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# Filtered-x RLS Algorithm based Active Noise Control of Impulsive Noise

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*Abstract*— **This paper presents active noise control (ANC) of impulsive noise modeled by non-Gaussian stable processes. The famous filtered-x least mean square (FxLMS) algorithm for ANC systems becomes unstable for impulsive noise as it is based on minimization of variance of error signal. As the family of RLS algorithms give better convergence than that of LMS algorithm, a new solution employing Filtered-x recursive least squares (FxRLS) algorithm is proposed for ANC of impulsive noise. Simulation results illustrate that the proposed solution has faster convergence and better stability than the existing techniques, for highly impulsive environments.**

***Keywords***— **Active noise control, FxLMS, FxRLS, impulsive noise, alpha stable distribution.**

## I. INTRODUCTION

The basic principle of Active noise control is destructive interference between acoustic waves [1,2]. Basically, the primary noise is mitigated in the region of the error microphone by producing and combining a noise having same amplitude but opposite phase of the primary noise source. This anti-phase noise is provided by the adaptive controller which uses an adaptive algorithm such as the filtered-x least mean square (FxLMS) algorithm or filtered-x recursive least square (FxRLS) algorithm. Fig. 1 shows a feed forward ANC system comprising of one reference microphone to obtain the reference noise, one cancelling loudspeaker (secondary speaker) and one error microphone to perceive the error between the reference noise and the cancelling noise.

Fig. 2 shows the filtered-x adaptive algorithm for single channel ANC system, where *P(z)* represents the primary path between the reference microphone and the error microphone, *S(z)* represents the secondary path between the cancelling speaker and the error microphone and the *S'(z)* is the secondary path estimate. Also *x'(n)* is the filtered reference input to the adaptive algorithm.

The FxLMS algorithm is a famous ANC algorithm as it has been successfully applied because of its low computational complexity and easy implementation [3] for ANC of broadband and narrow band noise [4]. The control of impulsive noise is a big challenge for researchers. Generally, impulsive noises occur with large amplitude but low probability. The statistics of impulsive noise can be modeled by symmetric alpha stable *(SαS)* distribution [5]. The main

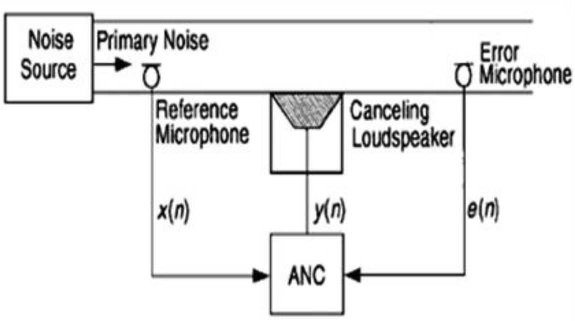



Fig. 1 Basic principle of feed forward ANC system

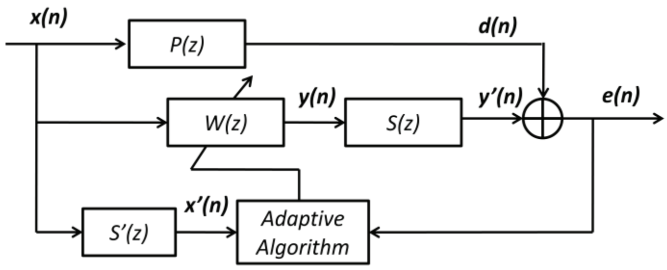



Fig. 2 Block diagram of single channel ANC system using the filtered-x adaptive algorithm

difference between the SαS distribution and the Gaussian distribution is that the density function of α-stable distribution has a heavy tail as compared to Gaussian density function. SαS distribution has characteristic function of the form

$$\varphi(t)=e^{-\gamma|t|^{\alpha}} \tag{1}$$

where $\alpha(0<\alpha\leq 2)$ controls the heaviness of its tail, called as characteristic exponent. If α is close to zero, then the distribution has a very heavy tail indicating highly impulsive noise. For α=2 the distribution becomes Gaussian and for α=1 it becomes Cauchy distribution. Here $\gamma(\gamma>0)$ is called as dispersion. In this paper, we have modeled impulsive noise using standard *SαS* distribution with γ =1. Fig. 3 shows the effect of varying α in its specified range.

