1 A machine is subject to failures of types i =1; 2; 3 at rates λ_1=1/24, λ_2=1/30, λ_3=1/84. A failure of type i takes an exponential amount of time with rate μ_1=1/3, μ_2=1/5, and μ_3=1/7. Formulate a Markov chain model with state space {0; 1; 2; 3} and find its stationary distribution.

$\vec{P} = (P_0, \dots, P_m)$

$\vec{P} R = 0$

$\sum P_i = 1$

$$R = \begin{bmatrix} -v_0 & q_{01} & q_{02} & \cdots \\ q_{10} & -v_1 & q_{12} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

$$\begin{array}{c|cccc|} & 0 & 1 & 2 & 3 \\ \hline 0 & -\frac{840}{73} & \lambda_1 & \lambda_2 & \lambda_3 \\ 1 & \mu_1 & -\mu_1 & 0 & 0 \\ 2 & \mu_2 & 0 & -\mu_2 & 0 \\ 3 & \mu_3 & 0 & 0 & -\mu_3 \end{array}$$

$\lambda_1 = \frac{1}{24}$ $\mu_1 = \frac{1}{3}$

$\lambda_2 = \frac{1}{30}$ $\mu_2 = \frac{1}{5}$

$\lambda_3 = \frac{1}{84}$ $\mu_3 = \frac{1}{7}$

$$v_0 = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} = \frac{840}{73}$$

$$\text{inv}(R^T)\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{array}{c|cccc|} & 0 & 1 & 2 & 3 \\ \hline 0 & -\frac{840}{73} & \lambda_1 & \lambda_2 & 1 \\ 1 & \mu_1 & -\mu_1 & 0 & 1 \\ 2 & \mu_2 & 0 & -\mu_2 & 1 \\ 3 & \mu_3 & 0 & 0 & 1 \end{array} \Rightarrow \sum P_i = 1$$

HW ~ 3 $\Rightarrow$ expm(Rt)

. 3. A service center consists of two servers, each working at an exponential rate of two services per hour. If customers arrive at a Poisson rate of three per hour, then, assuming a system capacity of at most three customers,
(a) what fraction of potential customers enter the system?
(b) what would the value of part (a) be if there was only a single server, and his rate was twice as fast (that is, $\mu = 4$)?

2 servers

each ~exp $\Delta = 2$ serv/hr

customer ~Poi $\Delta = 3$/hr