An example problem: Emotional states

Let's consider a simple example:
a robot that tracks the emotional states of a player.

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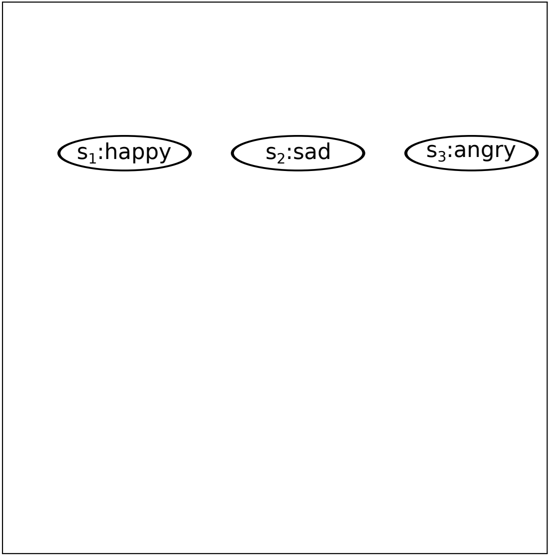

N - number of states

N $= 3$

states:

- s_1: happy
- s_2: sad
- s_3: angry

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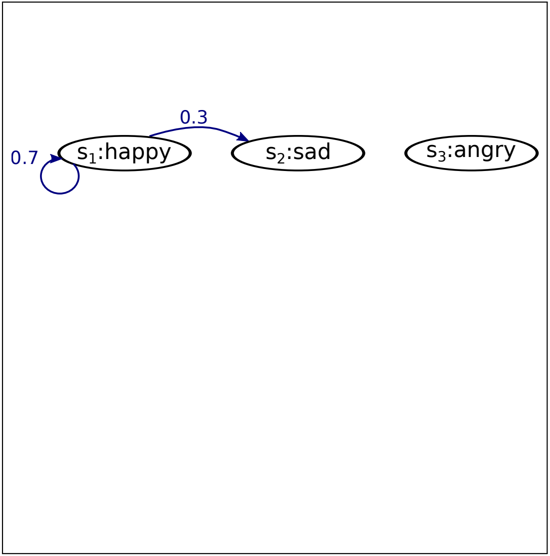

A - state transition probability distribution

$\mathbf{A} = \{a_{i,j}\},\ 1 \leq i,j \leq N$

$a_{i,j} = P(q_{t+1} = s_j | q_t = s_i)$

- $a_{\mathbf{1},1} = 0.7$
- $a_{\mathbf{1},2} = 0.3$
- $a_{\mathbf{1},3} = 0$

$$\sum_{j=1}^{N} a_{i,j} = 1, \quad 1 \leq i \leq N$$

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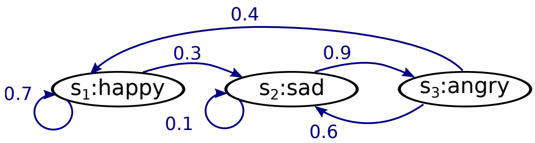

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$$\mathbf{A} = \begin{array}{c c} & \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.9 & 0.1 \\ 0.4 & 0.6 & 0 \end{pmatrix} \end{array}$$

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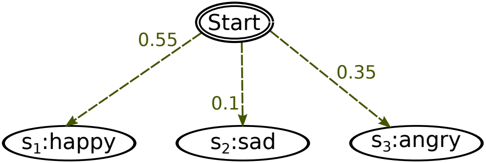

Π - initial state distribution

$$\mathbf{\Pi} = \{\pi_i\}, \quad 1 \leq i \leq N$$

$$\pi_i = P(q_1 = s_i)$$

$$\mathbf{\Pi} = \begin{array}{c} s_1 \quad\quad s_2 \quad\quad s_3 \\ \begin{pmatrix} 0.35 & 0.1 & 0.55 \end{pmatrix} \end{array}$$

