

# IIT JAM 2016

## Mathematical Statistics (MS)

**Actual Question Paper & Answer Key**



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<b>Special Instructions / Useful Data</b>	
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1, \dots, x_n) : x_i \in \mathbb{R}, i=1, \dots, n\}$
$P(A)$	Probability of an event $A$
i.i.d.	Independently and identically distributed
$Bin(n, p)$	Binomial distribution with parameters $n$ and $p$
$Poisson(\theta)$	Poisson distribution with mean $\theta$
$N(\mu, \sigma^2)$	Normal distribution with mean $\mu$ and variance $\sigma^2$
$Exp(\lambda)$	The exponential distribution with probability density function $f(x \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}, \lambda > 0$
$t_n$	Student's $t$ distribution with $n$ degrees of freedom
$\chi_n^2$	Chi-square distribution with $n$ degrees of freedom
$\chi_{n,\alpha}^2$	A constant such that $P(W > \chi_{n,\alpha}^2) = \alpha$ , where $W$ has $\chi_n^2$ distribution
$\Phi(x)$	Cumulative distribution function of $N(0,1)$
$\phi(x)$	Probability density function of $N(0,1)$
$A^C$	Complement of an event $A$
$E(X)$	Expectation of a random variable $X$
$Var(X)$	Variance of a random variable $X$
$B(m, n)$	$\int_0^1 x^{m-1} (1-x)^{n-1} dx, m > 0, n > 0$
$[x]$	The greatest integer less than or equal to real number $x$
$f'$	Derivative of function $f$
$\Phi(0.25) = 0.5987, \Phi(0.5) = 0.6915, \Phi(0.625) = 0.7341, \Phi(0.71) = 0.7612,$ $\Phi(1) = 0.8413, \Phi(1.125) = 0.8697, \Phi(2) = 0.9772$	

**SECTION – A**  
**MULTIPLE CHOICE QUESTIONS (MCQ)**

**Q. 1 – Q.10 carry one mark each.**

Q.1 Let

$$P = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -1 & -2 & 1 & 1 \\ 1 & 2 & -3 & -7 \\ 1 & 2 & -2 & -4 \end{bmatrix}.$$

Then rank of  $P$  equals

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Q.2 Let  $\alpha, \beta, \gamma$  be real numbers such that  $\beta \neq 0$  and  $\gamma \neq 0$ . Suppose

$$P = \begin{bmatrix} \alpha & \beta \\ \gamma & 0 \end{bmatrix},$$

and  $P^{-1} = P$ . Then

- (A)  $\alpha = 0$  and  $\beta\gamma = 1$
- (B)  $\alpha \neq 0$  and  $\beta\gamma = 1$
- (C)  $\alpha = 0$  and  $\beta\gamma = 2$
- (D)  $\alpha = 0$  and  $\beta\gamma = -1$

Q.3 Let  $m > 1$ . The volume of the solid generated by revolving the region between the  $y$ -axis and the curve  $xy = 4$ ,  $1 \leq y \leq m$ , about the  $y$ -axis is  $15\pi$ . The value of  $m$  is

- (A) 14
- (B) 15
- (C) 16
- (D) 17

Q.4 Consider the region  $S$  enclosed by the surface  $z = y^2$  and the planes  $z = 1, x = 0, x = 1, y = -1$  and  $y = 1$ . The volume of  $S$  is

- (A)  $\frac{1}{3}$
- (B)  $\frac{2}{3}$
- (C) 1
- (D)  $\frac{4}{3}$

Q.5 Let  $X$  be a discrete random variable with the moment generating function

$$M_X(t) = e^{0.5(e^t-1)}, t \in \mathbb{R}.$$

Then  $P(X \leq 1)$  equals

- (A)  $e^{-1/2}$       (B)  $\frac{3}{2} e^{-1/2}$       (C)  $\frac{1}{2} e^{-1/2}$       (D)  $e^{-(e-1)/2}$

Q.6 Let  $E$  and  $F$  be two independent events with

$$P(E|F) + P(F|E) = 1, \quad P(E \cap F) = \frac{2}{9} \text{ and } P(F) < P(E).$$

