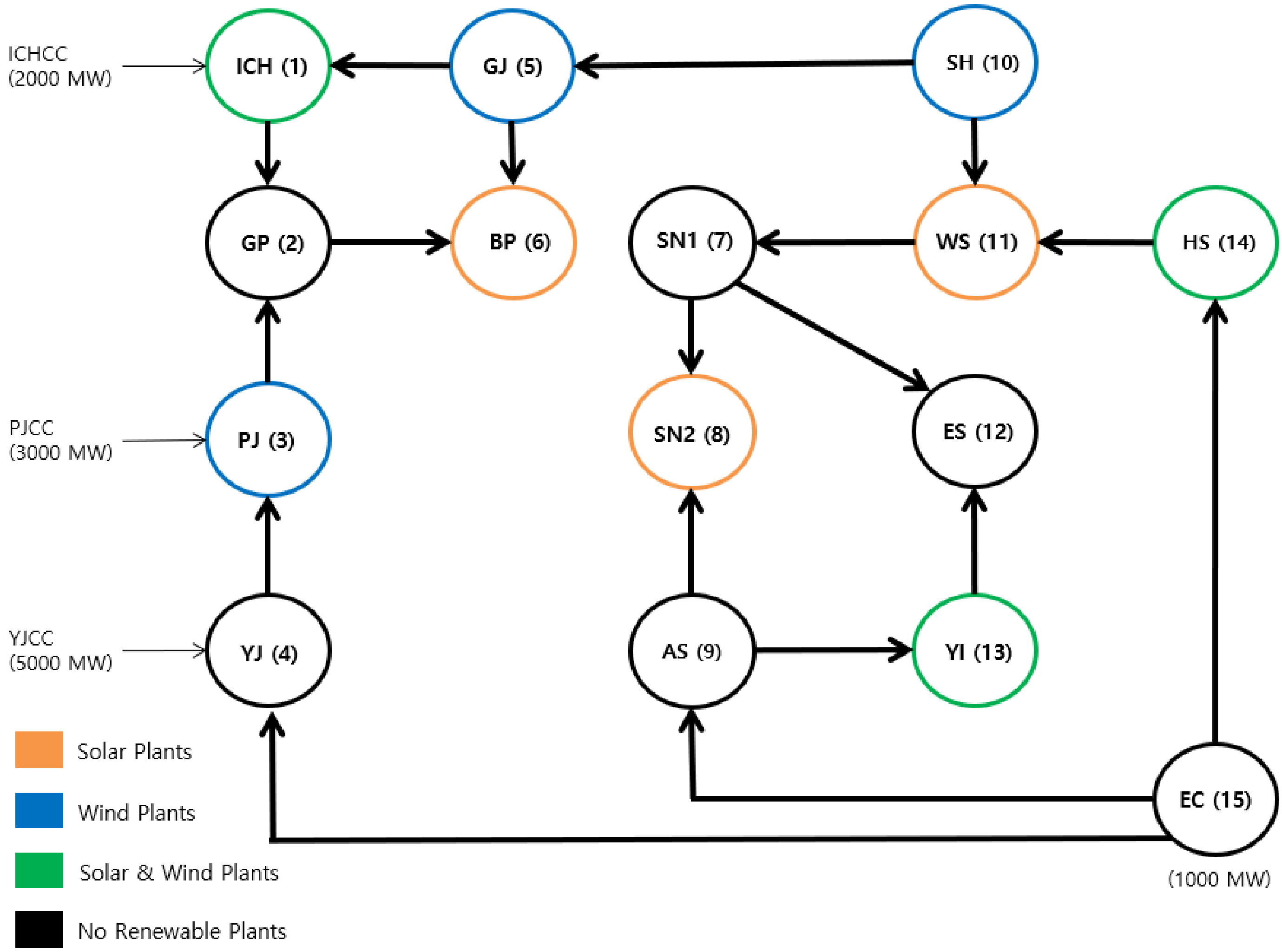

Fig. 2-4

The gernation limit of solar plants $S_{G,max}$ and wind plants $W_{G,max}$ are also predicted. Also, linear generation costs per hour for each power plants are also estimated. The generation limit and generation cost per hour are summarized in Table 2-3. It should be noted that locations that are inadequate for renewable power plants have zero generation limit. For example, bus 3 is only suitable for wind power plant. To indicate this fact, the generation limit of solar plant at bus 3 is zero while that of wind plant is positive (1500 MW).

Bus	Solar Max Gen $S_{G,max}$ (MW)	α_S	β_S	Wind Max Gen $W_{G,max}$ (MW)	α_W	β_W
1	500	300	0.9	3000	3600	0.3
2	0	0	0	0	0	0
3	0	0	0	1500	1500	0.1
4	0	0	0	0	0	0
5	0	0	0	2000	1500	0.2
6	3000	1600	0.9	0	0	0
7	0	0	0	0	0	0
8	1500	800	0.9	0	0	0
9	0	0	0	0	0	0
10	0	0	0	2600	1100	0.3
11	2700	1500	0.8	0	0	0
12	1000	1200	0.7	0	0	0

Bus	Solar Max Gen $S_{G,max}$ (MW)	α_S	β_S	Wind Max Gen $W_{G,max}$ (MW)	α_W	β_W
13	1000	800	0.9	7000	5000	0.1
14	1000	800	0.9	5700	1300	0.1
15	0	0	0	0.001	0	0

Table 2-3. Renewable Power Plants Information

The optimization for power grid in 2020 is a MCNF problem where all power plants have been constructed. Therefore, even if the a power plant is not operating, there are fixed cost for each power plants. Fixed generation costs for each power plants are modeled by coefficient \alpha in power grid in 2020 model.

In the power grid in 2030 model, the number of solar/wind power plants and their locations are also a variable that should be included in the optimization problem. When new renewable power plants are constructed, fixed cost such as maintenence cost are included in the objective function. Since each renewable power plants introduces additional fixed cost to the total generation cost, it is not optimal to build renewable power plants everywhere and not generate any power. Therefore, decision variable z_S, z_W are introduced to the optimization problem.

When solar power plant located at bus k is active, the power generated from that solar plant $S_G(k)$ is positive. In this case, the solar plant at bus k should be construted. As a result, the decision variable for the solar plant at bus k $z_S = 1$ and fixed cost for the solar plant in bus k must be included in the cost. This decision logic is shown by (28)

$$S_G(k) > 0 \Rightarrow z_S(k) = 1 \ (28)$$

where the contrapositive is given by (29)

$$z_S(k) = 0 \Rightarrow S_G(k) \leq 0. \ (29)$$

Since it is impossible to generate negative power, $SG_k \geq 0$ and (29) is equivalent to (30)

$$zS_k = 0 \Rightarrow S_G(k) = 0 \ (30)$$

which can be modeled into a constraint by big M method in MIP. The same logic can be applied to wind power plants with decision variable z_W and generated power W_G. The big M of each power generation is simply the maximum power generation capability of the renewable power plant.

2-7 Decision Variables and Constraints

The following is the list of decision variables and constraints that have been explained throughout section 2. Most of the variables and constraints are shown in matrix form which is the expression used in Julia code in section 3. Only a limited number of variables such as decision variables $z_S(k)$ and $z_W(k)$ are shown in elementwise form which is also consistant with the Julia code in section 3.

Given Values

$\overline{P_L}_{2020}$: Power consumed at each bus in 2020 (MW).

$\overline{P_L}_{2030}$: Power consumed at each bus in 2030 (MW). $\overline{P_L}_{2030} = 1.1 \cdot \overline{P_L}_{2020}$

A: Incidence matrix of the power grid.

$\overline{B'}$: $\overline{B}$ matrix without the row and column associated with the slack bus.

α: Fixed cost (coefficient) of a power plant.

β: Proportional cost coefficienct of a power plant.

$S_{G,max}(k)$: Power generation limit of solar power plant at bus k.

$W_{G,max}(k)$: Power generation limit of wind power plant at bus k.

N: Total number of buses.

Decision Variables and Matrices

$P_G(k)$: Power generated from convenrtional power plant at bus k in MW.

$\overline{P_G}$: Power generated from conventional power plants in MW.

$S_G(k)$: Power generated from solar power plant at bus k in MW.

$\overline{S_G}$: Power generated from solar power plants in MW.

$W_G(k)$: Power generated from wind power plant at bus k in MW.

$\overline{W_G}$: Power generated from wind power plants in MW.