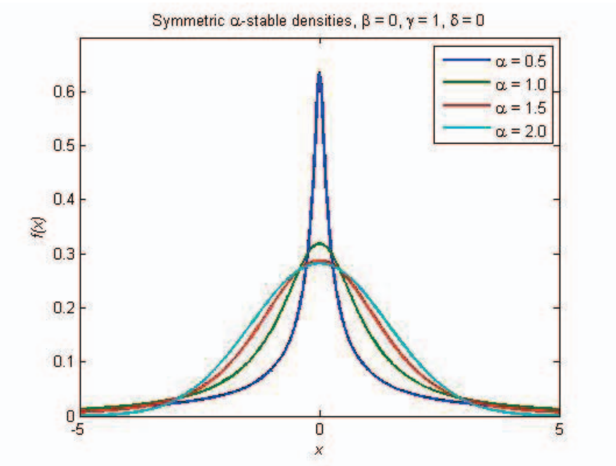



Fig. 3 PDFs of standard *SαS* process for various values of α

For stable distributions, it has been proved that only those moments that are of order less than the characteristic exponent are finite[5]. Hence, the second order moment (variance) is infinite. So, the famous filtered-x LMS algorithm which minimizes the mean square error of noise may become unstable and might not be the right choice for ANC of impulsive noise. Some solutions are proposed in literature to tackle this problem. In [5],the filtered-x least mean p-power (FxLMP) algorithm is presented, based on reducing a fractional lower order moment of error i.e. p($0<p<\alpha$) power of error, that exists for stable distribution. It gives better robustness as compared to FxLMS algorithm for active impulsive noise control. On the other hand, its convergence speed is very low especially when the noise is highly impulsive; also, it requires prior estimation of p depending on α, which is not an easy task for good results.

Another work is proposed in [6] by Sun et. al. which is a modification of FxLMS algorithm for impulsive ANC. In this proposed technique the reference signal is modified on the basis of its statistical properties i.e. if the magnitude of any sample is above a certain threshold value, the sample is ignored. The threshold is decided by the statistics of the signal. This technique gives stable and better performance in comparison to the FxLMS algorithm. However, it's not robust enough to give stable results when α is small. In [7], a further modification of Sun's algorithm is proposed by Akhtar et. al. in which the samples of reference signal as well as error signal are not ignored but are replaced by the threshold value. The main issue with the thresholding based algorithms [6,7] is the requirement of estimating appropriate thresholds which is not possible during online ANC operation.

In [8,9], two more algorithms are proposed by Akhtar et. al; one is Normalized step size FxLMS (NSS-FxLMS) algorithm which gives improved stability by normalizing the step size, and the other one is Data reusing based NSS-FxLMS (DRNSS-FxLMS) algorithm which provides better convergence speed by reusing the recent data, respectively. The main limitation of LMS based algorithms is their lower convergence rates than RLS in general and higher steady state mean square error as compared to RLS [10]. In [11], performance comparison of FxLMS, FxAPA and FxRLS ANC algorithms on fMRI noise validate the theoretical claims [10] that the FxRLS algorithm has a very high convergence rate.

Motivated by these findings, in this paper, we have investigated the performance of Filtered-x RLS algorithm for ANC of impulsive noise. Simulation results authenticate the better performance of FxRLS algorithm, in terms of convergence rate for impulsive ANC.

The rest of the paper is organized as follows. Section 2 gives overview of the existing techniques for impulsive ANC. Section 3 describes the proposed solution and the simulation results are discussed in Section 4. The concluding remarks are given in section 5.

## II. OVERVIEW OF EXISTING ALGORITHMS

### A. Normalized Step Size FxLMS algorithm

In Fig. 2, considering *W(z)* to be an FIR filter with L number of coefficients, the output of the filter is represented as

$$y(n) = w^T(n) * \boldsymbol{x}(n) \tag{2}$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu(n)e(n)\boldsymbol{x}'(n) \tag{3}$$

where *w(n)* the weights of the filter and *x'(n)* is the filtered reference input to the filter. The time varying normalized step size $\mu(n)$ is given as

$$\mu(n) = \frac{\bar{\mu}}{\delta + E_e(n) + \|x'(n)\|_2^2} \tag{4}$$

$E_e(n)$ is the energy of the residual error signal *e(n)* calculated as follows; with $\lambda$, called as forgetting factor, has value $0.9<\lambda<1$.

$$E_e(n) = \lambda E_e(n-1) + (1-\lambda)|e^2(n)| \tag{5}$$

The basic idea here is to freeze the adaptation when the noise amplitude is very high to prevent the algorithm from becoming unstable. However, freezing the adaptation slows down the convergence rate of the algorithm.

