Tarea 1
Programacion Declarativa

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1.-

2. a)

$$(f_2\ 0) \rightarrow_\beta (f_2\ 3)[(f_2 := \lambda n.\lambda x.\lambda y.yn] = ((\lambda n.\lambda x.\lambda y.yn)\ 0)$$
$$\rightarrow_\beta (\lambda x.\lambda y.yn)[n := 0] = (\lambda x.\lambda y.y\ 0) = 1$$

$$(f_2\ 3) \rightarrow_\beta (f_2\ 3)[(f_2 := \lambda n.\lambda x.\lambda y.yn] = ((\lambda n.\lambda x.\lambda y.yn)\ 3)$$
$$\rightarrow_\beta (\lambda x.\lambda y.yn)[n := 3] = (\lambda x.\lambda y.y\ 3) = 4$$

2.b)

$$(g_2\ 1) \rightarrow_\beta (g_2\ 1)[(g_2 := \lambda n.n\ 0\ (\lambda x.x)] = ((\lambda n.n\ 0\ (\lambda x.x))\ 1) \rightarrow_\beta (n\ 0\ (\lambda x.x))[n := 1]$$
$$= (n[n := 1]\ 0[n := 1]\ (\lambda x.x)[n := 1]) = (1\ 0\ (\lambda x.x)) = ((\lambda x.\lambda y.y\ 0)\ 0\ (\lambda x.x))$$
$$\rightarrow_\beta ((\lambda y.y\ 0)[x := 0]\ (\lambda x.x)) = ((\lambda y.y\ 0)\ (\lambda x.x)) \rightarrow_\beta (y\ 0)[y := \lambda x.x]$$
$$= (y[y := \lambda x.x]\ 0[y := \lambda x.x]) = ((\lambda x.x)\ 0) \rightarrow_\beta (x[x := 0]) = 0$$

$$(g_2\ 4) \rightarrow_\beta (g_2\ 4)[g_2 := \lambda n.n\ 0\ (\lambda x.x)] = ((\lambda n.n\ 0\ (\lambda x.x))\ 4) \rightarrow_\beta (n\ 0\ (\lambda x.x))[n := 4]$$
$$= (n[n := 4]\ 0[n := 4]\ (\lambda x.x)[n := 4]) = (4\ 0\ (\lambda x.x)) = ((\lambda x.\lambda y.y\ 3)\ 0\ (\lambda x.x))$$
$$\rightarrow_\beta ((\lambda y.y\ 3)[x := 0]\ (\lambda x.x)) = ((\lambda y.y\ 3)\ (\lambda x.x)) \rightarrow_\beta (y\ 3)[y := \lambda x.x]$$
$$= (y[y := \lambda x.x]\ 3[y := \lambda x.x]) = ((\lambda x.x)\ 3) \rightarrow_\beta (x[x := 3]) = 3$$

2.c)

$$(h_2\ 0) \rightarrow_\beta (h_2\ 0)[(h_2 := \lambda n.n\ \underline{true}\ (\lambda x.\underline{false})] = ((\lambda n.n\ \underline{true}\ (\lambda x.\underline{false}))\ 0)$$
$$\rightarrow_\beta (n\ \underline{true}\ (\lambda x.\underline{false}))[n := 0] = (n[n := 0]\ \underline{true}[n := 0]\ (\lambda x.\underline{false})[n := 0])$$
$$= (0\ \underline{true}\ (\lambda x.\underline{false})) = ((\lambda x.\lambda y.x)\ \underline{true}\ (\lambda x.\underline{false})) \rightarrow_\beta ((\lambda y.x)\ [x := \underline{true}]\ (\lambda x.\underline{false}))$$

$$= ((\lambda y.\underline{true})\ (\lambda x.\underline{false})) = \underline{true}$$

$$(h_2\ 5) \rightarrow_\beta (h_2\ 5)[(h_2 := \lambda n.n\ \underline{true}\ (\lambda x.\underline{false})] = ((\lambda n.n\ \underline{true}\ (\lambda x.\underline{false}))\ 5)$$

$$\rightarrow_\beta (n\ \underline{true}\ (\lambda x.\underline{false}))[n := 5] = (n[n := 5]\ \underline{true}[n := 5]\ (\lambda x.\underline{false})[n := 5])$$

$$= (5\ \underline{true}\ (\lambda x.\underline{false})) = ((\lambda x.\lambda y.y\ 4)\ \underline{true}\ (\lambda x.\underline{false})) \rightarrow_\beta ((\lambda y.y\ 4)\ [x := \underline{true}]\ (\lambda x.\underline{false}))$$

$$= ((\lambda y.y\ 4)\ (\lambda x.\underline{false})) \rightarrow_\beta ((y\ 4)[y := (\lambda x.\underline{false})] = (\lambda x.\underline{false}\ 4) = \underline{false}$$

2.d)

f_2 es una función que suma 1 al numero pasado como argumento (el numero tiene que ser un natural de Scott)

g_2 es una función que resta 1 al numero pasado como argumento (el numero tiene que ser un natural de Scott)

h_2 es una función quenos dice si el argumento es 0 el agumento es un numero natural de Scott

2.e)
tenemos que :
$\underline{true} = \lambda x.\lambda y.x$
$\underline{false} = \lambda x.\lambda y.y$
$\underline{if} = \lambda p.\lambda a.\lambda b\ p\ a\ b$
$\underline{h_2} = \lambda n.n\ \underline{true}\ (\lambda x.\underline{false})$
$f_2 = \lambda n.\lambda x.\lambda y.yn$
$g_2 = \lambda n.n\ 0\ (\lambda x.x)$

Sea F un combinador de punto fijo:
Definimos $sumaScott \rightleftharpoons Fg$ donde
$g \rightleftharpoons \lambda f.\lambda x.\lambda y.\underline{if}\ (h_2\ y)\ x\ (f\ (f_2\ x)\ (g_2\ y))$