$F(k)$: Power flow at transmission line k in MW.

$\overline{F}$: Power flow at each transmission line in MW.

$\theta(k)$: Voltage angle at bus k in radian.

$\overline{\theta}$: Voltage angle at each bus in radian.

$P_{inj}(k)$: Injected power at bus k in MW.

$\overline{P_{inj}}$: Injected power at each bus in MW.

$z_S(k)$: Binary decision variable for solar power plant at bus k. If $S_G(k) > 0$ then $z_S(k) = 1$, otherwise, $z_S(k) = 0$.

$z_W(k)$: Binary decision variable for wind power plant at bus k. If $W_G(k) > 0$ then $z_W(k) = 1$, otherwise, $z_W(k) = 0$.

Constraints

$\overline{P_{inj}} = \overline{P_G} + \overline{S_G} + \overline{W_G} - \overline{P_L}$: Injected power definition

$\overline{P'} = \overline{B'} \cdot \overline{\theta'}$: real power flow balance at k = 1~N-1 bus

$\sum_{n=1}^{N} P_{inj}(n) = 0$: Conservation of real power

$-\frac{\pi}{6} \leq \theta_k - \theta_n \leq -\frac{\pi}{6} \Longrightarrow -\frac{\pi}{6} \leq A^T \cdot \overline{\theta} \leq -\frac{\pi}{6}$: Power grid linearlization assumption (Power grid stability)

$F_m = B_{bus,k} \cdot (\theta_k - \theta_n)$ for all trasmission lines: bus k to bus n line (or line m) $\Longrightarrow$
$\overline{F} = Diag(B_{bus}) \cdot A^T \cdot \overline{\theta}$.

$\overline{P_G} \leq \overline{P_{G,max}}$

$S_G(k) \leq z_S(k) \cdot S_{G,max}(k)$: realization of if-then logic with big M method.

$W_G(k) \leq z_W(k) \cdot W_{G,max}(k)$: realization of if-then logic with big M method.

2-8 Optimization Problems

2-8-A Math model for Power grid 2020 (LP, MCNF)

Both LP, MCNF problem for power grid in year 2020 and MIP for power grid in year 2030 are shown in this section. The objective function for MCNF problem in year 2020 is given by (31)

$$C_{2020} = \sum_{k=1}^{N} (\alpha + \beta \cdot P_G(k)) \text{ (31)}$$

which is a linear sum of all generation cost per hour at every bus. Since coefficient α and β are zero in buses without power plant, it is possible to use gerneralized cost per hour expression as (31). Formal optimization problem for power grid in year 2020 is as follows.

$$\begin{aligned}
\min \ C_{2020} = & \sum_{k=1}^{N} (\alpha + \beta \cdot P_G(k)) \\
\text{s.t. } & \overline{P_{inj}} = \overline{P_G} - \overline{P_L}_{2020} \\
& \overline{P'} = \overline{B'} \cdot \overline{\theta'} \\
& \sum_{n=1}^{N} P_{inj}(n) = 0 \\
& \theta(N) = 0 \\
& A^T \cdot \overline{\theta} \leq \frac{\pi}{6} \\
& A^T \cdot \overline{\theta} \geq -\frac{\pi}{6} \\
& \overline{F} = Diag(B_{bus}) \cdot A^T \cdot \overline{\theta} \\
& \overline{P_G} \leq \overline{P_{G,max}} \\
& \overline{P_{inj}} \text{ : free variable} \\
& \overline{\theta} \text{ : free variable} \\
& \overline{P_G} \geq 0 \\
& \overline{F} \text{ : free variable}
\end{aligned}$$

Solving MCNF problem of power grid in 2020 is an LP since nonlinear term in the generation cost (26) is elminated and linear generation cost (27) is used instead.

2-8-A Math model for Power grid 2030 (MIP)

The objective function of MIP in year 2030 is given by (32)

$$C_{2030} = \sum_{k=1}^{N} (\alpha + \beta \cdot P_G(k)) + \sum_{k=1}^{N} (\alpha_S + \beta_S \cdot S_G(k)) \cdot z_S(k) + \sum_{k=1}^{N} (\alpha_W + \beta_W \cdot W_G(k)) \cdot z_W(k) \text{ (32)}$$

where binary variables $z_S(k)$ and $z_W(k)$ are used to model nonlinear fixed cost.

$$\min \ C_{2030} = \sum_{k=1}^{N}(\alpha + \beta \cdot P_G(k)) + \sum_{k=1}^{N}(\alpha_S + \beta_S \cdot S_G(k)) \cdot z_S(k) + \sum_{k=1}^{N}(\alpha_W + \beta_W \cdot W_G(k)) \cdot z_W(k)$$

$$\begin{aligned}
\text{s.t. } & \overline{P_{inj}} = \overline{P_G} + \overline{S_G} + \overline{W_G} - \overline{P_{L2030}} \\
& \overline{P'} = \overline{B'} \cdot \overline{\theta'} \\
& \sum_{n=1}^{N} P_{inj}(n) = 0 \\
& \theta(N) = 0 \\
& A^T \cdot \overline{\theta} \leq \frac{\pi}{6} \\
& A^T \cdot \overline{\theta} \geq -\frac{\pi}{6} \\
& \overline{F} = Diag(B_{bus}) \cdot A^T \cdot \overline{\theta} \\
& \overline{P_G} \leq \overline{P_{G,max}} \\
& S_G(k) \leq z_S(k) \cdot S_{G,max}(k) \text{ for all k = 1, 2,..., N} \\
& W_G(k) \leq z_W(k) \cdot W_{G,max}(k) \text{ for all k = 1, 2,..., N} \\
& \overline{P_{inj}} \text{ : free variable} \\
& \overline{\theta} \text{ : free variable} \\
& \overline{P_G} \geq 0 \\
& \overline{S_G} \geq 0 \\
& \overline{W_G} \geq 0 \\
& \overline{F} \text{ : free variable} \\
& z_S(k) \in \{0, 1\} \text{ for all k = 1, 2, ..., N} \\
& z_W(k) \in \{0, 1\} \text{ for all k = 1, 2, ..., N}
\end{aligned}$$

3. Solution

3-1. Data Setup

In [3]:

```
using JuMP, Gurobi, NamedArrays

# Useful Variables and Power Grid Data
lineNum = 18
busNum = 15
genNum = 5
fromBus = [1 2 3 4 5 5 7 7 9 9 10 10 11 13 14 15 15 15]
toBus =   [2 6 2 3 1 6 8 12 8 13 5 11 7 12 11 4 9 14]
Pb = 1000 # per unit base
xL = [0.02404, 0.05809, 0.06734, 0.03751, 0.26689, 0.26689, 0.08069, 0.02045, 0.04053, 0.0258, 0.062
busName = [:1, :2, :3, :4, :5, :6, :7, :8, :9, :10, :11, :12, :13, :14, :15]

#<Line Data Setting>#

# Line capacitiy limit (MW)
cap = [1300, 1500, 2000, 2000, 2100, 1000, 2700, 2000, 1500, 2000, 3300, 4000, 4000, 3000, 3000, 150

# Line reactance and suceptance (pu)
BL = zeros(lineNum)
for n=1:lineNum
    BL[n] = 1/xL[n]
end

diagBL = zeros(lineNum,lineNum) # Diagonal matrix of inverse reactance
for n=1:lineNum
    diagBL[n,n] = BL[n]
end

# Incidence matrix
A = zeros(busNum,lineNum) # incidence matrix

for n=1:lineNum
    A[fromBus[n],n] = 1
    A[toBus[n],n] = -1
end

# <Bus Data Setting> #

# Load at each bus (MW)
PL_2020 = [1275, 475.63, 794.5, 987.2, 1306.6, 726.4, 794.1, 858.6, 1556.4, 1184.2, 381.4, 997.8, 82
PL_2030 = 1.1*PL_2020 # Power demand increased by 10 % in 2030

# Bbus Matrix
Bbus = zeros(busNum, busNum)

for n=1:lineNum              # Bij entries (i~=j)
    Bbus[fromBus[n],toBus[n]] = BL[n]
    Bbus[toBus[n],fromBus[n]] = BL[n]
end

for n=1:busNum               # Bii entries
    Bbus[n,n] = -sum(Bbus[n,:])
end

# BbarPrime
BbarPrime = -Bbus[1:(busNum-1),1:(busNum-1)]

# Generation Limit
PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0, 6000, 0, 0, 0, 0, 10000]
```

```
# <Generation Cost>
alpha = [800 0 300 300 0 0 0 0 0 200 0 0 0 0 100]
beta = [1.4 0 2 2 0 0 0 0 0 0.2 0 0 0 0 0.1];
```