Then  $P(E)$  equals

- (A)  $\frac{1}{3}$       (B)  $\frac{1}{2}$       (C)  $\frac{2}{3}$       (D)  $\frac{3}{4}$

Q.7 Let  $X$  be a continuous random variable with the probability density function

$$f(x) = \frac{1}{(2+x^2)^{3/2}}, \quad x \in \mathbb{R}.$$

Then  $E(X^2)$

- |              |                    |
|--------------|--------------------|
| (A) equals 0 | (B) equals 1       |
| (C) equals 2 | (D) does not exist |

Q.8 The probability density function of a random variable  $X$  is given by

$$f(x) = \begin{cases} \alpha x^{\alpha-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}, \quad \alpha > 0.$$

Then the distribution of the random variable  $Y = \log_e X^{-2\alpha}$  is

- (A)  $\chi_2^2$       (B)  $\frac{1}{2}\chi_2^2$       (C)  $2\chi_2^2$       (D)  $\chi_1^2$

Q.9 Let  $X_1, X_2, \dots$  be a sequence of i.i.d.  $N(0,1)$  random variables. Then, as  $n \rightarrow \infty$ ,  $\frac{1}{n} \sum_{i=1}^n X_i^2$  converges in probability to

- (A) 0      (B) 0.5      (C) 1      (D) 2

Q.10 Consider the simple linear regression model with  $n$  random observations  $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ ,  $i = 1, \dots, n$ , ( $n > 2$ ).  $\beta_0$  and  $\beta_1$  are unknown parameters,  $x_1, \dots, x_n$  are observed values of the regressor variable and  $\varepsilon_1, \dots, \varepsilon_n$  are error random variables with  $E(\varepsilon_i) = 0$ ,  $i = 1, \dots, n$ , and for  $i, j = 1, \dots, n$ ,  $Cov(\varepsilon_i, \varepsilon_j) = \begin{cases} 0, & \text{if } i \neq j, \\ \sigma^2, & \text{if } i = j. \end{cases}$ . For real constants  $a_1, \dots, a_n$ , if  $\sum_{i=1}^n a_i Y_i$  is an unbiased estimator of  $\beta_1$ , then

- |   |   |
|---|---|
| (A) $\sum_{i=1}^n a_i = 0$ and $\sum_{i=1}^n a_i x_i = 0$ | (B) $\sum_{i=1}^n a_i = 0$ and $\sum_{i=1}^n a_i x_i = 1$ |
| (C) $\sum_{i=1}^n a_i = 1$ and $\sum_{i=1}^n a_i x_i = 0$ | (D) $\sum_{i=1}^n a_i = 1$ and $\sum_{i=1}^n a_i x_i = 1$ |

### Q. 11 – Q. 30 carry two marks each.

Q.11 Let  $(X, Y)$  have the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{2} y^2 e^{-x}, & \text{if } 0 < y < x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Then  $P(Y < 1 | X = 3)$  equals

- |                    |                    |                   |                   |
|--------------------|--------------------|-------------------|-------------------|
| (A) $\frac{1}{81}$ | (B) $\frac{1}{27}$ | (C) $\frac{1}{9}$ | (D) $\frac{1}{3}$ |
|--------------------|--------------------|-------------------|-------------------|

Q.12 Let  $X_1, X_2, \dots$  be a sequence of i.i.d. random variables having the probability density function

$$f(x) = \begin{cases} \frac{1}{B(6,4)} x^5 (1-x)^3, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let  $Y_i = \frac{X_i}{1-X_i}$  and  $U_n = \frac{1}{n} \sum_{i=1}^n Y_i$ . If the distribution of  $\frac{\sqrt{n}(U_n - 2)}{\alpha}$  converges to  $N(0,1)$  as  $n \rightarrow \infty$ , then a possible value of  $\alpha$  is

- |                |                |                |       |
|----------------|----------------|----------------|-------|
| (A) $\sqrt{7}$ | (B) $\sqrt{5}$ | (C) $\sqrt{3}$ | (D) 1 |
|----------------|----------------|----------------|-------|

Q.13 Let  $X_1, \dots, X_n$  be a random sample from a population with the probability density function

$$f(x|\theta) = \begin{cases} 4e^{-4(x-\theta)}, & x > \theta, \\ 0, & \text{otherwise} \end{cases}, \quad \theta \in \mathbb{R}.$$

