CS/ECE/ISyE 524 — Introduction to Optimization — Summer 2019

Optimization of Biochar-Based Carbon Manage Network

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1. Introduction

Climate change is regarded as one of the most important environmental issue; it has led to intensified research efforts to identify appropriate scientific and policy solutions at different levels. Carbon dioxide (CO_2) is an important heat-trapping (greenhouse) gas, thus, closely related to climate change, which is released through human activities such as deforestation and burning fossil fuels. Atmospheric carbon dioxide concentration recently reached 410 ppm, the highest level in recorded history (Kahn, 2017). Thus, there is now an urgent need to balance economic growth targets, especially in developing countries, with CO2 emissions constraints (Lee et al., 2017).

Biochar is the carbon-rich solid product derived from thermochemical processing of biomass. Its application to soil can sequester atmospheric carbon, leading to negative net emissions, while also improving agricultural yield. Biochar-based carbon management networks thus have the potential for scalable contributions to climate change mitigation efforts. However, it is necessary to allocate biochar of suitable quality to appropriate sinks, based on contaminant tolerance limits of soil. We interested in this project because this problem is closely related to my current research on CO_2 sequestration.

In this project, the network is first modelled as a bi-objective mixed integer linear programming model, with profit and carbon sequestration as the objective functions. The model calculates the optimum allocation of biochar such that their prescribed limits for the contaminants are met. A case study is solved to illustrate the practical use of the developed model, we use the data for Central Luzon region in the Philippines, which is one of the leading regions that produce agricultural crops and residual biomass resources in the country. The data is available in the following research paper https://www.sciencedirect.com/science/article/pii/S0959652618310369 (https://www.sciencedirect.com/science/article/pii/S0959652618310369). The results provide useful insights for the rational decision-maker to arrive at the most preferred solution among the Pareto optimal choices. In addition, we extend the bi-objective model to a tri-objective model by incorporating soil contamination level as another objective in our model.

The biochar planning problem, whose superstructure representation is given in the following Fig 1 can be stated formally as follows:

1. The biochar-based carbon management network is comprised of a set of pyrolysis plants selected as sources $i \in I(i = 1, 2, 3, \cdots, M)$ producing biochars that are to be allocated to a set of croplands chosen as sinks $j \in J(j = 1, 2, 3, \cdots, N)$ throughout a specified period, divided into time intervals $p \in P(p = 1, 2, 3, \cdots, T)$.
2. The limits of each flowrate and levels of undesirable impurities coming from source i $k \in K$ (metal contaminants, $k \in 1, 2, 3, \cdots, Q$) are prescribed.
3. Every sink j can only receive biochar up to a defined maximum flowrate, maximum storage capacity and maximum tolerance level for each impurity k.
4. For each potential source-sink pair, the carbon footprint (i.e.,from the handling, transportation and application) as well as the sequestration factor (i.e., direct and indirect benefits) per unit of biochar are known.
5. The problem is to determine how to allocate biochar from eachsource i to each sink j in each time interval p to maximize both CO_2 sequestration and profitability.

In the sections below, detailed math models, solution code, discussion, trade-off curves of bi-objective and tri-objective models and conclusion of the project will be displayed.

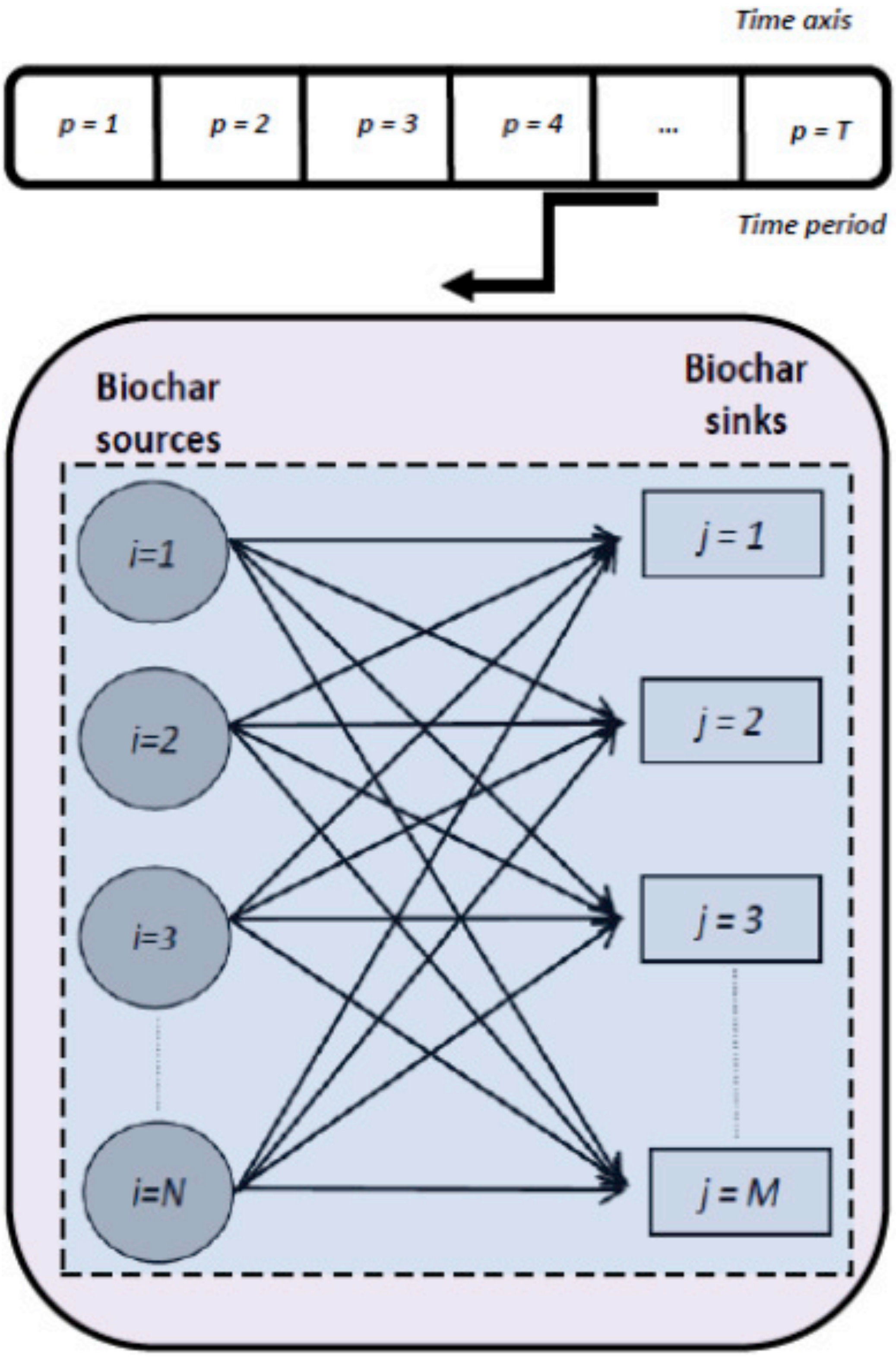


Fig 1 Source-sink superstructure for biochar network

2. Mathematical model

2.1 Bi-objective Model

Type of Model: a mixed integer linear programming model

Decision Variables:

x_{ijp} : Allocation of biochar for sink j coming form source i during the period p

b_{ip} : Binary variable for existence of production of source i in period p

s_{ip}: Biochar production rate of source i in period p

Parameters:

A_i: Sequestration factor of biochar at source i

B_{ij}: Transportation emissions factor for biochar from source i to sink j

Cf_i: Dimensionless conversion factor of bio-oil from source i

Bc: Biochar unit price

Bo: Bio-oil unit price

Pc: Cost of biochar production per unit mass

Cv: Cost of transportation

Vc: Capacity of vehicles needed to deliver the biochar to the sink

Ac: Application cost for each unit mass of biochar

L_j: Annual biochar storage capacity of sink j for each period

d_{ij}: Distance between source i and sink j

Q_{ikp}: Concentration of metal contaminant k in biochar produced by source i in period p

Q^*_{jk}: Concentration limit of metal contaminant k in biochar used in sink j

s^L_{ip}: Lower limit of biochar production rate of source i in period p

s^U_{ip}: Upper limit of biochar production rate of source i in period p

ψ: Risk aversion parameter of soil contamination level

Constraints:

Source biochar balance: $\sum_j x_{ijp} = s_{ip}, \ \forall i, p$

Sink storage limit: $\sum_i x_{ijp} \leq L_{jp}, \ \forall j, p$

Metal contamination limit: $\sum_i x_{ijp} Q_{ikp} \leq L_j Q^*_{jk} \psi, \ \forall j, k, p$

Source production limits using binary varibles:

$b_{ip} s^L_{ip} \leq s_{ip} \leq b_{ip} s^U_{ip}, \ \forall i, p$

$b_{ip} \in \{0, 1\}, \forall i, p$

Allocation decision variable constraint: $x_{ijp} \geq 0, \forall i, j, p$

Objectives:

One objective function is to maximize net CO_2 sequestration of network throughout whole periods.

Since net CO_2 sequestration = source CO_2 sequestration - transportation emissions,

we have the following expression for this objective:

Objective 1:

maximize $\sum_i \sum_j \sum_p (A_i - B_{ij}) x_{ijp}$

The second objective function is to maximize total profit.

Because total profit = profit by biochar + profit by bio-oil - production cost - transportation cost - application cost,

we have second objective modeled as follows:

Objective 2:

maximize $\sum_i \sum_j \sum_p (Bc + Bo \times Cf_i - Pc - 2d_{ij}Cv/Vc - Ac) x_{ijp}$

Full Math Model:

$$
\begin{aligned}
\max_x \ & \sum_i \sum_j \sum_p (A_{ij} - B_{ij}) x_{ijp} + \lambda \sum_i \sum_j \sum_p (Bc + Bo \times Cf_i - Pc - 2d_{ij}Cv/Vc - Ac) x_{ijp} \\
\text{s.t.} \ & \sum_j x_{ijp} = s_{ip}, \ \forall i, p \\
& \sum_i x_{ijp} \le L_{jp}, \ \forall j, p \\
& \sum_i x_{ijp} Q_{ikp} \le L_j Q^*_{jk} \psi, \ \forall j, k, p \\
& b_{ip} s^L_{ip} \le s_{ip} \le b_{ip} s^U_{ip}, \ \forall i, p \\
& b_{ip} \in \{0, 1\}, \ \forall i, p \\
& x_{ijp} \ge 0, \ \forall i, j, p
\end{aligned}
$$

2.2 Tri-objective Model

In this extended model we design, we try to incorporate soil metal contamination level as a new objective function instead of using risk aversion parameter ψ in the bi-objective model.

Type of Model: a mixed integer linear programming model

Decision Variables:

x_{ijp}: Allocation of biochar for sink j coming form source i during the period p

b_{ip}: Binary variable for existence of production of source i in period p

s_{ip}: Biochar production rate of source i in period p

Parameters:

A_i: Sequestration factor of biochar at source i

B_{ij}: Transportation emissions factor for biochar from source i to sink j

Cf_i: Dimensionless conversion factor of bio-oil from source i

Bc: Biochar unit price

Bo: Bio-oil unit price

Pc: Cost of biochar production per unit mass

Cv: Cost of transportation

Vc: Capacity of vehicles needed to deliver the biochar to the sink

Ac: Application cost for each unit mass of biochar

L_j: Annual biochar storage capacity of sink j for each period

d_{ij}: Distance between source i and sink j

Q_{ikp}: Concentration of metal contaminant k in biochar produced by source i in period p

Q^*_{jk}: Concentration limit of metal contaminant k in biochar used in sink j

s^L_{ip}: Lower limit of biochar production rate of source i in period p

s^U_{ip}: Upper limit of biochar production rate of source i in period p

Constraints:

Source biochar balance: $\sum_j x_{ijp} = s_{ip}, \ \forall i, p$

Sink storage limit: $\sum_i x_{ijp} \leq L_{jp}, \ \forall j, p$

Metal contamination limit: $\sum_i x_{ijp} Q_{ikp} \leq L_j Q_{jk}, \ \forall j, k, p$

Source production limits using binary varibles:

$b_{ip} s^L_{ip} \leq s_{ip} \leq b_{ip} s^U_{ip}, \ \forall i, p$

$b_{ip} \in \{0, 1\}, \forall i, p$

Allocation decision variable constraint: $x_{ijp} \geq 0, \forall i, j, p$

Objectives:

One objective function is to maximize net CO_2 sequestration of network throughout whole periods.

Since net CO_2 sequestration = source CO_2 sequestration - transportation emissions,

we have the following expression for this objective:

Objective 1:

maximize $\sum_i \sum_j \sum_p (A_i - B_{ij}) x_{ijp}$

The second objective function is to maximize total profit.

Because total profit = profit by biochar + profit by bio-oil - production cost - transportation cost - application cost,

we have second objective modeled as follows:

Objective 2:

maximize $\sum_i \sum_j \sum_p (Bc + Bo \times Cf_i - Pc - 2d_{ij}Cv/Vc - Ac) x_{ijp}$

Here we introduce a third new objective function to minimize the risk of hitting the metal contamination limit.

Objective 3:

minimize $\sum_j \sum_k \sum_p (\sum_i x_{ijp} Q_{ikp} - L_j Q_{jk})$ = maximize $\sum_j \sum_k \sum_p (L_j Q_{jk} - \sum_i x_{ijp} Q_{ikp})$

Full Math Model:

$$
\begin{aligned}
\max_x \; & \sum_i \sum_j \sum_p (A_{ij} - B_{ij}) x_{ijp} + \lambda \sum_i \sum_j \sum_p (Bc + Bo \times Cf_i - Pc - 2d_{ij}Cv/Vc - Ac) x_{ijp} \\
& + \eta \sum_j \sum_k \sum_p (L_j Q_{jk} - \sum_i x_{ijp} Q_{ikp}) \\
\text{s.t.} \; & \sum_j x_{ijp} = s_{ip}, \; \forall i, p \\
& \sum_i x_{ijp} \le L_{jp}, \; \forall j, p \\
& \sum_i x_{ijp} Q_{ikp} \le L_j Q_{jk}, \; \forall j, k, p \\
& b_{ip} s_{ip}^L \le s_{ip} \le b_{ip} s_{ip}^U, \; \forall i, p \\
& b_{ip} \in \{0, 1\}, \; \forall i, p \\
& x_{ijp} \ge 0, \; \forall i, j, p
\end{aligned}
$$

3. Solution

We implement both models using the data for Central Luzon region in the Philippines. To solve this multi-objective mixed integer programming (MIP) problem, we choose to use Gurobi optimizer, and the code is shown as follows.

