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Study of Causality Constraint on Feedforward Active Noise Control Systems

Xuan Kong and Sen M. Kuo

***Abstract*— Feedforward active noise control (ANC) systems function as adaptive system identification, thus able to control both broadband and narrowband noises. When the acoustic/electric delays in the noise cancelling subsystem exceed the acoustic delay of the primary path, the causality constraint will be violated. We present a performance analysis of the feedforward ANC system for ducts (one-dimensional acoustic field) under the noncausal condition. The bandwidth of the noise controllable by the ANC system decreases as the degree of noncausality increases. Noise with narrower bandwidth can be more effectively cancelled out by the ANC system with a given degree of noncausality. Analysis also shows that the convergence speed for the adaptive weight vector is independent of the degree of noncausality.**

I. INTRODUCTION

Broadband acoustic active noise control (ANC) has attracted much research interest in recent years (e.g., [1]–[3]). In the ANC system for ducts illustrated in Fig. 1, the primary noise is sensed by a reference microphone close to the noise source. An adaptive filter (AF) uses this reference signal $x[n]$ to generate a cancelling signal $y[n]$, which is fed to a secondary loudspeaker to cancel the primary noise. An error microphone measures the residual noise $e[n]$ and uses it to update the AF coefficients. The filtered-X least-mean-square (LMS) algorithm [4], [5] was proposed for updating the AF coefficients [8]. The discussion in this correspondence will be limited to the ANC system for narrow ducts (one-dimensional acoustic fields). Results may need modifications before they can be extended to a more general multidimensional field ANC system.

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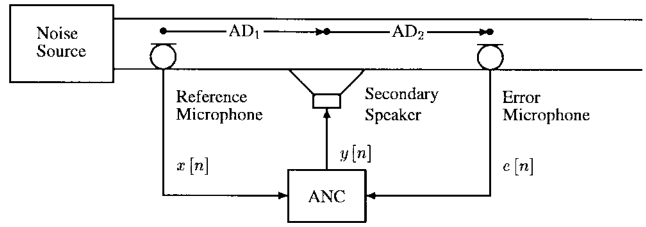

Fig. 1. Acoustic active noise control in ducts.

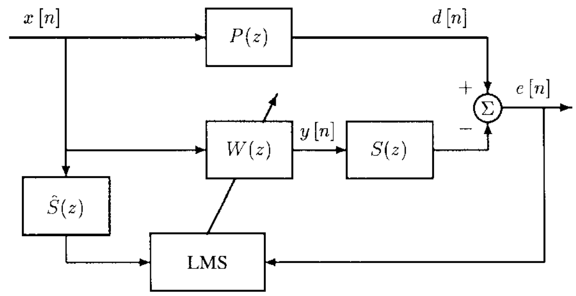

Fig. 2. Block diagram of ANC system using the filtered-X LMS algorithm.

As shown in Fig. 1, the acoustic delay AD_1 from the reference microphone to the secondary loudspeaker is proportional to the distance from the reference sensor to the secondary source. An additional acoustic delay AD_2 between the secondary loudspeaker and the error sensor is common for both primary noise and cancelling signal. Electric delay ED refers to the sum of the group delay of the AF and the total system delay in the antialiasing filter, A/D converter, D/A converter, reconstruction filter, loudspeaker, plus the processing time (one sampling period). Since the AF necessarily has a causal response, we must ensure that the acoustic delay AD_1 is greater than the electric delay ED. This condition is called the causality constraint, and it specifies the minimum length of a system for which broadband random noise can be effectively cancelled in a duct. When the electric delay ED is longer than the acoustic delay AD_1, the required response of the controller is noncausal, and hence unrealizable for broadband noise control. Therefore, to attenuate broadband noise, the standard approach to ANC in ducts is to use a long duct. However, installation constraints of commercial systems (such as automobile exhaust or air intake systems) usually limit the length of duct that can be used. There is an increasing demand for ANC systems to cancel unwanted noise in a duct with short length.

Fig. 2 shows the block diagram of the single-channel feedforward ANC system setup in Fig. 1. The primary path $P(z)$ models the propagation path between the reference sensor and error sensor (including AD_1 and AD_2), while $S(z)$ is the secondary path between the AF output and the error sensor (including a part of ED and AD_2). An estimate of the secondary path is denoted as $\hat{S}(z)$.