3-2. Power Grid in Year 2020

In [5]:

```
m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

@variable(m, PG[1:busNum] >=0)      # Power gernerated from conventional power plants

@variable(m, F[1:lineNum])          # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])       # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])        # Power injected to the bus, if the bus is sink, Pinj is negative

# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) + Pb base
@constraint(m, F .<= cap)                        # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)                       # since F is in per unit with 1000 MW, cap should be

# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/6)                    # angle differnence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/6)                    # angle should be smaller than 30 deg
@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] - PL_2020[1:(busNum)]) # definition of injected p
@constraint(m, sum(Pinj) == 0)                                                                      #
@constraint(m, theta[busNum]==0)                                                                    #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                             #

# Generation constriants
@constraint(m,PG .<= PGMax)

# Objective
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))
@objective(m, Min, conventionalGenCost)

optimize!(m)
println("\nThe minimum Gen Cost: ", value.(conventionalGenCost)," [\$/hour]\n")

println("\n Power Generation")
for k=1:busNum
    if(value.(PG[k]) >= 0.0001)
        println("Bus ",k ,": ", value.(PG[k])," [MW]\t")
    end
end

println("\n Power Flow (MW)")
for k=1:lineNum
        println("From Bus ", fromBus[k]," to ",toBus[k]," Power Flow: ",value.(F[k])," MW \t")
end
```

```
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The minimum Gen Cost: 8095.482068182921 [$/hour]



 Power Generation
Bus 1: 2000.0 [MW]
Bus 3: 1013.3363516752215 [MW]
Bus 10: 6000.0 [MW]
Bus 15: 3688.093648324778 [MW]

 Power Flow (MW)
From Bus 1 to 2 Power Flow: 864.3929126829171 MW
From Bus 2 to 6 Power Flow: 418.1374728080491 MW
```

```
From Bus 3 to 2 Power Flow: 29.374560125127573 MW
From Bus 4 to 3 Power Flow: -189.46179155009304 MW
From Bus 5 to 1 Power Flow: 139.3929126829139 MW
From Bus 5 to 6 Power Flow: 308.26252719195077 MW
From Bus 7 to 8 Power Flow: 871.5610086100014 MW
From Bus 7 to 12 Power Flow: 1629.8859007281135 MW
From Bus 9 to 8 Power Flow: -12.961008610003773 MW
From Bus 9 to 13 Power Flow: 195.51409927190616 MW
From Bus 10 to 5 Power Flow: 1754.2554398748653 MW
From Bus 10 to 11 Power Flow: 3061.544560125136 MW
From Bus 11 to 7 Power Flow: 3295.5469093381116 MW
From Bus 13 to 12 Power Flow: -632.0859007280915 MW
From Bus 14 to 11 Power Flow: 615.4023492129754 MW
From Bus 15 to 4 Power Flow: 797.7382084499085 MW
From Bus 15 to 9 Power Flow: 1738.9530906619025 MW
From Bus 15 to 14 Power Flow: 1151.4023492129757 MW
```

3-3. Power Grid in Year 2030 with 10 % Load Increase $\overline{P_{G}}_{2030}$

In [6]:

```
m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

@variable(m, PG[1:busNum] >=0)      # Power gernerated from conventional power plants

@variable(m, F[1:lineNum])          # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])       # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])        # Power injected to the bus, if the bus is sink, Pinj is negative

# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) + Pb base
@constraint(m, F .<= cap)                            # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)                           # since F is in per unit with 1000 MW, cap should be

# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/6)                        # angle differnence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/6)                        # angle should be smaller than 30 deg
@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] - PL_2030[1:(busNum)]) # definition of injected p
@constraint(m, sum(Pinj) == 0)                                                                        #
@constraint(m, theta[busNum]==0)                                                                      #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                               #

# Generation constraints
@constraint(m,PG .<= PGMax)

# Objective
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))
@objective(m, Min, conventionalGenCost)

optimize!(m)

# Output Result
println("\nThe minimum Gen Cost ", value.(conventionalGenCost),"[\$/hour]\n")

println("\n Power Generation")
for k=1:busNum
    if(value.(PG[k]) >= 0.0001)
        println("Bus ",k ,": ", value.(PG[k])," [MW]\t")
    end
end

println("\n Power Flow (MW)")
for k=1:lineNum
        println("From Bus ", fromBus[k]," to ",toBus[k]," Power Flow: ",value.(F[k])," MW \t")
end
```

```
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The minimum Gen Cost 12428.69047563747[$/hour]



 Power Generation
Bus 1: 2000.0 [MW]
Bus 3: 3000.0 [MW]
Bus 4: 227.1227240197211 [MW]
Bus 10: 6000.0 [MW]
Bus 15: 2744.450275980279 [MW]
```

```
 Power Flow (MW)
From Bus 1 to 2 Power Flow: 193.44021595238746 MW
From Bus 2 to 6 Power Flow: 973.7368409532032 MW
From Bus 3 to 2 Power Flow: 1303.4896250008146 MW
From Bus 4 to 3 Power Flow: -822.5603749991856 MW
From Bus 5 to 1 Power Flow: -404.05978404761305 MW
From Bus 5 to 6 Power Flow: -174.69684095320343 MW
From Bus 7 to 8 Power Flow: 1042.8394558652253 MW
From Bus 7 to 12 Power Flow: 1882.6474534728477 MW
From Bus 9 to 8 Power Flow: -98.37945586522801 MW
From Bus 9 to 13 Power Flow: 125.29254652712916 MW
From Bus 10 to 5 Power Flow: 858.5033749991835 MW
From Bus 10 to 11 Power Flow: 3838.8766250008175 MW
From Bus 11 to 7 Power Flow: 3798.9969093380696 MW
From Bus 13 to 12 Power Flow: -785.0674534728678 MW
From Bus 14 to 11 Power Flow: 379.66028433725114 MW
From Bus 15 to 4 Power Flow: 36.23690098109346 MW
From Bus 15 to 9 Power Flow: 1738.9530906619025 MW
From Bus 15 to 14 Power Flow: 969.2602843372514 MW
```

3-4. Power Grid in 2030 without Nuclear Power Plants

This problem is infeasible. It is verified from this problem that renewable power plants must be constructed to avoid blackouts in year 2030.