Parameters Initialization

In [161]:

```
#####################################################
## 3 sources and 4 sinks
## 3 metal elements
## 10 year periods
source = [:Aurora,:Nueva, :Tarlac]
sink = [:Pa,:Ba,:Bu,:Za]
metal = [:Na,:Mg,:Ca]
time = [:1,:2,:3,:4,:5,:6,:7,:8,:9,:10]
#######################################################
##distance between sources and sinks
using NamedArrays
dist = [257 303 238 232;93.5 173 99 265;65.2 123 122 210]
d = NamedArray(dist, (source,sink), ("source","sink"))

####################################################
## each source produces one type of biochar, hence there are 3 types of biochar
## sequestration factor for each type of biochar
A = Dict(zip(source,[3.43,2,3.25]))

##transportation emissions factor for biochar from source i to sink j
B = [1 2 1.5 0.4; 0.51 1.1 0.32 0.4; 1.2 1.3 2.1 0.9]
B = NamedArray(B,(source,sink),("source","sink"))
######################################################
## metal contamination content for each source
mat = [80 150 4600; 2900 10400 5000;300 1300 2600] #Q_{ijp}
Q = NamedArray(mat,(source,metal),("source","metal"))

######################################################
## min/max annual production rate for each source
########################################################
min_pro = Dict(zip(source,[6000,20000,10000])) #s^L
max_pro = Dict(zip(source,[8000,26000,13000])) #s^U

##########################################################
## Limiting biochar metal content for each sink
##
mat_lim = [750 2600 1250; 7250 26000 12500; 1500 5200 2500;2900 10400 5000] #Q*
Qs = NamedArray(mat_lim,(sink,metal),("sink","Metal"))

#######################################################
```

```
## Annual storage capacity
##
L = Dict(zip(sink,[6727,8460,21500,9483])) #L_{jp}

##########################################################
## Cost data for the case study
## Production cost
Pc = 100.5
## Capacity of vehicle
Vc = 25
##Specific vehicle transport cost
Cv = 1.5335
##Biochar application cost
Ac = 6.58
##Biochar price
Bc = 50
##Bio-oil price
Bo = 103
##Dimensionless conversion factor of bio-oil from each source
Cf = Dict(zip(source,[1.55,0.86,1.82]))

println("source metal contamination content:")
println(Q)
println("sink metal contamination limit:")
println(Qs)
println("sink storage limit:")
println(L)
```

```
source metal contamination content:
3×3 Named Array{Int64,2}
source ╲ metal │    :Na     :Mg     :Ca
───────────────┼───────────────────────
:Aurora        │     80     150    4600
:Nueva         │   2900   10400    5000
:Tarlac        │    300    1300    2600
sink metal contamination limit:
4×3 Named Array{Int64,2}
sink ╲ Metal │    :Na     :Mg     :Ca
─────────────┼───────────────────────
:Pa          │    750    2600    1250
:Ba          │   7250   26000   12500
:Bu          │   1500    5200    2500
:Za          │   2900   10400    5000
sink storage limit:
Dict(:Pa=>6727,:Ba=>8460,:Bu=>21500,:Za=>9483)
```

Optimization Code in General

In [159]:

```
using JuMP, Gurobi

##This function is designed to tune risk aversion parameter ψ for metal contaminati
##and trade-off parameters λ and η in latter discussion section
function solveOpt(ψ, Bc, λ, η)
    m = Model(solver=GurobiSolver(OutputFlag=0))

    @variable(m, x[source,sink,time]>=0)
    @variable(m, b[source,time], Bin)
    @variable(m, s[source,time])

    ## source production balance: source i produce s_i,p at time p
    @constraint(m, sip[i in source, p in time], s[i,p] == sum(x[i,j,p] for j in sin
    ## metal contanimation limit
    @constraint(m, mc[k in metal, j in sink, p in time], sum(x[i,j,p]*Q[i,k] for i i

    ## source annual production rate limit
    @constraint(m, max_prod[p in time, i in source],s[i,p] <= max_pro[i]*b[i,p] )
    @constraint(m, min_prod[p in time, i in source],s[i,p] >= min_pro[i]*b[i,p])
    ## sink storage capacity
    @constraint(m, sink_cap[j in sink,p in time],sum(x[i,j,p] for i in source) <= L

    ##Tarlac year of production 4-10
    #@constraint(m, year_prod[j in sink, p in time[1:3]],x[:Tarlac,j,p] == 0)
    @constraint(m, year_prod[p in time[1:3]],b[:Tarlac,p] == 0)

    ## three objective functions
    ## objective 1: CO2 sequestration
    @expression(m, y1, sum((A[i] - B[i,j])*x[i,j,p] for i in source,j in sink, p in
    ## objective 2: profit
    @expression(m, y2, sum((Bc + Bo*Cf[i] - Pc - 2*Cv/Vc*d[i,j] - Ac)*x[i,j,p] for i
    ## objective 3: risk of metal contamination level
    @expression(m, y3, sum(L[j]*Qs[j,k] - sum(x[i,j,p]*Q[i,k] for i in source) for k

    @objective(m, Max, y1 + λ*y2 + η*y3)
    solve(m)
    xsol = getvalue(x)
    y1_sol = getvalue(y1)
    y2_sol = getvalue(y2)
    y3_sol = getvalue(y3)
    return (xsol, y1_sol, y2_sol, y3_sol)
end
```

Out[159]:

```
solveOpt (generic function with 3 methods)
```

4. Results and discussion

4.1 Biochar Network Allocation

Based on the data from Central Luzon region in the Philippines, we first examine the biochar allocation from source to sink using bi-objective model. We select one general case by setting Bc = 50; ψ = 1.0; λ = 1.0; η = 0.0. Most sources and sinks have same situation on each year period except that Tarlac does not produce any biochar in first three years. This means the first three years have the same allocation and the remaining seven years have the same distributions. So we only present allocation distribution on first and fourth year as examples.

In [175]:

```
Bc = 50; ψ = 1.0; λ = 1.0; η = 0.0
(xopt, yopt1, yopt2, yopt3) = solveOpt(ψ, Bc, λ, η)
println("1st year biochar network allocation : ")
println("source/sink", '\t', "Pa",'\t',"Ba",'\t',"Bu", '\t',"Za")
for i in source
    println(i,'\t', '\t',round(xopt[i,:Pa,1]),'\t', round(xopt[i,:Ba,1]),'\t', round
end
println("4th year biochar network allocation: ")
println("source/sink", '\t', "Pa",'\t',"Ba",'\t',"Bu", '\t',"Za")
for i in source
    println(i,'\t', '\t', round(xopt[i,:Pa,4]),'\t', round(xopt[i,:Ba,4]),'\t', rour
end
```

...