If $\hat{S}(z) = S(z)$ and the adaptive gain μ used in the filtered-X least mean square (LMS) algorithm is sufficiently small so that $W(z)$

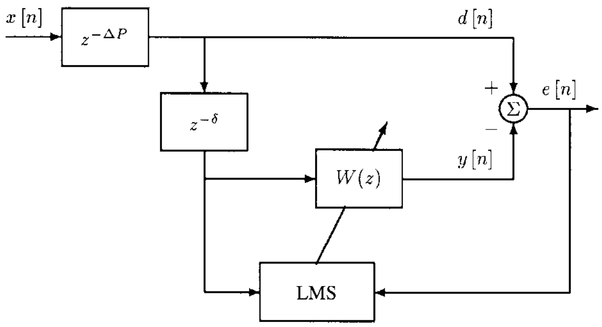

Fig. 3. Simplified feedforward ANC system, acting as an adaptive predictor.

changes slowly, then the systems $W(z)$ and $S(z)$ can be commuted. To allow a clear understanding of the causality issue, the transfer functions $P(z)$ and $S(z)$ are simplified as pure delays, $z^{-\Delta P}$ and $z^{-\Delta S}$, respectively. When $\delta = \Delta P - \Delta S \geq 0$, causality constraint is satisfied. The z-domain error signal $E(z)$ can be expressed as

$$E(z) = X(z)z^{-\Delta S}[z^{-\delta} - W(z)] \tag{1}$$

and the ANC system acts like a system identification scheme. When $\delta = \Delta S - \Delta P > 0$, the causality constraint is violated as we must have

$$E(z) = X(z)z^{-\Delta P}[1 - z^{-\delta}W(z)]. \tag{2}$$

The system is identical to an adaptive predictor scheme illustrated in Fig. 3. Clearly, for the same degree of noncausality δ, the AF $W(z)$ can make a better prediction of the signal $x[n]$ with a narrower bandwidth. For a given bandwidth of $x[n]$, the larger the delay δ is, the more difficult it is for the AF to predict accurately the primary noise $d[n]$. In the next section, we formalize the above intuitive observations via an analysis of the ANC system.

II. Performance Analysis

In this section, we analyze the performance of the ANC system outlined in Section I when the causality constraint is violated. The subject has been studied previously in both time domain [6] and in frequency domain [7]. The results presented here are in time domain with an emphasis on the adaptive algorithm performance. Investigation is first carried out under a simplified primary and secondary path condition in order to clearly demonstrate the effects of noncausality on the performance of the ANC system. The results are then generalized to a more realistic primary path model case.

A. Simplified Primary Path Model

A simple model for the primary path and secondary path is $P(z) = z^{-\Delta P}$ and $S(z) = z^{-\Delta S}$. When the causality constraint is violated, we have $\Delta S > \Delta P$. Such a condition requires the AF $W(z)$ to act like a prediction filter in order to minimize the residual error: $W(z) = z^{(\Delta S - \Delta P)} = z^{\delta}$. Without loss of generality, $\Delta P = 0$ is assumed in the following analysis. A commonly used model for the broadband primary noise is the autoregressive (AR) model:

$$x[n] = \sum_{l=1}^{L} a_l x[n-l] + \varepsilon[n] \tag{3}$$

where $\varepsilon[n]$ is white noise with zero mean and variance σ_ε^2 and a_l are the model parameters.

1) Moving-Average Interpretation: A moving average (MA) interpretation of the effects of noncausality is presented here. While the MA representation cannot be used in practice, it does provide insight of the results presented later.

An equivalent representation of the primary noise model (3) can be written as (see Fig. 3 with $\Delta P = 0$)

$$d[n] = x[n] = \sum_{k=0}^{\infty} w[k]\varepsilon[n-k] \tag{4}$$

with the corresponding AF output as

$$y[n] = \sum_{k=0}^{\infty} \hat{w}_n[k]\varepsilon[n-k-\delta] \tag{5}$$

where $w[k]$ and $\hat{w}_n[k]$ are the desired and estimated AF coefficients, respectively.

