Impermanent Loss and Trading Fees in AMMs

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1 Introduction

Providing liquidity to AMMs should make an economic sense - in other words, benefits of it should outweigh its costs. Liquidity provision costs come from the AMM rebalancing techniques - different type of AMMs have different pricing mechanism and it automatically changes liquidity providers' portfolio in order to match the market price. Benefits of liquidity provision come from accruing trading fees - traders are charged a percentage of swap value that benefits liquidity providers. Ideally, accrued fees should outweigh the portfolio loss. Of course, there is also a time factor - the longer liquidity stays in a pool, the more fees it gains, but there is also higher chance of large price moves that would lead to bigger losses.

2 Metrics for Liquidity Provider performance.

2.1 Impermanent Loss

Impermanent Loss - difference in value between holding the assets and adding them into the liquidity pool. Classic definition for LP losses:

$$IL = \frac{HoldValue - PoolValue}{HoldValue} \tag{1}$$

where $HoldValue$ is the value of assets if they were held outside of AMM pool and calculated as follows:

$$HoldValue = x_{t0} \times p_{xt} + y_{t0} \times p_{yt} \tag{2}$$

x_{t0} and y_{t0} are number of tokens x and y in pool at the start.

Pool value is calculated as follows:

$$PoolValue = x_t \times p_{xt} + y_t \times p_{yt} \tag{3}$$

where x_t and y_t are the number of tokens x and y respectively at the time t. As we can see, the main difference between $HoldValue$ and $PoolValue$ is the number of tokens. The number changes every time step to re-balance the pool according to the market price. Pool re-balancing depends on the mechanism of AMM.

2.1.1 Constant Product AMM

. The pricing curve in CPAMM is defined as follows:

$$x_t \times y_t = k \tag{4}$$

Since liquidity provider's deposits to the pool should always be 50/50 in denominated value (USD), then the following equilibrium condition is always true for CPAMM pools:

$$x_t \times P_{xt} = y_t \times P_{yt} \tag{5}$$

From here, we can express y in terms of x and re-write the expression for k:

$$k = x_t \times \frac{x_t \times P_{xt}}{P_{yt}} = x_t^2 \times \frac{P_{xt}}{P_{yt}} \tag{6}$$

Since k is the constant, we can find the number of tokens x after each price change as follows:

$$x_t = \sqrt{k \times \frac{P_{yt}}{P_{xt}}} \tag{7}$$

Using the same logic, we can find the number of token y in a pool at each time step:

$$y_t = \sqrt{k \times \frac{P_{xt}}{P_{yt}}} \tag{8}$$

Then we can calculate impermanent loss as shown above using x_t and y_t values found by equations 15 and 16.

2.1.2 Concentrated Liquidity AMM (Uniswap V3)

. In Uniswap V3 liquidity providers can provide assets in a certain price range (instead of providing along the entire price curve as in CPAMM) that makes their liquidity more capital efficient.

To find impermanent loss in UniV3, we need to find how AMM re-balances token numbers in a pool. In other words, same as before, we need to find x_t and y_t. Since LP positions are defined in a certain price range (p_a - lower price range and p_b - upper price range), there can be 3 different scenarios - 1. If price moves below the defined range $P \leq p_a$, LP position consists only of asset x. 2. If price moves above the defined range $P \geq p_b$, then LP position consists only of asset y. 3. If price is within the defined range $p_a < P < p_b$, then LP funds are traded as CPAMM curve with leverage.

For these 3 scenarios impermanent loss is calculated differently. We refer to the main equation of Uniswap V3 that describes the real reserves of LP position:

$$(x + \frac{L}{\sqrt{p_b}}) \times (y + L\sqrt{p_a}) = L^2 \tag{9}$$

From this equation we can define the x and y for each scenario. In first scenario $y = 0$, then we can re-write equation 9 as follows:

$$(x + \frac{L}{\sqrt{p_b}}) \times L\sqrt{p_a} = L^2 \tag{10}$$

And therefore, the x value can be expressed in the following form:

$$x = \frac{L}{\sqrt{p_a}} - \frac{L}{\sqrt{p_b}} \tag{11}$$

For the first scenario, $PoolValue = x \times p_{xt}$. And impermanent loss can be calculated using formula 9.

