Ch 6: Cont Time Markov Chains

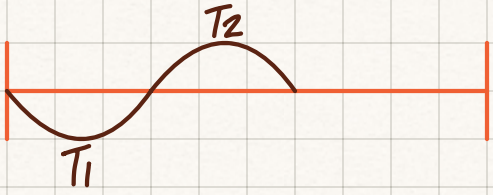
Stochastic process

$\{X(t), t \geq 0\}$, $(x_1, x_2, \ldots, x_n)$

$\forall t$, $X(t)$ = rand var

$N(t)$ = Poi process

Ex: customer service



$\{X_n, n = 0, 1, \ldots\}$ = discrete

Time MC

$P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \ldots, X_0 = i_0\} = P\{X_{n+1} = j \mid X_n = i\}$

State space: $S = 0, 1, \ldots, m$

$P = (P_{ij})_{i,j=0}^{m}$

Transition prob mat

$P_{ij} = P\{X_{n+1} = j \mid X_n = i\}$

$P_{ij}^{m} = P\{X_{n+m} = j \mid X_n = i\}$ $\quad (P^m)_{ij}$

Define: The process $\{X(t), t \geq 0\}$ is cont time MC if $\forall s, t \in \mathbb{R}$ &

$i, j \in \mathbb{Z}_+$ (non-neg int),

$P\{X(t+s) = j \mid \underset{X_n = i}{X(s) = i}, \underset{X_{n-1} = i_{n-1}}{X(u) = x(u)}, \underset{X_0 = i_0}{0 \leq u < s}\} =$

$X(u)_{X_{n+1} = j} \quad 0 \leq u < s$

$P\{X(t+s) = j \mid X(s) = i\}$

We assume that $P\{X(t+s) = j \mid X(s) = i\}$ is indep of (s)

$\parallel$

$P\{X(t) = j \mid X(0) = i\} \rightarrow$ MC is stationary or homogeneous

Let $X(s) = i$ for some s

$X(0) = i$

$\forall t \leq 5$

$X(s+t) = i$

T_i - time that process stays in i before making transition

(10-15 min)

$s = 10$

$P(T_i > 15 \mid T_i > 10) = P(T_i > 5)$

$T_i \sim Exp(\lambda) \Rightarrow E(T_i) = 1/\lambda$