8. Consider 2 machines, both of which have an $\sim exp$ lifetime w/ $\mu = 1/\lambda$. There's a single repairman that can service machines @ an $\sim exp(\mu)$. Set up Kolmogorov backward eqs; you need not solve them.

Birth & death

$P'_{0j}(t) = \lambda_0 P_{1j}(t) - \lambda_0 P_{0j}(t)$

$P'_{ij}(t) = \lambda_i P_{i+1,j}(t) + \mu_i P_{i-1,j}(t) - (\lambda_i + \mu_i) P_{ij}(t)$

M1 $\sim exp(\mu = 1/\lambda)$
M2 $\sim exp(\mu = 1/\lambda)$

Repair $\sim exp(\mu)$

$S = \{0, 1, 2\}$
0 = M1 & M2 ok
1 = M1 broke
2 = M2 broke

$$R: \begin{array}{c|ccc|} & 0 & 1 & 2 \\ 0 & -2\lambda & 2\lambda & 0 \\ 1 & \mu & -(\mu+\lambda) & \lambda \\ 2 & 0 & \mu & -\mu \end{array}$$

$\lambda + \lambda = 2\lambda$, λ

μ

$P'_{00}(t) = 2\lambda P_{10}(t) - 2\lambda P_{00}(t)$

$P'_{01}(t) = 2\lambda P_{10}(t) - 2\lambda P_{01}(t)$

$P'_{02}(t) = 2\lambda P_{10}(t) - 2\lambda P_{02}(t)$

$P'_{10}(t) = \lambda P_{20}(t) + \mu P_{00}(t) - (\lambda+\mu) P_{10}(t)$

$P'_{11}(t) = \lambda P_{21}(t) + \mu P_{01}(t) - (\lambda+\mu) P_{11}(t)$

$P'_{12}(t) = \lambda P_{22}(t) + \mu P_{02}(t) - (\lambda+\mu) P_{12}(t)$

$P'_{20}(t) = \lambda_2 P_{30}(t) + \mu P_{10}(t) - (\lambda_2+\mu) P_{20}(t)$
$= \mu P_{10}(t) - \mu P_{20}(t)$

$P'_{21}(t) = \mu P_{11}(t) - \mu P_{21}(t)$

$P'_{22}(t) = \mu P_{12}(t) - \mu P_{22}(t)$

9. The birth & death process w/ parameters:

$\lambda_n = 0$
$\mu_n = \mu$
$n > 0$

It's called a pure death process. Find $P_{ij}(t)$.

death rate constant $\Rightarrow P_{oi}$

pure birth $\Rightarrow \mu_n = 0, \forall n \geq 0; \lambda_n = \lambda \; \forall n \geq 0$

$\mu_t = \mu t$ as long as system isn't empty for any length t

$i \to i-1$ $\qquad P_{i,i-1} = \dfrac{\mu_i}{\lambda_i + \mu_i}$

$$P_{ij}(t) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-\mu t}(\mu t)^{i-j}}{(i-j)!}$$

3. Customers arrive @ full service one-pump gas station @ $\lambda = 20$ cars/hr. However, customers will go to another station if there are @ least 2 cars in the station, ie, one being served & one waiting. Suppose that service time for customers is ~exp($\mu = 6$ min)

a. Formulate MC model for # of cars @ gas station

$\lambda = 20$ cars/hr $\quad S = \{0, 1, 2\}$

$\mu = 6$ min

λ of service $= \frac{60}{6} = 10$/hr

$$R = \begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ 0 & -20 & 20 & 0 & = 0 \\ 1 & 10 & -30 & 20 & = 0 \\ 2 & 0 & 10 & -10 & = 0 \end{array}$$

b. Find $P(7)$

From Matlab

```
syms m 7
R=[-20 20 0;10 -30 20;0 10 -10]
expm(R*7)
```

```
>> HW2_Q3b

ans =

[(3*exp(- 30*t - 10*2^(1/2)*t))/7 + (3*exp(10*2^(1/2)*t - 30*t))/7 - (2^(1/2)*exp(- 30*t↙
- 10*2^(1/2)*t))/7 + (2^(1/2)*exp(10*2^(1/2)*t - 30*t))/7 + 1/7,            (2*2^(1/2)↙
*exp(10*2^(1/2)*t - 30*t))/7 - exp(10*2^(1/2)*t - 30*t)/7 - (2*2^(1/2)*exp(- 30*t - 10*2^↙
(1/2)*t))/7 - exp(- 30*t - 10*2^(1/2)*t)/7 + 2/7, (3*2^(1/2)*exp(- 30*t - 10*2^(1/2)*t))↙
/7 - (2*exp(10*2^(1/2)*t - 30*t))/7 - (2*exp(- 30*t - 10*2^(1/2)*t))/7 - (3*2^(1/2)*exp↙
(10*2^(1/2)*t - 30*t))/7 + 4/7]
[      (2^(1/2)*exp(10*2^(1/2)*t - 30*t))/7 - exp(10*2^(1/2)*t - 30*t)/14 - (2^(1/2)*exp↙
(- 30*t - 10*2^(1/2)*t))/7 - exp(- 30*t - 10*2^(1/2)*t)/14 + 1/7, (5*exp(- 30*t - 10*2^↙
(1/2)*t))/14 + (5*exp(10*2^(1/2)*t - 30*t))/14 + (3*2^(1/2)*exp(- 30*t - 10*2^(1/2)*t))↙
/14 - (3*2^(1/2)*exp(10*2^(1/2)*t - 30*t))/14 + 2/7,   (2^(1/2)*exp(10*2^(1/2)*t - 30*t))↙
/14 - (2*exp(10*2^(1/2)*t - 30*t))/7 - (2^(1/2)*exp(- 30*t - 10*2^(1/2)*t))/14 - (2*exp(-↙
30*t - 10*2^(1/2)*t))/7 + 4/7]
[(3*2^(1/2)*exp(- 30*t - 10*2^(1/2)*t))/28 - exp(10*2^(1/2)*t - 30*t)/14 - exp(- 30*t -↙
10*2^(1/2)*t)/14 - (3*2^(1/2)*exp(10*2^(1/2)*t - 30*t))/28 + 1/7,              (2^(1/2)↙
*exp(10*2^(1/2)*t - 30*t))/28 - exp(10*2^(1/2)*t - 30*t)/7 - (2^(1/2)*exp(- 30*t - 10*2^↙
(1/2)*t))/28 - exp(- 30*t - 10*2^(1/2)*t)/7 + 2/7, (3*exp(- 30*t - 10*2^(1/2)*t))/14 +↙
(3*exp(10*2^(1/2)*t - 30*t))/14 - (2^(1/2)*exp(- 30*t - 10*2^(1/2)*t))/14 + (2^(1/2)*exp↙
(10*2^(1/2)*t - 30*t))/14 + 4/7]

>>
```