# Liquidity Concentration

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## 1 Limit Order Book Liquidity

Order book is the continuous trading system for different types of assets - stocks, currencies, cryptocurrencies etc. There are market orders - immediately buy or sell order to execute at the market price, and limit orders - with the specified price at which one is willing to buy or sell the asset. Limit orders are stored in the book until they are excuted or canceled by liquidity providers. Centralized cryptocurrency exchanges use order books to trade assets, while for decentralized exchanges storing orders and executing trades in order book style would be too costly and they use Automated Market Making system to execute trades.

We use Limit Order Book cryptocurrency trading data from Binance to understand the demand on ask side for different price ranges. This would help us to improve the efficiency of liquidity providers capital by concentrating enough liquidity where the demand is. We queried the data from Binance in real time - order book snapshots were taken every 30 seconds for 5 hours. The example of order book snapshot and market depth for UNI-USDT pair is shown in Figure 1.

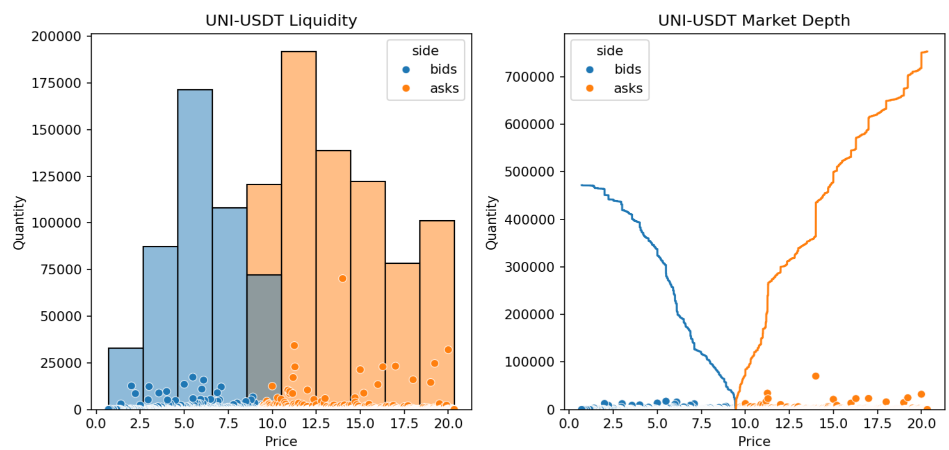


Figure 1: Example of Liquidity distribution and Market Depth snaphot for UNI-USDT pair in Binance.

### 1.1 Modelling ES as Limit Order Book

Limit order books reflect well the near term expectations of liquidity providers and traders about the price volatility. Here we try to model the AMM curve with automated liquidity concentration feature as an order book.

To convert the EulerSwap AMM curve to the order book we first need to do the following adjustments to the curve - set $x_c$ equal to the total liquidity on the ask side of order book (LOB), then set the $p$ equal to the minimum ask price in LOB and maximum price range in ES curve equal to the maximum ask price in LOB. After making these adjustments and keeping $c$ arbitrary for now, we can find the liquidity in every price range either using the binary search method or use the Equation derived in Whitepaper to find the centre of liquidity for every price within a range:

$$x_a = r_x \cdot \sqrt{\frac{1 - c_x}{p_a \cdot p^{-1} - c_x}}. \tag{1}$$

Then, after liquidity centre is found, the total amount of liquidity between two price ranges can be found by taking the difference. The example of ES curve represented as an orderbook is shown in Figure 3.

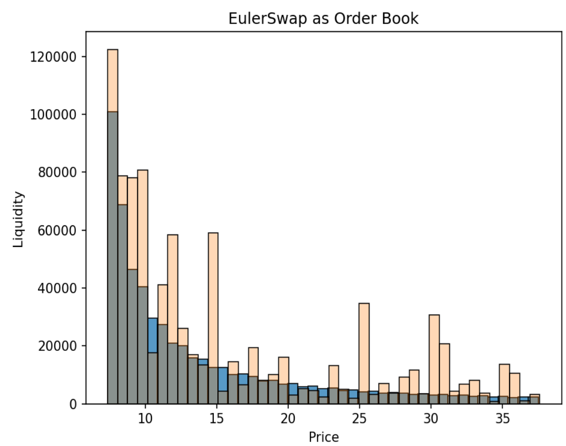


Figure 2: ES curve represented as an LOB - blue histogram is ES curve, pale orange is the LOB for UNI-USDT pair in Binance.

