Tarea 1
Programacion Declarativa

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1.-

a)

$$(f1\ 5\ 0)$$

$$((\lambda n.\lambda m.\lambda s.\lambda z.(m\ n)\ s\ z)\ 5)\ 0$$

$$\to_\beta (\lambda m.\lambda s.\lambda z.(m\ 5)\ s\ z)\ 0$$

$$\to_\beta (\lambda s.\lambda z.(0\ 5)\ s\ z)$$

$$\to_\beta \lambda s.\lambda z.((\lambda s.\lambda z.z)\ (\lambda s.\lambda z.s(s(s(s(s\ z))))))\ s\ z$$

$$\to_\beta \lambda s.\lambda z.(\lambda z.z)\ s\ z$$

$$\to_\beta \lambda s.\lambda z.s\ z$$

$$(f1\ 2\ 3)$$

$$((\lambda n.\lambda m.\lambda s.\lambda z.(m\ n)\ s\ z)\ 2)\ 3$$

$$\to_\alpha (((\lambda n.(\lambda m.(\lambda s.(\lambda z.(((m\ n)s)z)))))(\lambda a.(\lambda b.(a(a\ b)))))(\lambda c.(\lambda d.(c(c(c\ d))))))$$

$$\to_\beta ((\lambda m.(\lambda s.(\lambda z.(((m(\lambda a.(\lambda b.(a(a\ b)))))s)z))))(\lambda c.(\lambda d.(c(c(c\ d))))))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.((((\lambda c.(\lambda d.(c(c(c\ d)))))(\lambda a.(\lambda b.(a(a\ b)))))s)z)))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(((\lambda d.((\lambda a.(\lambda b.(a(a\ b))))((\lambda a.(\lambda b.(a(a\ b))))((\lambda a.(\lambda b.(a(a\ b))))d))))s)z)))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(((\lambda d.((\lambda a.(\lambda b.(a(a\ b))))((\lambda c.(\lambda e.(c(c\ e))))((\lambda f.(\lambda g.(f(f\ g))))d))))s)z)))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(((\lambda a.(\lambda b.(a(a\ b))))((\lambda c.(\lambda e.(c(c\ e))))((\lambda f.(\lambda g.(f(fg))))s)))z)))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.((\lambda b.(((\lambda c.(\lambda e.(c(c\ e))))((\lambda f.(\lambda g.(f(f\ g))))s))(((\lambda c.(\lambda e.(c(c\ e))))((\lambda f.(\lambda g.(f(f\ g))))s))b)))z)))$$

$$\rightarrow_\alpha (\lambda s.(\lambda z.((\lambda b.(((\lambda c.(\lambda e.(c(c\ e))))((\lambda f.(\lambda g.(f(f\ g))))s))(((\lambda a.(\lambda d.(a(a\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))b)))z)))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(((\lambda c.(\lambda e.(c(c\ e))))((\lambda f.(\lambda g.(f(f\ g))))s))(((\lambda a.(\lambda d.(a(ad))))((\lambda h.(\lambda i.(h(h\ i))))s))z))))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.((\lambda e.(((\lambda f.(\lambda g.(f(f\ g))))s)(((\lambda f.(\lambda g.(f(f\ g))))s)e)))(((\lambda a.(\lambda d.(a(a\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z))))$$

$$\rightarrow_\alpha (\lambda s.(\lambda z.((\lambda e.(((\lambda f.(\lambda g.(f(f\ g))))s)(((\lambda a.(\lambda b.(a(a\ b))))s)e)))(((\lambda c.(\lambda d.(c(c\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z))))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(((\lambda f.(\lambda g.(f(f\ g))))s)(((\lambda a.(\lambda b.(a(a\ b))))s)(((\lambda c.(\lambda d.(c(c\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z)))))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.((\lambda g.(s(s\ g)))(((\lambda a.(\lambda b.(a(a\ b))))s)(((\lambda c.(\lambda d.(c(c\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z)))))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(s(s(((\lambda a.(\lambda b.(a(a\ b))))s)(((\lambda c.(\lambda d.(c(c\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z))))))$$

$$\rightarrow_\beta (\lambda s.(\lambda z.(s(s((\lambda b.(s(s\ b)))(((\lambda c.(\lambda d.(c(c\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s(((\lambda c.(\lambda d.(c(c\ d))))((\lambda h.(\lambda i.(h(h\ i))))s))z)))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s((\lambda d.(((\lambda h.(\lambda i.(h(h\ i))))s)(((\lambda h.(\lambda i.(h(h\ i))))s)d)))z)))))))$$

