# A New Feedforward Hybrid Active Noise Control System

Yegui Xiao, *Member, IEEE*, and Jing Wang

***Abstract*—Performance of the conventional broadband active noise control (ANC) system may degrade severely if its primary and reference noise signals contain both wideband and narrowband components simultaneously. In this letter, we propose a new feedforward hybrid ANC system capable of reducing such primary noise signals. First, typical simulation results are provided to show the performance deterioration of the conventional system in the presence of mixture of wideband and narrowband components. Next, a new hybrid ANC system is proposed to tackle the problem. The new system consists of three subsystems, i.e., a sinusoidal noise canceller, a broadband and a narrowband ANC subsystem, which work in harmony. Extensive simulations are conducted to demonstrate the effectiveness of the proposed system.**

***Index Terms*—Broadband active noise control (ANC), FXLMS, hybrid ANC, narrowband ANC.**

## I. Introduction

SINCE the early 1970's, development of active noise control (ANC) systems has been attempted in force and many system structures and adaptive algorithms have been developed, see [1]–[3] and references therein. If one classifies ANC systems in terms of frequency characteristics of the noise signals targeted, there are two types. One is developed for wideband noise reduction. It is called broadband ANC (BANC) system. It may also be used for narrowband noise control under some conditions. The other one is designated particularly for narrowband noise suppression and is called narrowband ANC (NANC) system. It can cancel specific sinusoidal components specified in advance, but is totally powerless if the noise signal is of wideband nature. In addition to these two major feedforward ANC systems, several hybrid ANC systems have also been developed that usually consist of a feedforward and a feedback subsystem [1], [4]. The feedforward subsystem takes care of the wideband noise, while the feedback one focuses on reduction of periodic or narrowband noise which is uncorrelated with the reference noise and only resides in the primary noise. See [4] and references therein for details. We focus on the feedforward BANC in this work.

A conventional BANC system is depicted in Fig. 1, which is equivalent in structure to the ANC system shown in Fig. 4 of [1]

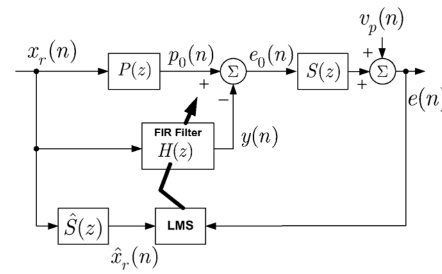


Fig. 1. Conventional broadband ANC system.

where the primary path contains the secondary path and the filtered-x LMS (FXLMS) algorithm is applied. It works quite well if the reference signal $x_r(n)$ only contains wideband noise and the FIR controller $H(z)$ is longer than the primary path $P(z)$ from the reference sensor to the cancelling speaker. However, in many real-life ANC applications to rotating machines such as factory cutters, fans, motors etc., the noise signal to be targeted has both wideband and narrowband (sinusoidal) components. Namely, one or more sinusoids usually exist in both the reference and primary noise signals. The frequencies of which may be calculated from a synchronization (Sync) signal acquired by a timing signal sensor such as tachometer [1]. These frequencies may become very low under some operational conditions. For such applications, the NANC does not have adequate power, as it only takes care of the sinusoidal components determined in advance, leaving the wideband component untouched. The BANC system in Fig. 1 may also lose its power for such applications, as it was originally designed to suppress wideband noise. To the best of our knowledge, the afore-mentioned scenario has almost not been investigated and discussed so far. Our numerous simulations have revealed that performance of the system in Fig. 1 may degrade in a serious way when the targeted noise consists of both wideband noise and low-frequency sinusoid(s).

To mitigate the performance deterioration, one may intuitively think about putting the conventional broadband and narrowband ANC systems together in a parallel form. We tried this and other ideas from different angles. Unfortunately, they did not work. This is because the BANC also attempts to target the sinusoids regardless of the fact that the NANC can do much better than it does in dealing with the sinusoids. Therefore, we need to design a new system structure on the basis of the conventional system. After a long trial-and-error process, we finally found a new ANC system with three subsystems. Our idea is to first introduce a sinusoidal noise canceller (SNC) as a subsystem to remove the sinusoids from the reference signal to let the BANC subsystem do what it is good at, and then place an NANC subsystem parallel to the broadband one to cancel the sinusoids in the primary noise. This way, both ANC

Manuscript received March 20, 2011; revised July 30, 2011; accepted August 02, 2011. Date of publication August 08, 2011; date of current version August 22, 2011. This work was supported in part by the National Natural Science Foundation of China (70872009). The associate editor coordinating the review of this manuscript and approving it for publications was Dr. Rudolf Rabenstein.