However, we can not set the $c$ in ES curve arbitrarily - to match the pattern of ask side orders, we need to find the $c$ that concentrates liquidity in AMM in the same pattern. The optimal $c$ can be found by creating ES curve with all values of $c$ from 0 to 1 and then deciding which $c$ value fitted the best. To decide on the best fit, we chose to use two different metrics - euclidean distance and histogram intersection.

Euclidean distance was calculated between liquidity in Binance vs liquidity in fitted EulerSwap curve for every price range for every value of c factor. Then, the c factor with the minimum distance was chosen as an optimal.

Histogram intersection is an alternative method to choose the best fitted histogram -

$$I = \frac{\sum_{j=1}^{n} min(I_j, M_j)}{\sum_{j=1}^{n} (M_j)} \tag{2}$$

where $I_j$ is the liquidity in LOB in price range $j$ and $M_j$ is the liquidity in fitted EulerSwap histogram at the same price range.

The dynamics of optimal $c$ according to two metrics is quite different - according to the euclidean distance, the $c$ doesn't change much within 5 hours, while according to the histogram intersection, it can fluctuate a lot in short time steps. To check which metric gives better fitting results, we plotted more histograms similar to the Figure 3 to explore visually and euclidean distance showed to be more accurate metric to decide on the optimal $c$.

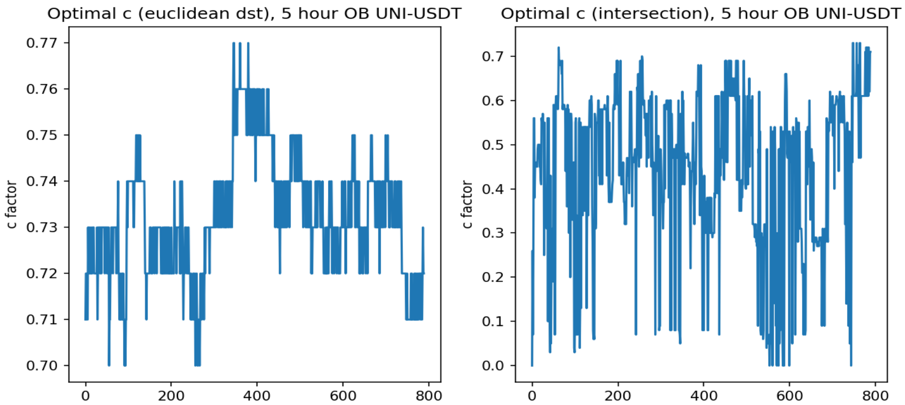


Figure 3: Optimal $c$ evolution in 5 hours according to different metrics.

# 2 Uniswap V3 Liquidity Dynamics

Unfortunately, queries on Binance API are limited to 5000 orders on each side of the book and there is no opportunity to query the historical order book snapshots. Looking only at the short time period LOB is not sufficient to model the concentration factor. Therefore, we decide to gather more information from the Uniswap V3 Liquidity dynamics for longer time periods. In Uniswap V3 liquidity providers can provide liquidity in a specific price ranges, optimizing their capital efficiency. The LP expectations on price moves in UniswapV3 is somewhat similar to the expectations of LPs in limit order books.

Figure 4 shows how liquidity providers distributed their capital - most of the capital is concentrated around current price. We are interested in fitting the EulerSwap curve to the right side of liquidity distribution histogram - price ranges equal or greater than current price $p$.

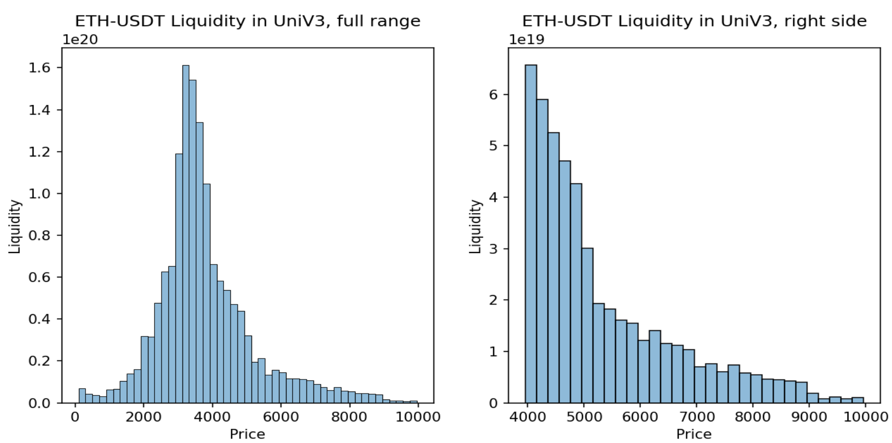


Figure 4: Example of liquidity distribution in ETH-USDT pool in Uniswap V3.

