THE CAPTURE CROSS-SECTION OF EARTH FOR ERRANT ASTEROIDS[1]

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ABSTRACT. The capture cross-section of Earth for an errant asteroid is hardly any larger than its geometric cross-section for asteroids approaching at speeds greater than 20 km s^{-1}. The capture cross-section increases greatly for asteroids with approach speeds less than 10 km s^{-1}. Asteroids moving in near-Earth circular orbits with radii between 0.9943 A.U. and 1.0057 A.U. will inevitably collide with Earth — indeed they must already have done so.

RÉSUMÉ. La section efficace de la Terre pour la capture d'un astéroïde est à peine plus grande que la coupe transversale géométrique lorsqu'il s'agit d'astéroïdes approchant à des vitesses plus élevées que 20 km s^{-1}. La coupe transversale de capture s'accroît grandement dans le cas des astéroïdes ayant des vitesses d'approche plus lentes que 10 km s^{-1}. Les astéroïdes se déplaçant sur des orbites circulaires pres de la Terre, et dont les rayons se trouvent entre 0,9943 U.A. et 1,0057 U.A., vont inévitablement entrer en collision avec la Terre; en effet, ils ont déjà frappé la Terre.

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It is by now well known that the collision of a sizable asteroid with Earth is a real possibility, and much recent research effort has been directed towards estimating the probability per millennium of collision of asteroids in various size ranges. The principal technical literature on the subject is given in the volume edited by Gehrels (1994), while two of the popular books on the subject are those by Lewis (1996) and Steel (1995). A short article by Binzel (1995) will also be found useful.

It is pointed out from time to time that the danger is larger than it may seem, because the gravitational attraction of Earth pulls in errant asteroids from an area that may be considerably larger than Earth's geometrical cross-section. I wanted to know whether that was really true. Are asteroids dragged in from a large area by Earth's gravitational field, so that the effective capture cross-section is much larger than Earth's geometric cross-section? Or will an asteroid collide with Earth only if it is headed almost directly towards Earth, Earth's gravitational field having little effect, the capture cross-section being little, if any, larger than Earth's geometric cross-section? The answer intuitively should depend on how fast the asteroid is moving, the capture cross-section presumably being larger for a slower-moving asteroid, but how do we calculate the answer quantitatively? It turns out that the answer can be calculated using only first-year physics, and is presumably well known to meteor scientists; but I felt that it might be of interest to those to whom it is less familiar.

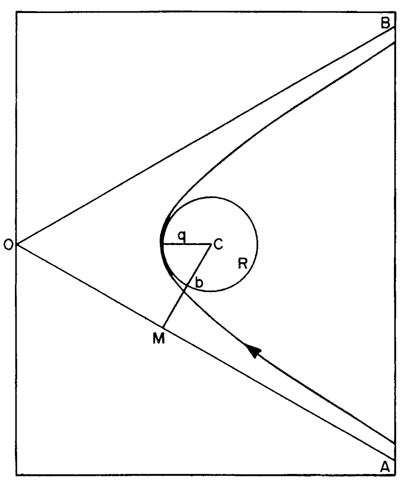

FIG. 1 — The impact parameter.

Figure 1 shows an asteroid approaching Earth from the bottom right hand corner of the figure moving along a hyperbolic orbit[2]. C is the centre of attraction — *i.e.* the centre of the Earth. The lines AO and OB are the asymptotes to the hyperbola, and it is assumed that initially, when the asteroid was a long way from Earth, it approached along the asymptote OA with initial speed v_0. The distance CM, of length b, is the least distance that the asteroid would reach from C if it carried on along the asymptote AO unaffected by the gravitational attraction of Earth, and is called the impact parameter. It may not be immediately obvious, but it is in fact the case, that the length b is also equal to the semi-transverse axis of the conjugate hyperbola and is the length b in the familiar equation for a hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 .$$

(The conjugate hyperbola is,

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 ;$$

it shares the same asymptotes as the former, but its axis is the y-axis rather than the x-axis.) The perihelion distance is q. The circle represents Earth, of radius R, and the figure is drawn so that a grazing collision occurs, the perihelion distance q being equal in that case to Earth's radius R. The asteroid misses Earth if $q > R$, and collides with Earth if $q < R$.

The geometric cross section of Earth is πR^2, its effective capture cross-section is πb^2, and we wish to find the ratio of the two as a function of initial speed v_0.

1 Based on a paper presented at the October 1996 meeting of the Meteorites and Impacts Advisory Committee • Comité consultatif sur les météorites et les impacts to the Canadian Space Agency, held in Saint-Hubert, Québec.

