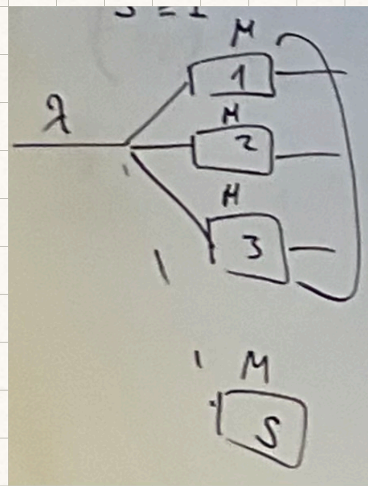
Multiple Server

M/M/S Queue

$S \geq 1$, $\{X(t), t \geq 0\}$: MC



$q_{n,n+1} = \lambda$

$$q_{n,n-1} = \begin{cases} \mu n, & n \leq S \\ \mu S, & n \geq S \end{cases}$$

balance eq

$\lambda \pi_{n-1} = \mu n \pi_n$

$$\begin{cases} \lambda \pi_{n-1} = \mu(n) \pi_n, & i \leq S \\ \lambda_{n-1} = \mu(S) \pi_n, & i \geq S \end{cases}$$

$$\pi_k = \begin{cases} \frac{c}{k!}\left(\frac{\lambda}{\mu}\right)^k, & k \leq S \\ \frac{c}{S!S^{k-S}}\left(\frac{\lambda}{\mu}\right)^k, & k \geq S \end{cases} \qquad \frac{c}{S!S^{-S}}\left(\frac{\lambda}{\mu S}\right)^k$$

Thm

If $\lambda < s\mu$

M/M/S queue is (+) recurrent

If $\lambda = s\mu$

M/M/S is 0 recurrent

If $\lambda > s\mu$

M/M/S is trans

Example

$s = 3 \qquad \lambda \sim s\mu$

$\lambda = 2 \qquad 2 \sim 3 \cdot 1 \Rightarrow$ (+) recurrent

$\mu = 1 \qquad 2 < 3$

Notation example

$$\pi_0 = c$$

$$\pi_1 = c\left(\frac{\lambda}{\mu}\right) = c\left(\frac{2}{1}\right) = 2c$$

$$\pi_2 = \frac{c}{2!}\left(\frac{\lambda}{\mu}\right)^2 = \frac{c}{2}(4) = 2c$$

$$\pi_3 = \frac{c}{3!}\left(\frac{\lambda}{\mu}\right)^3 = \frac{c}{6}(8) = \frac{4}{3}c$$

$$\pi_k = \frac{c(3^3)}{3!}\left(\frac{2}{3}\right)^k = \frac{27c}{6}\left(\frac{2}{3}\right)^k$$

$$\sum_{m=0}^{\infty} \frac{9}{2}c\left(\frac{2}{3}\right)^m\left(\frac{2}{3}\right)^3 = \frac{9}{2}\left(\frac{8}{27}\right)c\sum_{m=0}^{\infty}\left(\frac{2}{3}\right)^m$$

$$= \frac{4}{3}c\left(1\Big/\left(1-\frac{2}{3}\right)\right) = \frac{4}{3}c(3) = 4c$$

$$c + 2c + 2c + 4c = 1$$

$$9c = 1$$

$$c = \frac{1}{9}$$