Formule de folosit in slide-uri

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1 De început

- Markov assumption

$$P(q_t = s_j | q_{t-1} = s_i, q_{t-2} = s_k, \ldots) = P(q_t = S_j | q_{t-1} = s_i) \\ 1 \leq i, j \leq N \tag{1}$$

- Time independence

$$a_{i,j} = P(q_t = s_j | q_{t-1} = s_i) \tag{2}$$

$$a_{i,j} \geq 0 \tag{3}$$

$$\sum_{j=1}^{N} a_{i,j} = 1, \quad 1 \leq i \leq N \tag{4}$$

2 Parametrii modelului

- structure

$$\lambda = (A, B, \Pi) \tag{5}$$

- pi

$$\Pi = \{\pi_i\}, \quad 1 \leq i \leq N \tag{6}$$

$$\pi_i = P(q_1 = s_i) \tag{7}$$

- $A = \{a_{i,j}\}, \quad 1 \leq i, j \leq N$

$$a_{i,j} = P(q_{t+1} = s_j | q_t = s_i), \quad 1 \leq i, j \leq N \tag{8}$$

- $B = \{b_{j,k}\}, \ {}_{\substack{1 \leq j \leq N \\ 1 \leq l \leq M}}$

$$b_{j,k} = b_j(v_k) = P(o_t = v_k | q_t = s_j), \quad 1 \leq j \leq N, 1 \leq k, \leq M \tag{9}$$

- observațiile O

$$O = [o_1 o_2 \cdots o_T] \tag{10}$$

- variabilele de stare Q

$$Q = [q_1 q_2 \cdots q_T] \tag{11}$$

- stările $s_1, s_2, \ldots, s_N$

3 Niște probabilități

- Probabilitatea unei secvențe observate, dat fiind λ

$$P(O|Q,\lambda) = \prod_{t=1}^{T} P(o_t|q_t,\lambda) \tag{12}$$

4 Forward-Backward

4.1 Forward variables

- alpha definition

$$\alpha_{t,i} = P(o_1, o_2, \ldots, o_t, q_t = S_i|\lambda) \tag{13}$$

- alpha initialization

$$\alpha_{1,i} = \pi_i b_i(o_1), \quad 1 \le i \le N \tag{14}$$

- alpha induction

$$\alpha_{t+1,j} = \Big[\sum_{i=1}^{N} \alpha_{t,j} a_{i,j}\Big] b_j(o_{t+1}), \quad {\scriptstyle 1 \le t \le T-1, \atop 1 \le j \le N} \tag{15}$$

- alpha termination

$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_{T,i} \tag{16}$$

4.2 Backward variables

- beta (backward variable)

$$\beta_{t,i} = P(o_{t+1} o_{t+2} \cdots o_T | q_t = S_i, \lambda) \tag{17}$$

- beta initialization

$$\beta_{T,i} = 1, \quad 1 \le i \le N \tag{18}$$

- beta induction

$$\beta_{t,i} = \sum_{j=1}^{N} a_{i,j} b_j(o_{t+1}) \beta_{t+1,j}, \quad t = T-1, T-2, \ldots, 1, 1 \le i \le N \tag{19}$$

5 Viterbi

- Gamma

$$\gamma_{t,i} = P(q_t = s_i | O, \lambda) \tag{20}$$

$$\gamma_{t,i} = \frac{\alpha_{t,i}\beta_{t,i}}{P(O|\lambda)} = \frac{\alpha_{t,i}\beta_{t,i}}{\sum_{k=1}^{N} \alpha_{t,k}\beta_{t,k}} \tag{21}$$

- delta

$$\delta_{t+1,j} = [\max_i \delta_{t,i} \cdot a_{ij}] \cdot b_j(o_{t+1}) \tag{22}$$

6 Bau-Bau-Baum-Welch

- ksi definition

$$\xi_{t,i,j} = P(q_t = s_i, q_{t+1} = s_j | O, \lambda) \tag{23}$$

- ksi computation

$$\begin{aligned} \xi + t, i, j &= \frac{\alpha_{t,i} \cdot a_{i,j} \cdot b_j(o_{t+1}) \cdot \beta_{t+1,j}}{P(O|\lambda)} \\ &= \frac{\alpha_{t,i} \cdot a_{i,j} \cdot b_j(o_{t+1}) \cdot \beta_{t+1,j}}{\sum_{k=1}^{N} \sum_{l=1}^{N} \alpha_{t,k} \cdot a_{k,l} \cdot b_l(o_{t+1}) \cdot \beta_{t+1,l}} \end{aligned} \tag{24}$$