### B. Data-reusing based NSS-FxLMS algorithm

Referring to the Fig in [12], the DR algorithm computes error signal many times by reusing the input data. Therefore, it requires prior estimate of the desired signal *d(n)*, which is estimated as follows

$$d_1(n) = e(n) + y'(n) \tag{6}$$

where *y'(n)* is the output of the filter followed by the secondary path. The DR NSS-FxLMS algorithm calculates the error recursively by using the estimated desired output and the output of the DR based filter $y_1(n)$ as elaborated in [12].

$$e_1(n,i) = d_1(n-i+1) - y_1(n-i+1) \quad (7)$$
$$y_1(n) = x'^T(n-i+1)\boldsymbol{w_1}(n,i-1) \quad (8)$$

In [9,12], it has been shown through simulations that this algorithm is best among all other existing algorithms for ANC of impulsive noise.

## III. PROPOSED SCHEME

The algorithms discussed in previous section are variants of FxLMS algorithms which have been used in the past for impulsive ANC. But the basic problem with LMS is slow convergence and instability in presence of impulses, so there is a need to investigate adaptive filters other than LMS family for impulsive noise. Keeping this in mind, we have proposed a new solution for ANC of impulsive noise sources based on Adaptive filter of Least Square family i.e. FxRLS algorithm. The block diagram of FxRLS based ANC is depicted in Fig. 4. Table. 1 briefly describes the variables used in FxRLS algorithm. The FxRLS algorithm is summarized below in (10) to (14)

Initialization:

$$\mathbf{w}(0)= 0$$
$$\mathbf{P}(0) = \delta^{-1}\mathbf{I} \quad (9)$$

Algorithm:

$$\boldsymbol{\pi}(n) = \boldsymbol{P}(n-1)\boldsymbol{x}'(n) \quad (10)$$
$$\boldsymbol{k}(n) = \frac{\pi(n)}{\boldsymbol{x}'^{\boldsymbol{H}}(n)\pi(n)+\lambda} \quad (11)$$
$$\boldsymbol{P}(n) = \lambda^{-1}\boldsymbol{P}(n-1) - \lambda^{-1}\boldsymbol{k}(n)\boldsymbol{x}'^{\boldsymbol{H}}(n)\boldsymbol{P}(n-1) \quad (12)$$
$$\boldsymbol{\xi}(n) = d(n) - s(n) * (\,\boldsymbol{w}^H(n-1)\boldsymbol{x}'(n)\,) \quad (13)$$
$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \boldsymbol{k}(n)\xi(n) \quad (14)$$

Initialization of the FxRLS algorithm is a critical point for the convergence of the algorithm. The regularization parameter delta (δ) in (9) depends on SNR or in other words, variance of the input signal *x'(n)* [10]. When the noise level in the tap inputs is low, the FxRLS algorithm exhibits high convergence rate, provided that δ is chosen small enough; whereas, if noise level is very high it is preferable to initialize the algorithm with large value of δ [10]. The effect of varying delta (δ), from very small value to quite large values for impulsive noise is shown in section 4.