Y. Xiao is with the Department of Management and Information Systems, Prefectural University of Hiroshima, Hiroshima, Japan (e-mail: xiao@pu-hiroshima.ac.jp).

J. Wang is with School of Economics and Management, BeiHang University, Beijing, China (e-mail: pekwjing@public.bta.net.cn).

Digital Object Identifier 10.1109/LSP.2011.2164067

subsystems can work in harmony to achieve a common goal. This idea is simple, but has been found very effective.

Compared with the conventional ANC, the new system obviously costs more as both the SNC and the NANC subsystems are newly added. Since both the NANC and the SNC subsystems are updated by the LMS algorithm, the cost increase in our new system is expected to be moderate and will not skyrocket to a forbidden level. It is the sacrifice we have to make to benefit from its performance advantages.

Extensive simulations for many scenarios show that the new hybrid system, as anticipated, works as well as the conventional system does, even though both wideband and narrowband noise components reside in both the reference and primary noise signals.

## II. Conventional ANC System and Its Performance

In the conventional BANC system [1] of Fig. 1, the primary path, secondary path and its estimate are respectively expressed by

$$P(z) = \sum_{j=1}^{M_p} c_j z^{-j} \tag{1}$$

$$S(z) = \sum_{m=1}^{M} s_m z^{-m} \tag{2}$$

$$\hat{S}(z) = \sum_{m=1}^{\hat{M}} \hat{s}_m z^{-m} \tag{3}$$

where $M_p, M, \hat{M}$ are FIR model orders, and $c_j, s_m, \hat{s}_m$ are model coefficients, respectively. The reference signal is given by

$$\begin{aligned} x_r(n) &= x_f(n) + x_w(n) \\ &= \sum_{i=1}^{q} [a_{r,i} x_{a_i}(n) + b_{r,i} x_{b_i}(n)] + x_w(n) \end{aligned} \tag{4}$$

$$x_{a_i}(n) = \cos(\omega_i n), \quad x_{b_i}(n) = \sin(\omega_i n) \tag{5}$$

where $x_f(n)$ and $x_w(n)$ are narrowband and wideband components, respectively. $x_f(n)$ has $q$ frequencies $\{\omega_i\}_{i=1}^{q}$ that are usually derived from a synchronization signal, and $\{a_{r,i}, b_{r,i}\}_{i=1}^{q}$ are discrete Fourier coefficients (DFC). $x_w(n)$ is a zero-mean white or colored noise with variance $\sigma_w^2$. The primary noise is expressed by

$$p_0(n) = \sum_{i=1}^{q} [a_{p,i} x_{a_i}(n) + b_{p,i} x_{b_i}(n)] + p_w(n) \tag{6}$$

where $\{a_{p,i}, b_{p,i}\}_{i=1}^{q}$ are the DFCs of sinusoids due to $x_f(n)$, and $p_w(n)$ is the wideband primary noise component due to $x_w(n)$. The FXLMS algorithm is used to update the control filter weights $\{h_j(n)\}_{j=0}^{L-1}$, as follows:

$$h_j(n+1) = h_j(n) + \mu_h e(n) \hat{x}_r(n-j) \tag{7}$$

$$e(n) = \sum_{m=0}^{M-1} s_m e_0(n-m) + v_p(n) \tag{8}$$

$$e_0(n) = p_0(n) - \sum_{j=0}^{L-1} h_j(n) x_r(n-j) \tag{9}$$

where $\mu_h$ is a positive step size, $\hat{x}_r(n)$ is the reference signal filtered by the secondary-path estimate $\hat{S}(z)$, $v_p(n)$ is an additive white noise with zero-mean and variance $\sigma_p^2$ that models all noise elements that are independent of $x_r(n)$.