In second scenario $x = 0$ and equation 9 becomes as following:

$$\frac{L}{\sqrt{p_b}} \times (y + L\sqrt{p_a}) = L^2 \tag{12}$$

From here, we can express the y:

$$y = L \times (\sqrt{p_b} - \sqrt{p_a}) \tag{13}$$

In this case, $PoolValue = y \times p_{yt}$ and impermanent loss can be calculated using formula 9.

For the third scenario, when price is in the defined range, pricing follows CPAMM rules and therefore, the equilibrium condition shown in equation 13 holds true and can be defined for the Uniswap V3 position as follows:

$$x \times \frac{\sqrt{P} \times \sqrt{p_b}}{\sqrt{p_b} - \sqrt{P}} = \frac{y}{\sqrt{P} - \sqrt{p_a}} \tag{14}$$

The left and right sides of the equation are simply the L defined from equations 11 and 13 respectively. The reason L is defined this way is that when the price is in the range (p_a, p_b), we can divide the position into two sub-ranges - (P, p_b) is where x liquidity is and (p_a, P) is where y liquidity is. To get values for x and y we can use the equations 11 and 13.

2.1.3 Automated Liquidity Concentration AMM (Curve V2)

. In Curve V2 pricing curve is the weighted average of Constant Sum and Constant product AMMs. Similar to its stableswap formula, there is a parameter A - amplification factor that defines the shape of the curve (either more like CSAMM or CPAMM). To make the curve applicable for non-stable assets, new parameter K was introduced - which is amplification factor A with additional parameters K_0 and γ:

$$KD^{N-1} \sum x_i + \prod x_i = KD^N + (\frac{D}{N})^N \tag{15}$$

where K is defined as follows:

$$K = AK_0 \frac{\gamma^2}{(\gamma + 1 - K_0)^2} \tag{16}$$

Parameter K_0 checks the ratios between token reserves:

$$K_0 = \frac{\prod x_i N^N}{D^N} \tag{17}$$

As we can see, when assets have same price, $K_0 = 1$ and $K = A$ and we converge to the Stableswap formula.

As for impermanent loss in Curve V2, it is only impermanent for some period between pricing curve repegs. After repeg happens, loss becomes realized. Repeg is the event when D in the curve changes to reflect the total deposits fair to the current price:

$$D = Nx_{eq} \tag{18}$$

where the x_{eq} is the equilibrium point or in other words, the fair amount of tokens reflecting the current price. As we see from equations 15 and 17, D parametrises the curve and every time the shape of the curve changes, the loss becomes realized by liquidity providers.

The total value of portfolio is quantified as:

$$X_{cp} = (\prod \frac{D}{Np_i})^{\frac{1}{N}} \tag{19}$$

Every time D changes, the value of portfolio changes and LP can face losses.

2.1.4 Homotopy Curve AMM (EulerSwap)

. In EulerSwap liquidity provider has an option on how to provide liquidity to the pool - whether it is just one asset or both assets in different proportions. In EulerSwap liquidity ptovider's logic and motivation are somewhat different from the LPs in CPAMM-like markets: in CPAMM, LP provides assets to the pool in equal proportion and hopes for the profit from trading fees. In EulerSwap, there are more strategies and some of them can be seen as liquidity provision in order book style exchanges.

For instance, LP providing only token A to the AB pool is taking short position on A (in other words, he wants to sell his token A in exchange for token B and earn fees), because when his liquidity is used for swaps, his re-balanced position will consist of both token A (remainder after the swap) and token B (what he got in return for used A). Therefore, for single-sided LPs the classic definition of impermanent loss can not be applied, since they open position with a clear strategy and any price movements of assets is a trader's risk rather than impermanent loss that comes when AMM re-balances LP's portfolio to match the current price.

