
GRADUATE SCHOOL
OF ARTS AND SCIENCES
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1. Executive Summary

- The descriptive statistics highlight the profitability of the 15 large-cap value stocks considered. Annualized mean returns range from 7.674% to 14.874%. Annualized standard deviations range from 14.06% to 43.53%. Generalized error distributions and t-distributions are found to be the best fits for most of the assets.
- The MVP and tangency portfolio are computed which outperform the market in every aspect.
- The 5% Value at Risk and Expected Shortfalls are computed for a $100,000 investment in the portfolios constructed and in each asset. The VaRs of the assets range from $5,804 for JNJ to $19,874 for BAC.The Expected Shortfalls of the assets range from $7,601 for JNJ to $29,030 for BAC. There is considerable uncertainty in the risk measures, especially for the nonparametric estimation.
- Principal Component Analysis indicates the limited dimensionality reduction scope. Statistical factor analysis is performed and a few of the factors are interpreted to be a possible hedging portfolio/strategy in different sectors.
- t-Copula is the best fit for the joint distribution of the returns.

2. Descriptive Statistics

For this project, 15 large-cap value stocks traded in the United States are selected for analysis and portfolio construction. The monthly historical adjusted closes from November 2004 to November 2019 are used to calculate monthly returns for the past 15 years (180 months). The equities selected include: Berkshire Hathaway Inc Class B (BRK-B), JPMorgan Chase & Co (JPM), Johnson & Johnson (JNJ), Procter & Gamble (PG), Exxon Mobil Corp (XOM), AT&T Inc (T), Bank of America Corp (BAC), Walt Disney Co (DIS), Intel Corp (INTC), UnitedHealth Group Inc (UNH), Verizon Communications Inc (VZ), Chevron Corp (CVX), Merck & Co Inc (MRK), Wells Fargo & Co (WFC), and Pfizer Inc (PFE). S&P 500 is also included as the market benchmark.

The sample statistics (Means, standard deviations, Skewness Coefficients, Kurtosis Coefficients and beta of each asset) are summarized in Table 1.

UnitedHealth has the highest sample mean return and Bank of America has the highest sample standard deviation while ExxonMobil has the lowest sample mean return and Johnson and Johnson has the second lowest sample standard deviation after S&P 500. It can also be observed that only ExxonMobil hasn't outperformed the market in terms of sample mean returns emphasizing the importance of value stocks.

All stocks have a higher sample standard deviation than the market which is expected. Nine of the stocks show negative skewness. All stock returns show excess kurtosis due to fatter tails except Verizon and Chevron which are platykurtic. Bank of America, Wells Fargo, JPMorgan Chase, Disney and Intel have a beta greater than 1. This indicates that the stocks considered from the finance sector are aggressive.

The effect of the Financial Crisis from 2007 to 2009 can be seen in all the stocks and S&P 500 with returns as low as -52% for Bank of America, -35% for Wells Fargo and -30% for United Health. Berkshire's Class B shares saw a rise of 16% in January 2010 after a 50-for-1 stock split. Procter & Gamble shares saw a rise of 10.75% 10 months after its acquisition of Gillette Co. in a $57 billion deal.

Just prior to the time period considered i.e. in Feb 2005, ExxonMobil was the world's most valuable firm by market capitalization due to soaring oil prices, record-breaking profitability and deals with Qatar to produce liquefied natural gas. The fear of a renewed crude oil glut in 2018 and the suing of ExxonMobil by New York state, alleging that Exxon deceived investors about the risks to its business from climate change regulation led to a return of - 13.3% in Dec 2018. 2016 was a great year for AT&T despite tumbling 14% in Oct 2017(due to intense competition and the impact of recent hurricanes) it made an overall annual return of 23%. It gained 10.3% in June 2016 having integrated DirecTV successfully into its sales pitch and having added 9.5 million new

subscribers. An unveiling of the details of Disney's streaming service plans gave a jump of 23% to the stock in April 2019.

From the equity curve, it can be observed that investing $1 in a stock shows a sevenfold increase in the case of UnitedHealth and a fivefold increase in the case of Disney. JPMorgan Chase and Merck and Co show a fourfold increase. Intel and Berkshire Hathaway show a threefold increase. Bank of America and ExxonMobil are the only stocks which return less than the S&P 500 if 1 dollar were invested over the given time period. Disney and UnitedHealth have the highest Sharpe ratio while Bank of America and ExxonMobil have the lowest Sharpe ratio.