In [7]:

```
# Power Generation from Nuclear Plants are resticted to 0
# PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0, 6000, 0, 0, 0, 0, 10000]
  PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0,    0, 0, 0, 0, 0, 1000]


m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

@variable(m, PG[1:busNum] >=0)      # Power gernerated from conventional power plants

@variable(m, F[1:lineNum])          # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])       # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])        # Power injected to the bus, if the bus is sink, Pinj is negative

# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) + Pb base
@constraint(m, F .<= cap)                        # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)                       # since F is in per unit with 1000 MW, cap should be

# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/6)                 # angle differnence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/6)                 # angle should be smaller than 30 deg
@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] - PL_2030[1:(busNum)]) # definition of injected p
@constraint(m, sum(Pinj) == 0)                                                                      #
@constraint(m, theta[busNum]==0)                                                                    #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                             #

# Generation constriants
@constraint(m,PG .<= PGMax)

# Objective
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))
@objective(m, Min, conventionalGenCost)

optimize!(m)
println("\nThe minimum Gen Cost \$/h ", value.(conventionalGenCost),"\n")
println("\n Power Generation : ", value.(PG),"[MW]\n")
println("\n Power Injection: ", value.(Pinj),"[MW]\n")
println("\n Power Flow: ", value.(F),"[MW]\n")
println("\n angle: ", value.(theta),"[rad]\n")
```

```
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Result index of attribute MathOptInterface.VariablePrimal(1) out of bounds. There ar
e currently 0 solution(s) in the model.

Stacktrace:
 [1] check_result_index_bounds at C:\Users\samsung\.julia\packages\MathOptInterface
\bygN7\src\attributes.jl:133 [inlined]
 [2] get(::Gurobi.Optimizer, ::MathOptInterface.VariablePrimal, ::MathOptInterface.V
ariableIndex) at C:\Users\samsung\.julia\packages\Gurobi\7YNJV\src\MOI_wrapper.jl:20
50
 [3] get(::MathOptInterface.Bridges.LazyBridgeOptimizer{Gurobi.Optimizer}, ::MathOpt
Interface.VariablePrimal, ::MathOptInterface.VariableIndex) at C:\Users\samsung\.jul
ia\packages\MathOptInterface\bygN7\src\Bridges\bridge_optimizer.jl:752
 [4] get(::MathOptInterface.Utilities.CachingOptimizer{MathOptInterface.AbstractOpti
mizer,MathOptInterface.Utilities.UniversalFallback{MathOptInterface.Utilities.Model
{Float64}}}, ::MathOptInterface.VariablePrimal, ::MathOptInterface.VariableIndex) at
```

```
C:\Users\samsung\.julia\packages\MathOptInterface\bygN7\src\Utilities\cachingoptimiz
er.jl:568
 [5] _moi_get_result(::MathOptInterface.Utilities.CachingOptimizer{MathOptInterface.
AbstractOptimizer,MathOptInterface.Utilities.UniversalFallback{MathOptInterface.Util
ities.Model{Float64}}}, ::MathOptInterface.VariablePrimal, ::Vararg{Any,N} where N)
at C:\Users\samsung\.julia\packages\JuMP\YXK4e\src\JuMP.jl:847
 [6] get(::Model, ::MathOptInterface.VariablePrimal, ::VariableRef) at C:\Users\sams
ung\.julia\packages\JuMP\YXK4e\src\JuMP.jl:877
 [7] #value#28(::Int64, ::typeof(value), ::VariableRef) at C:\Users\samsung\.julia\p
ackages\JuMP\YXK4e\src\variables.jl:767
 [8] #value at .\none:0 [inlined]
 [9] #37 at C:\Users\samsung\.julia\packages\JuMP\YXK4e\src\aff_expr.jl:345 [inline
d]
 [10] value(::GenericAffExpr{Float64,VariableRef}, ::JuMP.var"#37#38"{Int64}) at
C:\Users\samsung\.julia\packages\JuMP\YXK4e\src\aff_expr.jl:159
 [11] _broadcast_getindex at C:\Users\samsung\.julia\packages\JuMP\YXK4e\src\aff_exp
r.jl:345 [inlined]
 [12] getindex at .\broadcast.jl:563 [inlined]
 [13] copy at .\broadcast.jl:829 [inlined]
 [14] materialize(::Base.Broadcast.Broadcasted{Base.Broadcast.DefaultArrayStyle{0},N
othing,typeof(value),Tuple{Base.RefValue{GenericAffExpr{Float64,VariableRef}}}}) at
.\broadcast.jl:819
 [15] top-level scope at In[7]:36
```

3-5. Power Grid in 2030 without Nuclear Plants and with Renewable Power Plants

In [8]:

```
# Power Generation from Nuclear Plants are resticted to 0
# PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0, 6000, 0, 0, 0, 0, 10000
  PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0,    0, 0, 0, 0, 0, 1000] # Maximum conventional power

# Solar Plant Costs and constraints
alphaS = [300 0 0 0 0 1600 0 800  0 0 1500 1200 800  800  0]      # alpha coefficient solar power g
betaS =  [0.9 0 0 0 0 0.9  0 0.9  0 0 0.8  0.7  0.9  0.9  0]      # beta coefficient for solar powe
SGMax =  [500 0 0 0 0 3000 0 1500 0 0 2700 1000 1000 1000 0]  # Maximum solar power at each bus in M

# Wind Plant Costs and constraints
alphaW = [3600 0 1500 0 1500 0 0 0 0 1100 0 0 5000 1300 0] # alpha coefficient wind power generation
betaW =  [0.30 0 0.10 0 0.20 0 0 0 0 0.30 0 0 0.10 0.10 0] # beta coefficient for wind power generat
WGMax =  [3000 0 1500 0 2000 0 0 0 0 2600 0 0 7000 5700 0] # Maximum wind power at each bus in MW

# Select Solver: Gurobi works fine
m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

# Decision Variables
@variable(m, PG[1:busNum] >=0)     # Power gernerated from conventional power plants

@variable(m, SG[1:busNum] >=0)     # Power generated from Solar Power Plants
@variable(m, WG[1:busNum] >=0)     # Power generated from Wind Power Plants

@variable(m, F[1:lineNum])         # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])      # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])       # Power injected to the bus, if the bus is sink, Pinj is negative

@variable(m,zS[1:busNum], Bin)     # Decision variable for solar power plant if zS[k] = 0 no solar p
@variable(m,zW[1:busNum], Bin)     # Decision variable for wind power plant if zW[k] = 0 no wind pla


# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) * Pb base in MW
@constraint(m, F .<= cap)                         # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)



# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/6)                   # angle differnence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/6)                   # angle should be smaller than 30 deg
@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] + SG[1:(busNum)] + WG[1:(busNum)]- PL_2030[1:(bus
@constraint(m, sum(Pinj) == 0)                                                                      #
@constraint(m, theta[busNum]==0)                                                                    #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                             #

# Generation constraints
@constraint(m,PG .<= PGMax)

# Renewable constaints (Big M)
@constraint(m, solarConstr[k in 1:busNum], SG[k] <= SGMax[k]*zS[k])
@constraint(m, windConstr[k in 1:busNum], WG[k] <= WGMax[k]*zW[k])


# Objective: Min generation cost
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))   # convention
@expression(m, solarGenCost, sum((alphaS[k] + beta[k]*SG[k])*zS[k] for k in 1:busNum))  # solar gene
@expression(m, windGenCost,  sum((alphaW[k] + betaS[k]*WG[k])*zW[k] for k in 1:busNum)) # wind gener
@expression(m, totalGenCost, conventionalGenCost + solarGenCost + windGenCost)          # total gene
```

```
@objective(m, Min, totalGenCost)

optimize!(m)

# Output Results
println("\nThe minimum Gen Cost ", value.(totalGenCost)," [\$/hour]\n")

println("\n Power Flow (MW)")
for k=1:lineNum
        println("From Bus ", fromBus[k]," to ",toBus[k]," Power Flow: ",value.(F[k])," MW \t")
end

println("\n Conventional Power Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(PG[k]) >= 0.00001)
        println("Bus ", k," (",value.(PG[k])," MW)","\t")
    end
end

println("\n Location of Solar Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(zS[k]) == 1)
        println("Bus ", k," (",value.(SG[k])," MW)","\t")
    end
end

println("\n Location of Wind Plants (Generated Power (MW))")
for k=1:busNum
    if(value.(zW[k]) == 1)
        println("Bus ", k," (",value.(WG[k])," MW)","\t")
    end
end
```

```
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The minimum Gen Cost 10210.121065706051 [$/hour]


 Power Flow (MW)
From Bus 1 to 2 Power Flow: -922.4323913592943 MW
From Bus 2 to 6 Power Flow: -1500.0 MW
From Bus 3 to 2 Power Flow: -54.374608640706356 MW
From Bus 4 to 3 Power Flow: -680.4246086407065 MW
From Bus 5 to 1 Power Flow: 258.55256170780984 MW
From Bus 5 to 6 Power Flow: -151.01795306710665 MW
From Bus 7 to 8 Power Flow: 216.35606628631217 MW
From Bus 7 to 12 Power Flow: 1308.7297099676625 MW
From Bus 9 to 8 Power Flow: -771.8960662863128 MW
From Bus 9 to 13 Power Flow: -300.78970996766293 MW
From Bus 10 to 5 Power Flow: 1544.7946086407032 MW
From Bus 10 to 11 Power Flow: -247.41460864070334 MW
From Bus 11 to 7 Power Flow: 2398.5957762539742 MW
From Bus 13 to 12 Power Flow: -211.14970996766306 MW
From Bus 14 to 11 Power Flow: 365.55038489467756 MW
From Bus 15 to 4 Power Flow: 405.49539135929365 MW
From Bus 15 to 9 Power Flow: 639.3542237460257 MW
From Bus 15 to 14 Power Flow: -44.849615105322464 MW
```

```
 Conventional Power Plants (Generated Power (MW)):
Bus 1 (221.5150469328937 MW)
Bus 15 (1000.0 MW)

 Location of Solar Plants (Generated Power (MW)):
Bus 6 (2450.0579530671066 MW)
Bus 8 (1500.0 MW)
Bus 11 (2700.0 MW)
Bus 13 (1000.0 MW)
Bus 14 (1000.0 MW)

 Location of Wind Plants (Generated Power (MW))
Bus 3 (1500.0 MW)
Bus 10 (2600.0 MW)
```

In [ ]:

```

```

4. Results and Discussion

4-1. Power Grid in Year 2020 Results

The minimum generation cost in year 2020 is $ 8095.48 /hour. The power flow at each transmission lines and generated power at each power plants are shown in Fig. 4-1. Arrows in Fig. 4-1 are adjusted to represent positive power flow. Active constraints at the optimal solution is colored red. In the case of year 2020 power grid, the generation limit of bus 1 and 10 are the active constraints. The capacitiy of transimission lines are slack and can carry higher power flow. Therefore in year 2020, increasing the transmission capability does not reduce the generation cost.