For two-side liquidity providers however, the classic definition of impermanent loss is still applicable - every time AMM re-balances their position, they might experience loss. How does EulerSwap mechanics re-balances LP portfolio? With every swap, in LP portfolio can be more tokens A or B depending on the direction of swap. For example, LP provided 100 tokens A and 100 tokens B (it does not have to be 50/50 provision), then 50 tokens A were swapped out for 25 of token B. Now LP has 50 A tokens and 125 B tokens. There is no loss due to re-balancing if the price did not change. But if price of token B went down, then LP is facing a loss. Impermanent loss in EulerSwap is basically depends on how big was the amount of swap and price change between assets A and B.

Using the same definition of impermanent loss as above for the other types of AMMs, we can calculate it for EulerSwap. Values of x_t and y_t if token x was swapped out from the main curve $f(x)$ for $f(x)$-side liquidity providers are as follows:

$$(x_t)_f = ((x_{t0})_f - (\Delta x)_f + (\Delta y)_f \cdot \frac{p_y}{p_x}) \tag{20}$$

And value of y_t for $f(x)$ liquidity providers is:

$$(y_t)_f = 0 \tag{21}$$

Values of x_t and y_t if token y was swapped out from the main curve $g(y)$ for $f(x)$-side liquidity providers is as follows:

$$(x_t)_f = (x_{t0})_f \tag{22}$$

And value of y_t for x liquidity providers is:

$$(y_t)_f = 0 \tag{23}$$

Values of x_t and y_t if token x was swapped out from the main curve $f(x)$ for y-side liquidity providers is as follows:

$$(x_t)_g = 0 \tag{24}$$

And value of y_t for y liquidity providers is:

$$(y_t) = (y_{t0})_g \tag{25}$$

Values of x_t and y_t if token y was swapped out from the main curve $g(y)$ for y-side liquidity providers is as follows:

$$(y_t)_g = (y_{t0})_g - ((y_{t0})_g - (\Delta y)_g + (\Delta x)_g \cdot \frac{p_x}{p_y})) \tag{26}$$

And value of x_{ty} for y liquidity providers is:

$$(x_t)_g = 0 \tag{27}$$

It should be noted that if we are calculating IL for a single user, then Δx is not an entire amount of swap happened in a pool, but rather the relative amount of it, proportional to the user's share in a pool.

If token y was swapped out, then equations 20 and 21 will be for y_t and x_t respectively. After x_t and y_t are found, the $PoolValue$ can be calculated as in equation 3 and impermanent loss can be found as in equation 1.

Here we show only impermanent loss for the negative returns of y token for x liquidity providers. Same dynamics can be expected for y liquidity providers when x token price moves down.

3 Trading fees

Size of trading fees in AMM should be found as an optimal balance between the two goals - 1. Benefit liquidity providers for supplying liquidity and 2. Minimize price impact for the trader.

Ideally, liquidity providers should be compensated for the impermanent loss and benefit from the accrual of fees paid by traders. However, one also should keep in mind that trading fees can't be too high - having too large of a price impact on trades would discourage swappers from using the AMM. As market conditions constantly change, the optimal balance between LP's and trader's benefit also changes. Having dynamic fee that automatically adjusts to the market conditions would help to find the balance between incentivising LPs to provide liquidity to the AMM and traders to actively use it.

Most of the AMM protocols implement flat trading fee - fixed percentage from the trade size. There are also some interesting trading fee models that aim to attract more liquidity providers and reduce price impact for traders.

3.1 Review of fee models and their performance.

3.1.1 Volume-based dynamic fees (Kyber Network (CPAMM))

Goal of this model is to increase fees when market is moving fast and to reduce the spread when market is quiet in order to attract more traders. In Kyber fees are adjusted based on on-chain trading volume in each pool.