This conventional system usually works very well if the primary noise contains white or colored wideband noise only. However, if the primary noise is a mixture of a wideband noise and some low-frequency sinusoids, the system may work very poorly. Here, typical simulation results are provided below to show how the system behaves in such a situation.

In the simulations, the primary and secondary paths, $P(z)$ and $S(z)$, are all lowpass filters with a cutoff frequency of $0.4\pi$. Their orders are 41 ($M_p$) and 21 ($M$), respectively. The estimate of the secondary path, $\hat{S}(z)$, is obtained by using the classical system identification configuration and the LMS algorithm, where model order $\hat{M} = 31$, step size $\mu_s = 0.001$, input white noise variance $\sigma^2 = 1$, observation additive noise variance $\sigma_{v_s}^2 = 0.1$, and iteration number $N_s = 10000$ are adopted. Three low-frequency sinusoids are included in $x_r(n)$ and $p_0(n)$. Their frequencies are $0.03\pi, 0.06\pi, 0.09\pi$. Their DFCs are $a_{r,1} = 2.0$, $a_{r,2} = 1.0$, $a_{r,3} = 0.5$, $b_{r,1} = -1.0$, $b_{r,2} = -0.5$, and $b_{r,3} = 0.1$. The variance of white noise $x_w(n)$ is 1.0. The variance of $v_p(n)$ is 0.1. Two cases are simulated; (a) $x_r(n) = x_w(n)$, and (b) $x_r(n) = x_w(n) + x_f(n)$. For comparison fairness, the reference signal power of (a) is adjusted to be the same as that of (b). Forty (40) independent runs are performed for each case. Two comparisons are provided here.

In Fig. 2, the controller $H(z)$ is an FIR filter with order $L = 51(> M_p(= 41))$ and the step size of the FXLMS is set to $\mu_h = 0.0007$ such that the system indicates relatively fast convergence. Fig. 3 shows the results for an FIR filter with order $L = 21(< M_p(= 41))$ and a step size $\mu_h = 0.0018$.

From Figs. 2, 3, and many other simulations for different settings, we have obtained the following insightful facts.

1) Low-frequency sinusoids in the reference noise signal generate spike-like impulses in the residual noise $e(n)$, if the system convergence is set relatively fast, even for sinusoids with larger frequencies such as $0.1\pi, 0.2\pi, 0.3\pi$. We call them "firework" noise as they sound like fireworks. This phenomenon appears for FIR control filters with any order that is either larger or smaller than that of the primary path.
2) Simulations conducted for reference signals containing $x_w(n)$ generated by an AR(1) model and $x_f(n)$ also reproduce the same phenomenon as shown in Figs. 2 and 3, as long as the adaptation is set relatively fast.
3) The locations and magnitudes of those spikes change from run to run, looking random.

As seen from the numerous simulations conducted, it seems that the existence of sinusoid(s) in both the reference and primary noise signals might put the conventional BANC system at an increased risk of diverging or becoming ineffective. This implies that the conventional BANC system will lose its effectiveness in real-world applications, if the primary noise contains both wideband and narrowband components, and a relatively fast adaptation is required. Therefore, a new system is proposed here that performs robustly in the presence of mixture of a wideband noise and sinusoid(s).

## III. A New Feedforward Hybrid ANC System

To overcome the difficulty the conventional BANC faces in cases that both wideband and narrowband noise components are present, we propose a new hybrid ANC system in Fig. 4,

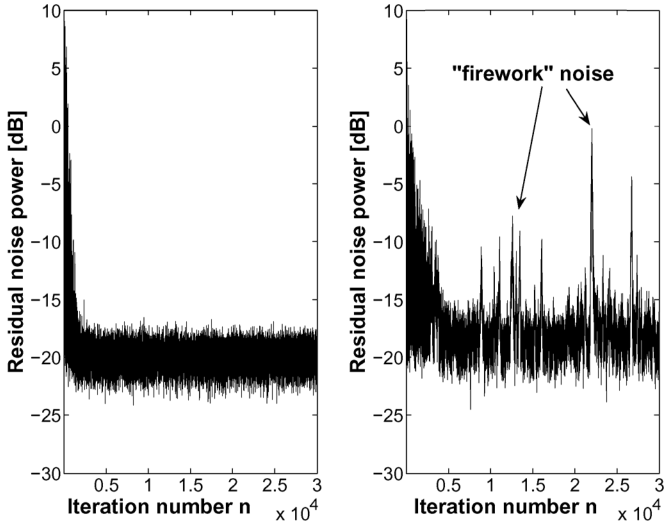


Fig. 2. Residual noise power of BANC with and without low-frequency sinusoids. Left: without, right: with sinusoids. ($L(=51) > M_p(=41)$, $\mu_h = 0.0007$, 40 runs).