Upon convergence of the LMS algorithm, the optimal AF conefficients for $\hat{w}[k]$ becomes $\hat{w}_{opt}[k] = w[k+\delta]$. The minimum mean square error (MSE) $\xi_{\min}$ is given by

$$\xi_{min} = E[(d[n] - y[n])^2] = \sigma_\varepsilon^2 \sum_{k=0}^{\delta-1} w^2[k]. \tag{6}$$

Obviously, $\xi_{\min}$ decreases as δ decreases.

The noise cancelling efficiency η is defined as

$$\eta = 1 - \xi_{\min}/\xi_{total} \tag{7}$$

to measure the effectiveness of the ANC system. The efficiency η reflects the percentage of the energy in the primary noise which the ANC system is able to reduce, and so provides a simple but quantitative means to measure the noise cancelling performance of the ANC system.

The total primary noise energy can be found as

$$\xi_{\text{total}} = E[d^2(n)] = \sigma_\varepsilon^2 \sum_{k=0}^{\infty} w_k^2. \tag{8}$$

Substituting (6) and (8) into (7), we have

$$\eta = 1 - \frac{\sigma_\varepsilon^2 \sum_{k=0}^{\delta-1} w_k^2}{\sigma_\varepsilon^2 \sum_{k=0}^{\infty} w_k^2} = \frac{\sum_{k=\delta}^{\infty} w_k^2}{\sum_{k=0}^{\infty} w_k^2}. \tag{9}$$

Thus, we conclude that the ANC system efficiency becomes lower as the degree of noncausality (δ) becomes higher.

2) Autoregressive Model Interpretation: A broadband noise source $x[n]$ with Q distinct spectrum peaks can be described by the following summation of second order processes:

$$X(z) = \sum_{q=1}^{Q} \frac{a_q + b_q z^{-1}}{(1 - \rho_q e^{j\theta_q} z^{-1})(1 - \rho_q e^{-j\theta_q} z^{-1})} \tag{10}$$

where $\rho_q < 1$ to ensure that $x[n]$ is stationary. In the following analysis, we concentrate on the ANC systems with a noise $x[n]$ whose spectrum is given by

$$X(z) = \frac{\sigma_\varepsilon^2}{(1 - \rho e^{j\theta} z^{-1})(1 - \rho e^{-j\theta} z^{-1})}. \tag{11}$$

The spectrum peak bandwidth decreases to zero as ρ approaches 1.

The AR model whose spectrum is given by (11) can be represented by the following MA model:

$$x[n] = \sum_{k=0}^{\infty} \rho^k \frac{\sin[(k+1)\theta]}{\sin\,\theta} \varepsilon[n-k]. \tag{12}$$

Substituting $w_k = \rho^k \sin[(k+1)\theta]/\sin\theta$ into (9) leads to

$$\eta(\rho, \delta) = \frac{\sum_{k=\delta}^{\infty} \rho^{2k}\{1-\cos[2(k+1)\theta]\}}{\sum_{k=0}^{\infty} \rho^{2k}\{1-\cos[2(k+1)\theta]\}}. \quad (13)$$

With a fixed ρ_0, it can be shown that the ANC efficiency $\eta(\rho_0, \delta)$ increases as the degree of noncausality δ decreases, and that for any fixed δ_0, $\eta(\rho, \delta_0)$ increases monotonely with ρ. These facts, together with $\eta(0, \delta_0) = 0$ and $\eta(1, \delta_0) = 1$, show that the narrower the bandwidth of the reference noise $x[n]$ is, the higher the efficiency of the ANC system can be.

3) Adaptive Algorithm Performance: Based on the AR representation of $x[n]$, the AF updates its parameters as follows:

$$\hat{\boldsymbol{w}}_{n+1} = \hat{\boldsymbol{w}}_n + \mu(x[n] - y[n])\boldsymbol{x}_{n-\delta} \quad (14)$$

