Recitation 1

Continuous MC, definition, transition probability

Problem 1.

Consider a birth and death process with birth rates $\lambda_i = (i + 1)\lambda$, $i \geq 0$, and death rates $\mu_i = i\mu$, $i \geq 0$.

(a) Determine the expected time to go from state 0 to state 4.

(b) Determine the expected time to go from state 2 to state 5.

(c) Determine the variances in parts (a) and (b).

Problem 2

In a birth and death process with birth parameter $\lambda_n = \lambda$, n=0,1,..., and death parameters $\mu_n = n\mu$ for n=0,1,... we have $P_{0,j}(t) = \frac{(\lambda p)^j e^{-\lambda p}}{j!}$

Where

$p = \frac{1}{\mu}(1 - e^{-\mu t})$.

Verify that these transition probabilities satisfy the forward equation with i=0.

Problem 3

(Jukes–Cantor Model). In this chain, the states are the four nucleotides A, C, G, T. Jumps, which correspond to nucleotide substitutions, occur according at rate $q_{ij} = \mu$ if $i \neq j$. Find the transition probability matrix $\mathbf{P}(t)$ using forward differential equation.

Problem 4

The nucleotides A and G are purines while C's and T's are pyrimidines. Kimura's model takes into account that mutations that do not change the type of base (called transitions) happen at a different rate than those that do (called transversions), so the transition matrix P

$$R = \begin{pmatrix} -(\alpha+2\beta) & \alpha & \beta & \beta \\ \alpha & -(\alpha+2\beta) & \beta & \beta \\ \beta & \beta & -(\alpha+2\beta) & \alpha \\ \beta & \beta & \alpha & -(\alpha+2\beta) \end{pmatrix}$$

Find $\mathbf{P}(t)$ using forward differential equation.