

NONLINEAR PLS METHOD FOR SUBCRITICAL FLOWS OVER SHARP-CRESTED RECTANGULAR SIDE WEIRS

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ABSTRACT

Side weirs are widely used for level control and flow regulation in hydraulic engineering applications such as irrigation, land drainage, and sewer systems. These hydraulic structures allow a part of the flow to spill laterally when the surface of the flow in the main channel rises above the weir crest. In fact, they can be served for adjustment and diverting of flow with minimum energy loss. Previous investigators have extensively studied the hydraulic performance of sharp-crested rectangular side weirs and introduced different linear and non-linear equations for estimation of their De Marchi coefficient of discharge C_M in subcritical flow. In this study, based on over 178 experimental data available from previous investigators and through using the multivariable nonlinear partial least square (PLS) method, a new accurate nonlinear equation is presented for discharge coefficient C_M of sharp-crested rectangular side weirs. The obtained empirical equation relating C_M with the relevant geometric and hydraulic dimensionless parameters Fr_1 , p/h_1 , L/B in a rectangular open channel. Comparison between results of the new presented equation and the measured data shows that the new presented empirical equation can predict the discharge of diverted flow over side weirs with higher accuracy.

Keywords: Side weir; Rectangular weir; Discharge coefficient; PLS method.

1 INTRODUCTION

It is likely that the flow discharge exceeds the capacity of a channel or river and, thus, control structures such as side weir, should be employed to protect the system from overflowing. Usually side weirs, with different geometries, are installed in the channel side wall, parallel to the flow direction and at a desired height so that when the water level rises to the weir height, some portion of the flow would be deviated laterally over the side weir.

According to the themes of the succeeding investigations on side weirs, the studies can be classified into four major categories. A number of former studies have focused on identifying the side weirs' discharge coefficient with sub- and/or supercritical approach flow (Jalili and Borghei 1996, Borghei et al. 1999, Azimi et al. 2014 and Crispino et al. 2015). The second group have focused on a progressing subject, e.g. increasing the efficiency of the side weirs, by applying the modified shapes including oblique, triangular, trapezoidal, elliptical and labyrinth side weirs (Parvaneh and Borghei 2009; Parvaneh et al. 2010; Borghei and Parvaneh 2011; Borghei et al. 2013; Emiroglu et al. 2014; Nezami et al. 2015; Aydin and Emiroglu 2016; Aydin and Kayisli 2016; Parvaneh et al. 2016; and Parvaneh et al. 2017). Some others have commonly focused on the effects of the main channel cross-section (e.g. trapezoidal, circular, parabolic or triangular channels) on the efficiency of the side weirs of different geometries (Cheong 1991; Uyumaz 1992; Vatankhah 2012; Vatankhah 2013; and Azimi and Shabanlou 2015). More recent studies have simulated/predicted the side weir discharge coefficient applying computational fluid dynamics and soft computing methods (Emiroglu et al. 2010; Aydin 2012; Vatankhah 2012; Aydin 2015; Parsaie and Haghiabi 2015; and Zaji et al. 2016).

Most of the aforementioned studies were subjected to determine the discharge coefficient of side weirs. In practice, the side weir discharge coefficient is a major factor to be considered. For a given opening length along the side-wall of the main channel, applying the modified shapes such as; triangular and asymmetric labyrinth side weirs results in the increase of the discharge coefficient. Former studies indicated that the angle of flow diversion due to the use of oblique side weirs γ and vertex angle of triangular and trapezoidal side weirs δ along with the upstream approach flow Froude number Fr_1 significantly affect the side weir discharge coefficient (Ura et al. 2001; Parvaneh et al. 2011; Parvaneh et al. 2012; Borghei et al. 2013; Emiroglu et al. 2014; Parvaneh et al. 2016). Until quite recently, approximate methods based on experiments conducted over a limited range of

the many variables involved have been used. In many cases, the use of such methods caused very substantial errors in the calculated spill discharge. Partial least square method PLS, artificial neural networks ANN, fuzzy logic FL and adaptive neuro-fuzzy inference system ANFIS methods are recently widely used for determination of side weir flow discharge coefficient (Ramamurthy et al. 2006; Aydin 2015; and Bonakdari et al. 2015). Borghei and Parvaneh (2011), and Borghei et al. (2013) showed that efficiency of the labyrinth side weirs could be improved by applying a modified shape of oblique-triangular labyrinth side weir. The main objective of the present study is to develop a precise semi-analytical approach for determination of the discharge coefficient of sharp-crested rectangular side weirs in subcritical flow using nonlinear PLS method.