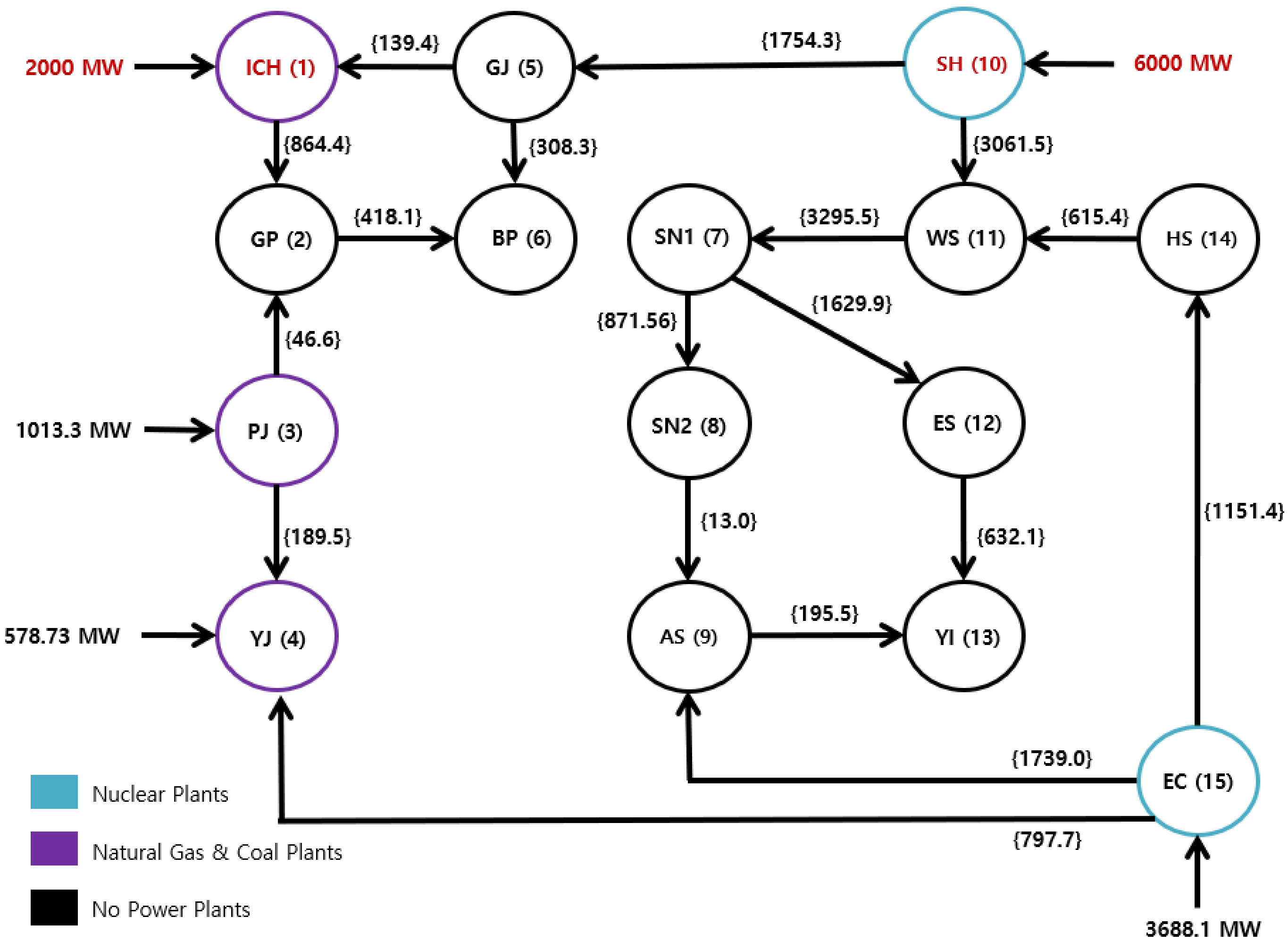

Fig. 4-1

4-2. Power Grid in Year 2030 Results

The minimum generation cost in year 2030 without nuclear plants and with renewable power plants is $\$ 10210.12$ / hour which is $\$ 2114.64$ higher than that of grid in year 2020. However, this is not a fair comparison since the load increased by 10% from year 2020 to year 2030. The MCNF problem with year 2020 power grid is solved with the increase load and the generation cost is $\$ 12428.69$ /hour which is $\$ 2218.57$ /hour more expensive than that of the power grid in year 2030.

The power flow at each transmission lines and generated power at each power plants are shown in Fig. 4-2. Similar to Fig. 4-1, arrows in Fig. 4-2 are adjusted to represent positive power flow. Active constraints at the optimal solution is colored red. Compared to year 2020 grid case, there are more active constraints at the optimial solution in the year 2030 grid. This is because renewable power plants in year 2030 grid is optimized to minimize the generation cost. If the optimal solution has multiple renewable power plants that are slack, it means that the capacity of the transmission line associated with the slack renewable power plant is constraining the optimal solution. This phenomenon is observed with bus 6 connected to line 2. Although the solar plant in bus 6 is slack, the power flow in the line 2 is at it limit. If the capacity of line 2 increases to 3000 MW, the total generation cost reduces to $\$ 9340.20$ /hour while the solar plant in bus 6 meets its limit.