with $y[n] = \hat{\boldsymbol{w}}_n^T \boldsymbol{x}_{n-\delta} = \sum_{l=0}^{L-1} \hat{w}_l[n]x[n-l-\delta]$ and $\boldsymbol{x}_{n-\delta} = [x[n-\delta]x[n-\delta-1]\cdots x[n-\delta-L+1]]^T$. The optimal weight vector $\hat{\boldsymbol{w}}_{opt}$ is given by $\hat{\boldsymbol{w}}_{opt} = \boldsymbol{R}_x^{-1}\hat{\boldsymbol{r}}_\delta$, where $\boldsymbol{R}_x$ is a Toeplitz matrix generated by $\hat{\boldsymbol{r}}_\delta$ with $\delta = 0$ and $\hat{\boldsymbol{r}}_\delta = (\gamma_x[\delta]\gamma_x[\delta+1]\cdots\gamma_x[\delta+L-1])^T$ with $\gamma_x[l] = E(x[n]x[n+l])$.

From (14), the weight error vector $\tilde{\boldsymbol{w}}_n = \hat{\boldsymbol{w}}_n - \hat{\boldsymbol{w}}_{opt}$ follows

$$\tilde{\boldsymbol{w}}_{n+1} = \tilde{\boldsymbol{w}}_n + \mu(x[n] - \tilde{\boldsymbol{w}}_n^T\boldsymbol{x}_{n-\delta} - \hat{\boldsymbol{w}}_{opt}^T\boldsymbol{x}_{n-\delta})\boldsymbol{x}_{n-\delta}. \quad (15)$$

A technique called averaging analysis [10] will be applied to the analysis of the system (15). Averaging analysis relates important functionals of a system like (15) to a much simpler but related system called averaged system. For example, it was established in [10] that the trajectory of (15) can be made arbitrarily close to the trajectory of the following averaged system by reducing μ:

$$\bar{\boldsymbol{w}}_{n+1} = (\boldsymbol{I} - \mu\boldsymbol{R}_x)\bar{\boldsymbol{w}}_n. \quad (16)$$

In practice, the adaptive gain μ used is small enough to allow a good approximation of $\tilde{w}_n$ by $\bar{\boldsymbol{w}}_n$.

The convergence speed of the homogeneous system (16) is determined by the eigenvalues of $\boldsymbol{R}_x$. Since the composition of $\boldsymbol{R}_x$ is independent of δ, one can conclude that the speed of the exponential convergence for the weight vector $\hat{\boldsymbol{w}}_n$ remains the same regardless of the degree of the noncausality δ. This is very important because operators of ANC systems would like to know how quickly the adaptation will occur even though the accurate knowledge of the degree of noncausality is not available. Empirical study of the ANC system by Elliot *et al.* [11] found that the maximum achievable convergence speed is affected by the delay in the secondary path (δ) via

$$\mu_{\max} \propto \frac{1}{L+\delta}.$$

The result here does not deal with this bound, but rather considers the convergence speed with a given $\mu < \mu_{\max}$.

The minimum residual error, once the system converges, is given by

$$\xi = E[e^2(n)] = E[e_*^2(n)] + E[(\tilde{\boldsymbol{w}}_n^T\boldsymbol{x}_{n-\delta}\boldsymbol{x}_{n-\delta}^T\tilde{\boldsymbol{w}}_n)] \equiv \xi_* + \xi_{ex} \quad (17)$$

where $e_*[n] = x[n] - \hat{\boldsymbol{w}}_{opt}^T\boldsymbol{x}_{n-\delta}$. The minimum MSE ξ_* is given by $(\tilde{\boldsymbol{r}}_\delta = \boldsymbol{r}_\delta/\sigma_x^2)$

$$\xi_*(\delta, \rho) = E\{e_*^2[n]\} = \sigma_x^2(1 - \hat{\boldsymbol{w}}_{opt}^T\tilde{\boldsymbol{r}}_\delta). \quad (18)$$

It will be shown that the excessive MSE ξ_{ex} remains the same even when the causality constraint is violated. So the noise cancelling efficiency $\eta(\rho, \delta)$ due to noncausality δ is

$$\eta(\rho, \delta) = 1 - \xi_*/\sigma_x^2 = \sigma_x^2\tilde{\boldsymbol{r}}_\delta^T\boldsymbol{R}_x^{-1}\tilde{\boldsymbol{r}}_\delta. \quad (19)$$

For a reference noise of the form $x[n] = \rho x[n-1] + \varepsilon[n]$, the noise cancelling efficiency is given by

$$\eta(\rho, \delta) = \rho^{2\delta}\tilde{\boldsymbol{r}}_0^T\tilde{\boldsymbol{R}}_x^{-1}\tilde{\boldsymbol{r}}_0 \quad (20)$$

where $\tilde{\boldsymbol{R}}_x$ is a Toeplitz matrix generated by $\tilde{\boldsymbol{r}}_0 = [1\ \rho\ \rho^2 \cdots \rho^{L-1}]^T$. The above equation indicates that when the degree of noncausality increases, the noise cancelling efficiency decreases. Also, as $\rho\ (<1)$ increases, the efficiency η increases.