Borghei et al. (1999) thoroughly studied the hydraulic characteristics of sharp-crested rectangular side weirs (Figure 1). They conducted over 178 experimental tests in subcritical flow (Table 1) and to analyze the results, they applied the spatially varied flow approach. Borghei et al. (1999) introduced Eq. [1] for estimation of the De Marchi coefficient of discharge C_M for sharp-crested rectangular side weirs.

$$C_M = 0.7 - 0.48(Fr_1) - 0.3\left(\frac{p}{h_1}\right) + 0.06\left(\frac{L}{B}\right) \quad [1]$$

Where C_M = De Marchi coefficient of discharge (-); Fr_1 = Froude number at the upstream end (-), p = weir height (m); h_1 = flow depth at the upstream end of the side weir (m); L = opening length of side weir (m); and B = channel width (m).

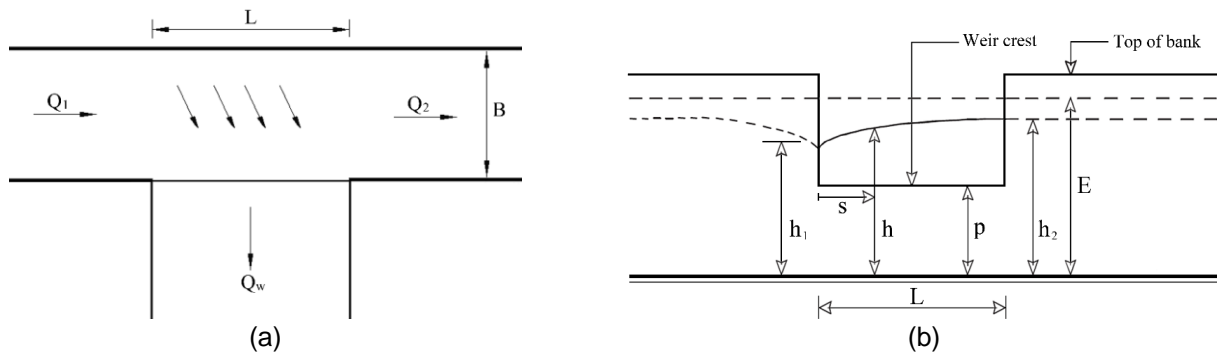


Figure 1. Schematic view of a sharp-crested rectangular side weir; a) plan, b) section.

The current study aims to contribute to the literature by introducing a new equation for discharge coefficient C_M of sharp-crested rectangular side weirs through using the multivariable nonlinear partial least square (PLS) method. Finally the accuracy of presented equation will be evaluated using the experimental data and corresponding equations in the literature.

Table 1. Range of the test variables

L (cm)	p (cm)	S_0 (%)	p/h_1 (-)	Q_1 (L/s)	Fr_1 (-)	Number of runs
20	1	-0.50, 0.00, 0.50, 1.25, 2.50	0.026 - 0.056	44 - 83	0.34 - 0.63	18
	10	-0.50, 0.00, 0.50, 1.25, 2.50	0.229 - 0.648	44 - 89	0.22 - 0.80	29
	19	-0.50, 0.00, 0.50, 1.25, -	0.440 - 0.826	37 - 97	0.19 - 0.74	21
30	1	-0.50, 0.00, 0.50, 1.25, -	0.034 - 0.050	45 - 84	0.52 - 0.58	12
	10	-0.50, 0.00, 0.50, 1.25, 2.50	0.261 - 0.608	44 - 91	0.29 - 0.77	28
	19	-0.50, 0.00, 0.50, 1.25, 2.50	0.388 - 0.847	44 - 91	0.20 - 0.75	17
45	10	-0.50, 0.00, 0.50, 1.25, 2.50	0.304 - 0.641	45 - 84	0.39 - 0.78	16
75	10	-0.50, 0.00, 0.50, - , -	0.399 - 0.500	46 - 80	0.52 - 0.79	8
	19	-0.50, 0.00, 0.50, 1.25, 2.50	0.528 - 0.868	44 - 91	0.29 - 0.76	29

