Wald equation: N = stopping time
X_i = iid

$$\mathbb{E}\left(\sum_{k=1}^{N} X_k\right) = \mathbb{E}N(\mathbb{E}X)$$

$$\mathbb{E}(X_i) = \mathbb{E}X$$

1. W/L = 50/50 1st - W → Quit

X = cum wins 1st - L
2nd
3rd
↓ Quit

$1 = 0.5$
$-1 = 0.5$

$N = 1, = 3 \} = \frac{1}{2}$

a. $\boxed{X = \sum(W) \quad \mathbb{E}N(\mathbb{E}W) = 0 \quad \mathbb{E}W = 0}$

W - Quit
L . . - Quit
-1 -1 -1
-1 -1 1
-1 1 1

X	-3	-1	1
P	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2} + \frac{1}{8}$

a

2. interarrival of customers @ taxi stand
   - independent
   - $\sim F(\mu_F)$

rand fare $\sim G(\mu_G)$

$W(t)$ = tot fares paid up to time t

$$\lim_{t \to \infty} \frac{\mathbb{E}(W(t))}{t} = \frac{\text{reward/cycle}}{\text{length/cycle}} = \frac{\mu_G}{\mu_F}$$

3. emergency $\sim P(\lambda = 0.5/\text{hr})$

sleep $\sim$ 36 min (0.6 hrs) since last emergency

ex: emergency - 1:00
- 1:17 ~ 36 min 1:53

a. $\frac{\mu_F}{\mu_F + \mu_G} = \frac{0.5}{0.5 + 0.6} = \frac{0.5}{0.11} = 4.5454$

$r_i = (z_i - 0.6)^+$ $\qquad \frac{\mathbb{E}(r_i)}{\mathbb{E}(z_i)} = \frac{2e^{-0.3}}{2} = e^{-0.3}$

$$r = \begin{cases} 0 & , z < 0.6 \\ z - 0.6 & , z > 0.6 \end{cases}$$ cycle = time bet emergency

$\mathbb{E}r = \mathbb{E}(r \mid z < 0.6) P(z < 0.6) + \mathbb{E}(r \mid z > 0.6) P(z > 0.6)$

$= 0 + 2(e^{-0.5(0.6)}) = 2e^{-0.3}$

b. s_i = sleep

μ_i = awake $\qquad \mathbb{E}(\mu_i) = ?$

$$\frac{\mu_F}{\mu_F + \mu_G} \Rightarrow \frac{\mathbb{E}(s_i)}{\mathbb{E}(s_i) + \mathbb{E}(\mu_i)} = \frac{2}{2 + \mathbb{E}(\mu_i)} = e^{-0.3}$$

$$\Rightarrow 1 + \frac{2}{\mathbb{E}(\mu_i)} = e^{-0.3}$$

$$\frac{2}{\mathbb{E}(\mu_i)} = e^{-0.3} - 1$$

$$\mathbb{E}(\mu_i) = \frac{2}{e^{-0.3} - 1}$$

$$\frac{2}{2 + \mathbb{E}(\mu_i)} = e^{-0.3} \Rightarrow 2 = e^{-0.3}(2 + \mathbb{E}\mu_i)$$

$$\Rightarrow \frac{2}{e^{-0.3}} = 2 + \mathbb{E}\mu_i$$

$$\Rightarrow \mathbb{E}\mu_i = \frac{2}{e^{-0.3}} - 2$$

$\mathbb{E}(\mu_i) = \mathbb{E}(\mu_i \mid z < 0.6) P(z < 0.6) + \mathbb{E}(\mu_i \mid z > 0.6) P(z > 0.6)$

$$= \int_0^{0.6} x\, 0.5 e^{-0.5x}\, dx + 0.6\, P(z > 0.6) + \sim$$

$$= 0.5 \int_0^{0.6} x e^{-0.5x}\, dx + \sim$$

$$= x(-e^{-0.5x})\Big|_0^{0.6} + \int_0^{0.6} e^{-0.5x}\,dx + 2$$

$$= -0.6e^{-0.3} + 2(-e^{-0.5x})\Big|_0^{0.6} = -0.6e^{-0.3} - 2e^{-0.3} + 2 + 0.6e^{-0.3}$$

$$= 2(1-e^{-0.3})$$

5a. Machine lifetime $\sim U(0,2)$

In long-run, % of time is machine in use < 1yr

$$\text{long run} < c = \frac{\mathbb{E}(\min(X,c))}{\mathbb{E}X} = \int_0^c (1-F(x))\,d \Big/ \mathbb{E}X$$

$$= \int_0^1 \left(1-\frac{x}{2}\right)dx \Big/ 1 = x\Big|_0^1 - \frac{x^2}{4}\Big|_0^1 = 1-\frac{1}{4} = \frac{3}{4}$$