2 The Earth and the asteroid are each, of course, in elliptical orbits about the Sun. What we are concerned with here is the trajectory of an asteroid relative to the Earth when it is so close that the Earth's gravitational attraction is much greater than that of the Sun. The attraction of the Earth is ten times the attraction of the Sun, for example, when an asteroid is about 13 Earth radii from the centre of the Earth. In the present context the asteroid's "hyperbolic" orbit refers to a geocentric reference frame, as does the "initial speed v_0." For precise ephemeris computations we do not neglect the influence of either the Sun or the Moon; we are concerned here only with order-of-magnitude effects.

We shall suppose that the perigee speed of the asteroid is v_p. Energy and angular momentum are conserved, so:

$$v_0^2 = v_p^2 - \frac{2GM}{q} \tag{1}$$

and

$$v_0 b = v_p q \, . \tag{2}$$

Here G is the universal gravitational constant and M is the mass of the Earth. Elimination of v_p from the equations, together with the replacement of q by R, gives the required ratio,

$$\frac{b^2}{R^2} = 1 + \frac{2GM}{Rv_0^2} \, , \tag{3}$$

which can also be written as,

$$\frac{b^2}{R^2} = 1 + \frac{v_e^2}{v_0^2} \, , \tag{4}$$

where $v_e = \sqrt{2GM/R)} \cong 11.2$ km s^{-1} is the escape speed from the surface of the Earth.

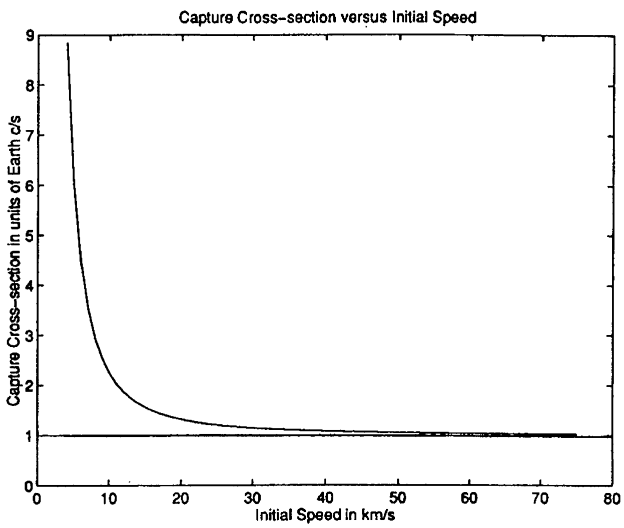

FIG. 2 — The capture cross-section (in units of Earth's geometric cross-section) as a function of the initial speed.

The relation is shown in figure 2. Starting at the right hand side of the figure, we see that for high initial speeds (50 to 80 km s^{-1}) the effective capture cross section of Earth is hardly any greater than its geometric cross-section. It is not much greater even for initial speeds as low as 20 km s^{-1}. For initial speeds between 10 and 20 km s^{-1}, however, the capture cross-section does start to become a bit larger than the geometric cross-section, while for speeds less than 10 km s^{-1} the capture cross section rapidly becomes very much larger than the geometric cross-section.

Long period comets are sometimes said to be a particular danger to Earth because they are unpredictable and impact at high speed. They are indeed unpredictable, which does add an element of fear to the problem, and certainly their high speed would result in severe and perhaps catastrophic damage in the event of an impact. But their admittedly high speed also means that Earth presents a small target, so that the likelihood of collision is correspondingly reduced.

There is one class of asteroids, however, that do approach Earth closely and with small relative speeds, and they must surely be considered to be likely impactors. Beginning in 1991 the Spacewatch team has discovered a number of small objects, tens of metres in size or less, that move around the Sun in orbits very similar to that of Earth. David Balam has helped to track some of them at this observatory and at the Dominion Astrophysical Observatory. The first and most famous of them is 1991 VG, which has the distinction, at eight metres, of being the smallest astronomical object ever discovered. The semi-major axis of its orbit is 1.04 A.U. and its eccentricity is a mere 0.067 — though such a small eccentricity is enough to bring it inside Earth's orbit at perihelion (perihelion distance of the osculating orbit = 0.97 A.U.). It was in fact speculated at the time of discovery that it might be a stray bit of artificial space debris, though the general view is that it is a natural object.