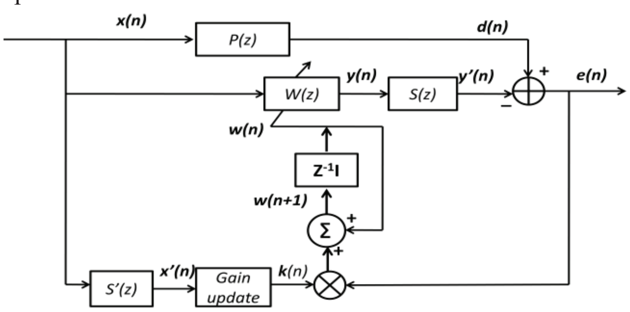



Fig. 4 Block diagram of proposed ANC for impulsive noise

TABLE I
LIST OF VARIABLES

| *Variables* | *Description* |
|---|---|
| *S(z)* | Secondary path of the cancelling signal |
| *S'(z)* | Estimated secondary path |
| *P(z)* | Primary path of the reference signal |
| *d(n)* | Desired signal to be cancelled |
| *y'(n)* | Generated signal following secondary path |
| *x(n)* | Reference noise vector |
| *x'(n)* | Filtered reference noise vector |
| *w(n)* | Adaptive filter coefficients |
| *P(n)* | Inverse of the correlation matrix |
| *k(n)* | Gain of RLS algorithm |
| δ | Regularization parameter |
| $\lambda$ | Forgetting factor |

$\lambda$ is the forgetting factor ($0.9 < \lambda < 1$). Appropriate selection of the forgetting factor also plays important role in the convergence of the algorithm. Good convergence is achieved by setting $\lambda$ closer to 1 as shown in section 4. The simulation results show that the FxRLS algorithm surpass the existing algorithms for ANC of impulsive noise in terms of fast convergence of the algorithm.

## IV. COMPUTER SIMULATIONS AND DISCUSSION

In this section, simulation results are presented to validate the effectiveness of the FxRLS algorithm for ANC of impulsive noise in comparison with the NSS-FxLMS algorithm and Data reusing based NSS-FxLMS algorithm. The data for the acoustic paths is taken from the disk attached with [3]. Using this data FIR filter configuration is used for *P(z)* and *S(z)* with order 256 and 128, respectively. It is assumed that the estimated secondary path filter *S'(z)* is exactly the same as *S(z)*. The length of *w(n)* is selected to be 192 with FIR filter configuration for *W(z)*. The performance metric used is mean noise reduction (MNR), defined as

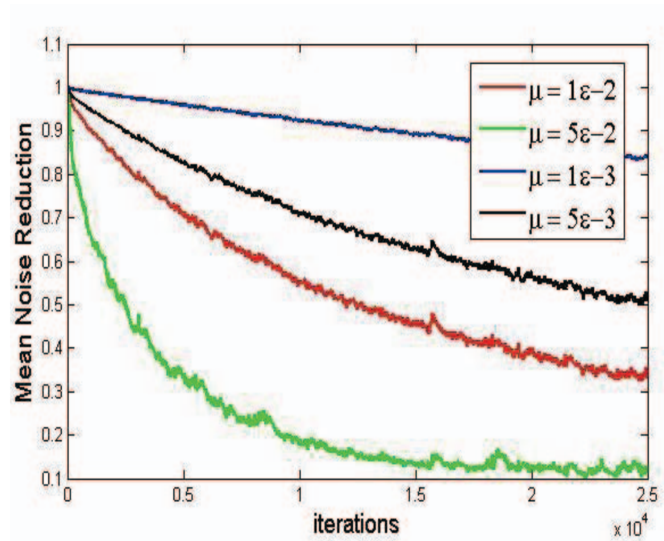
$$\mathrm{MNR(n)} = \mathbb{E}\left\{\frac{A_e(n)}{A_d(n)}\right\} \quad (15)$$

where $\mathbb{E}\{.\}$ represents ensemble averaging, $A_e(n)$ and $A_d(n)$ are estimated absolute values,of residual error signal *e(n)* and desired signal *d(n)*, respectively. These are estimated using low pass estimator of the form

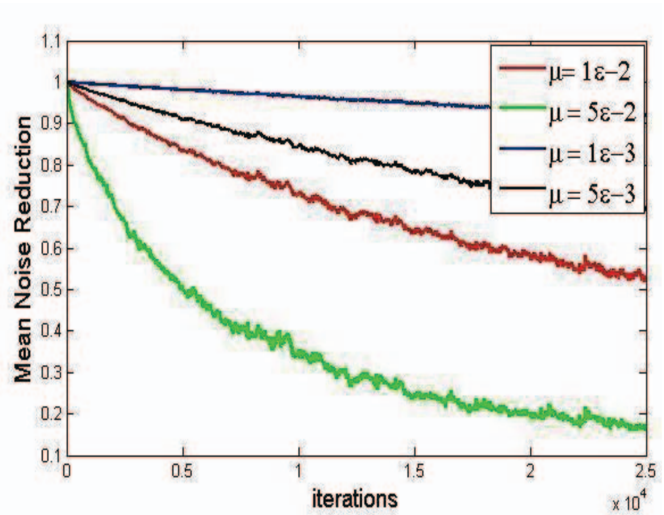
$$A_r(n) = \lambda A_r(n-1) + (1-\lambda)|r(n)| \quad (16)$$

