Optimization Issues

Starting with the following,

$$\Psi(\lambda, h, g) = \frac{1}{x_{max} - x_{min}}(\sum_x [\ln \frac{1}{1-q_x} - \frac{h}{g} e^{gx}(e^g - 1) - \lambda])^2$$

For simplicity, Set

$$R = x_{max} - x_{min} \ , \quad Q = \sum_x \ln \frac{1}{1-q_x} \ , \quad H = \frac{h}{g}(e^g - 1)$$

So,

$$\Psi(\lambda, h, g) = \frac{1}{R}(Q - R\lambda - H \sum_x e^{gx})^2$$

Note that λ here has minimal interaction with any of the obesrvations. It's actually quite trivial to show that to minimize Ψ on λ we set λ to minimum of its range (i.e. $\lambda \to 0$). This is infact what we get when we run optimization functions in R or visualize Ψ through brute force.

Additionally, if we took the exponent of equation (10) in your paper:

$$\ln(\frac{1}{1-q_x}) = \frac{h}{g}(e^g - 1)e^{gx} + \lambda$$

We get an expression analgous to:

$$y = ae^{bx} + c$$

This represent the model that we would need to solve, and unfourtinately the gradient of this expression at $c = \lambda$ is just 1, making it impossible to optimize.