$$\to_\alpha (\lambda s.(\lambda z.(s(s(s(s((\lambda d.(((\lambda h.(\lambda i.(h(h\ i))))s)(((\lambda a.(\lambda b.(a(a\ b))))s)d)))z)))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s(((\lambda h.(\lambda i.(h(h\ i))))s)(((\lambda a.(\lambda b.(a(a\ b))))s)z))))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s((\lambda i.(s(s\ i)))(((\lambda a.(\lambda b.(a(a\ b))))s)z))))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s(s(s(((\lambda a.(\lambda b.(a(a\ b))))s)z)))))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s(s(s((\lambda b.(s(s\ b)))z)))))))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s(s(s(s(s(s(s\ z))))))))))$$

b)

$$(g1\ 0)$$

$$((\lambda n.\lambda s.\lambda z.((((n\ (\lambda h1.\lambda h2.(h2\ (h1\ s))))\ (\lambda u.z))\ (\lambda u.u))))\ (\lambda s.\lambda z.z))$$

$$\to_\alpha ((\lambda n.(\lambda s.(\lambda z.(((n(\lambda h1.(\lambda h2.(h2(h1\ s)))))(\lambda u.z))(\lambda a.a)))))(\lambda b.(\lambda c.c)))$$

$$\to_\beta (\lambda s.(\lambda z.((((\lambda b.(\lambda c.c))(\lambda h1.(\lambda h2.(h2(h1\ s)))))(\lambda u.z))(\lambda a.a))))$$

$$\to_\beta (\lambda s.(\lambda z.(((\lambda c.c)(\lambda u.z))(\lambda a.a))))$$

$$\to_\beta (\lambda s.(\lambda z.((\lambda u.z)(\lambda a.a))))$$

$$\to_\beta (\lambda s.(\lambda z.z))$$

$(g1\ 3)$

$$((\lambda n.\lambda s.\lambda z.((((n\ (\lambda h1.\lambda h2.(h2\ (h1\ s))))\ (\lambda u.z))\ (\lambda u.u))))\ (\lambda s.\lambda z.(s\ (s\ (s\ z)))))$$

$$\to_\alpha ((\lambda n.(\lambda s.(\lambda z.(((n(\lambda h1.(\lambda h2.(h2(h1\ s)))))(\lambda u.z))(\lambda a.a)))))(\lambda b.(\lambda c.(b(b(b\ c))))))$$

$$\to_\beta (\lambda s.(\lambda z.((((\lambda b.(\lambda c.(b(b(b\ c)))))(\lambda h1.(\lambda h2.(h2(h1\ s)))))(\lambda u.z))(\lambda a.a))))$$

$$\to_\beta (\lambda s.(\lambda z.(((\lambda c.((\lambda h1.(\lambda h2.(h2(h1\ s))))((\lambda h1.(\lambda h2.(h2(h1\ s))))((\lambda h1.(\lambda h2.(h2(h1\ s))))c))))(\lambda u.z))(\lambda a.a))))$$

$$\to_\alpha (\lambda s.(\lambda z.(((\lambda c.((\lambda h1.(\lambda h2.(h2(h1\ s))))((\lambda a.(\lambda b.(b(a\ s))))((\lambda d.(\lambda e.(e(d\ s))))c))))(\lambda u.z))(\lambda f.f))))$$

$$\to_\beta (\lambda s.(\lambda z.(((\lambda h1.(\lambda h2.(h2(h1\ s))))((\lambda a.(\lambda b.(b(a\ s))))((\lambda d.(\lambda e.(e(d\ s))))(\lambda u.z))))(\lambda f.f))))$$

$$\to_\beta (\lambda s.(\lambda z.((\lambda h2.(h2(((\lambda a.(\lambda b.(b(a\ s))))((\lambda d.(\lambda e.(e(d\ s))))(\lambda u.z)))s)))(\lambda f.f))))$$

$$\to_\beta (\lambda s.(\lambda z.((\lambda f.f)(((\lambda a.(\lambda b.(b(a\ s))))((\lambda d.(\lambda e.(e(d\ s))))(\lambda u.z)))s))))$$

$$\to_\beta (\lambda s.(\lambda z.(((\lambda a.(\lambda b.(b(a\ s))))((\lambda d.(\lambda e.(e(d\ s))))(\lambda u.z)))s)))$$