If  $T_n = \min\{X_1, \dots, X_n\}$ , then

- (A)  $T_n$  is unbiased and consistent estimator of  $\theta$
- (B)  $T_n$  is biased and consistent estimator of  $\theta$
- (C)  $T_n$  is unbiased but NOT consistent estimator of  $\theta$
- (D)  $T_n$  is NEITHER unbiased NOR consistent estimator of  $\theta$

Q.14 Let  $X_1, \dots, X_n$  be i.i.d. random variables with the probability density function

$$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

If  $X_{(n)} = \max\{X_1, \dots, X_n\}$ , then  $\lim_{n \rightarrow \infty} P(X_{(n)} - \log_e n \leq 2)$  equals

- |                   |                     |
|-------------------|---------------------|
| (A) $1 - e^{-2}$  | (B) $e^{-e^{-0.5}}$ |
| (C) $e^{-e^{-2}}$ | (D) $e^{-e^2}$      |

Q.15 Let  $X$  and  $Y$  be two independent  $N(0,1)$  random variables. Then  $P(0 < X^2 + Y^2 < 4)$  equals

- |                  |                  |                  |              |
|------------------|------------------|------------------|--------------|
| (A) $1 - e^{-2}$ | (B) $1 - e^{-4}$ | (C) $1 - e^{-1}$ | (D) $e^{-2}$ |
|------------------|------------------|------------------|--------------|

Q.16 Let  $X$  be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{8}, & 0 \leq x < 2, \\ \frac{x^2}{16}, & 2 \leq x < 4, \\ 1, & x \geq 4. \end{cases}$$

Then  $E(X)$  equals

- |                     |                     |                     |                     |
|---------------------|---------------------|---------------------|---------------------|
| (A) $\frac{12}{31}$ | (B) $\frac{13}{12}$ | (C) $\frac{31}{21}$ | (D) $\frac{31}{12}$ |
|---------------------|---------------------|---------------------|---------------------|

Q.17 Let  $X_1, \dots, X_n$  be a random sample from a population with the probability density function

$$f(x) = \frac{1}{2\theta} e^{-|x|/\theta}, \quad x \in \mathbb{R}, \theta > 0.$$

For a suitable constant  $K$ , the critical region of the most powerful test for testing  $H_0 : \theta = 1$  against  $H_1 : \theta = 2$  is of the form

- |  |  |
|--|--|
| (A) $\sum_{i=1}^n  X_i  > K$           | (B) $\sum_{i=1}^n  X_i  < K$           |
| (C) $\sum_{i=1}^n \frac{1}{ X_i } < K$ | (D) $\sum_{i=1}^n \frac{1}{ X_i } > K$ |

Q.18 Let  $X_1, \dots, X_n, X_{n+1}, X_{n+2}, \dots, X_{n+m}$  ( $n > 4, m > 4$ ) be a random sample from  $N(\mu, \sigma^2)$ ;  $\mu \in \mathbb{R}, \sigma > 0$ . If  $\bar{X}_1 = \frac{1}{n} \sum_{i=1}^n X_i$  and  $\bar{X}_2 = \frac{1}{m-2} \sum_{i=n+1}^{n+m-2} X_i$ , then the distribution of the random variable

$$T = \frac{X_{n+m} - X_{n+m-1}}{\sqrt{\sum_{i=1}^n (X_i - \bar{X}_1)^2 + \sum_{i=n+1}^{n+m-2} (X_i - \bar{X}_2)^2}}$$

is

(A)  $t_{n+m-2}$

(B)  $\sqrt{\frac{2}{n+m-1}} t_{n+m-1}$

(C)  $\sqrt{\frac{2}{n+m-4}} t_{n+m-4}$

(D)  $t_{n+m-4}$

Q.19 Let  $X_1, \dots, X_n$  ( $n > 1$ ) be a random sample from a  $Poisson(\theta)$  population,  $\theta > 0$ , and  $T = \sum_{i=1}^n X_i$ . Then the uniformly minimum variance unbiased estimator of  $\theta^2$  is