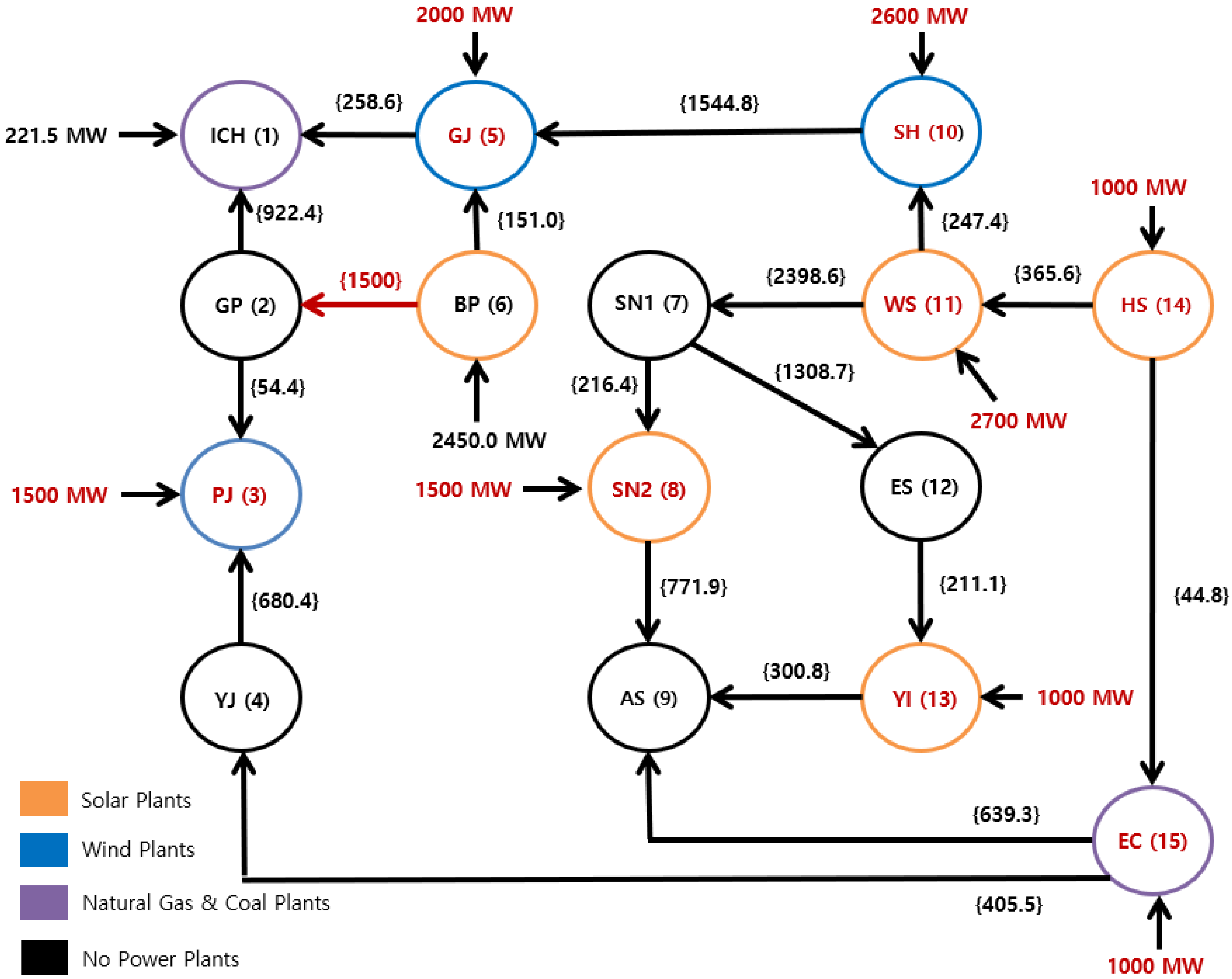

Fig. 4-2

4-3. Limitations of the Model and Sensitivity to Assumptions

One of the most critical assumption made in the linear power grid model are as follows:

(i) Bus voltages are almost constant: $|\widetilde{V_k}| = 1p.u.$

(ii) The voltage angle difference between two connected buses are less than 30^o: $-\frac{\pi}{6} \leq \theta_k - \theta_n \leq \frac{\pi}{6}$

(iii) Power grid highly inductive and almost lossless: $G_{kn} \approx 0$

The (i) is strictly controlled by the power grid engineers and it is enforced by standard regulation. However, (ii) and (iii) are questionable. Therefore in this section, it will be verified that that both assumptions hold and the linear power grid model is a decent approximation for more complicated nonlinear power grid model.

4-3.A. Voltage Angle Assumption

The voltage angle constraint is extended from $-\frac{\pi}{6} \leq \theta_k - \theta_n \leq \frac{\pi}{6}$ to $-\frac{\pi}{3} \leq \theta_k - \theta_n \leq \frac{\pi}{3}$. The year 2030 power grid problem is solved again with this changed assumption.

It is verified that extending the voltage range does not change the optimal generation cost value. As a result, it is concluded that the voltage angle constraint is not active and $-\frac{\pi}{6} \leq \theta_k - \theta_n \leq \frac{\pi}{6}$ assumption is valid.

In [42]:

```
cap = [1300, 1500, 2000, 2000, 2100, 1000, 2700, 2000, 1500, 2000, 3300, 4000, 4000, 3000, 3000, 150

# Power Generation from Nuclear Plants are resticted to 0
# PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0, 6000, 0, 0, 0, 0, 10000
  PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0,    0, 0, 0, 0, 0, 1000] # Maximum conventional power

# Solar Plant Costs and constraints
alphaS = [300 0 0 0 0 1600 0 800  0 0 1500 1200 800  800  0]       # alpha coefficient solar power g
betaS =  [0.9 0 0 0 0 0.9  0 0.9  0 0 0.8  0.7  0.9  0.9  0]       # beta coefficient for solar powe
SGMax =  [500 0 0 0 0 3000 0 1500 0 0 2700 1000 1000 1000 0]  # Maximum solar power at each bus in M

# Wind Plant Costs and constraints
alphaW = [3600 0 1500 0 1500 0 0 0 0 1100 0 0 5000 1300 0] # alpha coefficient wind power generation
betaW =  [0.30 0 0.10 0 0.20 0 0 0 0 0.30 0 0 0.10 0.10 0] # beta coefficient for wind power generat
WGMax =  [3000 0 1500 0 2000 0 0 0 0 2600 0 0 7000 5700 0] # Maximum wind power at each bus in MW

# Select Solver: Gurobi works fine
m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

# Decision Variables
@variable(m, PG[1:busNum] >=0)     # Power gernerated from conventional power plants

@variable(m, SG[1:busNum] >=0)     # Power generated from Solar Power Plants
@variable(m, WG[1:busNum] >=0)     # Power generated from Wind Power Plants

@variable(m, F[1:lineNum])         # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])      # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])       # Power injected to the bus, if the bus is sink, Pinj is negative

@variable(m,zS[1:busNum], Bin)     # Decision variable for solar power plant if zS[k] = 0 no solar p
@variable(m,zW[1:busNum], Bin)     # Decision variable for wind power plant if zW[k] = 0 no wind pla


# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) * Pb base in MW
@constraint(m, F .<= cap)                        # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)



# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/3)                  # angle differnence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/3)                  # angle should be smaller than 60 deg
@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] + SG[1:(busNum)] + WG[1:(busNum)]- PL_2030[1:(bus
@constraint(m, sum(Pinj) == 0)                                                                     #
@constraint(m, theta[busNum]==0)                                                                   #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                            #

# Generation constriants
@constraint(m,PG .<= PGMax)

# Renewable constaints (Big M)
@constraint(m, solarConstr[k in 1:busNum], SG[k] <= SGMax[k]*zS[k])
@constraint(m, windConstr[k in 1:busNum], WG[k] <= WGMax[k]*zW[k])



# Objective: Min generation cost
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))   # convention
@expression(m, solarGenCost, sum((alphaS[k] + beta[k]*SG[k])*zS[k] for k in 1:busNum))  # solar gene
```

```
@expression(m, windGenCost,  sum((alphaW[k] + betaS[k]*WG[k])*zW[k] for k in 1:busNum)) # wind gener
@expression(m, totalGenCost, conventionalGenCost + solarGenCost + windGenCost)          # total gene

@objective(m, Min, totalGenCost)

optimize!(m)

# Output Results
println("\nThe minimum Gen Cost ", value.(totalGenCost)," [\$/hour]\n")

println("\n Power Flow (MW)")
for k=1:lineNum
        println("From Bus ", fromBus[k]," to ",toBus[k]," Power Flow: ",value.(F[k])," MW \t")
end

println("\n Conventional Power Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(PG[k]) >= 0.00001)
        println("Bus ", k," (",value.(PG[k])," MW)","\t")
    end
end

println("\n Location of Solar Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(zS[k]) == 1)
        println("Bus ", k," (",value.(SG[k])," MW)","\t")
    end
end

println("\n Location of Wind Plants (Generated Power (MW))")
for k=1:busNum
    if(value.(zW[k]) == 1)
        println("Bus ", k," (",value.(WG[k])," MW)","\t")
    end
end
```

```
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The minimum Gen Cost 10210.121065706051 [$/hour]


 Power Flow (MW)
From Bus 1 to 2 Power Flow: -922.4323913592943 MW
From Bus 2 to 6 Power Flow: -1500.0 MW
From Bus 3 to 2 Power Flow: -54.374608640706356 MW
From Bus 4 to 3 Power Flow: -680.4246086407065 MW
From Bus 5 to 1 Power Flow: 258.5525617078098 4 MW
From Bus 5 to 6 Power Flow: -151.01795306710665 MW
From Bus 7 to 8 Power Flow: 216.35606628631217 MW
From Bus 7 to 12 Power Flow: 1308.7297099676625 MW
From Bus 9 to 8 Power Flow: -771.8960662863128 MW
From Bus 9 to 13 Power Flow: -300.78970996766293 MW
From Bus 10 to 5 Power Flow: 1544.7946086407032 MW
From Bus 10 to 11 Power Flow: -247.41460864070334 MW
From Bus 11 to 7 Power Flow: 2398.5957762539742 MW
From Bus 13 to 12 Power Flow: -211.14970996766306 MW
From Bus 14 to 11 Power Flow: 365.5503848946775 6 MW
From Bus 15 to 4 Power Flow: 405.49539135929365 MW
From Bus 15 to 9 Power Flow: 639.3542237460257 MW
```

```
From Bus 15 to 14 Power Flow: -44.849615105322464 MW

 Conventional Power Plants (Generated Power (MW)):
Bus 1 (221.5150469328937 MW)
Bus 15 (1000.0 MW)

 Location of Solar Plants (Generated Power (MW)):
Bus 6 (2450.0579530671066 MW)
Bus 8 (1500.0 MW)
Bus 11 (2700.0 MW)
Bus 13 (1000.0 MW)
Bus 14 (1000.0 MW)

 Location of Wind Plants (Generated Power (MW))
Bus 3 (1500.0 MW)
Bus 10 (2600.0 MW)
```