Table 1: 1st year biochar network allocation

source/sink	Pa	Ba	Bu	Za
Aurora	0.0	0.0	0.0	8000.0
Nueva	1682.0	8460.0	10750.0	1483.0
Tarlac	0.0	0.0	0.0	0.0

Table 2: 4th year biochar network allocation

source/sink	Pa	Ba	Bu	Za
Aurora	0.0	0.0	0.0	8000.0
Nueva	0.0	0.0	0.0	0.0
Tarlac	3234.0	8460.0	1306.0	0.0

4.2 Effect of ψ on CO_2 sequestration and profit

ψ is an important parameter incorporated in the model to account for the uncertainty arising on the difficulty of finding the level of impurity that can be tolerated by the receiving soil. This risk aversion parameter quantifies the willingness of the decision-maker to allow contamination in the soil. Here we set λ = 1.0, η = 0.0 to discuss effect of ψ on CO_2 sequestration and profit using bi-objective model.

In [176]:

```
λ = 1.0;  Bc = 50; η = 0.0;
N = 9
ysol1 = zeros(N)
ysol2 = zeros(N)
for (i,ψ) in enumerate(range(0.2, 0.1, 9))
    (x1, ysol1[i], ysol2[i], ysol3) = solveOpt(ψ, Bc, λ, η)
end
```

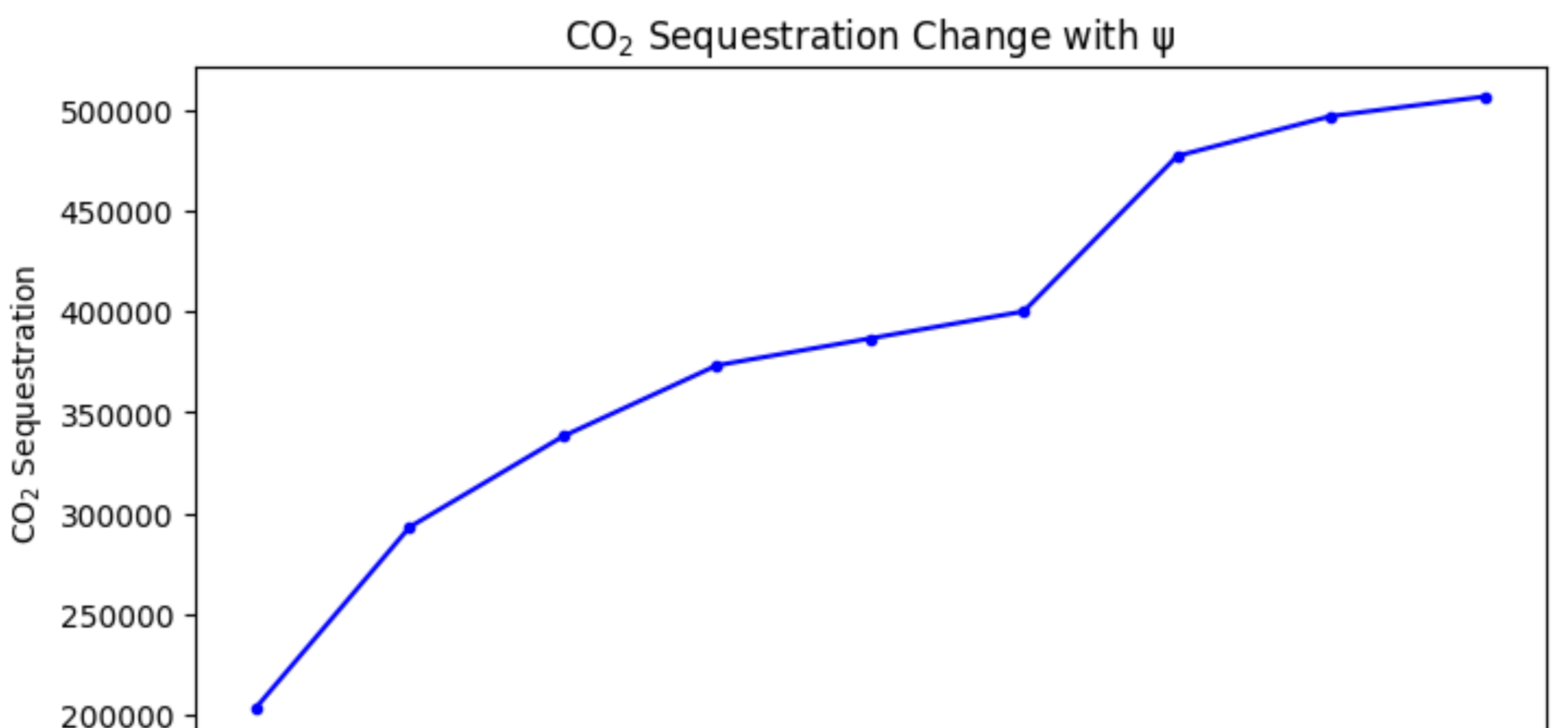
...

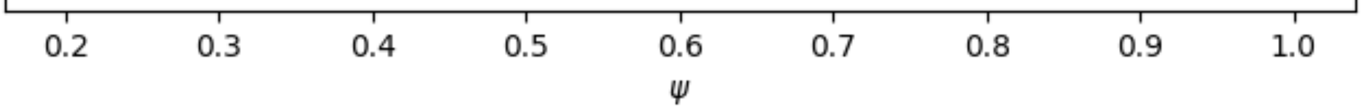
In [177]:

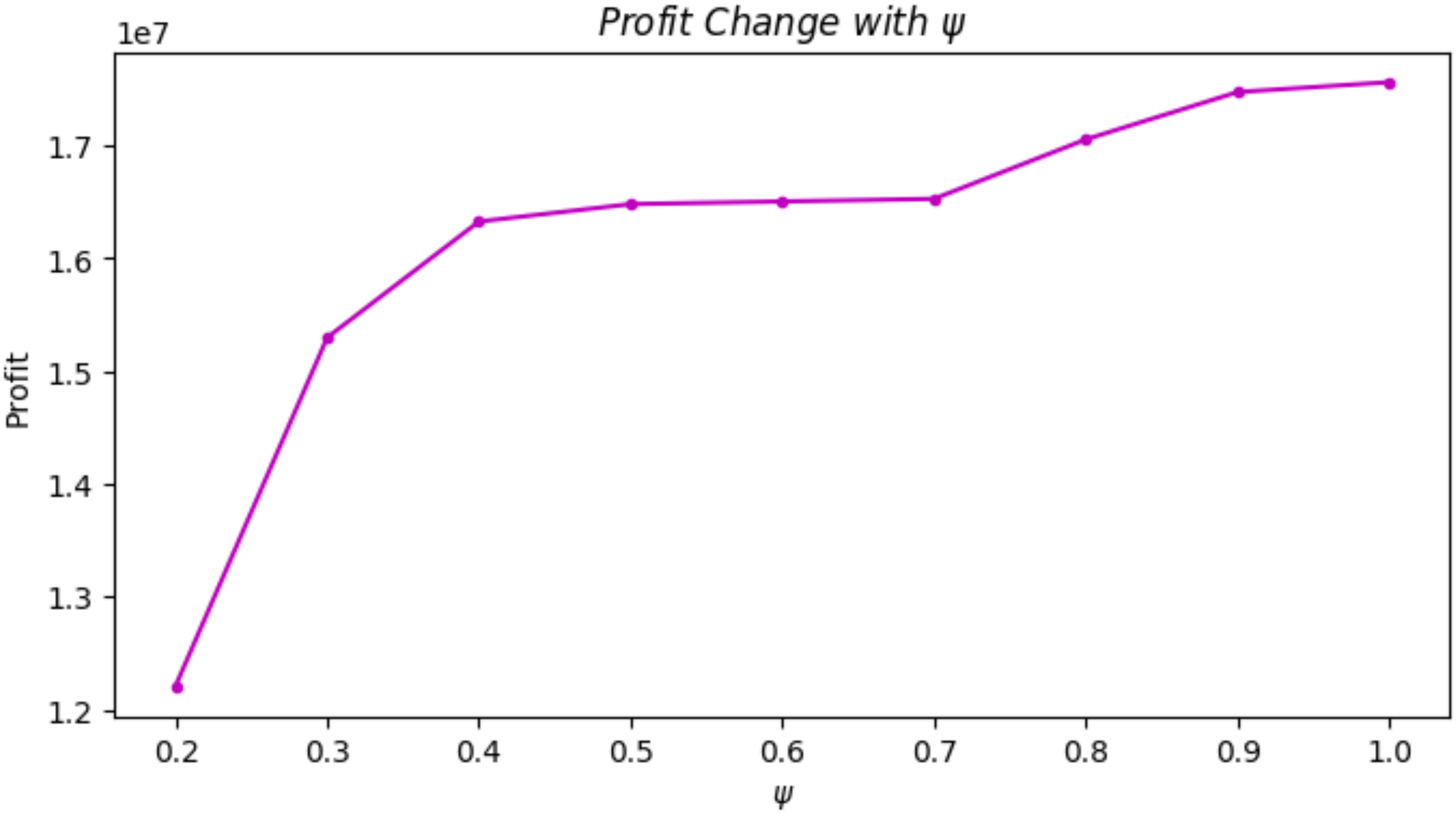
```
using PyPlot

t = range(0.2, 0.1, 9)
figure(figsize=(8,4))
plot( t, ysol1, "b.-" )
xlabel(L"\psi")
ylabel(L"\sf CO_2 \ Sequestration");
title(L"\sf CO_2 \ Sequestration\ Change\ with\  \psi")

figure(figsize=(8,4))
plot( t, ysol2, "m.-" )
xlabel(L"\psi")
ylabel("Profit");
title(L"Profit \ Change\ with\  \psi")
```







Out[177]:

```
PyObject Text(0.5,1,'$Profit \\ Change\\ with\\  \\psi$')
```

From the above two plots we can see that both CO_2 sequestration and profit increase as ψ becomes larger. The profit objective increases more rapidly when ψ is in the range 0.2-0.4. At ψ is about 0.72, the increasement rate of both objectives with respect to ψ has increased rapidly.

In [180]:

```
println("\psi", '\t', "CO2 sequestration",'\t', "Profit")
for (i,ψ) in enumerate(t)
    println(ψ, '\t', round(ysol1[i]), '\t', round(ysol2[i]))
end
```

...

ψ	CO2 sequestration	Profit
0.2	203802.0	1.2206324e7
0.3	293234.0	1.5298122e7
0.4	338206.0	1.632734e7
0.5	373305.0	1.6481907e7
0.6	386681.0	1.650527e7
0.7	400057.0	1.6528632e7
0.8	477062.0	1.7055723e7
0.9	496772.0	1.747593e7
1.0	506459.0	1.7563295e7

4.3 Sensitivity analysis of biochar price

Here we discuss the bi-objective case with $\lambda = 1.0$ and $\psi = 1.0$. We alter the biochar unit price Bc to see how two objectives CO_2 sequestration and profit will change.

In [182]:

```
λ = 1.0; ψ = 1.0; η = 0.0
Npts = 10
ysol1 = zeros(Npts)
ysol2 = zeros(Npts)
for (i, Bc) in enumerate(range(10, 10, 10))
    (x1, ysol1[i], ysol2[i], ysol3) = solveOpt(ψ, Bc, λ, η)
end
```

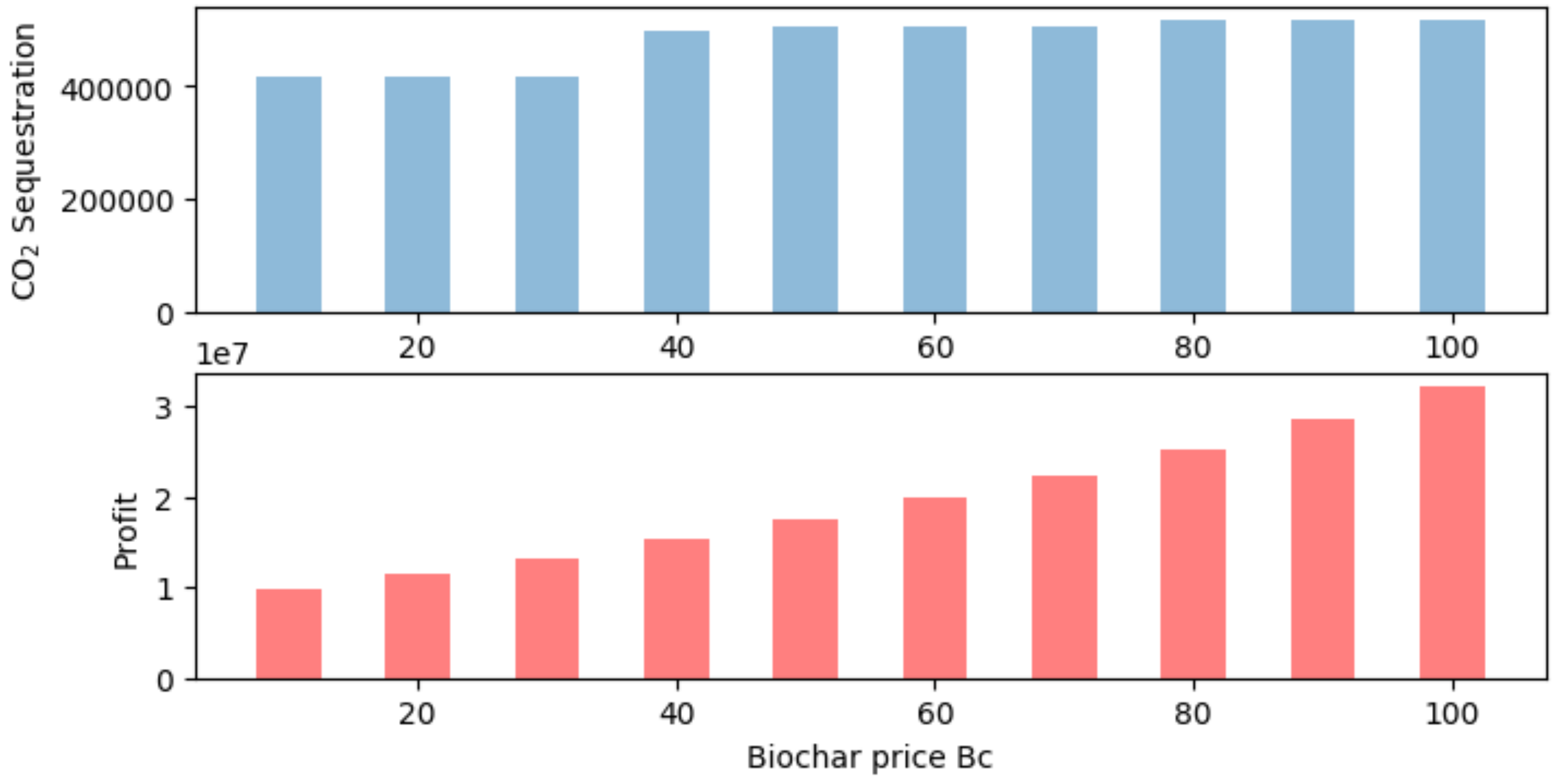
...