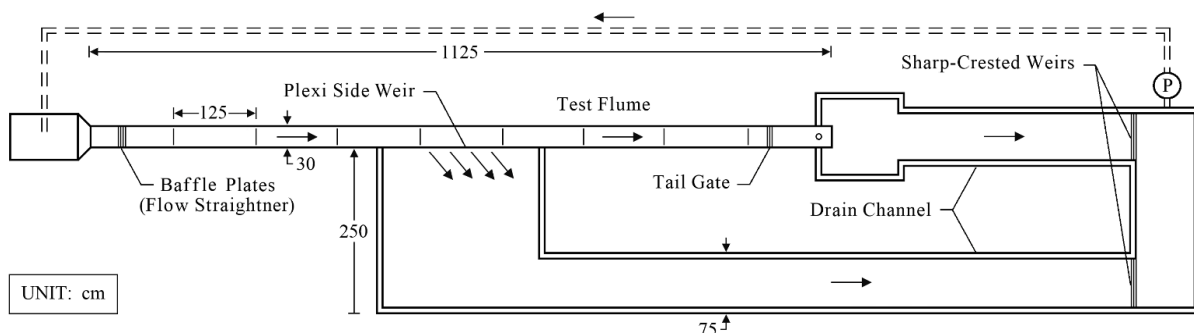


Figure 2. Schematic view of the experimental setup

2 DATA ANALYSIS

Considering Figure 1, the De-Marchi approach, by assuming constant energy in the main channel, along the side weir length, leads to the following equations:

$$s = \frac{3B}{2C_M} \Phi(h, E, p) + \text{Const.} \quad [2]$$

In which $\Phi(h, E, p)$ is;

$$\Phi(h, E, p) = \frac{2E - 3p}{E - p} \sqrt{\frac{E - h}{h - p}} - 3 \sin^{-1} \sqrt{\frac{E - h}{E - p}} \quad [3]$$

Where " s ", " B ", " p " and " h " are the flow direction, channel width, side weir height and water depth respectively as showed in Figure 1. " E " is the specific energy and " C_M " is the discharge coefficient of the side weir. Thus, the relation between C_M and other hydraulic variables of the flow would be;

$$C_M = \frac{3B}{2L} (\Phi_2 - \Phi_1) \quad [4]$$

Where, Φ_1 and Φ_2 are accounted for immediately upstream and downstream of the weir respectively, and C_M has to be found experimentally.

3 APPLICATION OF PLS METHOD

Partial least square (PLS) is a robust method to estimate and fit multivariable statistical data. In effect, PLS is used to determine a dependent variable in terms of independent variables for nonlinear problems. Ramamurthy et al. (2006) used this method in hydraulic engineering for the first time. As this method has received less attention in hydraulic engineering practice, in the present study, this method is employed to estimate discharge coefficient C_M for sharp-crested rectangular side weirs in terms of relevant dimensionless parameters.

Based on the restudy of the existing data (Borghesi et al. 1999), presented in Table. 1, the dependent variable C_M for rectangular side weirs to be a function of several independent variables (Fr_1 , p/h_1 , L/B therefore:

$$C_M = g_1(Fr_1) \times g_2\left(\frac{p}{h_1}\right) \times g_3\left(\frac{L}{B}\right) = \prod_{i=1}^3 g_i(x_i) \quad [5]$$

$$g_i(x_i) = a_{i0} + a_{i1}x_i + a_{i2}x_i^2 + a_{i3}x_i^3 + a_{i4}x_i^4 = \sum_{j=0}^4 a_{ij} x_i^j \quad [6]$$

$$g_1(Fr_1) = a_{10} + a_{11}(Fr_1) + a_{12}(Fr_1)^2 + a_{13}(Fr_1)^3 + a_{14}(Fr_1)^4 \quad [7]$$

$$g_2\left(\frac{p}{h_1}\right) = a_{20} + a_{21}\left(\frac{p}{h_1}\right) + a_{22}\left(\frac{p}{h_1}\right)^2 + a_{23}\left(\frac{p}{h_1}\right)^3 + a_{24}\left(\frac{p}{h_1}\right)^4 \quad [8]$$

