Math 742

Rewards in the renewal process.
Regenerative process.

13. In each game played one is equally likely to either win or lose 1. Let X be your cumulative winnings if you use the strategy that quits playing if you win the first game, and plays two more games and then quits if you lose the first game.
(a) Use Wald's equation to determine E[X].
(b) Compute the probability mass function of X and use it to find E[X].

2)The times between the arrivals of customers at a taxi stand are independent and have a distribution F with mean F. Assume an unlimited supply of cabs, such as might occur at an airport. Suppose each customer pays a random fare with distribution G and mean μ_G. Let W(t) be the total fares paid up to time t. Find

· $\lim_{t\to\infty} \frac{E(W(t))}{t}$

3)A young doctor is working at night in an emergency room. Emergencies come in at times of a Poisson process with rate 0.5 per hour. The doctor can only get to sleep when it has been 36 minutes (.6 hours) since the last emergency. For example, if there is an emergency at 1:00 and a second one at 1:17 then she will not be able to get to sleep until at least 1:53, and it will be even later if there is another emergency before that time.
(a) Compute the long-run fraction of time she spends sleeping by formulating a renewal reward process in which the reward in the ith interval is the amount of time she gets to sleep in that interval.
(b) The doctor alternates between sleeping for an amount of time s_i and being awake for an amount of time ui. Use the result from (a) to compute Eui.

45)Each time a certain machine breaks down it is replaced by a new one of the same type. In the long run, what percentage of time is the machine in use less than one year old if the life distribution of a machine is
(a) uniformly distributed over (0, 2)?
(b) exponentially distributed with mean 1?

4) Each time the frozen yogurt machine at the mall breaks down, it is replaced by a new one of the same type. (a)What is the limiting age distribution for the machine in use if the lifetime of a machine has a gamma(2,λ) distribution, i.e., the sum of two exponentials with mean 1/λ. (b) Find the answer to (a) by thinking about a rate one Poisson process in which arrivals are alternately colored red and blue.