Mathematical Equations for Homomorphic Filtering in Frequency Domain: A Literature Survey

Conference Paper · January 2012

1 author:

Sami A. M. Saleh
Universiti Sains Malaysia

2012 International Conference on Information and Knowledge Management (ICIKM 2012)
IPCSIT vol.45 (2012) © (2012) IACSIT Press, Singapore

Mathematical Equations for Homomorphic Filtering in Frequency Domain: A Literature Survey

Sami Abdulla Mohsen Saleh+, and Haidi Ibrahim

School of Electrical and Electronic Engineering, Engineering Campus, Universiti Sains Malaysia, 14300 Nibong Tebal, Penang, Malaysia

Abstract. Homomorphic filtering technique is one of the important ways used for digital image enhancement, especially when the input image is suffers from poor illumination conditions. This filtering technique has been used in many different imaging applications, including biometric, medical, and robotic vision. Homomorphic filtering works in frequency domain, by applying a high-pass type filter to reduce the significance of low frequency components. However, in literatures, there are several versions of mathematical equation used to present this filter. Therefore, this paper will review some of these equations.

Keywords: digital image processing, digital image enhancement, homomorphic filtering

1. Introduction

Images are sometimes been acquired under poor illumination. Under this condition, the same uniform region will appears brighter on some areas and darker on others. This undesired situation will leads to several severe problem in computer vision based system. The pixels might be misclassified, leading to wrong segmentation results, and therefore contribute to inaccurate evaluation or analysis from the system. Therefore, it is very crucial to process this type of images first before they are fed into the system.

One of the popular methods used to enhance or restore the degraded images by uneven illumination is by using homomorphic filtering. This technique uses illumination-reflectance model in its operation. This model consider the image is been characterized by two primary components. The first component is the amount of source illumination incident on the scene being viewed $i(x,y)$. The second component is the reflectance component of the objects on the scene $r(x,y)$. The image $f(x,y)$ is then defined as [1]-[3]:

$$f(x,y) = i(x,y)r(x,y) \tag{1}$$

In this model, the intensity of $i(x,y)$ changes slower than $r(x,y)$. Therefore, $i(x,y)$ is considered to have more low frequency components than $r(x,y)$. Using this fact, homomorphic filtering technique aims to reduce the significance of $i(x,y)$ by reducing the low frequency components of the image. This can be achieved by executing the filtering process in frequency domain. In order to process an image in frequency domain, the image need first to be transformed from spatial domain to frequency domain. This can be done by using transformation functions, such as Fourier transform. However, before the transformation is taking place, logarithm function has been used to change the multiplication operation of $r(x,y)$ with $i(x,y)$ in (1) into addition operation.

In general, homomorphic filtering can be implemented using five stages, as stated as follows:

STAGE 1: Take a natural logarithm of both sides to decouple $i(x,y)$ and $r(x,y)$ components

+ Corresponding author. Tel.: (006)(04) 599 5822; fax: (006)(04) 594 1023.
E-mail address: eng_sami2007@yahoo.com.

$$z(x,y) = \ln i(x,y) + \ln r(x,y) \tag{2}$$

STAGE 2: Use the Fourier transform to transform the image into frequency domain:

$$\Im\{z(x,y)\} = \Im\{\ln i(x,y)\} + \Im\{\ln r(x,y)\} \tag{3}$$

or

$$Z(u,v) = F_i(u,v) + F_r(u,v) \tag{4}$$

where $F_i(u,v)$ and $F_r(u,v)$ are the Fourier transforms of $\ln i(x,y)$ and $\ln r(x,y)$ respectively.

STAGE 3: High pass the $Z(u,v)$ by means of a filter function $H(u,v)$ in frequency domain, and get a filtered version $S(u,v)$ as the following:

$$S(u,v) = H(u,v)Z(u,v) = H(u,v)F_i(u,v) + H(u,v)F_r(u,v) \tag{5}$$

STAGE 4: Take an inverse Fourier transform to get the filtered image in the spatial domain:

$$s(x,y) = \Im^{-1}\{S(u,v)\} = \Im^{-1}\{H(u,v)F_i(u,v) + H(u,v)F_r(u,v)\} \tag{6}$$

STAGE 5: The filtered enhanced image $g(x,y)$ can be obtained by using the following equations:

$$g(x,y) = \exp\{s(x,y)\} \tag{7}$$

However, in literatures, there are several variations of equations that have been used to present $H(u,v)$ in (5). Therefore, this paper will survey some of them.

2. Homomorphic Filtering equations

The typical filter for homomorphic filtering process has been introduced in [1]- [5]. This filter has circularly symmetric curve shape, centred at $(u,v)=(0,0)$ coordinates in frequency domain. This filter is modified from Gaussian highpass filter, which is known as Difference of Gaussian (DoG) filter. The transfer function for DoG filter is defined as:

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - \exp\left\{-c\left(\frac{D(u,v)}{D_0}\right)^2\right\}\right] + \gamma_L \tag{8}$$

where constant c has been introduced to control the steepness of the slope, D_0 is the cut-off frequency, $D(u,v)$ is the distance between coordinates (u,v) and the centre of frequency at (0,0). For this filter, three important parameters are needed to be set by the user. They are the high frequency gain γ_H, the low frequency gain γ_L, and the cut-off frequency D_0. If γ_H is set greater than 1, and γ_L is set lower than 1, the filter function tends to decrease the contribution made by the illumination (which occupies mostly the low frequency components) and amplify the contribution made by the reflectance (which occupies most of the high frequency components). At the end, the net result will be a simultaneous dynamic range compression and contrast enhancement. The value of the low frequency gain should be set such as $\gamma_L=0.5$, to halve the spectral energy of the illumination, and the value of high frequency gain is set such as $\gamma_H=2$ to double the spectral energy of the reflectance components [1]. In [6], the value of c is suggested to be equal to 0.5. In practice, all these three parameter values are often determined empirically and there is no clear way to choose the exact suitable values for these parameters.