$$g_3\left(\frac{L}{B}\right) = a_{30} + a_{31}\left(\frac{L}{B}\right) + a_{32}\left(\frac{L}{B}\right)^2 + a_{33}\left(\frac{L}{B}\right)^3 + a_{34}\left(\frac{L}{B}\right)^4 \quad [9]$$

$$C_M = \prod_{i=1}^3 \left(\sum_{j=0}^4 a_{ij} x_i^j \right) \quad [10]$$

And SSE can be defined as

$$\delta^2 = \sum_{k=1}^s (C_M - C_{Mk})^2 \quad [11]$$

Through using the least square method and taking derivative with respect to a_{ij} , we may have:

$$\frac{\partial}{\partial a_{ij}} \left[\sum_{k=1}^s (C_M - C_{Mk})^2 \right] = 0 \quad [12]$$

his equation can be reduced to Eq. [13].

$$\sum_{k=1}^s \left[(C_M - C_{Mk}) \frac{\partial C_M}{\partial a_{ij}} \right] = 0 \quad [13]$$

By using Eqs. [5] and [6]:

$$\frac{\partial C_M}{\partial a_{tj}} = \prod_{\substack{i=1 \\ i \neq t}}^3 g_i \frac{\partial g_t}{\partial a_{tj}} = \prod_{\substack{i=1 \\ i \neq t}}^3 g_i x_t^j \quad (j = 0, 1, 2, 3, 4) \quad [14]$$

Substituting Eq. [14] in Eq. [13] yields:

$$\sum_{k=1}^s \left[(C_M - C_{Mk}) \left(\prod_{\substack{i=1 \\ i \neq t}}^3 g_i x_t^j \right) \right] = 0 \quad [15]$$

Using Eqs. [5] and [6] in Eq. [7] and simplifying Eq. [15], Eq. [16] is obtained:

$$a_{ij} = \frac{\sum_{k=1}^s \left\{ (C_{Mk}) \left[\left(\prod_{\substack{i=1 \\ i \neq t}}^3 g_i \right) x_t^j \right]_k - \left[\left(\prod_{\substack{i=1 \\ i \neq t}}^3 g_i \right)^2 \left(\prod_{\substack{p=0 \\ p \neq j}}^4 a_{tp} x_t^p \right) x_t^j \right]_k \right\}}{\sum_{k=1}^s \left[\left(\prod_{\substack{i=1 \\ i \neq t}}^3 g_i \right) x_t^j \right]_k^2} \quad [16]$$

Where $t=1, 2, 3$; $j=0, 1, 2, 3, 4$.

Solving Eq. [16] iteratively, the constants a_{tj} can be obtained. Hence, the relationship among the variables is established. Using Eqs. [7], [8], and [9] in Eq. [5] and ignoring the constants a_{tj} with negligible values, Eq. [17] is obtained for predicting the discharge coefficient C_M of sharp-crested rectangular side weirs.

$$C_M = [0.664 - 0.241(Fr_1) - 0.096(Fr_1)^2 - 0.071(Fr_1)^3 - 0.309(Fr_1)^4] \times [14.379 - 6.509(p/h_1) + 0.959(p/h_1)^2 - 0.478(p/h_1)^3 - 4.689(p/h_1)^4] \times [-0.646 + 2.530(L/B) - 3.121(L/B)^2 + 1.590(L/B)^3 - 0.280(L/B)^4] \quad [17]$$

Figure 3 shows the comparison of calculated discharge coefficients C_M for sharp-crested rectangular side weirs using Eq. [17], versus the measured data. In Figure 3b most of the data are within the range of $\pm 7\%$ with NRMSE of 0.3690 for sharp-crested rectangular side weirs. Table 2 shows a better comparison between the accuracy of Eq. [17] with Eq. [1] in predicting the discharge coefficient C_M for rectangular side weirs.