(A)  $\frac{T(T-1)}{n^2}$

(B)  $\frac{T(T-1)}{n(n-1)}$

(C)  $\frac{T(T-1)}{n(n+1)}$

(D)  $\frac{T^2}{n^2}$

- Q.20 Let  $X$  be a random variable whose probability mass functions  $f(x|H_0)$  (under the null hypothesis  $H_0$ ) and  $f(x|H_1)$  (under the alternative hypothesis  $H_1$ ) are given by

$X = x$	0	1	2	3
$f(x H_0)$	0.4	0.3	0.2	0.1
$f(x H_1)$	0.1	0.2	0.3	0.4

For testing the null hypothesis  $H_0: X \sim f(x|H_0)$  against the alternative hypothesis  $H_1: X \sim f(x|H_1)$ , consider the test given by: Reject  $H_0$  if  $X > \frac{3}{2}$ .

If  $\alpha$  = size of the test and  $\beta$  = power of the test, then

- (A)  $\alpha = 0.3$  and  $\beta = 0.3$
- (B)  $\alpha = 0.3$  and  $\beta = 0.7$
- (C)  $\alpha = 0.7$  and  $\beta = 0.3$
- (D)  $\alpha = 0.7$  and  $\beta = 0.7$

- Q.21 Let  $X_1, \dots, X_n$  be a random sample from a  $N(2\theta, \theta^2)$  population,  $\theta > 0$ . A consistent estimator for  $\theta$  is

- (A)  $\frac{1}{n} \sum_{i=1}^n X_i$
- (B)  $\left( \frac{5}{n} \sum_{i=1}^n X_i^2 \right)^{1/2}$
- (C)  $\frac{1}{5n} \sum_{i=1}^n X_i^2$
- (D)  $\left( \frac{1}{5n} \sum_{i=1}^n X_i^2 \right)^{1/2}$

- Q.22 An institute purchases laptops from either vendor  $V_1$  or vendor  $V_2$  with equal probability. The lifetimes (in years) of laptops from vendor  $V_1$  have a  $U(0, 4)$  distribution, and the lifetimes (in years) of laptops from vendor  $V_2$  have an  $Exp(1/2)$  distribution. If a randomly selected laptop in the institute has lifetime more than two years, then the probability that it was supplied by vendor  $V_2$  is

- (A)  $\frac{2}{2+e}$
- (B)  $\frac{1}{1+e}$
- (C)  $\frac{1}{1+e^{-1}}$
- (D)  $\frac{2}{2+e^{-1}}$

Q.23 Let  $y(x)$  be the solution to the differential equation

$$x^4 \frac{dy}{dx} + 4x^3y + \sin x = 0; \quad y(\pi) = 1, \quad x > 0.$$

Then  $y\left(\frac{\pi}{2}\right)$  is

(A)  $\frac{10(1+\pi^4)}{\pi^4}$

(B)  $\frac{12(1+\pi^4)}{\pi^4}$

(C)  $\frac{14(1+\pi^4)}{\pi^4}$

(D)  $\frac{16(1+\pi^4)}{\pi^4}$

Q.24 Let  $a_n = e^{-2n} \sin n$  and  $b_n = e^{-n} n^2 (\sin n)^2$  for  $n \geq 1$ . Then

(A)  $\sum_{n=1}^{\infty} a_n$  converges but  $\sum_{n=1}^{\infty} b_n$  does NOT converge

(B)  $\sum_{n=1}^{\infty} b_n$  converges but  $\sum_{n=1}^{\infty} a_n$  does NOT converge

(C) both  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  converge

(D) NEITHER  $\sum_{n=1}^{\infty} a_n$  NOR  $\sum_{n=1}^{\infty} b_n$  converges

Q.25 Let

$$f(x) = \begin{cases} x \sin^2(1/x), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x(\sin x) \sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then

(A)  $f$  is differentiable at 0 but  $g$  is NOT differentiable at 0

(B)  $g$  is differentiable at 0 but  $f$  is NOT differentiable at 0

(C)  $f$  and  $g$  are both differentiable at 0

(D) NEITHER  $f$  NOR  $g$  is differentiable at 0

Q.26 Let  $f : [0, 4] \rightarrow \mathbb{R}$  be a twice differentiable function. Further, let  $f(0) = 1$ ,  $f(2) = 2$  and  $f(4) = 3$ . Then