B. General Primary Path Model

A more realistic model for an ANC system is to allow an arbitrary primary path transfer function while maintaining a pure delay as the secondary path model. The adaptive algorithm for estimating the AF $\hat{\boldsymbol{w}}_n$ remains the same as (14), except $x[n]$ is replaced by $d[n]$ defined by

$$d[n] = \sum_{k=0}^{L-1} w_k x[n-k] = \boldsymbol{w}_*^T\boldsymbol{x}_n. \quad (21)$$

At the steady state, the adaptive algorithm will converge to

$$\hat{\boldsymbol{w}}_{opt} = \boldsymbol{R}_x^{-1}\boldsymbol{R}_\delta\boldsymbol{w}_* \quad (22)$$

where $\boldsymbol{R}_\delta = [\gamma_x(j-i-\delta)]_{(L+1)\times(L+1)}$ [where $\gamma_x(\cdot)$ is the autocorrelation function for input $x[n]$]. Of course, when $\delta = 0$, we have $\hat{\boldsymbol{w}}_{opt} = \boldsymbol{w}_*$ as expected.

The error vector $\tilde{\boldsymbol{w}}_n = \hat{\boldsymbol{w}}_n - \hat{\boldsymbol{w}}_{opt}$ obeys the following difference equation:

$$\tilde{\boldsymbol{w}}_{n+1} = \tilde{\boldsymbol{w}}_n + \mu(\boldsymbol{w}_*^T\boldsymbol{x}_n - \tilde{\boldsymbol{w}}_n^T\boldsymbol{x}_{n-\delta} - \hat{\boldsymbol{w}}_{opt}^T\boldsymbol{x}_{n-\delta})\boldsymbol{x}_{n-\delta}. \quad (23)$$

Averaging analysis [10] yields the corresponding averaged system

$$\bar{\boldsymbol{w}}_{n+1} = (\boldsymbol{I} - \mu\boldsymbol{R}_x)\bar{\boldsymbol{w}}_n. \quad (24)$$

Since the eigenvalues of $(\boldsymbol{I} - \mu\boldsymbol{R}_x)$ are independent of δ, the speed of the convergence for the averaged system (24) remains the same regardless the degree of the noncausality. Recall that the original system (23) follows closely the trajectory of the averaged system for small μ; we can conclude that the speed of the ANC system to adapt itself to the changes in the noise cancelling environment is not a function of the degree of noncausality.

Once the algorithm converges, the filter weight vector will fluctuate around the optimal solution $\hat{\boldsymbol{w}}_{opt}$. The excessive MSE is given by (see [9] for a derivation) $\xi_{ex}(n) = \mathrm{tr}[\boldsymbol{R}_x\boldsymbol{K}(n)]$, where $\boldsymbol{K}(n) = E[\tilde{\boldsymbol{w}}_n\tilde{\boldsymbol{w}}_n^T]$ is the steady state weight error correlation matrix satisfying

$$\boldsymbol{K}(n+1) = (\boldsymbol{I} - \mu\boldsymbol{R}_x)\boldsymbol{K}(n)(\boldsymbol{I} - \mu\boldsymbol{R}_x) + \mu^2\boldsymbol{R}_x\xi_*. \quad (25)$$

Thus, one can conclude that the only possible dependence of the excessive MSE ξ_{ex} on δ will be through ξ_*. Thus, the misadjustment defined as

$$\mathcal{M} = \xi_{ex}/\xi_*$$

will be independent of the degree of noncausality.