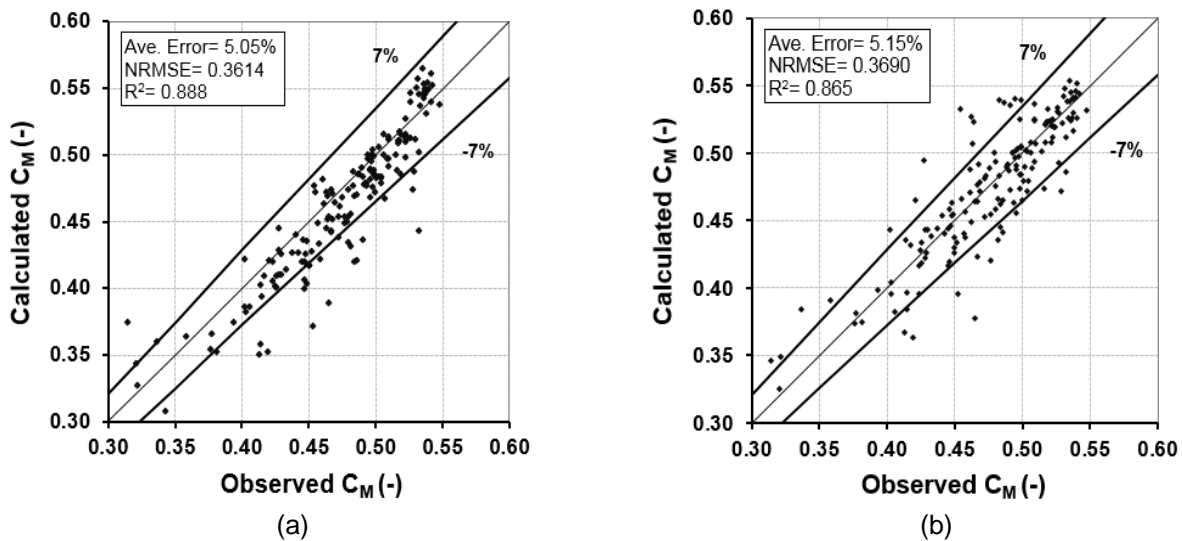


Figure 3. Comparison of measured C_M versus calculated C_M using; a) Eq. [1], b) Eq. [17]

Table 2. The accuracy of Eq. [17], compared with the presented linear equation by Borghei et al. (1999), Eq. [1], for sharp-crested rectangular side weirs.

Accuracy	Eq. [1]	Eq. [17]
Ave. Error	5.05%	5.15%
NRMSE	0.3614	0.3690
R^2	0.888	0.865

4 CONCLUSION

This study presents the relation between the discharge coefficient C_M of a sharp-crested rectangular side weir and its geometric and hydraulic dimensionless parameters in a rectangular open channel (Fr_1 , p/h_1 , L/B) on the basis of the multivariable nonlinear PLS method. The experimental results from over 178 laboratory tests are utilized to extract the De-Marchi discharge coefficient for the studied rectangular side weir. The non-linear Eq. [17] with NRMSE= 0.3690, and $R^2= 0.865$, is introduced to calculate the discharge coefficient of sharp-crested rectangular side weirs. In comparison with the previous equations, the predicted results for the rectangular side weir discharge coefficient show an acceptable agreement with the experimental results. The nonlinear PLS method can also be applied to many other hydraulic cases characterized by a large number of variables.

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NOTATION

The following symbols are used in the present study:

- a_{ij} = constants
- B = width of main channel
- C_M = discharge coefficient for side weir
- E = specific energy
- f = dependent variables
- Fr = Froude number in the channel
- g = acceleration due to gravity
- g_i = polynomials
- h = flow depth at section s
- L = length (width) of side weir
- p = weir height
- Q = discharge in the main channel
- Q_w = discharge over side weir
- q = discharge over side weir per unit length
- s = distance from beginning of side weir
- V = velocity in main channel
- x_i = independent variables

Subscripts

- 1 upstream condition
- 2 downstream condition

REFERENCES

- Aydin, M. C. (2012). "CFD simulation of free-surface flow over triangular labyrinth side weir." *Advances in Engineering Software*, 45(1), 159–166.
- Azimi, H., and Shabanlou, S. (2015). "The flow pattern in triangular channels along the side weir for subcritical flow regime." *Flow Meas. Instrum.*, 46, 170–178.
- Borghei, S., Jalili, M., and Ghodsian, M. (1999). "Discharge Coefficient for Sharp-Crested Side Weir in Subcritical Flow." *J. Hydraul. Eng.*, 125(10), 1051–1056.
- Borghei, S. M., Nekooie, M. A., Sadeghian, H., and Jalili, M. R. (2013). "Triangular labyrinth side weirs with one and two cycles." *Proc. Inst. Civ. Eng., Water Manage.*, 166(1), 27–42.
- Borghei, S. M., and Parvaneh, A. (2011). "Discharge characteristics of a modified oblique side weir in subcritical flow." *Flow Meas. Instrum.*, 22(5), 370–376.
- Cheong, H.-F. (1991). "Discharge coefficient of lateral diversion from trapezoidal channel." *J. Irrig. Drain. Eng.*, 117(4), 461–475.
- Crispino, G., Cozzolino, L., Della Morte, R., and Gisonni, C. (2015). "Supercritical low-crested bilateral weirs: Hydraulics and design procedure." *Journal of Applied Water Engineering and Research*, 3(1), 35–42.