- (A) there does NOT exist any  $x_1 \in (0, 2)$  such that  $f'(x_1) = \frac{1}{2}$
- (B) there exist  $x_2 \in (0, 2)$  and  $x_3 \in (2, 4)$  such that  $f'(x_2) = f'(x_3)$
- (C)  $f''(x) > 0$  for all  $x \in (0, 4)$
- (D)  $f''(x) < 0$  for all  $x \in (0, 4)$

Q.27 Let  $f(x, y) = x^2 - 400xy^2$  for all  $(x, y) \in \mathbb{R}^2$ . Then  $f$  attains its

- (A) local minimum at  $(0, 0)$  but NOT at  $(1, 1)$
- (B) local minimum at  $(1, 1)$  but NOT at  $(0, 0)$
- (C) local minimum both at  $(0, 0)$  and  $(1, 1)$
- (D) local minimum NEITHER at  $(0, 0)$  NOR at  $(1, 1)$

Q.28 Let  $y(x)$  be the solution to the differential equation

$$4 \frac{d^2y}{dx^2} + 12 \frac{dy}{dx} + 9y = 0, \quad y(0) = 1, \quad y'(0) = -4.$$

Then  $y(1)$  equals

- (A)  $-\frac{1}{2} e^{-3/2}$
- (B)  $-\frac{3}{2} e^{-3/2}$
- (C)  $-\frac{5}{2} e^{-3/2}$
- (D)  $-\frac{7}{2} e^{-3/2}$

Q.29 Let  $g : [0, 2] \rightarrow \mathbb{R}$  be defined by

$$g(x) = \int_0^x (x-t)e^t dt.$$

The area between the curve  $y = g''(x)$  and the  $x$ -axis over the interval  $[0, 2]$  is

- (A)  $e^2 - 1$
- (B)  $2(e^2 - 1)$
- (C)  $4(e^2 - 1)$
- (D)  $8(e^2 - 1)$

Q.30 Let  $P$  be a  $3 \times 3$  singular matrix such that  $P\vec{v} = \vec{v}$  for a nonzero vector  $\vec{v}$  and

$$P \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/5 \\ 0 \\ -2/5 \end{bmatrix}.$$

Then

- (A)  $P^3 = \frac{1}{5}(7P^2 - 2P)$   
(B)  $P^3 = \frac{1}{4}(7P^2 - 2P)$   
(C)  $P^3 = \frac{1}{3}(7P^2 - 2P)$   
(D)  $P^3 = \frac{1}{2}(7P^2 - 2P)$

**SECTION - B**  
**MULTIPLE SELECT QUESTIONS (MSQ)**

**Q. 31 – Q. 40 carry two marks each.**

Q.31 For two nonzero real numbers  $a$  and  $b$ , consider the system of linear equations

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b/2 \\ a/2 \end{bmatrix}.$$

Which of the following statements is (are) TRUE?

- (A) If  $a = b$ , the solutions of the system lie on the line  $x + y = 1/2$
- (B) If  $a = -b$ , the solutions of the system lie on the line  $y - x = 1/2$
- (C) If  $a \neq \pm b$ , the system has no solution
- (D) If  $a \neq \pm b$ , the system has a unique solution

Q.32 For  $n \geq 1$ , let

$$a_n = \begin{cases} n 2^{-n}, & \text{if } n \text{ is odd,} \\ -3^{-n}, & \text{if } n \text{ is even.} \end{cases}$$

Which of the following statements is (are) TRUE?

- (A) The sequence  $\{a_n\}$  converges
- (B) The sequence  $\{|a_n|^{1/n}\}$  converges
- (C) The series  $\sum_{n=1}^{\infty} a_n$  converges
- (D) The series  $\sum_{n=1}^{\infty} |a_n|$  converges

Q.33 Let  $f : (0, \infty) \rightarrow \mathbb{R}$  be defined by

$$f(x) = x \left( e^{1/x^3} - 1 + \frac{1}{x^3} \right).$$

Which of the following statements is (are) TRUE?