From the QQ plots and box plots,it can be observed that Johnson and Johnson, Procter & Gamble, ExxonMobil, Intel and Pfizer look the closest to a normal distribution,though there are many outliers observed in all of them. Using AIC and BIC, it can be observed that GED and skewed GED is the best fit for half of them while t distribution and skewed t distribution are the best fit for the rest. Wells Fargo has very heavy tails which can explain why nu is less than 1 in the GED fit making the tail weights more extreme than the double-exponential distribution.

3. Individual Asset Risk Management

The estimated 5% Values at Risk (VaR) and expected shortfalls (ES) for individual assets assuming $100,000 investment over a month investment horizon are summarized in Table 2. Among all 15 risky assets included in the portfolio construction, BAC has the highest VaR and expected shortfall using both parametric (based on normal distribution) and nonparametric methods. Even though it is not the worst performer in terms of returns in the given period, it has significantly higher standard deviation than the rest, and that is reflected in the VaR and ES with parametric methods. On the other hand, JNJ has the lowest VaR and ES using both methods. It is also the asset with the lowest risk.

The estimated standard errors (SE) and 95% confidence intervals (CI) for 5% VaR and expected shortfall are also included in Table 2. Note that due to the random sampling in bootstrap method, the estimated standard errors and confidence intervals will vary across simulations.

4. Portfolio Theory and Asset Allocation

With the 15 risky assets discussed above, portfolios are constructed with and without the constraint of short sales.

4.1. Efficient Frontier & Efficient Portfolio

The plots of the efficient frontier using the Markowitz approach and the efficient portfolio are presented in Section 8 and 9 in Appendix, where the plot in Section 8 allows short sales and the plot in Section 9 prohibits short sales. By comparing the two plots, we observe that due to the additional constraint on short sales, the efficient frontier in Section 9 is more bent than the one Section 8. The slope of the efficient portfolio line (i.e. Sharpe ratio) is also smaller. From these plots, we discuss 4 portfolios in more details: minimum variance portfolio (MVP), portfolio with target expected return of 6% per year with only risky assets, tangency portfolio, and portfolio with target expected return of 6% per year with T-bills and tangency portfolio. The details about asset weights, performance, and risk profiles for each portfolio are summarized in Table 3 and Table 4, where Table 3 includes portfolios allowing short selling and Table 4 includes portfolios prohibiting short selling.

4.2. Portfolio 1: Minimum Variance Portfolio (MVP)

MVP is the portfolio that combines different assets to minimize the volatility of the overall portfolio, and they are labelled as Portfolio 1 in Table 3 and 4. If short sales are allowed, this portfolio shorts BAC, CVX, and MRK, and longs all other assets. The annualized expected return is 10.10%, and the annualized risk is 0.1032. MVP risk is smaller than the risk of all individual assets due to the diversification benefit.

The expected return of this portfolio is also higher than 7 individual assets. In addition, this portfolio has better performance than S&P 500 in the given period, with both higher return and lower risk.

If short sales are prohibited, this portfolio assigns zero weights in JPM, BAC, DIS, CVX, and WFC. The annualized expected return is 9.61% and annualized risk is 0.1074. The risk remains lower than all assets, and is slightly higher than the portfolio above due to the additional constraints on short sales. The expected return remains higher than 7 individual assets. This portfolio also has better performance than S&P 500.

4.3. Portfolio 2: Portfolio with target expected return of 6% per year with only risky assets

This portfolio seeks the combination of 15 individual assets such that the portfolio would have a monthly expected return of 0.5%. If short sales are allowed, this portfolio shorts JPM, BAC, DIS, UNH, CVX, and MRK, and longs all other risky assets. Due to the fact that the selected risky assets in this portfolio all have annualized returns higher than 6%, this portfolio shorts the top performing assets and longs the assets with lower returns. It is a less desirable portfolio than MVP, since it generates lower return while bearing higher risk. Compared with S&P 500, it has a lower return, but also lower risk.

Since all risky assets have annualized returns higher than 6%, if short sales are not allowed, a target expected return of 6% cannot be achieved. The minimum return reachable in this situation would be 6.426% per year, when the portfolio assigns 100% weight to XOM only, and 0% weights to all other assets.