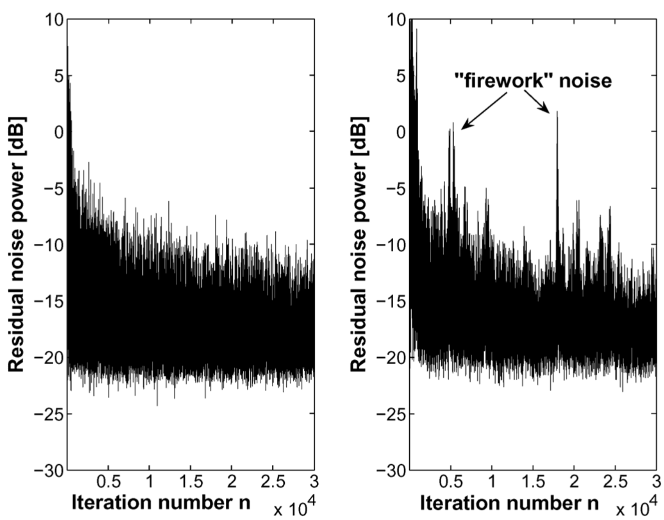


Fig. 3. Residual noise power of BANC with and without sinusoids. Left: without, right: with sinusoids. ($L(=21) < M_p(=41)$, $\mu_h = 0.0018$, other conditions the same as in Fig. 2).

which is capable of dealing with both components simultaneously. This system has a sinusoidal noise canceller and two controllers, with one designated for wideband noise and the other placed for narrowband noise.

## A. Sinusoidal Noise Canceller (SNC)

The reference noise signal $x_r(n)$ in (4) is not used to directly feed the BANC controller $H(z)$, as its sinusoidal component may destroy the ability of $H(z)$ to reduce the wideband component in the primary noise. Instead, the reference signal is pre-filtered by an adaptive Fourier analyzer or sinusoidal noise canceller (SNC) with variable step size parameter. The SNC removes the sinusoids residing in $x_r(n)$ such that $H(z)$ could work effectively. The output of the SNC is given by

$$x(n) = x_r(n) - \sum_{i=1}^{q} \left[\hat{a}_{c,i}(n)x_{a_i}(n) + \hat{b}_{c,i}(n)x_{b_i}(n)\right]. \quad (10)$$

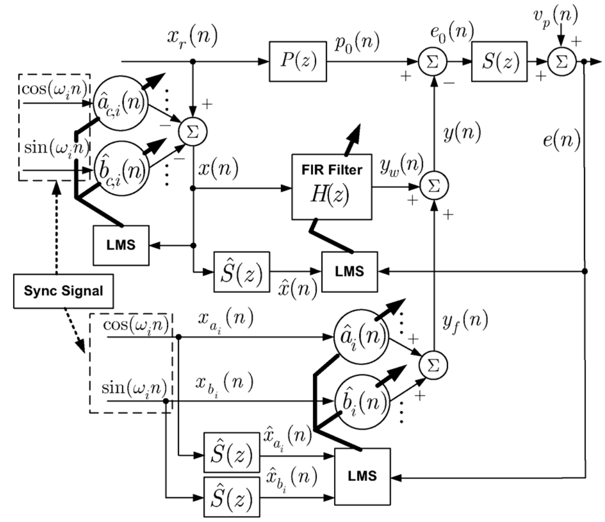


Fig. 4. Proposed feedforward hybrid ANC system.