## 2.1 Modelling ES as concentrated liquidity AMM

Similar to how EulerSwap curve was modelled to replicate the liquidity in Binance LOB, we use the equation 1 to fit the EulerSwap curve to the Uniswap V3 liquidity histogram. Figure 5 is an example of snapshot where EulerSwap curve with properly chosen $c$ factor can replicate the liquidity distribution in UniV3.

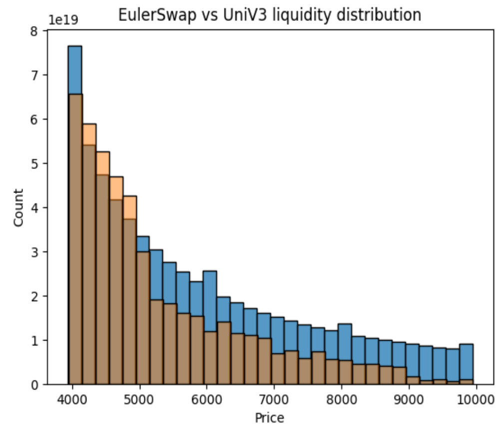


Figure 5: ES curve represented as an LOB - blue histogram is ES curve, pale orange is the liquidity for ETH-USDT pair in UniswapV3.

Using similar approach as with fitting ES to the Binance LOB, we calculate euclidean distance for all values of c factor and choose the c which has the minimal distance as the optimal one. Then we obtain time series of optimal c factor and explore it together with other information of pool's assets - price, volatility and liquidity.

## 2.2 Analysis of Liquidity Dynamics in Uniswap V3

First, we just visually inspect time series of optimal $c$ factor that we found after fitting ES to the UniV3 liquidity histograms and price time series. Interestingly, from Figure 6 it seems that the liquidity concentration moves in a similar pattern as price. Plotting the volatility of price and $c$ factor showed clearly that these two series are highly correlated - 75% and the relation seem to be linear.

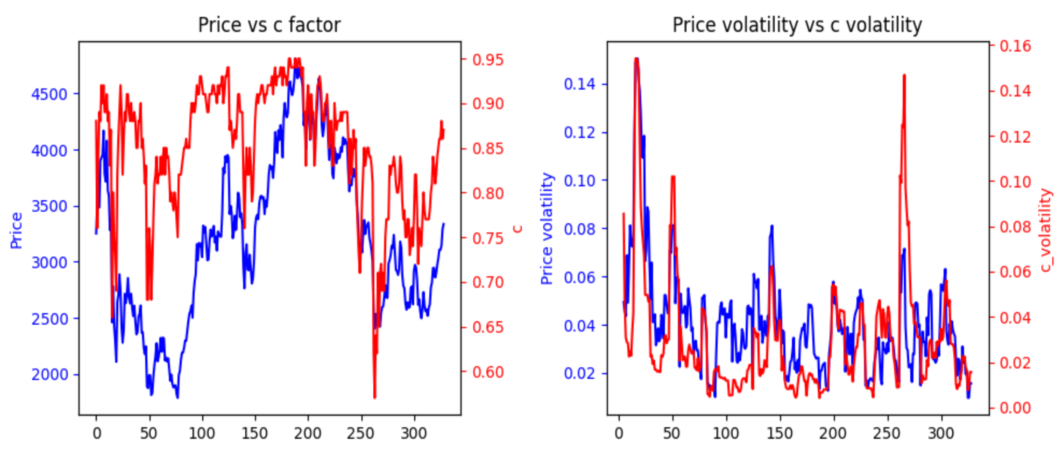


Figure 6: Price and optimal $c$ factor time series.

We further investigate into the relation between these two time series and build a linear regression model between the change in price and in $c$ factor:

Regression results show the adequate linear dependence between two time series - $R^2$ is 59%, which is not very high but regression residuals were found to be random with no heteroscedasticity.