III. Simulation Results

Simulation results are presented here to verify the theoretical development in Section II. The following reference noise model is used for all simulations:

$$x[n] = 2\rho\cos\theta x[n-1] - \rho^2 x[n-2] + \varepsilon[n], \qquad \theta = \pi/4 \quad (26)$$

where $\varepsilon[n]$ is a white Gaussian noise with zero mean and unit variance. The secondary path model is also fixed at $S(z) = z^{-\delta}$, where the delay parameter δ is to be chosen.

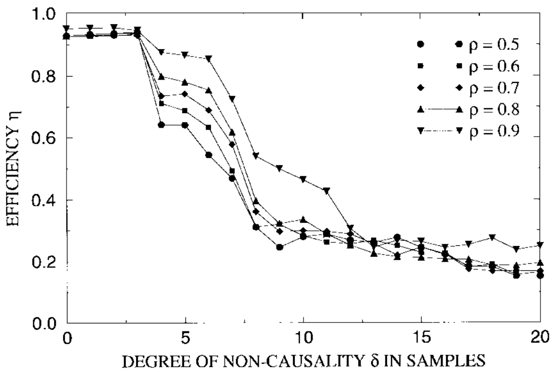

Fig. 4. Performance of the ANC system as a function of the bandwidth of the primary noise and the degree of noncausality. General primary path is used.

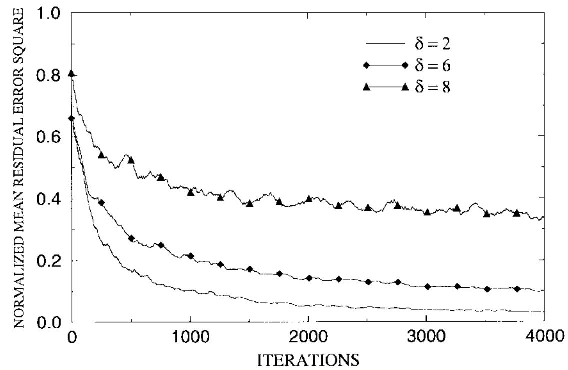

Fig. 5. Convergence speed of the ANC system is independent of the degree of the noncausality. General primary path is used.

The primary path model is

$$P(z) = \frac{\sum_{i=0}^{24} b_i z^{-i}}{1 + \sum_{j=1}^{24} a_i z^{-i}}. \tag{27}$$

The model is obtained via measurements of an actual ANC system (for a list of the model parameters, see files s_p.asc and s_z.asc on the disk of [2]). A close inspection of the model parameters reveals that the first four coefficients in the numerator are very small. This suggests that the primary path has a delay of $\Delta P = 4$. So the secondary path delay δ $(= \Delta S)$ will not have any appreciable effect on the ANC system performance up to $\delta = 4$.

Fig. 4 shows the efficiency of the ANC system $(L = 30, \mu = 0.01/\sigma_x^2)$ under various conditions. Each curve represents a simulation condition for a specific bandwidth of the reference signal $x[n]$ (via the specification of ρ). Various degrees of noncausality δ are tested. For a specific ρ and δ, the estimated variance of the residual error $e[n]$ was calculated based on the last 1000 iterations of a total of 5000 iterations. The estimated variances were then averaged over 10 realizations with identical initial conditions and independent realizations of $\varepsilon[n]$. The results confirm the two important conclusions drawn in Section II: 1) the ANC system efficiency decreases exponentially as a function of the degree of noncausality δ, and 2) the efficiency decreases as the bandwidth of the reference signal increases.

Fig. 5 shows that the convergence speed of the ANC system $(L = 30)$ is independent of the degree of the noncausality (δ). Each curve represents the averaged results over ten independent realizations of the adaptive algorithm. Indeed, the speed of the convergence is the same for different levels of noncausality as measured by δ. The adaptive gain for all instances is set at $\mu = 0.005/\sigma_x^2$.

IV. Conclusions

In this paper, an analysis is presented for the feedforward ANC system regarding its noise cancelling performance when the causality constraint is violated. Under such a noncausal system condition, the feedforward ANC system is required to predict the primary noise based on available reference input. It is shown that the smaller the degree of the noncausality, the better the ANC system performance. When the noise has a narrower bandwidth at its spectrum peaks, the noise cancelling results of the ANC are more effective.

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