4-3.B. Lossless Power Grid Assumption

For simple analysis, let's assume that the resistance of a transmission line is proportional to the reactance X_L. In general, the line resistance is at least ten time smaller than that of line reactance X_L. The sum of all admittance seen from the voltage bus k is the diagonal component of B_{bus}. As a result, the power loss in the transmission line is lumped into the loss in each voltage buses and defined as (33)

$$P_{loss} = -0.1 \cdot \frac{1}{B_{busk,k}} \ (33)$$

Due to the conservation of power, the injected power $\overline{P_{inj}}$ should equal to the the sum of all power generation, load and loss as shown in (34)

$$\overline{P_{inj}} = \overline{P_G} + \overline{S_G} + \overline{W_G} - \overline{P_{L2030}} - \overline{P_{loss}} \ (34)$$

As a result, the $\overline{P_{inj}}$ constraint is changed into (34) to include the impact of the power loss in the power grid. As expected, the influence of power loss is marginal and the generation cost is exactly the same as the result obtained from the lossless power grid model.

In [51]:

```
# Power Generation from Nuclear Plants are resticted to 0
# PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0, 6000, 0, 0, 0, 0, 10000
  PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0,    0, 0, 0, 0, 0, 1000] # Maximum conventional power

# Solar Plant Costs and constraints
alphaS = [300 0 0 0 0 1600 0 800  0 0 1500 1200 800  800  0]       # alpha coefficient solar power g
betaS =  [0.9 0 0 0 0 0.9  0 0.9  0 0 0.8  0.7  0.9  0.9  0]       # beta coefficient for solar powe
SGMax =  [500 0 0 0 0 3000 0 1500 0 0 2700 1000 1000 1000 0]  # Maximum solar power at each bus in M

# Wind Plant Costs and constraints
alphaW = [3600 0 1500 0 1500 0 0 0 0 1100 0 0 5000 1300 0] # alpha coefficient wind power generation
betaW =  [0.30 0 0.10 0 0.20 0 0 0 0 0.30 0 0 0.10 0.10 0] # beta coefficient for wind power generat
WGMax =  [3000 0 1500 0 2000 0 0 0 0 2600 0 0 7000 5700 0] # Maximum wind power at each bus in MW


# Select Solver: Gurobi works fine
m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

# Decision Variables
@variable(m, PG[1:busNum] >=0)     # Power gernerated from conventional power plants

@variable(m, SG[1:busNum] >=0)     # Power generated from Solar Power Plants
@variable(m, WG[1:busNum] >=0)     # Power generated from Wind Power Plants

@variable(m, F[1:lineNum])         # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])      # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])       # Power injected to the bus, if the bus is sink, Pinj is negative

@variable(m,zS[1:busNum], Bin)     # Decision variable for solar power plant if zS[k] = 0 no solar p
@variable(m,zW[1:busNum], Bin)     # Decision variable for wind power plant if zW[k] = 0 no wind pla

@variable(m,Ploss[1:busNum] >=0)   # Fictitious power loss

# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) * Pb base in MW
@constraint(m, F .<= cap)                         # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)



# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/6)                   # angle diffrenence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/6)                   # angle should be smaller than 30 deg

#@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] + SG[1:(busNum)] + WG[1:(busNum)]- PL_2030[1:(bu
@constraint(m, constr[k in 1:busNum], Ploss[k] == -0.1/Bbus[k,k]) # Fictitious power loss model
@constraint(m, lossConstr[k in 1:busNum], Pinj[k] == PG[k] + SG[k] + WG[k] - PL_2030[k] - Ploss[k])
@constraint(m, sum(Pinj) == 0)                                                                     #
@constraint(m, theta[busNum]==0)                                                                   #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                            #


# Generation constriants
@constraint(m,PG .<= PGMax)

# Renewable constaints (Big M)
@constraint(m, solarConstr[k in 1:busNum], SG[k] <= SGMax[k]*zS[k])
@constraint(m, windConstr[k in 1:busNum], WG[k] <= WGMax[k]*zW[k])
```

```
# Objective: Min generation cost
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))   # conventior
@expression(m, solarGenCost, sum((alphaS[k] + beta[k]*SG[k])*zS[k] for k in 1:busNum))  # solar gene
@expression(m, windGenCost,  sum((alphaW[k] + betaS[k]*WG[k])*zW[k] for k in 1:busNum)) # wind gener
@expression(m, totalGenCost, conventionalGenCost + solarGenCost + windGenCost)          # total gene

@objective(m, Min, totalGenCost)

optimize!(m)

# Output Results
println("\nThe minimum Gen Cost ", value.(totalGenCost)," [\$/hour]\n")

println("\n Power Flow (MW)")
for k=1:lineNum
        println("From Bus ", fromBus[k]," to ",toBus[k]," Power Flow: ",value.(F[k])," MW \t")
end

println("\n Conventional Power Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(PG[k]) >= 0.00001)
        println("Bus ", k," (",value.(PG[k])," MW)","\t")
    end
end

println("\n Location of Solar Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(zS[k]) == 1)
        println("Bus ", k," (",value.(SG[k])," MW)","\t")
    end
end

println("\n Location of Wind Plants (Generated Power (MW))")
for k=1:busNum
    if(value.(zW[k]) == 1)
        println("Bus ", k," (",value.(WG[k])," MW)","\t")
    end
end
```

```
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The minimum Gen Cost 10210.160798330759 [$/hour]


 Power Flow (MW)
From Bus 1 to 2 Power Flow: -922.4175255040091 MW
From Bus 2 to 6 Power Flow: -1500.0 MW
From Bus 3 to 2 Power Flow: -54.38811694428978 MW
From Bus 4 to 3 Power Flow: -680.4357078614094 MW
From Bus 5 to 1 Power Flow: 258.54125247060443 MW
From Bus 5 to 6 Power Flow: -151.02792326890025 MW
From Bus 7 to 8 Power Flow: 216.35924969708003 MW
From Bus 7 to 12 Power Flow: 1308.7338544894255 MW
From Bus 9 to 8 Power Flow: -771.8965518207397 MW
From Bus 9 to 13 Power Flow: -300.79042052241766 MW
From Bus 10 to 5 Power Flow: 1544.7776055575796 MW
From Bus 10 to 11 Power Flow: -247.39934505073816 MW
From Bus 11 to 7 Power Flow: 2398.6043779496426 MW
```

```
From Bus 13 to 12 Power Flow: -211.15228585632394 MW
From Bus 14 to 11 Power Flow: 365.54489298016733 MW
From Bus 15 to 4 Power Flow: 405.4877561772601 MW
From Bus 15 to 9 Power Flow: 639.3545256960435 MW
From Bus 15 to 14 Power Flow: -44.85193169190005 MW

 Conventional Power Plants (Generated Power (MW)):
Bus 1 (221.54342737911293 MW)
Bus 15 (1000.0 MW)

 Location of Solar Plants (Generated Power (MW)):
Bus 6 (2450.0726939132783 MW)
Bus 8 (1500.0 MW)
Bus 11 (2700.0 MW)
Bus 13 (1000.0 MW)
Bus 14 (1000.0 MW)

 Location of Wind Plants (Generated Power (MW))
Bus 3 (1500.0 MW)
Bus 10 (2600.0 MW)
```

If the power loss increases by changing the (33) into (35),

$$P_{loss} = -20000 \cdot \frac{1}{B_{busk,k}} \ (35)$$

the generation cost increases to $\$ \ 19651.59$ / hour and bus 14 needs both solar and wind power plants.