$$\to_\beta (\lambda s.(\lambda z.((\lambda b.(b(((\lambda d.(\lambda e.(e(d\ s))))(\lambda u.z))s)))s)))$$

$$\to_\beta (\lambda s.(\lambda z.(s(((\lambda d.(\lambda e.(e(d\ s))))(\lambda u.z))s))))$$

$$\to_\beta (\lambda s.(\lambda z.(s((\lambda e.(e((\lambda u.z)s)))s))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s((\lambda u.z)s)))))$$

$$\to_\beta (\lambda s.(\lambda z.(s(s\ z))))$$

c)

$$(h1\ 1)$$

$$(\lambda n.fst\ (n\ ss\ zz))\ 1$$

$$\to_\beta\ fst(1\ (\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 0))$$

$$\to_\beta\ fst((\lambda.s.z.(s\ z))\ (\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 0))$$

$$\to_\beta\ fst((\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 0))$$

$$\to_\beta\ fst(pair\ (snd\ (pair\ 0\ 0))\ (suc\ (snd\ (pair\ 0\ 0))))$$

$$\to_\beta\ fst(pair\ 0\ 1)$$

$$\to_\beta\ 0$$

$$(h1\ 2)$$

$$\to_\beta\ (\lambda n.fst\ (n\ ss\ zz))\ 2$$

$$\to_\beta\ fst(2\ (\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 0))$$

$$\to_\beta\ fst((\lambda.s.z.(s\ (s\ z)))\ (\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 0))$$

$$\to_\beta\ fst(((\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ ((\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 0))))$$

$$\to_\beta\ fst(((\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ ((pair\ (snd\ (pair\ 0\ 0))\ (suc\ (snd\ (pair\ 0\ 0)))))))$$

$$\to_\beta\ fst((\lambda p.pair\ (snd\ p)\ (suc\ (snd\ p)))\ (pair\ 0\ 1))$$

$$\to_\beta\ fst(pair\ (snd\ (pair\ 0\ 1))\ (suc\ (snd\ (pair\ 0\ 1))))$$

$$\to_\beta\ fst(pair\ 1\ (suc\ 1))$$

$$\to_\beta fst(pair\ 1\ 2)$$

$$\to_\beta 1$$

f1 es la función potencia: n^m
g1 es la función que resta 1 a un número natural.
h1 es una función que resta 1 a un número natural.

2. a)

$$(f_2\ 0) \to_\beta (f_2\ 3)[(f_2 := \lambda n.\lambda x.\lambda y.yn] = ((\lambda n.\lambda x.\lambda y.yn)\ 0)$$
$$\to_\beta (\lambda x.\lambda y.yn)[n := 0] = (\lambda x.\lambda y.y\ 0) = 1$$

$$(f_2\ 3) \to_\beta (f_2\ 3)[(f_2 := \lambda n.\lambda x.\lambda y.yn] = ((\lambda n.\lambda x.\lambda y.yn)\ 3)$$
$$\to_\beta (\lambda x.\lambda y.yn)[n := 3] = (\lambda x.\lambda y.y\ 3) = 4$$

2.b)

$$(g_2\ 1) \to_\beta (g_2\ 1)[(g_2 := \lambda n.n\ 0\ (\lambda x.x)] = ((\lambda n.n\ 0\ (\lambda x.x))\ 1) \to_\beta (n\ 0\ (\lambda x.x))[n := 1]$$
$$= (n[n := 1]\ 0[n := 1]\ (\lambda x.x)[n := 1]) = (1\ 0\ (\lambda x.x)) = ((\lambda x.\lambda y.y\ 0)\ 0\ (\lambda x.x))$$
$$\to_\beta ((\lambda y.y\ 0)[x := 0]\ (\lambda x.x)) = ((\lambda y.y\ 0)\ (\lambda x.x)) \to_\beta (y\ 0)[y := \lambda x.x]$$
$$= (y[y := \lambda x.x]\ 0[y := \lambda x.x]) = ((\lambda x.x)\ 0) \to_\beta (x[x := 0]) = 0$$