In [183]:

```julia
using PyPlot

t = range(10, 10, 10)
fig = figure(figsize=(8,4))
ax1 = fig[:add_subplot](2,1,1);
bar(t,ysol1, align="center",width=5,alpha=0.5)
#xlabel("Biochar price Bc")
ylabel(L"\sf CO_2 \ Sequestration");
#title(L"\sf CO_2 \ Sequestration\ Change\ with\  Bc")
ax2 = fig[:add_subplot](2,1,2);
bar(t,ysol2, color="r",align="center",width=5,alpha=0.5)
xlabel("Biochar price Bc")
ylabel("Profit");
#title("Profit Change with Bc")


println("Bc", '\t', "CO2 sequestration",'\t',"Profit")
for (i,Bc) in enumerate(t)
    println(Bc, '\t', ysol1[i], '\t', ysol2[i])
end
```




```
Bc      CO2 sequestration       Profit
10      414801.04807692295      9.741733951523084e6
20      414801.04807692295      1.1451733951523082e7
30      414801.04807692295      1.316173395152308e7
40      499340.4705769229       1.5231037329578081e7
50      506458.87057692284      1.7563295449778035e7
60      506458.87057692284      1.9944537949778043e7
70      506458.8705769229       2.2325780449778043e7
80      515610.25057692284      2.515325641640807e7
90      515610.25057692284      2.862753141640807e7
100     515610.25057692284      3.2101806416408077e7
```

From the above plot and table, we can clearly see that profit increases with the increase of Bc because Bc is involved in the profit objective calculation. However, if Bc does not change too much (for example, from 10 to 30), CO_2 sequestration objective value would remain the same. CO_2 sequestration objective value would increase with large increment on Bc such that the feasible region of dual problem changes. What we find here agrees with sensitivity analysis of dual problem we learn in class.

4.4 Trade-off Curve with Bi-Objective

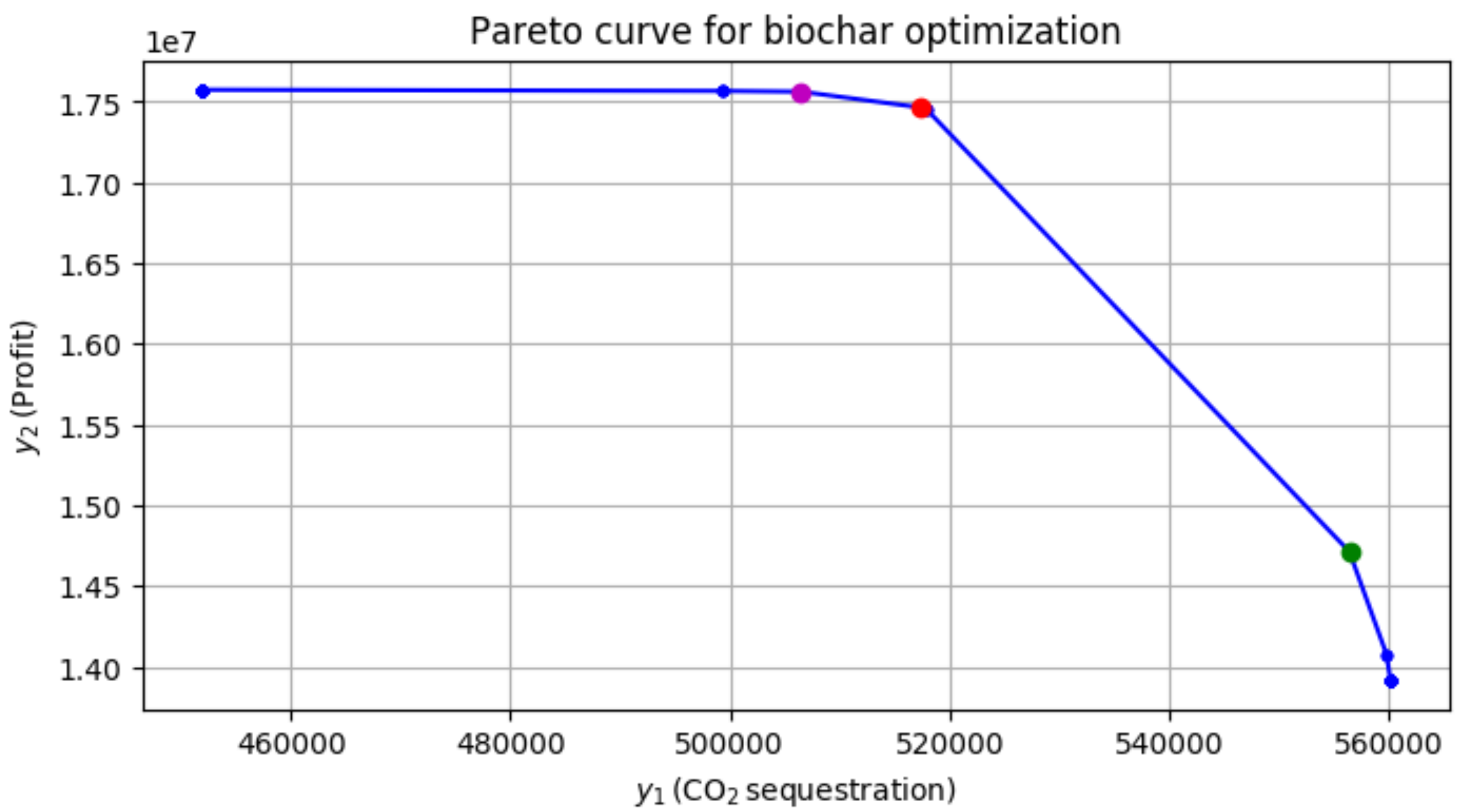
In [191]:

```
ψ = 1.0; Bc = 50; η = 0.0
Npts = 101
yopt1 = zeros(Npts)
yopt2 = zeros(Npts)
for (i,λ) in enumerate(logspace(-5,5,Npts))
    (xopt, yopt1[i], yopt2[i], ysol3) = solveOpt(ψ, Bc, λ, η)
end
```

...