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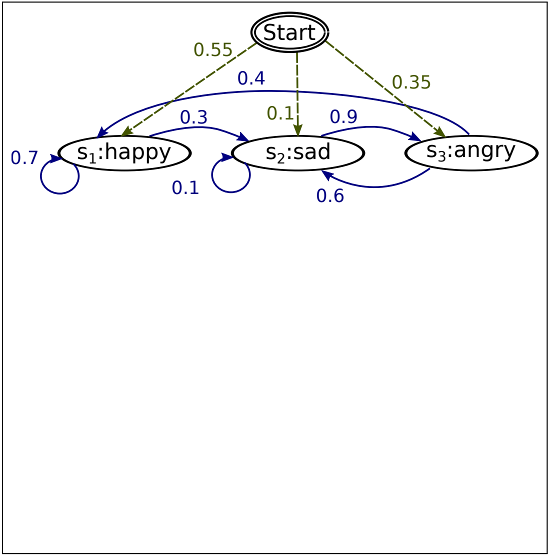

$$A = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{pmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.9 & 0.1 \\ 0.4 & 0.6 & 0 \end{pmatrix} \end{array}$$

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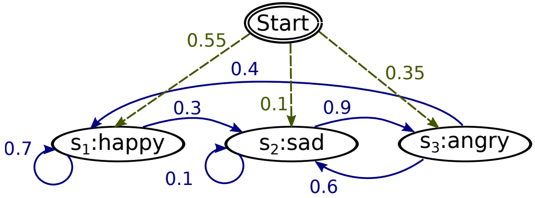

sad → sad →angry →sad → angry → happy

$$A = \begin{array}{cc} & \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.9 & 0.1 \\ 0.4 & 0.6 & 0 \end{pmatrix} \end{array}$$

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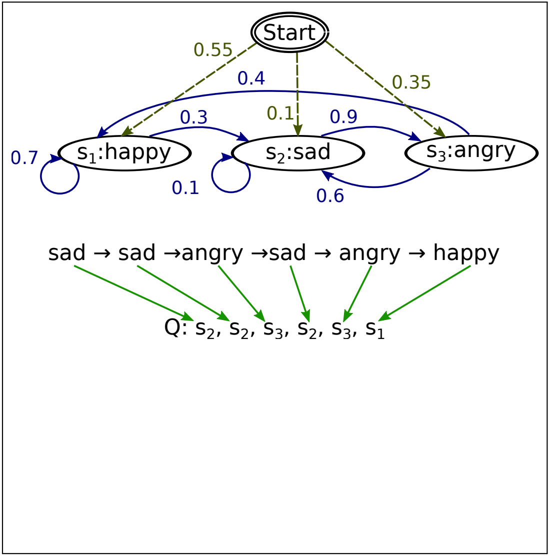

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$$Q = [q_1 q_2 \cdots q_T]$$

$$P(Q|A,\Pi) =$$

$$= \pi_{q_1} a_{q_1,q_2} \cdots a_{q_{T-1},q_T}$$

$$P(s_2, s_2, s_3, s_2, s_3, s_1|A,\Pi) =$$

$$= \pi_2 \cdot a_{2,2} \cdot a_{2,3} \cdot a_{3,2} \cdot a_{2,3} \cdot a_{3,1}$$

$$= 0.1 \cdot 0.3 \cdot 0.1 \cdot 0.9 \cdot 0.6 \cdot 0.9 \cdot 0.4$$

$$= 0.0005832$$

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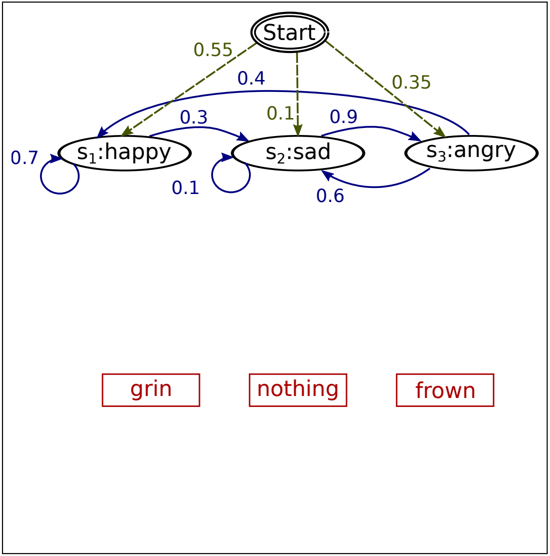

M - number of distinct observable values

M $= 3$

values:

- v_1: grin
- v_2: nothing
- v_3: frown

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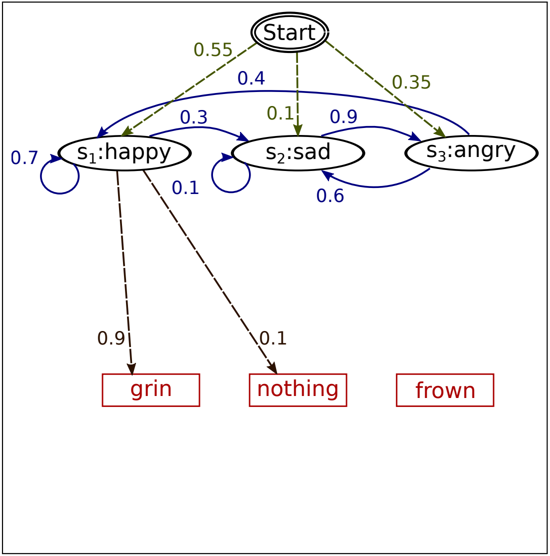

B - observation values probability distribution

$\mathbf{B} = \{b_{j,k}\}_{1 \leq j \leq N, 1 \leq k, \leq M}$

$$b_{j,k} = b_j(v_k)$$
$$= P(o_t = v_k | q_t = s_j)$$

- $b_{\mathbf{1},1} = b_{\mathbf{1}}(grin) = 0.9$
- $b_{\mathbf{1},2} = b_{\mathbf{1}}(nothing) = 0.1$
- $b_{\mathbf{1},3} = b_{\mathbf{1}}(frown) = 0$

$$\sum_{k=1}^{M} b_{j,k} = 1, \quad 1 \leq j \leq N$$

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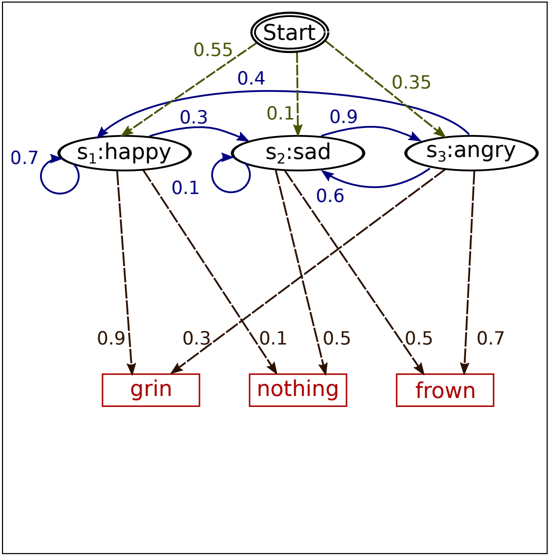

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$$\mathbf{B} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} \begin{array}{ccc} grin & notg & frown \end{array} \\ \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.5 & 0.5 \\ 0.3 & 0 & 0.7 \end{pmatrix} \end{array}$$

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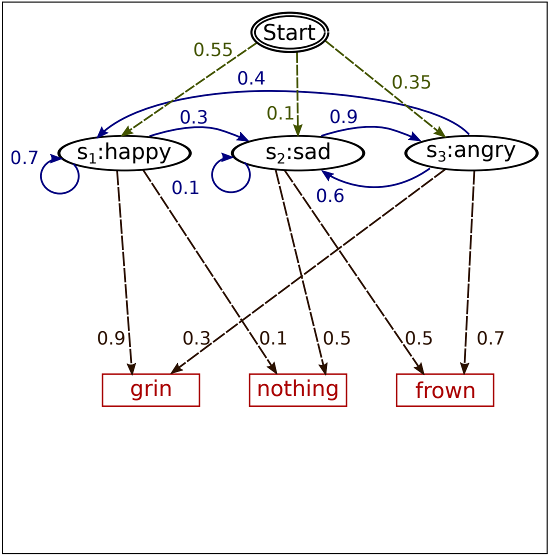