There is a base fee pre-defined by AMM and a dynamic part which is a sigmoid function:

$$Z = \frac{m(r_\sigma - n)}{\sqrt{p + (r_\sigma - n)^2}} \tag{28}$$

where m is the maximum fee, n is the breakpoint to encourage or discourage trades, p is how fast fees grow after the breakpoint and r_σ is ratio between short-term and long-term traded volume.

Base fee and parameters for the dynamic part are determined based on the cross-volatility of the assets in a trading pair. Pools are divided into 4 groups - stable asset pairs, strongly correlated, correlated and weakly/uncorrelated assets. Base fee and maximum fee vary depending on the type of pool - higher values for less correlated assets.

3.1.2 Tier-based percentage fee (CEXs)

Centralized exchanges implement the tier-based fee model where small trades have larger fee percentage, while big trades are charged less. This can be a solution for reducing the price impact for a trader - the bigger the trade, the bigger the price impact and charging less fee would reduce the final impact.

3.1.3 Dynamic fee based on the divergence from oracle price.

This model is implemented in Curve V2 and it serves two purposes - boosting LP profit and insentivizing the pool re-balancing to match the oracle price. There are three parameters in a fee model - f_{mid} is the minimum fee values, when the current pool state fully matches the oracle price. f_{out} - the maximum fee when pool's state is highly imbalanced and γ_{fee} - decides how quickly fees increase with the imbalance.

The final model of the dynamic is an weighted average between minimum f_{mid} and maximum f_{out} fee values:

$$fee = g \times f_{mid} + (1 - g) \times f_{out} \tag{29}$$

The weight g is decided based on the pool imbalance during the period N and parameter γ_{fee}:

$$g = \frac{\gamma_{fee}}{\gamma_{fee} + 1 - \frac{\prod x_i}{(\sum x_i/N})^N} \tag{30}$$

3.1.4 Volatility-adjusted fee (Hydraswap)

In this model, the velocity of AMM is calculated first as ration of traded volume vs TVL. Then, the LP wealth in no fee environment is estimated assuming that the square root of price movements follow the geometric Brownian motion. Then, the fee model is cinstructed as the function of the AMM velocity and expected LP loss. However, there are still some open questions - which model to use as a predictor for future price movements (EWMA is suggested), how often the model need to be re-calibrated, how to calculate the velocity - for each pool or across entire AMM and how big should be fee ranges.

3.1.5 Flow-driven dynamic fees

There are few models proposed based on the trading flow identification - main idea is to have higher fee for the toxic flow (arbitrageurs) and lower fee for uninformed flow (traders). Couple methods were proposed to identify the toxic flow - based on the trade size, on the number of swaps originated from the wallet. However, in practice these ideas are not easy to implement on-chain.

3.1.6 Dynamic fee predictor model (Crocswap)

This model was built on an idea that given certain fee tiers (0.05%, 1% and 0.3% un Uniswap e.g.) it is possible to predict the best performing fee tier in the next time interval. The general idea of the model is to look back at the past time intervals to see which fee tier has accumulated more fees. Then, using all the available historical data determine the expected highest pay-off. This simple model has been later extended to make predictions possible for more time intervals ahead. The extended model is a linear regression model between chosen predictors and fee-tier.

3.2 Dynamic fee model for Euler Swap

As it was shown in Section 2, different AMM pricing curves would result in different values of impermanent loss for LPs. Therefore, it makes sense to tailor dynamic fee model for the given type of AMM.

When pool is at the point (1, 1) on a normalized EulerSwap curve - it is a balanced pool. When the point moves away from the centre (after swaps) it leads to the re-balance of LP's initial liquidity provision. Depending on the new point on a curve, LPs share in a pool could worth less than initially provided. The dynamic part of the fee model is designed to compensate for the risk of LP's share rebalancing due to the swaps and volatility.