The reference noise, *x(n)* is modeled by standard $S\alpha S$ distribution with α=1.65(Case I), α=1.45(Case II) and α=0.95 (Case III). Here case I represents a less impulsive case, more towards Gaussian distribution. Case II represents more impulsive environment whereas case III represents highly impulsive environment. The simulation results are presented here by using 25 realizations for the ensemble averaging process.

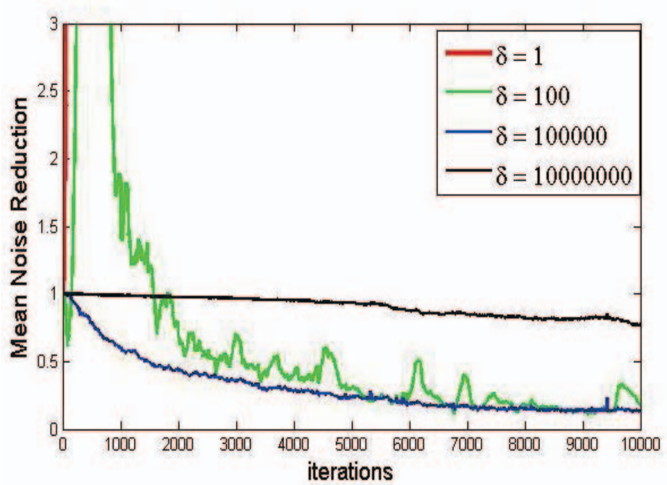
The effect of varying controlling parameters of NSS-FxLMS algorithm, DR NSS-FxLMS algorithm and FxRLS algorithm for case II are given in Fig. 5. The results show that for NSS-FxLMS and DR NSS-FxLMS algorithms $\mu = 5\text{x}10^{-2}$ and for FxRLS algorithm δ = 100000, λ= 0.99999 are the most suitable values. Similarly, extensive simulations are performed to find the best values of these controlling parameters for case I and case III as well.



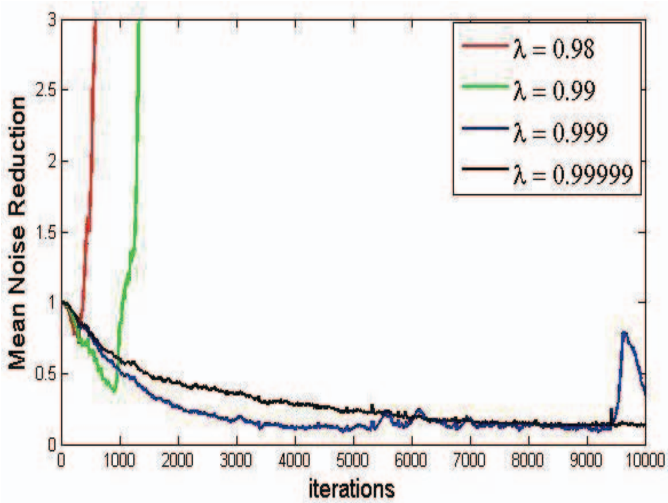

(a)




(b)




(c)




(d)

Fig. 5 Performance of various algorithms for case II (α=1.45). (a) NSS-FxLMS and (b) DR NSS-FxLMS with varying step size (μ), (c) FxRLS with varying delta (δ), and (d) FxRLS with varying lambda (λ)

On the basis of the best results for the respective algorithms the performance comparisons for case I, II and III is shown in Fig. 6, 7 and 8 respectively. For case I, $\mu = 5\text{x}10^{-2}$ for NSS-FxLMS and DR NSS-FxLMS algorithm while δ = 1000 and λ = 0.99999 for FxRLS algorithm. Similarly, for case III, $\mu = 5\text{x}10^{-2}$ for NSS-FxLMS and DR NSS-FxLMS algorithm and δ = 10000000 and λ = 0.99999 for FxRLS algorithm. It can been seen from Fig 6-8, that FxRLS algorithm converges at 5000 iterations to its steady state, whereas the NSSFxLMS and DR-NSSFxLMS algorithms achieve steady state after 15000 iterations. These simulation results assure the superior performance of the proposed solution for ANC of impulsive noise than the existing algorithms in terms of convergence speed and better noise mitigation potential.