Work in [7] evaluated the properties of homomorphic filter by comparing it with some other advanced image enhancement method. Two equations have been used for homomorphic filtering process in that paper. They are defined by the following equations:

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - \exp\left\{-a\left(u^2 + v^2\right)\right\}\right] + \gamma_L \tag{9}$$

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - \frac{1}{1 + \left[(u^2 + v^2)/a\right]^n}\right] + \gamma_L \tag{10}$$

Equation (9) is the Gaussian high-pass filter, and (10) is the modified Butterworth high-pass filter. In (10), the term $[(u^2+v^2)/a]$ determines the transition point and Butterworth power coefficient n determines the steepness of the transition slope. Empirically, when applying the homomorphic filtering process onto different types of image, the author found that a maximal amplification value $(\gamma_H - \gamma_L) \geq 1.5$ is too much for many images. Furthermore, the author also found that the modified high-pass Butterworth equation is better than Gaussian high-pass filter for the use in homomorphic filtering process because it allows an independent setting of the transition point from the transition slope.

Another homomorphic filter equation has been used in [2]. The transfer function of this filter is defined as:

$$H(u,v) = \frac{1}{1 + \exp\{-a(D(u,v) - D_0)\}} + A \tag{11}$$

In this equation, the high frequency gain and the low frequency gain are given as:

$$\gamma_H = 1 + A \qquad \text{and} \qquad \gamma_L = \frac{1}{1 + \exp\{aD_0\}} + A \tag{12}$$

The suggested values for this transfer function are a=1, D_0=128, and A=10.

A simple modification to the standard homomorphic filter has been proposed in [8]. This modification has significantly improves the performance of face recognition. This filter is using the following Butterworth equation:

$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v)\right]^{2n}} \tag{13}$$

The suggested value of D_0 is 0.25 and n is 1. The work in [8] split the original image into several sub-images. It then filters each sub image individually. The enhanced image is found by combining these filtering results. For the face images, the enhancement result $g_R(x,y)$ is defined as:

$$g_R(x,y) = 0.5\left[g_V(x,y) + 0.75.g_H(x,y)\right] \tag{14}$$

where $g_V(x,y)$ is the filtered result from image that has been split vertically, and $g_H(x,y)$ is the filtered result from image that has been split horizontally. The constant 0.75 in (14) has been chosen based on several experimental results.

The author in [6] has proposed illumination normalization method by using homomorphic filtering in two steps. On the first step, the author processed face images by using a modified homomorphic filter based on DoG filter to eliminate the effect of uneven illumination. In the second state, the author has applied histogram equalization method to enhance the contrast of the image. In this paper, the process of parameter's selection is as the following:

- Selection of high frequency gain, γ_H:
  - Apply homomorphic filtering to input face images using $H(u,v)$ as filter function, set γ_H =2 and vary γ_L from 0 to 1.
  - Execute histogram equalization process.
  - Perform face recognition using Eigenfaces method.
  - Compare the recognition rate from different γ_L.
- Selection of low frequency gain, γ_L:

- Apply homomorphic filtering to input face images using $H(u,v)$ as filter function, set γ_L =0.02 and vary γ_H from 1 to 3.
- Execute histogram equalization process.
- Perform face recognition using Eigenfaces method.
- Compare the recognition rate from different γ_H.

3. Summary

This paper surveys several homomorphic filter equations that are available in literature. Although the shape of the filter is similar, the performance of each filter might be different from each other. Therefore, in our future work, we will try to see the advantages and disadvantages in each filter.

4. Acknowledgements

This work was supported in part by the Universiti Sains Malaysia's Short Term Research Grant with account number 304/PELECT/60311013.

5. References

[1] R. C. Gonzalez and R. E. Woods, *Digital Image Processing*: Prentice Hall, 2008.

[2] M. Petrou and C. Petrou, *Image Processing: The Fundamentals*: Wiley, 2010.

[3] B. Horn, *Robot Vision*: The MIT Press, 1986.

[4] J. R. Parker, *Algorithms for Image Processing and Computer Vision*: Wiley Publishing, 2010.

[5] S. E. Umbaugh, *Computer Imaging: Digital Image Analysis and Processing*: CRC Press, 2005.

[6] C.-N. Fan and F.-Y. Zhang, "Homomorphic filtering based illumination normalization method for face recognition," *Pattern Recognition Letters,* vol. 32, pp. 1468-1479, 2011.

[7] A. Holger G, "Butterworth equations for homomorphic filtering of images," *Computers in Biology and Medicine,* vol. 28, pp. 169-181, 1998.

[8] K. Delac, et al., "Sub-image homomorphic filtering technique for improving facial identification under difficult illumination conditions," Proc. 13th International Conference on Systems, Signals and Image Processing, 2006, pp. 95-98.