Recitation 2

1 A machine is subject to failures of types i =1; 2; 3 at rates λ_1=1/24, λ_2=1/30, λ_3=1/84. A failure of type i takes an exponential amount of time with rate μ_1=1/3, μ_2=1/5, and μ_3=1/7. Formulate a Markov chain model with state space {0; 1; 2; 3} and find its stationary distribution.

2.A small computer store has room to display up to three computers for sale. Customers come at times of a Poisson process with rate 2 per week to buy a computer and will buy one if at least 1 is available. When the store has only one computer left it places an order for two more computers. The order takes an exponentially distributed amount of time with mean 1 week to arrive. Of course, while the store is waiting for delivery, sales may reduce the inventory to 1 and then to 0. (a) Write down the matrix of transition rates Rij and solve $\vec{P}R = 0$ to find the stationary distribution. (b) At what rate does the store make sales?

. *3.* A service center consists of two servers, each working at an exponential rate of two services per hour. If customers arrive at a Poisson rate of three per hour, then, assuming a system capacity of at most three customers,
(a) what fraction of potential customers enter the system?
(b) what would the value of part (a) be if there was only a single server, and his rate was twice as fast (that is, μ = 4)?

4 Potential customers arrive at a full-service, one-pump gas station at a Poisson rate of 20 cars per hour. However, customers will only enter the station for gas if there are no more than two cars (including the one currently being attended to) at the pump. Suppose the amount of time required to service a car is exponentially distributed with a mean of five minutes.
(a) What fraction of the attendant's time will be spent servicing cars?
(b) What fraction of potential customers are lost?

5. Two people are working in a small office selling shares in a mutual fund. Each is either on the phone or not. Suppose that calls come into the two brokers at rate λ_1=λ_2=1 per hour, while the calls are serviced at rate μ_1=μ_2=3.
(a) Formulate a Markov chain model for this system with state space (0; 1; 2; 12) where the state indicates who is on the phone. (b) Find the stationary distribution.
(c) Suppose they upgrade their telephone system so that a call to one line that is busy is forwarded to the other phone and lost if that phone is busy. Find the new stationary probabilities. (d) Compare the rate at which calls are lost in the two systems.