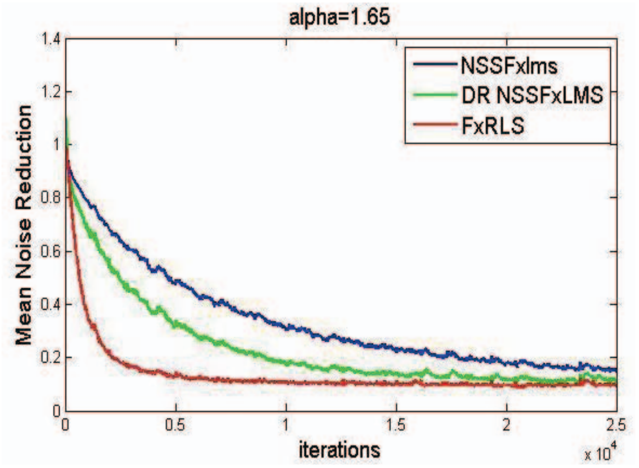



Fig. 6 Performance comparison between various algorithms for case I (α=1.65)

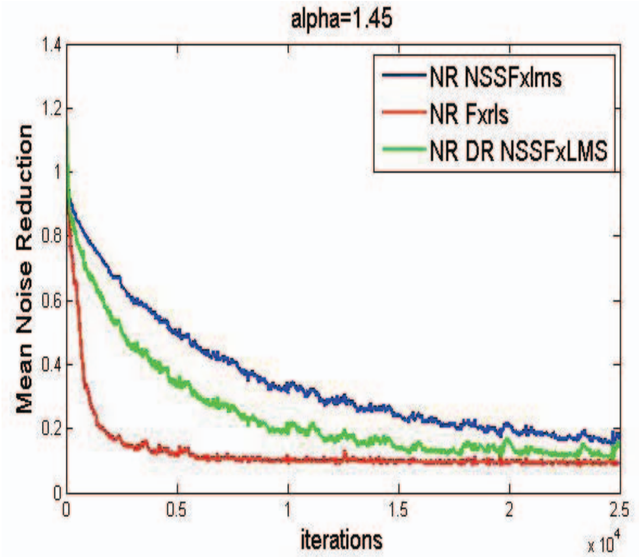



Fig. 7 Performance comparison between various algorithms for case II (α=1.45)

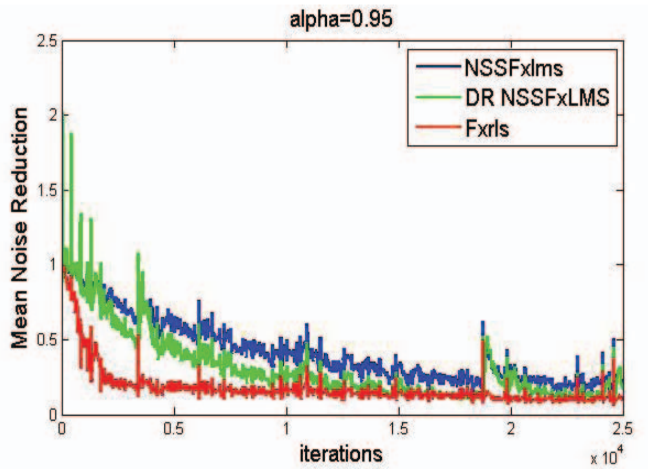



Fig. 8 Performance comparison between various algorithms for case III (α=0.95)

## V. CONCLUSION

In this paper, FxRLS algorithm based ANC for impulsive noise is presented. The main advantage of the proposed solution for impulsive ANC is its fast convergence because of the use of adaptive filter of Least Squares family. Computer simulations have been carried out to validate that the proposed scheme has improved noise cancellation potential and better convergence speed than that of the other investigated algorithms.

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