- (A)  $\lim_{x \rightarrow \infty} f(x)$  exists
- (B)  $\lim_{x \rightarrow \infty} x f(x)$  exists
- (C)  $\lim_{x \rightarrow \infty} x^2 f(x)$  exists
- (D) There exists  $m > 0$  such that  $\lim_{x \rightarrow \infty} x^m f(x)$  does NOT exist.

**Q.34** For  $x \in \mathbb{R}$ , define  $f(x) = \cos(\pi x) + \lceil x^2 \rceil$  and  $g(x) = \sin(\pi x)$ . Which of the following statements is (are) TRUE?

- (A)  $f(x)$  is continuous at  $x=2$
- (B)  $g(x)$  is continuous at  $x=2$
- (C)  $f(x)+g(x)$  is continuous at  $x=2$
- (D)  $f(x)g(x)$  is continuous at  $x=2$

**Q.35** Let  $E$  and  $F$  be two events with  $0 < P(E) < 1$ ,  $0 < P(F) < 1$  and  $P(E|F) > P(E)$ . Which of the following statements is (are) TRUE?

- (A)  $P(F|E) > P(F)$
- (B)  $P(E|F^c) > P(E)$
- (C)  $P(F|E^c) < P(F)$
- (D)  $E$  and  $F$  are independent

**Q.36** Let  $X_1, \dots, X_n$  ( $n > 1$ ) be a random sample from a  $U(2\theta-1, 2\theta+1)$  population,  $\theta \in \mathbb{R}$ , and  $Y_1 = \min\{X_1, \dots, X_n\}$ ,  $Y_n = \max\{X_1, \dots, X_n\}$ . Which of the following statistics is (are) maximum likelihood estimator(s) of  $\theta$ ?

- (A)  $\frac{1}{4}(Y_1 + Y_n)$
- (B)  $\frac{1}{6}(2Y_1 + Y_n + 1)$
- (C)  $\frac{1}{8}(Y_1 + 3Y_n - 2)$
- (D) Every statistic  $T(X_1, \dots, X_n)$  satisfying  $\frac{(Y_n - 1)}{2} < T(X_1, \dots, X_n) < \frac{(Y_1 + 1)}{2}$

**Q.37** Let  $X_1, \dots, X_n$  be a random sample from a  $N(0, \sigma^2)$  population,  $\sigma > 0$ . Which of the following testing problems has (have) the region  $\left\{ (x_1, \dots, x_n) \in \mathbb{R}^n : \sum_{i=1}^n x_i^2 \geq \chi_{n, \alpha}^2 \right\}$  as the most powerful critical region of level  $\alpha$ ?

- (A)  $H_0 : \sigma^2 = 1$  against  $H_1 : \sigma^2 = 2$
- (B)  $H_0 : \sigma^2 = 1$  against  $H_1 : \sigma^2 = 4$
- (C)  $H_0 : \sigma^2 = 2$  against  $H_1 : \sigma^2 = 1$
- (D)  $H_0 : \sigma^2 = 1$  against  $H_1 : \sigma^2 = 0.5$

**Q.38** Let  $X_1, \dots, X_n$  be a random sample from a  $N(0, 2\theta^2)$  population,  $\theta > 0$ . Which of the following statements is (are) TRUE?

- (A)  $(X_1, \dots, X_n)$  is sufficient and complete
- (B)  $(X_1, \dots, X_n)$  is sufficient but NOT complete
- (C)  $\sum_{i=1}^n X_i^2$  is sufficient and complete
- (D)  $\frac{1}{2n} \sum_{i=1}^n X_i^2$  is the uniformly minimum variance unbiased estimator for  $\theta^2$

**Q.39** Let  $X_1, \dots, X_n$  be a random sample from a population with the probability density function

$$f(x|\theta) = \begin{cases} \theta e^{-\theta x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}, \quad \theta > 0.$$

Which of the following is (are)  $100(1-\alpha)\%$  confidence interval(s) for  $\theta$ ?

