Units in the Hodgkin-Huxley spiking neuron model.

Austin Soplata

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1 Assumed derivation

First, we will derive the units for a typical channel of $g_{ion}(V_{cell} - V_{revpot})$ type, assuming the units given in page 519 of *Hodgkin, A. L., & Huxley, A. F. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. The Journal of Physiology, 117(4), 500.* The units given for g_{ion} are $\frac{milliSiemens}{cm^2}$ or $\frac{mS}{cm^2}$, voltage and reversal potentials are in mV, and current are in $\frac{\mu A}{cm^2}$. The identities used include, from wikipedia, $Siemens = \frac{Ampere}{Volt}$.

$$g(V - V_{rev}) = \frac{mS * mV}{cm^2} = \frac{1}{cm^2}\frac{\cancel{mS}}{1}\frac{1S}{1000\cancel{mS}}\frac{\cancel{mV}}{1}\frac{1V}{1000\cancel{mV}} = \frac{SV}{cm^2 1E6} = \frac{A}{cm^2 1E6} = \frac{\cancel{A}}{cm^2\cancel{1E6}}\frac{\cancel{1E6}\mu A}{\cancel{A}} = \frac{1\mu A}{cm^2} \tag{1.1}$$

Next, we will show the same equivalence for the more important $C\frac{dV}{dt}$ contribution to the neuronal current in the model, where C is in $\frac{microFarads}{cm^2}$ or $\frac{\mu F}{cm^2}$, dV is still in mV, and dt, as with any of the time variables in the model equations, is in $milliseconds$ or ms. This uses the $F = \frac{A*s}{V}$ identity.

$$C\frac{dV}{dt} = \frac{\mu F * mV}{cm^2 * ms} = \frac{1}{cm^2}\frac{1\cancel{\mu F}}{1}\frac{1F}{1E6\cancel{\mu F}}\frac{\cancel{mV}}{1}\frac{1V}{\cancel{1000}\cancel{mV}}\frac{1}{\cancel{ms}}\frac{\cancel{1000}\cancel{ms}}{s} = \frac{A * \cancel{s}}{cm^2 1E6\cancel{V}}\frac{\cancel{V}}{\cancel{s}} = \frac{\cancel{A}}{cm^2\cancel{1E6}}\frac{\cancel{1E6}\mu A}{\cancel{A}} = \frac{1\mu A}{cm^2} \tag{1.2}$$

Thus, capacitance C_m is in units of $\frac{\mu F}{cm^2}$
Voltages, including dV, are in units of mV
Time, including dt and time constants, is in units of milliseconds, or ms
Conductances are in units of milliSiemens over cm^2, or $\frac{mS}{cm^2}$
And finally, current is in units of $\frac{\mu A}{cm^2}$

Separately, there are a number of different ways of treating "external" inputs into the cell like applied current and synaptic currents, that may or may not require the geometry of the cell to be specified. For what seems like most modern simple models that aren't, say, trying to reconstruct realistically-sized networks or even large (>500 neurons) networks (e.g. Traub et al. 2005), the trend appears to be strictly leaving out geometry and specifying applied and synaptic currents in total-area-independent units like those of Na, K, and L in the original HH equations: $\frac{\mu A}{cm^2}$ and $\frac{mS}{cm^2}$ for applied current and synaptic conductance, respectively.

If a paper is 1. recent (e.g. > 2000 whatever), 2. does not specify the geometric size of the neuron, and 3. does not do justice to explicitly specify synaptic conductance units, then one should probably assume the synaptic conductances are total-area-independent as well, i.e. in $\frac{mS}{cm^2}$. Following this same line of thinking, since one will not be able to specify any absolute current if the geometry is unknown, applied current will also be total-area-independent, in units of $\frac{\mu A}{cm^2}$ and the like. Examples of this include Borgers et al. 2008 and Wang et al. 1996.

Alternatively, Destexhe at least uses $\frac{mS}{cm^2}$ for typical non-synaptic conductances like g_{Na}, but absolute conductance in mS, or nS etc. for synaptic conductances; in this case, one must explicitly specify the surface area of the cell in order to calculate and take into account the current afforded by the synapse. Similarly, if the geometry/surface area is specified, then the applied current can be specified in absolute terms, e.g. nA etc. Examples of this include Destexhe et al. 1996, Destexhe et al. 1993.