In [192]:

```
using PyPlot
figure(figsize=(8,4))
plot( yopt1, yopt2, "b.-" )
plot(yopt1[31], yopt2[31], "g.", markersize=12) #λ=0.01
plot(yopt1[41], yopt2[41], "r.", markersize=12) #λ=0.1
plot(yopt1[51], yopt2[51], "m.", markersize=12) #λ=1.0
xlabel(L"y_1\, (\sf CO_2\, sequestration)")
ylabel(L"y_2\, (\sf Profit )");
title("Pareto curve for biochar optimization")
grid()
```



With Bc and ψ fixed, we observe from the Pareto curve (blue curve in the above plot) that, when we decrease the "weight" of profit in the bi-objective model, CO_2 sequestration increases in the beginning while the profit stays the same, Then profit decrease while CO_2 sequestration increase.Based on that we can select the best trade-off point where max profit is achieved and CO_2 sequestration is reasonably large. In the above plot, the green, red, and magenta dot represent the result with λ=0.01, 0.1 and 1.0 respectively. This means we can probably choose λ from 0.1 to 1.0 to get optimal profit and CO_2 sequestration simultaneously.

Effect of ψ on bi-objective tradeoff curve

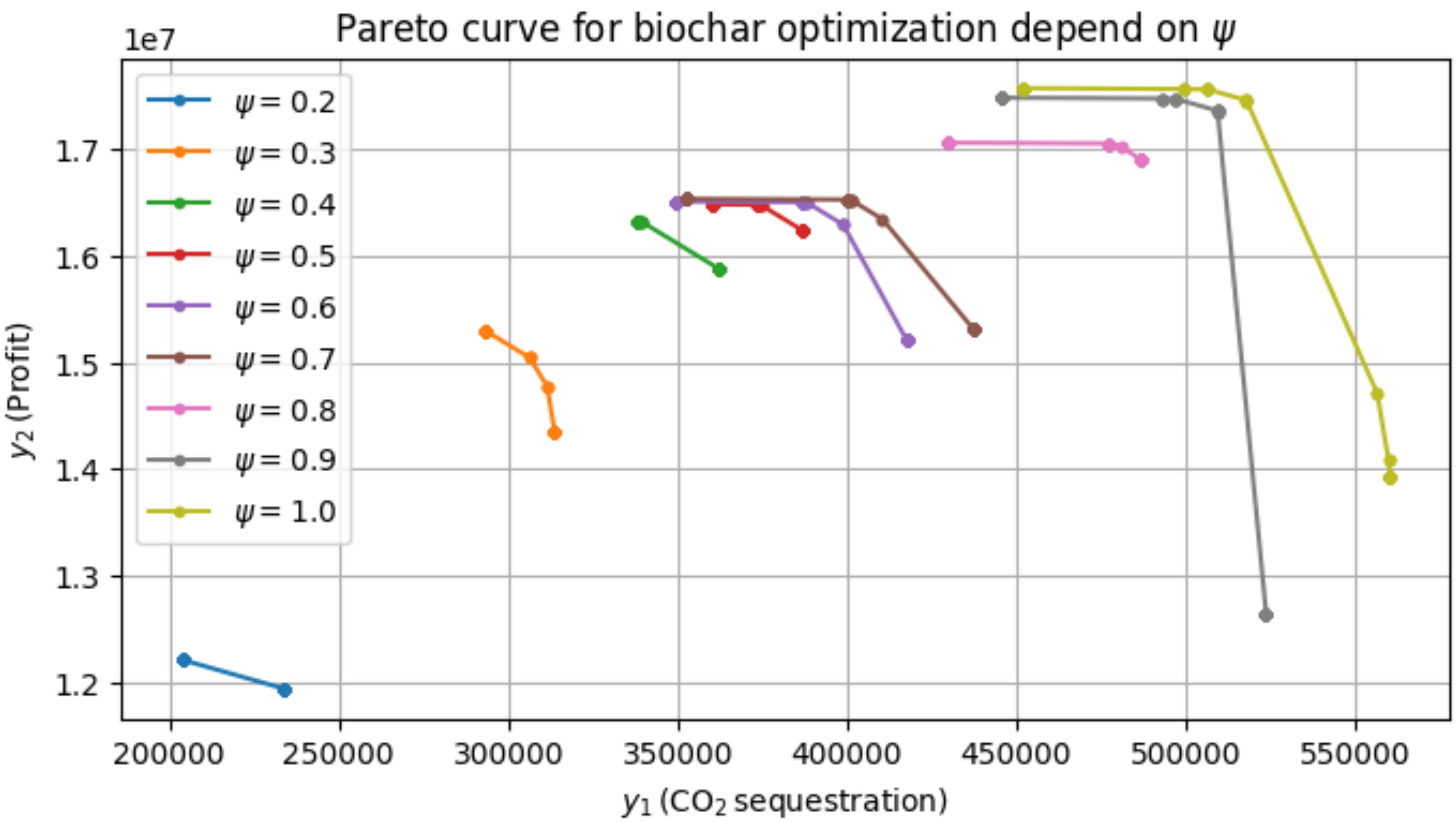
In [193]:

```
Bc = 50; η = 0.0
N=9; Npts = 101
t = range(0.2, 0.1, N)
yopt1 = zeros(N, Npts)
yopt2 = zeros(N, Npts)
for (i,ψ) in enumerate(t)
    for (j,λ) in enumerate(logspace(-5,5,Npts))
        (xopt, yopt1[i,j], yopt2[i,j], ysol3) = solveOpt(ψ, Bc, λ, η)
    end
end
```

...

In [194]:

```
using PyPlot
figure(figsize=(8,4))
plot(yopt1[1,:], yopt2[1,:], ".-", label=L"\psi = 0.2" )
plot(yopt1[2,:], yopt2[2,:], ".-", label=L"\psi = 0.3" )
plot(yopt1[3,:], yopt2[3,:], ".-", label=L"\psi = 0.4" )
plot(yopt1[4,:], yopt2[4,:], ".-", label=L"\psi = 0.5" )
plot(yopt1[5,:], yopt2[5,:], ".-", label=L"\psi = 0.6" )
plot(yopt1[6,:], yopt2[6,:], ".-", label=L"\psi = 0.7" )
plot(yopt1[7,:], yopt2[7,:], ".-", label=L"\psi = 0.8" )
plot(yopt1[8,:], yopt2[8,:], ".-", label=L"\psi = 0.9" )
plot(yopt1[9,:], yopt2[9,:], ".-", label=L"\psi = 1.0" )
xlabel(L"y_1\, (\sf CO_2\, sequestration)")
ylabel(L"y_2\, (\sf Profit )");
title(L"Pareto curve for biochar optimization depend on $\psi$")
legend()
grid()
```



The Pareto curves show similar patterns: when we decrease the "weight" of profit in the bi-objective model, CO_2 sequestration increases in the beginning while the profit stays the same, Then profit decrease while CO_2 sequestration increase. One observation when we increase ψ is that it shifts the curve to upper-right corner, which means that larger profit and CO_2 sequestration can be obtained when we increase ψ. This is reasonable because larger ψ means we have more tolerance for soil contamination level, Thus both objective can be improved simultaneously.