4.4. Portfolio 3: Tangency Portfolio

The tangency portfolio is the portfolio on the efficient frontier that has the highest Sharpe ratio. If short sales are allowed, this portfolio longs all assets other than XOM, BAC and PFE. These three assets are also have the lowest individual Sharpe ratios. The tangency portfolio have annualized return of 16.36%, annualized risk of 0.1350, and a monthly Sharpe ratio 0.3220. Even though its return is slightly lower than UNH, the best performer, its Sharpe ratio is much greater than all individual assets. With similar risk levels, it more than doubles the return, as well as the Sharpe ratio, of S&P 500 index.

If short sales are not allowed, the tangency portfolio assigns zero weights to XOM, BAC, PEF, and WFC, the four assets with the lowest individual Sharpe ratios. The Sharpe ratio is 0.2645, which is slightly lower than the tangency portfolio without constraint, but remains above all individual assets. Its annualized return and risk are 12.52% and 0.1225 respectively. Therefore, it is a better performer than S&P 500 in the given period, with higher return, lower risk, and higher Sharpe ratio.

4.5. Portfolio 4: Portfolio with Target Expected Return of 6% Per Year with T-Bills and Tangency Portfolio

This portfolio combines the tangency portfolio with T-bills to seek monthly return of 0.5%. Without the constraints on short sales, this portfolio assigns 68.79% of the weight to T-bills, and the remaining to the tangency portfolio. Due to the zero risk of the T-bills, the annualized risk decreased significantly to 0.0421. For the portfolio with short sales constraints, the weight of the T-bills is 58.11%, and the annualized risk is 0.0513, slightly higher than the one without constraints.

5. Portfolio Risk Management

The estimated 5% Values at Risk (VaR) and expected shortfalls (ES) for portfolios assuming $100,000 investment over a month investment horizon are summarized in Table 3 and 4. The estimated standard errors (SE) and 95% confidence intervals for 5% VaR and expected shortfall are also presented in the same tables.

Due to the inclusion of T-bills, which provides guaranteed return and no risk, Portfolio 4 has much lower Value at Risk and expected shortfall than the other three portfolios and individual assets using both methods. Among the three portfolios with zero weights in T-bills, Portfolio 1 (MVP) has smaller VaR and expected shortfall than the other two. In general, portfolios constructed in Section 4 have lower VaRs and expected shortfalls than individual risky assets as a result of diversification.

6. Principal Component Analysis

Based on the correlation matrix, Disney and JPMorgan Chase show maximum correlation with Wells Fargo.The energy sector stocks, Chevron and ExxonMobil, being rivals in the energy sector are highly correlated. Bank of America and JPMorgan Chase are also highly correlated. Verizon appears to have low correlation with the financial sector stocks. Since most of the correlations are low,diversification can reduce risk with these assets. From Table 5, it can be assessed that one needs eleven principal components to explain 95% of the variance.

Factor Analysis:
The unrotated factors can be interpreted as follows : The first factor appears to be an overall index of the 15 stocks with the largest weight assigned to the financial sector-Bank of America,Wells Fargo and JPMorgan. The second factor has the largest loadings on AT&T and Verizon and so it may be interpreted as a factor more focused on telecommunications. The third factor approximately mimics a hedge portfolio that is long on the energy sector and short on the financial sector.The fourth factor approximately mimics a hedge portfolio that is long on the pharmaceutical sector and short on the financial sector. The fifth,sixth and seventh factor seem less interpretable. The rotated factors seem less interpretable as they are all positive with no particular sector/asset dominating.

7. Copulas

Using the AIC and BIC criteria, the t-Copula is found to be the best fit with the estimate of degrees of freedom equal to 7.6602. Gaussian copula has a correlation matrix, but no tail dependence. The additional parameter in t-Copula, degrees of freedom, allows the model to capture the tail dependence (i.e. the joint riskiness) among different assets. Therefore, the better fit of t-Copula implies that in our dataset, there exists joint fat tails and an increased probability of joint extreme events compared with the Gaussian copula. This is indicative of the market conditions where extreme returns are very plausible.

8. Conclusion

In conclusion, the 15 large-cap value stocks in this project have performed reasonably well compared to the market for the past 15 years, but at the cost of higher variance. However, holding a minimum variance portfolio or a tangency portfolio would still exceed the annualized market returns by more than 2% while lowering the standard deviation.The Values at Risk and Expected Shortfalls on a $100,000 investment in these portfolios are also low indicating that an investor holding a portfolio with similar weights during this period would have had a substantial profit. Principal Component Analysis highlights the fact that dimensionality reduction is difficult here: 11 components are required for accurate representation. Factor Analysis presents ideas about different ways to invest.