In [52]:

```
# Power Generation from Nuclear Plants are resticted to 0
# PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0, 6000, 0, 0, 0, 0, 10000
  PGMax = [2000, 0, 3000, 5000, 0, 0, 0, 0, 0,    0, 0, 0, 0, 0, 1000] # Maximum conventional power

# Solar Plant Costs and constraints
alphaS = [300 0 0 0 0 1600 0 800  0 0 1500 1200 800  800  0]       # alpha coefficient solar power g
betaS =  [0.9 0 0 0 0 0.9  0 0.9  0 0 0.8  0.7  0.9  0.9  0]       # beta coefficient for solar powe
SGMax =  [500 0 0 0 0 3000 0 1500 0 0 2700 1000 1000 1000 0]  # Maximum solar power at each bus in M

# Wind Plant Costs and constraints
alphaW = [3600 0 1500 0 1500 0 0 0 0 1100 0 0 5000 1300 0] # alpha coefficient wind power generation
betaW =  [0.30 0 0.10 0 0.20 0 0 0 0 0.30 0 0 0.10 0.10 0] # beta coefficient for wind power generat
WGMax =  [3000 0 1500 0 2000 0 0 0 0 2600 0 0 7000 5700 0] # Maximum wind power at each bus in MW


# Select Solver: Gurobi works fine
m = Model(Gurobi.Optimizer)
set_optimizer_attribute(m,"OutputFlag",false)

# Decision Variables
@variable(m, PG[1:busNum] >=0)     # Power gernerated from conventional power plants

@variable(m, SG[1:busNum] >=0)     # Power generated from Solar Power Plants
@variable(m, WG[1:busNum] >=0)     # Power generated from Wind Power Plants

@variable(m, F[1:lineNum])         # Power flow, it can be bidirectional so the polarity can be nega
@variable(m, theta[1:busNum])      # voltage angle at each bus in radian
@variable(m, Pinj[1:busNum])       # Power injected to the bus, if the bus is sink, Pinj is negative

@variable(m,zS[1:busNum], Bin)     # Decision variable for solar power plant if zS[k] = 0 no solar p
@variable(m,zW[1:busNum], Bin)     # Decision variable for wind power plant if zW[k] = 0 no wind pla

@variable(m,Ploss[1:busNum] >=0)   # Fictitious power loss

# Line flow constraints
@constraint(m, F .== Pb*diagBL*transpose(A)*theta) # equation (23) * Pb base in MW
@constraint(m, F .<= cap)                          # real power flow: -line capacitiy =< F <= line capa
@constraint(m, F .>= -cap)



# Bus constraints
@constraint(m, transpose(A)*theta .<= pi/6)                  # angle differnence should be less tha
@constraint(m, transpose(A)*theta .>= -pi/6)                  # angle should be smaller than 30 deg

#@constraint(m, Pinj[1:(busNum)] .== PG[1:(busNum)] + SG[1:(busNum)] + WG[1:(busNum)]- PL_2030[1:(bu
@constraint(m, constr[k in 1:busNum], Ploss[k] == -20000/Bbus[k,k]) # Fictitious power loss model
@constraint(m, lossConstr[k in 1:busNum], Pinj[k] == PG[k] + SG[k] + WG[k] - PL_2030[k] - Ploss[k])
@constraint(m, sum(Pinj) == 0)                                                                  #
@constraint(m, theta[busNum]==0)                                                                #
@constraint(m, Pinj[1:(busNum-1)] .== Pb*BbarPrime*theta[1:(busNum-1)])                         #


# Generation constriants
@constraint(m,PG .<= PGMax)

# Renewable constaints (Big M)
@constraint(m, solarConstr[k in 1:busNum], SG[k] <= SGMax[k]*zS[k])
@constraint(m, windConstr[k in 1:busNum], WG[k] <= WGMax[k]*zW[k])
```

```
# Objective: Min generation cost
@expression(m, conventionalGenCost, sum( alpha[k] + beta[k]*PG[k] for k in 1:busNum))   # convention
@expression(m, solarGenCost, sum((alphaS[k] + beta[k]*SG[k])*zS[k] for k in 1:busNum))  # solar gene
@expression(m, windGenCost,  sum((alphaW[k] + betaS[k]*WG[k])*zW[k] for k in 1:busNum)) # wind gener
@expression(m, totalGenCost, conventionalGenCost + solarGenCost + windGenCost)          # total gene

@objective(m, Min, totalGenCost)

optimize!(m)

# Output Results
println("\nThe minimum Gen Cost ", value.(totalGenCost)," [\$/hour]\n")

println("\n Power Flow (MW)")
for k=1:lineNum
        println("From Bus ", fromBus[k]," to ",toBus[k]," Power Flow: ",value.(F[k])," MW \t")
end

println("\n Conventional Power Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(PG[k]) >= 0.00001)
        println("Bus ", k," (",value.(PG[k])," MW)","\t")
    end
end

println("\n Location of Solar Plants (Generated Power (MW)):")
for k=1:busNum
    if(value.(zS[k]) == 1)
        println("Bus ", k," (",value.(SG[k])," MW)","\t")
    end
end

println("\n Location of Wind Plants (Generated Power (MW))")
for k=1:busNum
    if(value.(zW[k]) == 1)
        println("Bus ", k," (",value.(WG[k])," MW)","\t")
    end
end
```

```
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The minimum Gen Cost 19651.593639405743 [$/hour]


 Power Flow (MW)
From Bus 1 to 2 Power Flow: 648.3699305724149 MW
From Bus 2 to 6 Power Flow: -1500.0 MW
From Bus 3 to 2 Power Flow: -1353.6665900841958 MW
From Bus 4 to 3 Power Flow: -1497.9000140231562 MW
From Bus 5 to 1 Power Flow: 521.2500583914441 MW
From Bus 5 to 6 Power Flow: 253.16887562311564 MW
From Bus 7 to 8 Power Flow: 975.3559985628069 MW
From Bus 7 to 12 Power Flow: 1448.2015520989157 MW
From Bus 9 to 8 Power Flow: -991.3207304552342 MW
From Bus 9 to 13 Power Flow: -753.4681505653057 MW
From Bus 10 to 5 Power Flow: 1066.9501090623003 MW
From Bus 10 to 11 Power Flow: -117.46874077438588 MW
```

```
From Bus 11 to 7 Power Flow: 3551.8201780105364 MW
From Bus 13 to 12 Power Flow: -1036.8949317549127 MW
From Bus 14 to 11 Power Flow: 1622.8248760883134 MW
From Bus 15 to 4 Power Flow: 280.8277198594897 MW
From Bus 15 to 9 Power Flow: 266.8589591947993 MW
From Bus 15 to 14 Power Flow: -1477.6503980596144 MW

 Conventional Power Plants (Generated Power (MW)):
Bus 1 (1970.690617721135 MW)
Bus 15 (1000.0 MW)

 Location of Solar Plants (Generated Power (MW)):
Bus 6 (3000.0 MW)
Bus 8 (1500.0 MW)
Bus 11 (2700.0 MW)
Bus 12 (1000.0 MW)
Bus 13 (1000.0 MW)
Bus 14 (1000.0 MW)

 Location of Wind Plants (Generated Power (MW))
Bus 3 (1500.0 MW)
Bus 5 (2000.0 MW)
Bus 10 (2600.0 MW)
Bus 14 (3325.140860662392 MW)
```

5. Conclusion

The minimal generation cost problem defined in a complicated nonlinear power grid model is linearilized and solved as a LP MCNF problem. Constant voltage, small angle difference, and negligible loss in the transmission line are assumped when linearlizing the power grid model. Unlike ordinary LP MCNF problems, flow in each edges (transmission lines) are defined as a free variable. This difference is caused by the fact that power can flow in either direction of the transmission line. The negative flow is interpreted as a power flow in the opposite direction. The LP MCNF problem became a MIP by adding binary decision variables to the model. Binary decision variables are added to determine the location to construct renewable power plants. Big M method is used to represent the binary logic in MIP.

It has been shown that the MIP has more active constraints than that of the LP MCNF. In general, the generation limit of each power plants are the major constraints and capacity of transmission lines remain slack. As a result, it is concluded that more investments should be made to increase the power plant's generation limit rather increasing the capacity of transmission line.

The small angle assumption and the the negligible loss assumptions are verified by relaxing the angle constraint and introducing fictitious loss to the model. It has been shown that assumptions made in this project is reasonable.

For future work, it will be interesting to approximate the quadratic generation cost function as a piecewise linear function instead of the linear function. Although the linear function worked in this project, it may not be true for general power grid systems. Unlike constant voltage, small angle, and low loss assumption, there are no physical reason that the γ is smaller than α when γ is multiplied by P_G^2. Therefore, to further improve the model developed in this project, addtional effort is required on the generation cost function.

6. References

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