- (A)  $\left( \frac{\chi_{2n,1-\alpha/2}^2}{2 \sum_{i=1}^n X_i}, \frac{\chi_{2n,\alpha/2}^2}{2 \sum_{i=1}^n X_i} \right)$
- (B)  $\left( 0, \frac{\chi_{2n,\alpha}^2}{2 \sum_{i=1}^n X_i} \right)$
- (C)  $\left( \frac{\chi_{2n,1-\alpha/2}^2}{\sum_{i=1}^n X_i}, \frac{\chi_{2n,\alpha/2}^2}{\sum_{i=1}^n X_i} \right)$
- (D)  $\left( \frac{2 \sum_{i=1}^n X_i}{\chi_{2n,\alpha/2}^2}, \frac{2 \sum_{i=1}^n X_i}{\chi_{2n,1-\alpha/2}^2} \right)$

**Q.40** The cumulative distribution function of a random variable  $X$  is given by

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{10} \left( x^2 - \frac{7}{3} \right), & 2 \leq x < 3, \\ 1, & x \geq 3. \end{cases}$$

Which of the following statements is (are) TRUE?

- (A)  $F(x)$  is continuous everywhere
- (B)  $F(x)$  increases only by jumps
- (C)  $P(X = 2) = \frac{1}{6}$
- (D)  $P\left(X = \frac{5}{2} \mid 2 \leq X \leq 3\right) = 0$

**SECTION – C**  
**NUMERICAL ANSWER TYPE (NAT)**

**Q. 41 – Q. 50 carry one mark each.**

- Q.41 Let  $X_1, \dots, X_{10}$  be a random sample from a  $N(3,12)$  population. Suppose  $Y_1 = \frac{1}{6} \sum_{i=1}^6 X_i$  and  $Y_2 = \frac{1}{4} \sum_{i=7}^{10} X_i$ . If  $\frac{(Y_1 - Y_2)^2}{\alpha}$  has a  $\chi^2$  distribution, then the value of  $\alpha$  is \_\_\_\_\_

- Q.42 Let  $X$  be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{2x}{9}, & 0 < x < 3, \\ 0, & \text{otherwise.} \end{cases}$$

Then the upper bound of  $P(|X - 2| > 1)$  using Chebyshev's inequality is \_\_\_\_\_

- Q.43 Let  $X$  and  $Y$  be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} e^{-(x+y)}, & -\infty < x, y < 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then  $P(X < Y) = _____$

- Q.44 Let  $X$  and  $Y$  be continuous random variables with the joint probability density function

$$f(x, y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2}, \quad (x, y) \in \mathbb{R}^2.$$

Then  $P(X > 0, Y < 0) = _____$

- Q.45 Let  $Y$  be a  $Bin\left(72, \frac{1}{3}\right)$  random variable. Using normal approximation to binomial distribution, an approximate value of  $P(22 \leq Y \leq 28)$  is \_\_\_\_\_

Q.46 Let  $X$  be a  $\text{Bin}(2, p)$  random variable and  $Y$  be a  $\text{Bin}(4, p)$  random variable,  $0 < p < 1$ . If

$$P(X \geq 1) = \frac{5}{9}, \text{ then } P(Y \geq 1) = \underline{\hspace{2cm}}$$

Q.47 Consider the linear transformation

$$T(x, y, z) = (2x + y + z, x + z, 3x + 2y + z).$$

The rank of  $T$  is \_\_\_\_\_

Q.48

$$\text{The value of } \lim_{n \rightarrow \infty} n \left[ e^{-n} \cos(4n) + \sin\left(\frac{1}{4n}\right) \right] \text{ is } \underline{\hspace{2cm}}$$

Q.49

Let  $f : [0, 13] \rightarrow \mathbb{R}$  be defined by  $f(x) = x^{13} - e^{-x} + 5x + 6$ . The minimum value of the function  $f$  on  $[0, 13]$  is \_\_\_\_\_

Q.50

Consider a differentiable function  $f$  on  $[0, 1]$  with the derivative  $f'(x) = 2\sqrt{2x}$ . The arc length of the curve  $y = f(x)$ ,  $0 \leq x \leq 1$ , is \_\_\_\_\_

