Recitation 4 Math 742

1. Suppose in Example 7.4 that Beverly does not keep any surplus batteries on hand, and so each time a failure occurs she must go and buy a new battery. If the amount of time it takes for her to get a new battery is uniformly distributed over (0, 1), then what is the average rate that Beverly changes batteries?

2. Suppose that potential customers arrive at a single-server bank in accordance with a Poisson process having rate λ. However, suppose that the potential customer will enter the bank only if the server is free when he arrives. That is, if there is already a customer in the bank, then our arriver, rather than entering the bank, will go home. If we assume that the amount of time spent in the bank by an entering customer is a random variable having distribution G, then $\lim_{t\to\infty} \frac{N(t)}{t} = \frac{1}{\mu} = \frac{1}{\mu_G + \frac{1}{\lambda}}$ = rate

(a) what is the rate at which customers enter the bank? $\mu = \mu_G + \frac{1}{\lambda}$

(b) what proportion of potential customers actually enter the bank?

3

Consider a renewal process with mean interarrival time μ. Suppose that each event of this process is independently "counted" with probability p. Let $N_C(t)$ denote the number of counted events by time t, $t > 0$. p $1-p$ $\frac{\Delta N_c}{\Delta N} = p$ $\frac{p}{\mu}$

(a) Is $N_C(t), t \geq 0$ a renewal process?

(b) What is $\lim_{t\to\infty} N_C(t)/t$?

$$\lim_{t\to\infty} \frac{N_c(t)}{t} = \Delta \cdot N_c(t) = p \cdot \Delta(N) = \frac{p}{\mu}$$

4.

Let $U_1, \ldots, U_n, \ldots$ be independent uniform (0, 1) random variables. Let

$$N = \min\{n : U_n > .8\} \quad (U_1, \ldots, U_n, \ldots) \sim U(0,1)$$

and let $S = \sum_{i=1}^{N} U_i$.

(a) Find $E[S]$ by conditioning on the value of U_1. $E(S) = E(E(S|U_1))$

(b) Find $E[S]$ by conditioning on N. $E(S|U_1 = n)$

5. A worker sequentially works on jobs. Each time a job is completed, a new one has begun. Each job, independently, takes a random amount of time, having distribution F to complete. However, independently of this, shocks occur according to a Poisson process with rate λ. Whenever a shock occurs, the worker discontinues working on the present job and starts a new one. In the long run, at what rate are jobs completed? job $\sim F$ shock $\sim Poi(\lambda)$

1. $\mathcal{U}(30, 60)$ - battery life time
$\mathcal{U}(0,1)$ - time to buy battery

$\mu = \mu_1 + \mu_2$

$\frac{1}{\mu_1 + \mu_2}$

$N(t) = N_1(t) + N_2(t)$

$\underset{\text{renewal}}{N_1(t)}$ $\underset{\text{renewal}}{N_2(t)}$

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2. $(U_1, \ldots, U_n, \ldots) \sim \mathcal{U}(0,1)$
$N = \min\{n : U_n > 0.8\}$
$S = \sum_{i=1}^{n} U_i$

a) $\mathbb{E}S$, conditioning U_1

$$\mathbb{E}S = \mathbb{E}[\mathbb{E}(S|U_1)] = \int_0^1 \mathbb{E}(S|U = u) f_u(u)\,du$$

$$\mathbb{E}(S|U_1) = \begin{cases} u + \mathbb{E}S, & u \le 0.8 \\ u, & u > 0.8 \end{cases}$$

$$\mathbb{E}S = \int_0^{0.8} u + \mathbb{E}S\,du + \int_{0.8}^1 u\,du =$$

b) $\mathbb{E}S = \mathbb{E}(\mathbb{E}(S|N))$
$\mathbb{E}(S|N = n)$