$$(g_2\ 4) \to_\beta (g_2\ 4)[g_2 := \lambda n.n\ 0\ (\lambda x.x)] = ((\lambda n.n\ 0\ (\lambda x.x))\ 4) \to_\beta (n\ 0\ (\lambda x.x))[n := 4]$$
$$= (n[n := 4]\ 0[n := 4]\ (\lambda x.x)[n := 4]) = (4\ 0\ (\lambda x.x)) = ((\lambda x.\lambda y.y\ 3)\ 0\ (\lambda x.x))$$
$$\to_\beta ((\lambda y.y\ 3)[x := 0]\ (\lambda x.x)) = ((\lambda y.y\ 3)\ (\lambda x.x)) \to_\beta (y\ 3)[y := \lambda x.x]$$
$$= (y[y := \lambda x.x]\ 3[y := \lambda x.x]) = ((\lambda x.x)\ 3) \to_\beta (x[x := 3]) = 3$$

2.c)

$$(h_2\ 0) \to_\beta (h_2\ 0)[(h_2 := \lambda n.n\ \underline{true}\ (\lambda x.\underline{false})] = ((\lambda n.n\ \underline{true}\ (\lambda x.\underline{false}))\ 0)$$
$$\to_\beta (n\ \underline{true}\ (\lambda x.\underline{false}))[n := 0] = (n[n := 0]\ \underline{true}[n := 0]\ (\lambda x.\underline{false})[n := 0])$$
$$= (0\ \underline{true}\ (\lambda x.\underline{false})) = ((\lambda x.\lambda y.x)\ \underline{true}\ (\lambda x.\underline{false})) \to_\beta ((\lambda y.x)\ [x := \underline{true}]\ (\lambda x.\underline{false}))$$
$$= ((\lambda y.\underline{true})\ (\lambda x.\underline{false})) = \underline{true}$$

$$(h_2\ 5) \to_\beta (h_2\ 5)[(h_2 := \lambda n.n\ \underline{true}\ (\lambda x.\underline{false})] = ((\lambda n.n\ \underline{true}\ (\lambda x.\underline{false}))\ 5)$$
$$\to_\beta (n\ \underline{true}\ (\lambda x.\underline{false}))[n := 5] = (n[n := 5]\ \underline{true}[n := 5]\ (\lambda x.\underline{false})[n := 5])$$

$= (5\ \underline{true}\ (\lambda x.\underline{false})) = ((\lambda x.\lambda y.y\ 4)\ \underline{true}\ (\lambda x.\underline{false})) \rightarrow_\beta ((\lambda y.y\ 4)\ [x := \underline{true}]\ (\lambda x.\underline{false}))$

$= ((\lambda y.y\ 4)\ (\lambda x.\underline{false})) \rightarrow_\beta ((y\ 4)[y := (\lambda x.\underline{false})] = (\lambda x.\underline{false}\ 4) = \underline{false}$

2.d)

f_2 es una función que suma 1 al numero pasado como argumento (el numero tiene que ser un natural de Scott)

g_2 es una función que resta 1 al numero pasado como argumento (el numero tiene que ser un natural de Scott)

h_2 es una función quenos dice si el argumento es 0 el agumento es un numero natural de Scott

2.e)
tenemos que :
$\underline{true} = \lambda x.\lambda y.x$
$\underline{false} = \lambda x.\lambda y.y$
$\underline{if} = \lambda p.\lambda a.\lambda b\ p\ a\ b$
$h_2 = \lambda n.n\ \underline{true}\ (\lambda x.\underline{false})$
$f_2 = \lambda n.\lambda x.\lambda y.yn$
$g_2 = \lambda n.n\ 0\ (\lambda x.x)$

Sea F un combinador de punto fijo:
Definimos $sumaScott \rightleftharpoons Fg$ donde
$g \rightleftharpoons \lambda f.\lambda x.\lambda y.\underline{if}\ (h_2\ y)\ x\ (f\ (f_2\ x)\ (g_2\ y))$

3.-
Sea F un combinador de punto fijo y
$iszero =_{def} \lambda m.m\ (\lambda x.false)\ true$

a)Una función que dados n y m calcule n^m.
$pot = Fg$
$g = \lambda f.\lambda n.\lambda m.if\ (iszero\ n)\ then\ 1\ else\ n * (f\ n\ (m-1))$

b) Una función que decida si un natural de Scott es impar.
$imp = Fg$
$g =$
$\lambda f.\lambda n.if\ (h_2\ 0)\ then\ false\ else\ (if\ (h_2\ (g_2\ n))\ then\ true\ else\ (f\ (g_2\ (g_2\ n))))$