

## 9. Table Doubling, Karp Rabin

Link: <https://www.youtube.com/watch?v=BRO7mVIFt08>

How to choose  $m$ ?

- We want  $m = \Theta(n)$ 
  - $\alpha = \Theta(1)$
- Ideal start small:  $m = 8$
- grow/shrink as necessary

Grow table:

- $m \rightarrow m'$
- Allocate memory and rehash
- Make table of size  $m'$ 
  - Build new hash  $f'$
  - Rehash:
    - For each item in  $T$ :
      - $T'.insert(item)$
- Will take  $\Theta(n+m+m')$  which is  $\Theta(n)$ 
  - $n$  is the amount of  $k,v$  in linked list,  $m$  is spots in original hash table, and  $m'$  is writing them to the new hash table
- $m' = 2m$  aka table doubling
  - Grows in  $\Theta(1 + 2 + 4 + 8 + \dots + n) \leftarrow$  Grows in a geometric fashion
  - $\Theta(n)$
  - Some of them are restructured at  $\log n$  speed while others grow at  $n$  due to a quick insertion in table with no collision

Amortization:

- Operation takes " $T(n)$  amortized"
  - If  $k$  operations
  - Take  $\leq k * T(n)$  time
  - Think of meaning
  - $T(n)$  on average where average over all operations
- Table doubling:
  - $K$  inserts
  - Take  $\Theta(k)$  time
  - $\Theta(1)$  amortized/insert
  - Also,  $k$  inserts and deletes take  $O(k)$
- Table shrink
  - Slow method: If  $m = n/2$  then shrink by  $m/2$

- The issue is that if you have 8 keys in a table size of 8 and add a 9th. You double your table size. Yet, if you minus 1 out, you shrink again. So you're running an expensive operation of shrinking and growing the table when alternating between 8 and 9.  $\Theta(n)$  per operation
- If  $m = n/4$  then shrink  $\rightarrow m/2$ 
  - Amortized time  $\rightarrow$  constant  $\Theta(1)$

## String Searching

### Simple (Naive) Algorithm:

- any( $s == t[i:i + \text{len}(s)]$  for  $i$  in range( $\text{len}(t) - \text{len}(s)$ ))
- Runs in  $O(|s| * |t|)$ , can be quadratic

### Rolling Hash ADT:

- `r.append(c)`:
  - Add char  $c$  to end of  $x$
- `r.skip(c)`: delete first char of  $x$  (assuming it is  $c$ )
- $r$  maintains a string  $x$ 
  - `r()`: hash value of  $x$   $x = h(x)$

### Karp Rabin algorithm:

- Uses rolling hashes
- For  $c$  in  $s$ : `rs.append(c)`
- For  $c$  in  $t[:\text{len}(s)]$ :
  - `rt.append(c)`
- If `hs() == ht():...`
- For  $i$  in range( $\text{len}(s), \text{len}(t)$ ):
  - `rt.skip(t[i - \text{len}(s)])`
  - `rt.append(t[i])`
  - If `rs() == rt()`
    - # This does not mean the strings are equal, there could be a collision hash
    - Check whether  $s == t[i - \text{len}(s) + 1 : i + 1]$
    - If equal
      - Found match
    - Else:
      - Happens with probability  $\leq 1/|s|$
      - $O(1)$  expected time
- It all amounts to linear time  $O(|s| + |t| + \# \text{match } |s|)$  in expectation
- Use a random prime number  $\geq |s|$

- Treat  $x$  as multidigit number  $u$  in base  $a$  (alphabet size)
- The picture below shows the operation and code for recalculating the hash value when appending and skipping