λ - parameters of the model

$\lambda = (A, B, \Pi)$

A - state transition probability distribution

B - observation values probability distribution

Π - initial state distribution

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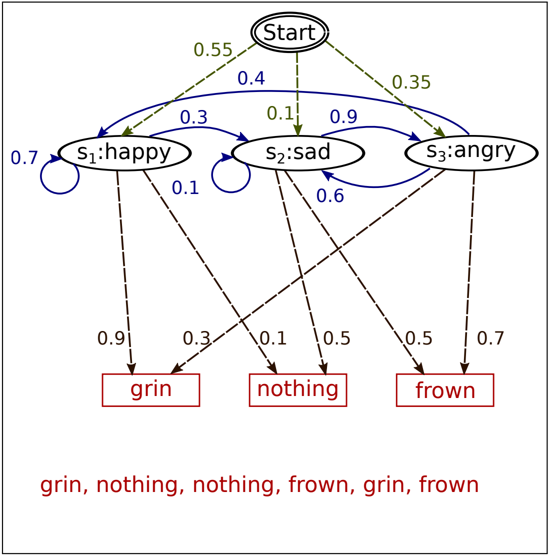

O - observation sequence

T - length of observation sequence

$$O = [o_1 o_2 \cdots o_T]$$

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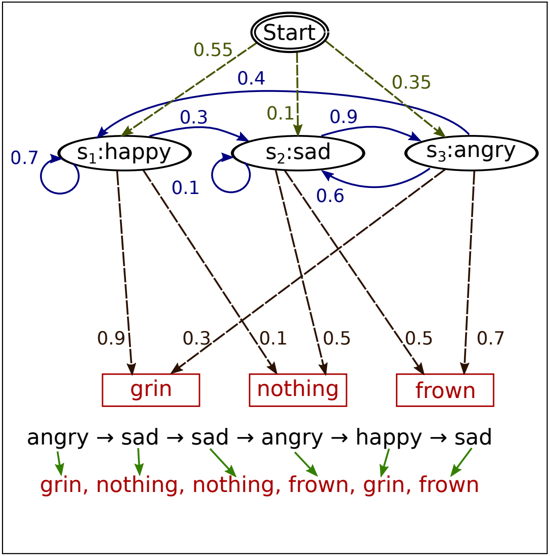

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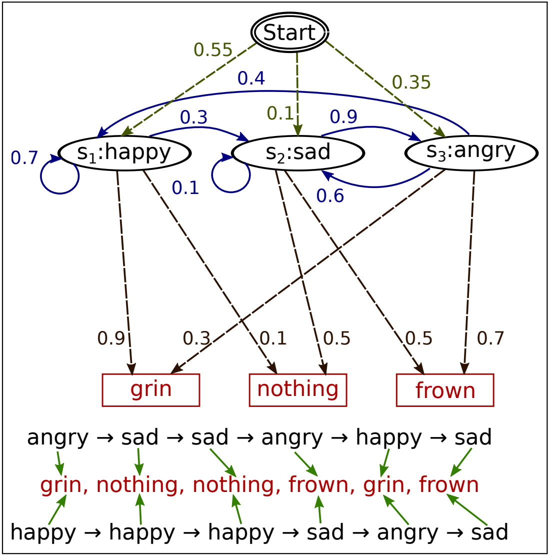

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- Example inspired from:

  R. Zubek (2006). “Introduction to hidden markov models”. In: *AI Game Programming Wisdom* 3, pp. 633–646

Restating the three fundamental HMM Problems

Estimation

... Iuam de la Alex

$$P(O|Q,\lambda) = \prod_{t=1}^{T} P(o_t|q_t,\lambda) = \prod_{t=1}^{T} b_{q_t}(o_t) = b_{q_1}(o_1)\cdot\ldots\cdot b_{q_T}(o_T) \quad (1)$$

$$P(Q|\lambda) = \pi_{q_1}\prod_{t=2}^{T} a_{q_{t-1},q_t} = \pi_{q_1}\cdot a_{q_1,q_2}\cdot a_{q_2,q_3}\cdot\ldots\cdot a_{q_{T-1},q_T} \quad (2)$$

$$\begin{aligned} P(O|\lambda) &= \sum_{all\ Q} P(O,Q|\lambda) = \sum_{all\ Q} P(O,|Q,\lambda)\cdot P(Q,\lambda) \\ &= \sum_{all\ Q}\left(\pi_{q_1}\cdot b_{q_1}(o_1)\cdot\prod_{t=2}^{T} b_{q_t}(o_t)a_{q_{t-1},q_t}\right) \end{aligned} \quad (3)$$