The LMS algorithm with variable step size parameter is utilized to update the weights $\{\hat{a}_{c,i}(n), \hat{b}_{c,i}(n)\}_{i=1}^{q}$ of the SNC:

$$\hat{a}_{c,i}(n+1) = \hat{a}_{c,i}(n) + \mu_c(n)x(n)x_{a_i}(n) \quad (11)$$

$$\hat{b}_{c,i}(n+1) = \hat{b}_{c,i}(n) + \mu_c(n)x(n)x_{b_i}(n). \quad (12)$$

The variable step size parameter is calculated by

$$\mu_c(n) = \alpha\mu_c(n-1) + (1-\alpha)\mu_{c,min}, \quad \mu_c(0) = \mu_{c,max} \quad (13)$$

where $\alpha$ is a constant defined within (0, 1], with typical values like 0.975, 0.980, etc., $\mu_{c,max}$ is the maximum or initial value of the step size $\mu_c(n)$, while $\mu_{c,min}$ is its minimum or steady-state value. $\mu_{c,max}$ is set to a large value such that the SNC converges fast enough in the early stage of adaptation. $\mu_{c,min}$ is set small to guarantee the steady-state performance of the SNC. This way, the SNC will converge very fast in the early stage of adaptation, quickly and precisely removing the sinusoids residing in $x_r(n)$ to make it easier for the BANC subsystem $H(z)$ that follows to play its role effectively. The SNC with constant step size is investigated in detail in [5].

## B. Broadband ANC Subsystem

Output of the controller $H(z)$ is expressed by

$$y_w(n) = \sum_{j=0}^{L-1} h_j(n)x(n-j). \quad (14)$$

The controller is updated by an FXLMS algorithm [1]

$$h_j(n+1) = h_j(n) + \mu_h e(n)\hat{x}(n-j) \quad (15)$$

where

$$e(n) = \sum_{m=0}^{M-1} s_m\left[p_0(n-m) - y(n-m)\right] + v_p(n) \quad (16)$$

$$\hat{x}(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m x(n-m) \quad (17)$$

$$y(n) = y_w(n) + y_f(n) \quad (18)$$

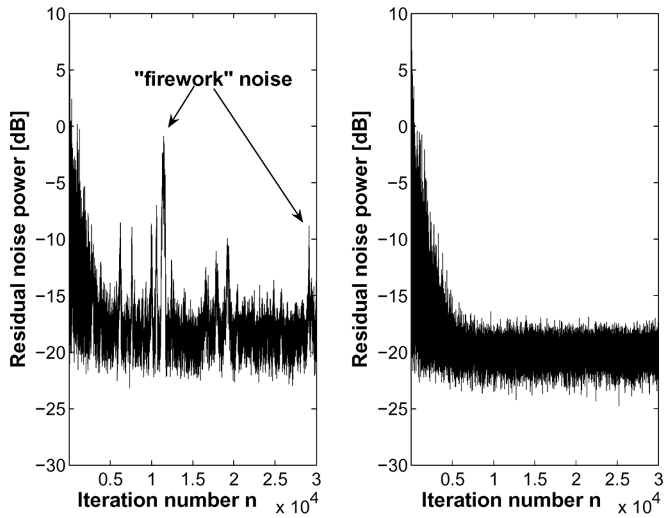


Fig. 5. Residual noise power of conventional and proposed hybrid ANC with both wideband noise and sinusoids. Left: conventional, right: proposed. (Simulation conditions the same as in Fig. 2).

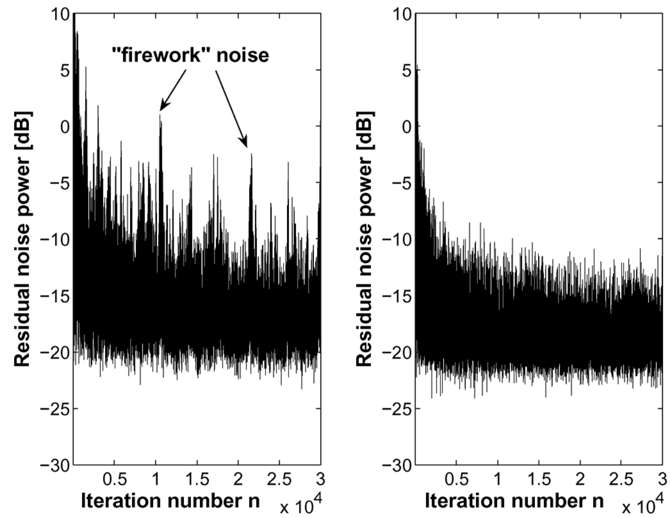


Fig. 6. Residual noise power of conventional and proposed hybrid ANC with both wideband noise and sinusoids. Left: conventional, right: proposed. (Simulation conditions the same as in Fig. 3).