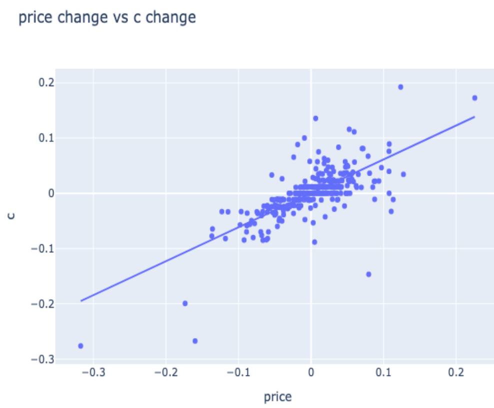


Figure 7: Linear Regression between price change and $c$ factor change.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:               c_change   R-squared:                       0.591
Model:                            OLS   Adj. R-squared:                  0.589
Method:                 Least Squares   F-statistic:                     464.3
Date:                Mon, 27 May 2024   Prob (F-statistic):           2.15e-64
Time:                        16:50:21   Log-Likelihood:                 704.23
No. Observations:                 324   AIC:                            -1404.
Df Residuals:                     322   BIC:                            -1397.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0003      0.002      0.180      0.857      -0.003       0.003
return         0.6444      0.030     21.548      0.000       0.586       0.703
==============================================================================
Omnibus:                       60.579   Durbin-Watson:                   2.412
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              720.298
Skew:                          -0.263   Prob(JB):                    3.88e-157
Kurtosis:                      10.286   Cond. No.                         19.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
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Figure 8: Linear Regression results statistics

This means that overall the price increase lead to a more concentrated liquidity around it in UniV3. While price decrease would lead to more flattened liquidity concentration.

Concentrating liquidity in a similar pattern as UniV3 liquidity providers does not necessarily mean the effective capital allocation. We want to know how much liquidity gets actually utilized during different price movements and whether concentrating more/less during price increase/decrease is an efficient strategy. For this, we additionally look at the time series of capital utilization - how much liquidity in a pool gets utilized by trades in general and within the certain price range.

From Figure 9 we can see that there are times when liquidity concentrated around current price is not sufficient for trades - during some periods (with the increased price volatility), liquidity concentration drops while the demand for trades increases. This makes sense - when prices are fluctuating, LPs would prefer to distribute their liquidity across wider price ranges, while trading activity increases. However, if more liquidity was concentrated at the current price during this time, the more profitable it would have been for both traders (having lower slippage) and LPs (gaining more fees). In fact, we did not even need highly concentrated liquidity - the maximum utilization was 30%. Concentrating just 20-30% of available liquidity around current price could have been more efficient and still safe - since the rest of liquidity (70-80%) would have been distributed across wider price ranges.

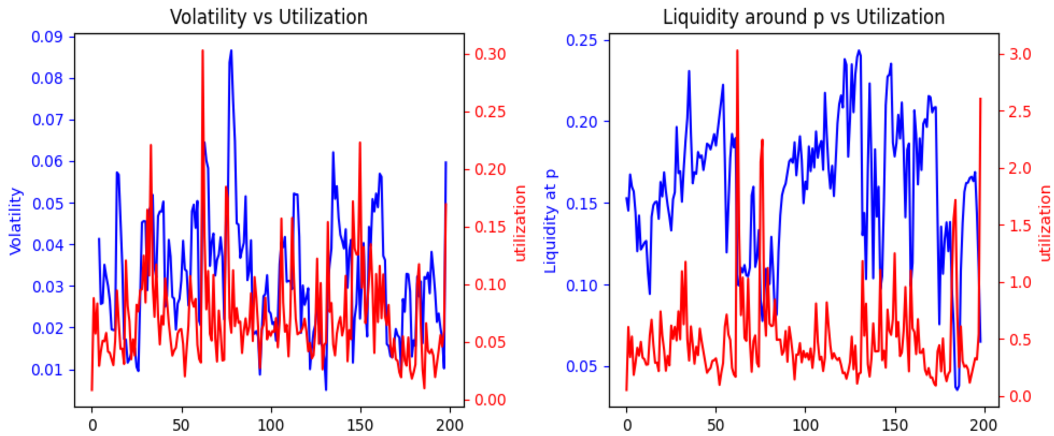


Figure 9: Price volatility and utilization time series. Amount of liquidity (percentage from the total in pool) and utilization of it.

# 3 Liquidity Concentration in EulerSwap

The optimally chosen concentration factor is expected to

1. Low slippage and therefore, attract more trades
2. Smooth trade execution

## 3.1 Constraints.

In EulerSwap we concentrate liquidity around the peg price - peg price stays constant until the re-peg condition is met. Re-peg happens when the difference between peg price and oracle price is equal or greater than $n\%$. Oracle in Euler Swap is the TWAP (or EMA) of marginal prices over certain time period. This means that in order to peg price to keep up with the broader market price, the arbitrage should happen smoothly and bring the peg price close to the market price. The higher the concentration factor, the more difficult it is to arbitrage the price - the larger arbitrage trades would be needed. In case of low volatile asset pairs, this is not a big concern, but if we have a pool with highly volatile pair, the smooth arbitrage would be very important. Therefore, we take this as a constraint to limit the maximum value of concentration factor for the given pool depending on the volatility.