Fee model for EulerSwap consists of two parts - base fee and dynamic fee. Base fee depends on the c - concentration parameter of the curve. Dynamic part of the fee aims to reduce the price impact for traders and compensate for the loss LPs face with every curve re-peg. As it was discussed before, there is an impermanent loss that LPs can face depending on the direction of swap and price movement. To address the realized loss for LPs, we should take into account both the amount swapped and the price change. For example, if token x was swapped out from the pool in exchange for token y and price of token x went up, it means there is a loss for LP, since their position has more of cheaper token y after the swap. In opposite situation, when price of token y went up, LPs do not have any loss. This gives a room to decrease fees for traders without hurting LPs.

Overall, there are two types of pools - stable pair pools (stablecoins, LSDs) and volatile pair pools. For stable pair pools impermanent loss for LPs is minimal, but there is a small chance of assets' de-pegging. While in volatile pools, main risk for LPs is an impermanent loss. We propose two types of dynamic fee model for two types of pools.

1. Model for Stable pools. In ideal scenario, price of token x and y are close to each other and there is little to none of volatility. Therefore, since there is no loss due to volatility for LPs, we can minimise trading fees to encourage more trades that will be profitable for both traders and LPs. However, in bad scenario, if de-peg happens between x and y, it's important to quickly react and discourage speculations. Fees can serve as an additional security layer during de-peg event - raising trading fees quickly proportional to the de-peg, can help to minimise de-peg consequences such as draining liquidity in a pool.

Proposed trading fee model for Stable pools is as follows:

$$Fee = base + dynamic \tag{31}$$

where $base$ is the base fee and it is a function of concentration parameter - the higher the c, the lower the base fee. For stable pools, concentration parameter c is high (close to 1) and therefore, base fee is small.

Dynamic part of the fee increases as a sigmoid function proportional to the distance from the centre point (1, 1):

$$dynamic = \frac{m \times (n - d)}{\sqrt{p + (n - d)^2}} \tag{32}$$

where $d = abs(\frac{x - x_0}{x0})$, and n is a break point - when we want to start increasing the fees. For example, if $n = 0.1$, we start increasing fees as soon as center point moves away from (1,1) by 10%. m - is the maximum dynamic fee, p - is the scaling factor that determines how quick fees will grow with the distance.

For stable pools, it makes sense to allow some volatility and set n little lower than 1, but once break point is reached, setting p high increases fees dramatically. This dynamic fee behaviour addresses risks in stable pools while making trading fees attractive for swappers.

2. Model for Volatile pools. As it was shown in Impermanent Loss section, LPs in volatile pools can suffer impermanent loss or not (IL=0), depending on the swap direction. We can use this fact to adjust fees so they are minimal when LPs IL is zero, and proportional to the loss when there's LP loss. For this, we would need to check at every time step (every swap) the return of the used (swapped out) asset and swap size. Overall, the fee model consists of base and dynamic parts, same as for Stable pools, but dynamic part is defined differently. First, we check if the return for the swapped out asset was positive. If it was, it means LP's position after the swap worth less than before. Therefore, we should compensate for the loss and we calculate dynamic part of the fee as follows:

$$dynamic = amount \times \alpha \times return \tag{33}$$

where *amount* is the relative amount of the swap to the reserves on the side where swap happened, α is the multiplier that defines how big should be the fee in response to the return.

If return was negative, then LP's IL is zero, and we can set $dynamic = 0$, making trade cheaper for swappers.