$y_w(n)$ and $y_f(n)$ are wideband and narrowband secondary sources, respectively. $y_f(n)$ will be given later.

### C. Narrowband ANC Subsystem

The narrowband portion in the primary noise $p_0(n)$ is compensated by a typical NANC subsystem [1] which is a linear combiner with two control filter weights for each targeted frequency. Again, the FXLMS algorithm is used to update the control filter weights:

$$\hat{a}_i(n+1) = \hat{a}_i(n) + \mu_N e(n)\hat{x}_{a_i}(n) \quad (19)$$

$$\hat{b}_i(n+1) = \hat{b}_i(n) + \mu_N e(n)\hat{x}_{b_i}(n) \quad (20)$$

where

$$y_f(n) = \sum_{i=1}^{q} \left[\hat{a}_i(n)x_{a_i}(n) + \hat{b}_i(n)x_{b_i}(n)\right] \quad (21)$$

$$\hat{x}_{a_i}(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m x_{a_i}(n-m) \quad (22)$$

$$\hat{x}_{b_i}(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m x_{b_i}(n-m) \quad (23)$$

and $\mu_N$ is another positive step size.

### D. Computational Complexity

In the SNC subsystem, approximately $4q+2$ multiplications are required for update. The number of multiplications needed in BANC subsystem is $2L+\hat{M}+1$. The NANC subsystem requires $2q\hat{M}+2q+1$ multiplications. Clearly, the SNC will not cause serious cost increase as the number of frequencies $q$ in many real applications with rotating machines is 2, 3, or at most around 10. However, the NANC subsystem may significantly increase the computational cost compared to that of the conventional BANC system, especially when the estimated secondary-path order $\hat{M}$ is not small. This may pose a practical concern that needs further research effort.

### E. Simulations

To demonstrate the effectiveness of the proposed hybrid ANC system, extensive simulations are performed. Typical results are shown in Figs. 5 and 6. User parameters of the SNC are $\alpha = 0.980$, $\mu_{c,max} = 0.05$, $\mu_{c,min} = 0.00001$. The step size $\mu_N$ is 0.01. Other simulation conditions are the same as in Figs. 2 and 3, respectively. It has been found from both Figs. 5, 6, and many other simulations that the proposed hybrid system is very effective in reducing both wideband and narrowband components in the primary noise signal. The firework noise no longer exists in the residual noise of the new system.

## IV. Conclusions

In this letter, representative simulations are first provided to show the performance deterioration of the conventional BANC system in the presence of wideband noise accompanied by some low-frequency sinusoids. Then, a new hybrid ANC system is proposed that effectively suppresses both wideband and narrowband components. Numerous simulations for various scenarios have been conducted to confirm the effectiveness of the new system. Future topics include system modifications for robustness, reduction of computational cost and so on.

## References

[1] S. M. Kuo and D. R. Morgan, "Active noise control: A tutorial review," *Proc. IEEE*, vol. 87, no. 6, pp. 943–973, Jun. 1999.

[2] M. Zhang, H. Lan, and W. Ser, "On comparison of online secondary-path modeling methods with auxiliary noise," *IEEE Trans. Speech Audio Process.*, vol. 13, no. 4, pp. 618–628, Jul. 2005.

[3] A. Carini and S. Malatini, "Optimal variable step-size NLMS algorithms with auxiliary noise power scheduling for feedforward active noise control," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 16, no. 8, pp. 1383–1395, Nov. 2008.

[4] M. Akhtar and W. Mitsuhashi, "Improving performance of hybrid active noise control systems for uncorrelated narrowband disturbances," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 19, no. 7, pp. 2058–2066, Sep. 2011.

[5] Y. Xiao, Y. Tadokoro, and K. Iwamoto, "Real-valued LMS Fourier analyzer for sinusoidal signals in additive noise," *Signal Process.*, vol. 69, pp. 131–147, 1998.