- Emiroglu, M. E., Kisi, O., and Bilhan, O. (2010). "Predicting discharge capacity of triangular labyrinth side weir located on a straight channel by using an adaptive neuro-fuzzy technique." *Advances in Engineering Software*, 41(2), 154–160.
- Emiroglu, M. E., Aydin, M. C., and Kaya, N. (2014). "Discharge characteristics of a trapezoidal labyrinth side weir with one and two cycles in subcritical flow." *J. Irrig. Drain. Eng.*, 140(5), 1–13.
- Gonzalez, J., Rojas, H., Ortega, J., and Prieto, A. (2002). "A new clustering technique for function approximation." *IEEE Trans. Neural Netw.*, 13(1), 132–142.
- Jalili, M. and Borghei, S. (1996). "Discussion: Discharge Coefficient of Rectangular Side Weirs." *J. Irrig. Drain. Eng.*, 122(2), 132–132.
- Nezami, F., Farsadizadeh, D., and Nekooie, M. A. (2015) "Discharge coefficient for trapezoidal side weir." *Alexandria Engineering Journal*, 54(3), 595–605.
- Parvaneh, A., and Borghei, S. M. (2009). "Oblique side weir." *Proc. 33rd IAHR-ASCE Congress*, 9-14 August. University of British Columbia, Vancouver, Canada. Press, No.11310, pp. 1046-1053.
- Parvaneh, A., Borghei, S. M., and Jalili, M. R. (2010). "Determination of discharge coefficient for oblique side weirs using the Partial Least Square method." *Proc. ISHPF 2010 Congress*, 13-15 September. Nanjing Hydraulic Research Institute, Nanjing, China. Press, Volume 2, pp. 171-177.
- Parvaneh, A., Borghei, S. M., and Jalili Ghazizadeh, M. R. (2011). "Discussion of discharge capacity of labyrinth side weir located on a straight channel." *Journal of Irrigation and Drainage Engineering*, 137(11), 743–745.
- Parvaneh, A., Borghei, S., and Jalili Ghazizadeh, M. R. (2012). "Hydraulic performance of asymmetric labyrinth side weirs located on a straight channel." *Journal of Irrigation and Drainage Engineering*, 138(8), 766–772.
- Parvaneh, A., Kabiri-Samani, A., and Nekooie, M. A. (2016). "Discharge coefficient of triangular and asymmetric labyrinth side weirs using the nonlinear PLS method." *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)IR.1943-4774.0001081.
- Parvaneh, A., Kabiri-Samani, A., and Parvaneh, M. (2017). "Discussion of prediction of discharge capacity over two-cycle labyrinth side weir using ANFIS" *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)IR.1943-4774.0001137.
- Parvaneh, A., Yang, J., Javaheri, A., and Kabiri-Samani, A. R. (2016). "Discussion: Triangular labyrinth side weirs with one and two cycles" *Proc. Inst. Civ. Eng. Water Manage.*, 10.1680/jwama.15.00032.
- Ramamurthy, A. S., Qu, J., and Vo, D. (2006). "Nonlinear pls method for side weir flows." *J. Irrig. Drain. Eng.*, 132(5), 486–489.
- Ura, M., Kita, Y., Akiyama, J., Moriyama, H., and Kumar, J. A. (2001). "Discharge coefficient of oblique side-weirs." *J. Hydrosoci. Hydraul. Eng.*, 19(1), 85–96.
- Uyumaz, A. (1992). "Side weir in triangular channel." *J. Irrig. Drain. Eng.*, 118(6), 965–970.
- Vatankhah, A. R. (2013). "Water surface profile along a side weir in a parabolic channel." *Flow Meas. Instrum.*, 32, 90–95.