### **Q. 51 – Q. 60 carry two marks each.**

Q.51 Let  $m$  be a real number such that  $m > 1$ . If

$$\int_1^m \int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dy dx dz = e - 1,$$

then  $m = \underline{\hspace{2cm}}$

Q.52 Let

$$P = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}.$$

The product of the eigen values of  $P^{-1}$  is \_\_\_\_\_

Q.53 The value of the real number  $m$  in the following equation

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2 + y^2) dy dx = \int_{m\pi}^{\pi/2} \int_0^{\sqrt{2}} r^3 dr d\theta$$

is \_\_\_\_\_

Q.54 Let  $a_1 = 1$  and  $a_n = 2 - \frac{1}{n}$  for  $n \geq 2$ . Then

$$\sum_{n=1}^{\infty} \left( \frac{1}{a_n^2} - \frac{1}{a_{n+1}^2} \right)$$

converges to \_\_\_\_\_

Q.55 Let  $X_1, X_2, \dots$  be a sequence of i.i.d. random variables with the probability density function

$$f(x) = \begin{cases} 4x^2 e^{-2x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$$

and let  $S_n = \sum_{i=1}^n X_i$ . Then  $\lim_{n \rightarrow \infty} P\left(S_n \leq \frac{3n}{2} + \sqrt{3n}\right)$  is \_\_\_\_\_

Q.56 Let  $X$  and  $Y$  be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} \frac{c x^2}{y^3}, & 0 < x < 1, y > 1, \\ 0, & \text{otherwise} \end{cases}$$

where  $c$  is a suitable constant. Then  $E(X) =$  \_\_\_\_\_

Q.57 Two points are chosen at random on a line segment of length 9 cm. The probability that the distance between these two points is less than 3 cm is \_\_\_\_\_

Q.58 Let  $X$  be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x+1}{2}, & -1 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then  $P\left(\frac{1}{4} < X^2 < \frac{1}{2}\right) = \underline{\hspace{2cm}}$

Q.59 If  $X$  is a  $U(0,1)$  random variable, then  $P\left(\min(X, 1-X) \leq \frac{1}{4}\right) = \underline{\hspace{2cm}}$

Q.60 In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly  $k$  children is  $(0.5)^k$ ;  $k = 1, 2, \dots$ . A child is either a male or a female with equal probability. The probability that such a family consists of at least one male child and at least one female child is  $\underline{\hspace{2cm}}$

**END OF THE QUESTION PAPER**

JAM 2016: Mathematical Statistics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	C	1
2	MCQ	A	1
3	MCQ	C	1
4	MCQ	D	1
5	MCQ	B	1
6	MCQ	C	1
7	MCQ	D	1
8	MCQ	A	1
9	MCQ	C	1
10	MCQ	B	1
11	MCQ	B	2
12	MCQ	C	2
13	MCQ	B	2
14	MCQ	C	2
15	MCQ	A	2
16	MCQ	D	2
17	MCQ	A	2
18	MCQ	C	2
19	MCQ	A	2
20	MCQ	B	2
21	MCQ	D	2
22	MCQ	A	2
23	MCQ	D	2
24	MCQ	C	2
25	MCQ	B	2
26	MCQ	B	2
27	MCQ	D	2
28	MCQ	B	2
29	MCQ	A	2
30	MCQ	A	2

JAM 2016: Mathematical Statistics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;B;D	2
32	MSQ	A;C;D	2
33	MSQ	A;B;C;D	2
34	MSQ	B;D	2
35	MSQ	A;C	2
36	MSQ	A;B;C;D	2
37	MSQ	A;B	2
38	MSQ	B;C;D	2
39	MSQ	A;B	2
40	MSQ	C;D	2

JAM 2016: Mathematical Statistics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	4.9 to 5.1	1
42	NAT	0.49 to 0.51	1
43	NAT	0.49 to 0.51	1
44	NAT	0.24 to 0.26	1
45	NAT	0.52 to 0.62	1
46	NAT	0.80 to 0.81	1
47	NAT	1.9 to 2.1	1
48	NAT	0.24 to 0.26	1
49	NAT	4.9 to 5.1	1
50	NAT	2.1 to 2.2	1
51	NAT	3.9 to 4.1	2
52	NAT	-0.51 to -0.49	2
53	NAT	0.24 to 0.26	2
54	NAT	0.74 to 0.76	2
55	NAT	0.97 to 0.98	2
56	NAT	0.74 to 0.76	2
57	NAT	0.5 to 0.6	2
58	NAT	0.20 to 0.21	2
59	NAT	0.49 to 0.51	2
60	NAT	0.3 to 0.4	2