To find what would be the maximum c factor at which the arbitrage can not be performed efficiently, we do the following:

For example, from Figure 10 we can see that if the expected maxium price change is 10%, and we don't want more than half of the pool's liquidity to be used for arbitrage, then maximum value of $c$ should not exceed 0.65. The same approach can be applied for different type of pools to cap the value of $c$.

## 3.2 Model for dynamic concentration factor

One of the desirable properties of concentration factor dynamics is to attract more trades by lowering the slippage. To achieve this, we can adjust $c$ by increasing it in times of low trading volume and

**Algorithm 1:** Maximum concentration factor

**Input:** $\sigma$, $L$

**Output:** arbitrage amount

1 Set the maximum price change, according to the given $\sigma$

2 Simulate trade volumes up to 100% of liquidity pool size

3 Calculate arbitrage amount $a$ needed for every trade

4 Calculate ratio

$$r = \frac{a}{L}$$

5 Repeat for every value of $c$

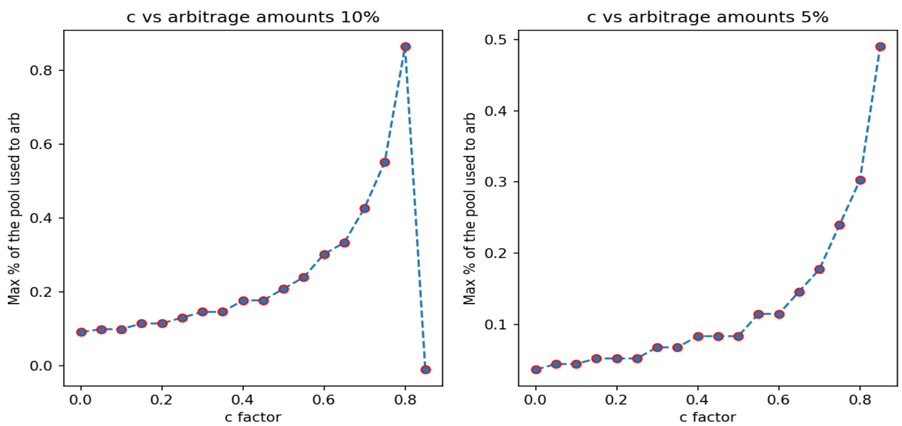


Figure 10: Example of arbitrage amount needed for different c, when maximum price change is 5% and 10%.

decreasing when volume is too high. One way of adjusting $c$ depending on the trade volume is by implementing the following equation:

$$c_t = \frac{c_{max} \cdot \Delta U}{\sqrt{p + \Delta U^2}} \tag{3}$$

where $\Delta U = U_{max} - U_t$, $U_{max}$ is the maximum utilization of liquidity in a pool we want to achieve and $U_t$ is the utilization at current time step. $p$ defines the steepness of curve - how fast $c$ when $U_t$ is smaller than $U_{max}$.

Figure 11 shows how $c$ would change depending on the current trade volume - with the small value of $p$ concentration factor increases sharply once the trade volume is lower than the maximum. The bigger the $p$ - the slower $c$ is increased and also it should be noted that this model is asymptotic and with the larger values of $p$ it never reaches the $c_max$ even with zero trading volume.

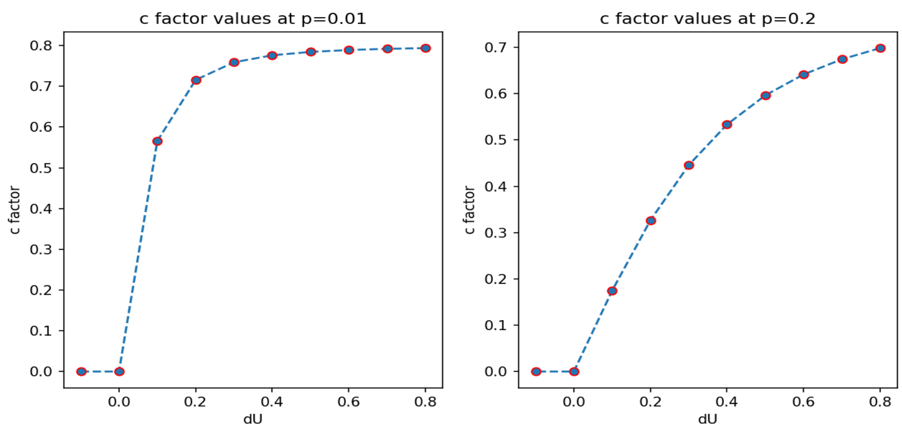


Figure 11: Behavior of dynamic $c$ model depending on the value of $p$