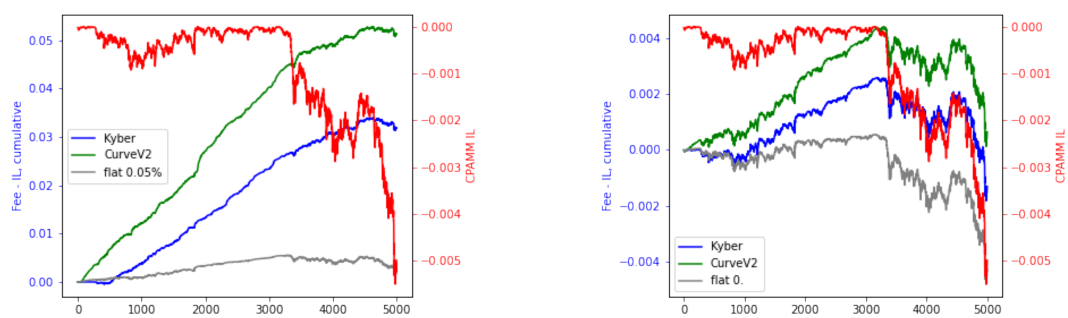
3.3 Performance of fee models

To measure the fee model's performance, we compare liquidity provider's performance with and without fees. We check the impermanent loss against the price move in USDC-ETH pool. Then, we implement few different types of model with the parameters taken from their whitepaper. To be able to fairly assess the model's performance, we need to keep in mind that fee is the percentage from the swapped amount, while impermanent loss is the percentage loss from the LP position. IL purely depends on the price move (volatility), while absolute value of the fee depends on the traded volume. Impermanent Loss after fee is calculated as follows:

$$Performance = IL + \frac{fee \cdot V}{TVL} \tag{34}$$

where IL is the pure impermanent loss without considering fees, V is the volume of swap and TVL is pool depth.

Figure 1 shows the performance of few different types of fee models in CPAMM pool for different values of TVL.



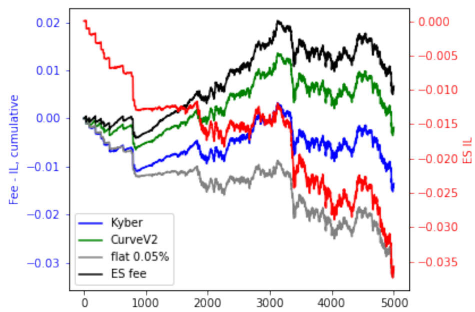
(a) TVL=15mln USD

(b) TVL=150mln USD

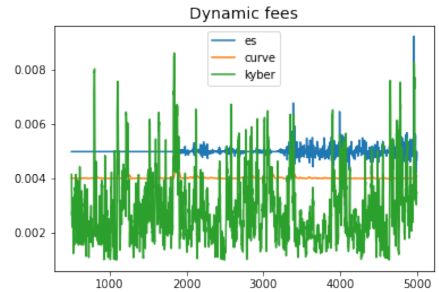
Figure 1: Dynamic fees (Kyber, Curve V2 and flat fee) performance in Constant Product AMM.

Figure 2 a) shows the performance of different fee models for Euler Swap pricing curve. Figure 2 b) shows the overall dynamics of different fee models.

Overall, Curve V2 dynamic volatility-based fee model seems to outperform the volume-based Kyber model. The main reason for that is that the minimum fee of Curve model stays constant and accumulated effect of it helps to reduce the LP performance in a long run. While in Kiber model the base fee is reduced during the low trading periods, even later in periods of higher trading volume the cumulative fees are significantly less than those in Curve model. Also, volume-based Kyber dynamic



(a) Dynamic fees performance in ES, TVL=100mln



(b) Evolution of dinamic fees

Figure 2: Dynamic fees (Kyber, Curve V2, EulerSwap and flat fee) performance in Euler Swap curve.

fee is more volatile, which can be a disadvantage for traders.

Euler Swap dynamic fee model takes into account both the traded volume and volatility, since the impermanent loss for this curve depends on both. Fee is low during positive price return of asset that was traded out from the active side of ES curve, which makes it better for traders, but at the same it accumulates nicely and is able to compensate for LP losses in a long run. During the negative price returns, fees are increased to boost the LP performance.