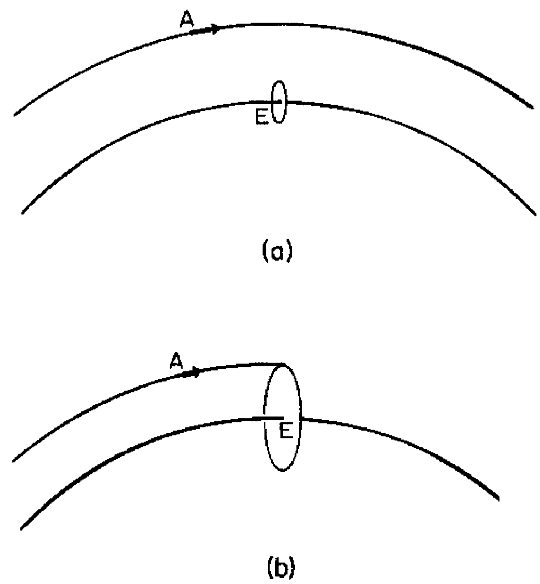

FIG. 3 — Earth has a small capture cross-section for an asteroid in a nearby orbit. If the asteroid's orbit is a little bit closer, the capture cross-section is much larger.

How Earthlike does the orbit of an asteroid have to be in order for the asteroid to strike Earth? The situation is depicted in figure 3, which is not drawn to scale, but which is intended to show the principles. The circle E represents Earth's orbit, and the circle A represents the orbit of an asteroid. The figure is drawn in a co-rotating reference frame in which the Earth is at rest; in such a frame the asteroid moves clockwise as seen from the ecliptic north pole. The small ellipse represents the effective capture cross-section. In part (a) the asteroid easily clears the Earth's capture cross-section and is therefore safe from capture. In part (b) the asteroid's orbit is supposed to be a little bit closer to that of Earth. Not only that but, because of the closer similarity of the orbits, the relative speed of approach of the asteroid is a little less, and consequently Earth's capture cross-section is much greater, and the asteroid is captured.

Let V be the speed of Earth in its orbit in the non-rotating reference frame, and let a be the radius of Earth's orbit. Let $a + \delta a$ represent the radius of the asteroid's orbit (I assume both orbits to be circular and co-planar) and $V + \delta V$ the speed of the asteroid. Kepler's third law requires that,

$$\delta V = -\frac{V\delta a}{2a} . \tag{5}$$

But δV, the differential speed between the asteroid and the Earth, is in the previous notation v_0, the initial approach speed of the asteroid to Earth. The parameter δa, the difference in radii between the two orbits, is in the previous notation the impact parameter b. On substitution of equation (5) into equation (3) using the new notation, we find that the condition for an asteroid to be captured is that the radius of its orbit should differ from the radius of Earth's orbit by less than δa, where

$$(\delta a)^4 - R^2(\delta a)^2 - \frac{8GmRa^2}{V^2} = 0 , \tag{6}$$

or a = 0.0057 A.U. = 130 Earth radii = 2.2 times the mean distance to the Moon.

From equation (5) we can calculate that the orbital speed of an asteroid moving in a circular orbit that differs in orbital radius from that of the Earth's orbit by that amount will differ from the orbital speed of the Earth by only 0.085 km s^{-1}. Its impact speed at the subsequent grazing collision, which is readily calculated from equations (2) and (4) bearing in mind the change in notation, is 11.4 km s^{-1}, just a little more than the escape velocity of 11.2 km s^{-1} from the Earth's surface. The two equations also yield the interesting result that,

$$v_p = \frac{Vb^2}{2aR} , \tag{7}$$

which gives the perigee speed in terms of the impact parameter — the smaller the impact parameter, the smaller the perigee speed.

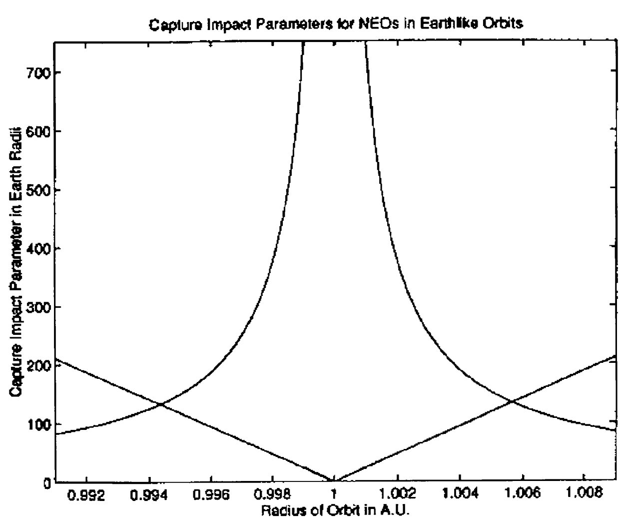

FIG. 4 — The capture cross-section for an asteroid as a function of the size of the asteroids orbit.

The curved lines in figure 4 show the radius of Earth's capture cross-section (impact parameter) as a function of the radius of an asteroid's orbit. The straight lines show the difference in radii between an asteroid's orbit and Earth's orbit. Wherever the curved line lies above the straight line, capture will occur. Earth sweeps out a toroidal volume of radius 0.0057 A.U. and removes any asteroids moving in circular orbits within that volume. The cross-sectional area of the toroidal volume is 17,300 times the geometrical cross-section of the Earth. Any asteroid moving in a circular orbit with a radius between 0.9943 A.U. and 1.0057 A.U. will inevitably collide with Earth. Or, to rephrase the statement: Any asteroid that moved in a circular orbit with a radius between 0.9943 A.U. and 1.0057 A.U. has already collided with Earth.

