Ch 6.

6.2. Continuous Markov chains

Let $\{X(t),\ t \geq 0\}$ be a continuous time stochastic process.

$X(t)$ is a random variable $\forall t$.

Def. The process $\{X(t),\ t \geq 0\}$ is a continuous time Markov chain if

$\forall\ s, t \geq 0$ and $i, j \in \mathbb{Z}_+$ (non neg integers),

$x(u) \quad 0 \leq u < s$

$$P\{X(t+s) = j \mid X(s) = i,\ X(u) = x(u),\ 0 \leq u < s\} =$$
$$= P\{X(t+s) = j \mid X(s) = i\}$$

In other words, a continuous-time Markov chain is a stochastic process having the Markovian property that the conditional distribution of the future $X(t+s)$ given the present $X(s)$ and the past $X(u)$ $0 \leq u < s$, depends only on the present and is independent of the past.

If in addition

$P\{X(t+s) = j \mid X(s) = i\}$ is independent of s, then the continuous time MC is said to have stationary or homogeneous transition probabilities.

We will consider only MC with stationary probabilities

Let $X(s) = i$ for some s, e.g. $s=0$.

$P(X(s+t) = i) - ?$ for $\forall t \le 5$

process stay at i

- What is the probability (process does not leave state i during the following 5 min)

Denote T_i the amount of time that the process stays in i before making transition to a different state.

Assume $s = 10$

$$P(T_i > 15 \mid T_i > 10) = P(T_i > 5)$$

In general

$$P(T_i > s+t \mid T_i > s) = P(T_i > t).$$

It is memoryless property $\Rightarrow T_i \sim Exp(\lambda)$

λ - is a parameter

Another approach to define continuous time MC. It is a stochastic process such that each time it enters state i:

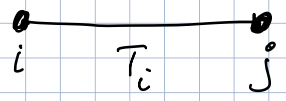
(i) The amount of time it spends in that state before making a transition into a different state $T_i \sim Exp(v_i)$

$$ET_i = \frac{1}{v_i}$$

(ii) P_{ij} - probability that process enters state j after the state i:

1) $P_{ii} = 0$

2) $\sum_{\forall j} P_{ij} = 1 \quad \forall i$



P_{ij} (as in discrete time MC)

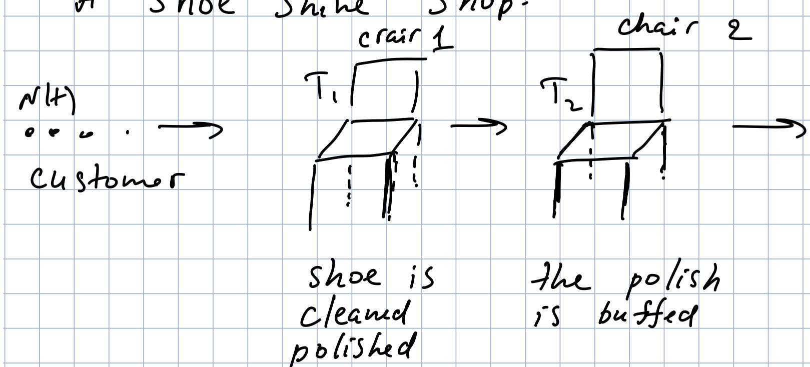
$T_i \sim \mathrm{Exp}$

T_i is indep. from ..

next state visited.

Example.

A Shoe Shine Shop.



T_1 ~~-~~ service ch. 1
T_2 time ch 2.

$T_1 \sim \mathrm{Exp}(\mu_1)$ $\qquad$ $T_2 \sim \mathrm{Exp}(\mu_2)$

$ET_1 = 1/\mu_1$ $\qquad$ $ET_2 = 1/\mu_2$

Arrival of customers $\{N(t), t \geq 0\}$ — Poisson (rate $= \lambda$)

The potential customer will enter only if no other cust. present.

Model: Continuous MC

State space: 0 system empty

1 a customer is in chair "1"

2 a customer is in chair "2"

$v_0 = \lambda \qquad v_1 = \mu_1 \qquad v_2 = \mu_2$

$P_{01} = P_{12} = P_{20} = 1$