4.5 Trade-off Curve with Tri-Objective

In [ ]:

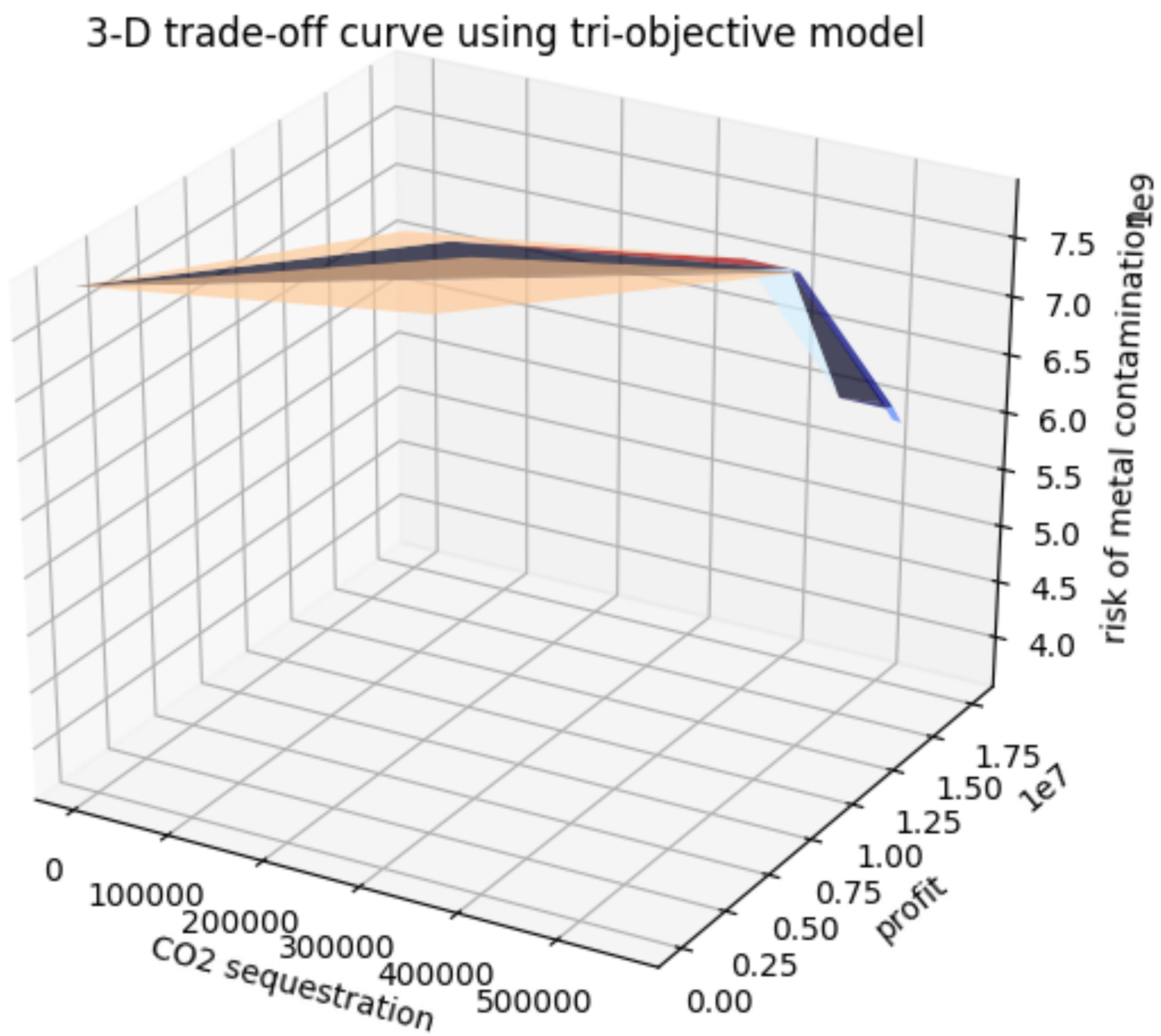
```
ψ = 1.0; Bc = 50;
N = 31
yopt1 = zeros(N,N)
yopt2 = zeros(N,N)
yopt3 = zeros(N,N)
for (i,λ) in enumerate(logspace(-5,5,N))
    for (j,η) in enumerate(logspace(-5,5,N))
        (xopt, yopt1[i,j], yopt2[i,j], yopt3[i,j]) = solveOpt(ψ, Bc, λ, η)
    end
end
```

In [157]:

```
# Plot tradeoff curve using tri-objective model

fig = figure(figsize=(6,5));
surf(yopt1,yopt2,yopt3, cmap=ColorMap("flag"), alpha=0.7)

#scatter3D(yopt1[13,16], yopt2[13,16], yopt3[13,16], color=:red, marker="o", s=40) #
#scatter3D(yopt1[16,16], yopt2[16,16], yopt3[16,16], color=:red, marker="o", s=40) #
xlabel("CO2 sequestration"); ylabel("profit"); zlabel("risk of metal contamination")
title("3-D trade-off curve using tri-objective model")
tight_layout()
```



This plot shows the 3-D tradeoff contour for tri-objective model. The different colors denote different tradeoff planes. Hence, this tradeoff plot is composed of different connected planes. There is a peak point shown in 3D trade-off plot. Based on this, we can select the best trade-off point where max profit is achieved, CO_2 sequestration is reasonably large and the metal contamination level are relatively small.

5. Conclusion

In this project, we have demonstrated the work of optimizing biochar-based carbon management network with multi objectives using mixed integer linear programming method. Our solution is based on data for Central Luzon region in the Philippines. Discussions on import risk aversion parameter ψ for soil contamination tolerance are provided to illustrate the effect of ψ on both objectives and help decison-maker to wisely choose contamination tolerance level. Sensitivity analysis of biochar market price are included in our study to provide more insight. We also plot the Pareto curve to analyze the trade-off between CO_2 sequestration and profit. Furthermore, we extend the bi-objective model into a tri-objective model to see the effect of incorporating soil contamination level into overall objective function.

For the ongoing research of this project, more case studies could be done to test the generality of the model. More detailed discussions of parameters would provide more helpful insight. In addition, environmental pollution cost could be included to construst a more realistic model such that we can ultilize this model to shed more light on practical applications of biochar-based carbon network.

6. References

1. Tan, R.R, 2018. Bi-objective optimization of biochar-based carbon management networks. J. Clean. Prod. 4.
2. Tan, R.R., 2016. A multi-period source-sink mixed integer linear programming model for biochar-based carbon sequestration systems. Sustainable Production and Consumption 8.
3. Kahn, B., 2017. We just breached the 410 parts per million threshold. www. climatecentral.org/news/we-just-breached-the-410-parts-per-millionthreshold-21372. (Accessed 12 May 2017).
4. Lee, C.T., Hashim, H., Ho, C.S., Fan, Y.Van, 2017. Sustaining the low-carbon emission development in Asia and beyond: sustainable energy , water , transportation and low-carbon emission technology. J. Clean. Prod. 146.