Does that mean that Earth, having already gathered up all such asteroids, is now safe? Not at all. Any asteroid in an orbit only a little outside the limits could be perturbed into the danger zone, and indeed 1991 VG, in its slightly elliptical orbit, crosses the danger zone twice per year. I have not studied the future of 1991 VG in detail; to do so would require acknowledgment of the existence of the Moon. Until it has been proved otherwise, however, 1991 VG and other rocks of similar ilk must be regarded as dangerous. Fortunately the known ones are not large enough to cause a global catastrophe, but a hit in the right place could be quite tiresome.

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THE INNISFREE METEORITE AND THE CANADIAN CAMERA NETWORK

Meteorite research in Canada received a great stimulus as a result of events that began with the fall of the Bruderheim meteorite near Edmonton, Alberta, on March 4, 1960. One recent consequence of the Bruderheim event was the photographic recording and prompt recovery of the Innisfree meteorite, which fell in Alberta on February 5, 1977, only 116 km from the much larger Bruderheim fall.

The fall and recovery of the large shower of stone meteorites north of the small town of Bruderheim has been described by Folinsbee & Bayrock (1961). Many fragments, totalling over 300 kg, of this typical hypersthene chondrite (petrologic type L6) were recovered within an elliptic area nearly 4×6 km in extent. Such an abundant supply has made possible a very extensive study of the meteorite and provided exchange material for the improvement of the meteorite collections in Canada. The total recovered mass for Bruderheim is the largest for any Canadian meteorite.

The experience gained in field searches and laboratory study of Bruderheim was put to good use when three other meteorite falls occurred within 500 km of Edmonton in the next seven years. Those meteorites were Peace River in 1963 (Folinsbee & Bayrock 1964), Revelstoke in 1965 (Folinsbee *et al.* 1967) and Vilna in 1967 (Folinsbee *et al.* 1969). The four events brought to 10 the number of Canadian meteorite falls for which the date of fall was known and suggested that several falls per decade might be expected if reports of bright fireballs were pursued intensively. By coincidence, the next fall in Canada, the Innisfree meteorite, occurred ten years to the day (almost to the hour) after the Vilna event. Several other meteorites classified as “finds” rather than “falls” were recovered during the decade, but information on the date of fall is not available in those cases.

The Bruderheim event had other, less obvious, results. The collection of the fallen meteorites and their acquisition for scientific study had been handled efficiently because of the presence and interest of amateur astronomers within the Edmonton Centre of the Royal Astronomical Society of Canada and geologists at the University of Alberta and the Research Council of Alberta. In many parts of Canada the event might have gone unnoticed. That realization led directly to the establishment in 1960 of the Associate Committee on Meteorites by the National Research Council of Canada to stimulate interest in all aspects of meteorite research. The Committee consists of about fifteen members chosen to give broad geographic coverage across Canada. One of the Committee’s first actions was to organize a national system for the prompt reporting of fireballs to the regional representatives and the central office in Ottawa, a system which worked well for Innisfree.

Another early action of the Associate Committee was to recommend the creation of a network of photographic stations in western Canada to record bright fireballs. The aim of such networks is to provide accurate data on the atmospheric trajectory, from which the probable impact point of meteorites on the ground can be calculated with more confidence and less delay than by the collection and analysis of eyewitness reports. Secondly, the data from the upper end of the luminous path can be used to calculate the orbit of the meteoroid around the Sun before it collided with Earth. The observational data may also be used for statistical studies of the rate of occurrence of bright meteors, orbital statistics or large meteoroids in the solar system, etc.

Photographs of the atmospheric flight of the Pribram meteorite in Czechoslovakia had been secured in 1959 with conventional meteor cameras (Ceplecha 1961). In 1961 the International Astronomical Union approved a resolution calling for the construction of photographic fireball networks and the first two networks were established about 1964 in Czechoslovakia (Ceplecha & Rajchl 1965) and the central plains of the United States (McCrosky & Boeschenstein 1965). The Czechoslovakian network soon expanded in co-operation with West Germany and is known as the European Network. The Canadian Meteorite and Observation and Recovery Project (MORP) became the third such project (Halliday 1973), followed more recently by networks in the United Kingdom (Hindley 1975) and in the U.S.S.R. (Zotkin *et al.* 1976).

Ian Halliday, Alan T. Blackwell and Arthur A. Griffin
from *Journal*, Vol. 72, pp. 15–17, February, 1978.