Math 760

Chapter 5 HW

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3.

(a) Use expression (5-15) to evaluate T^2 for the data in Exercise 5.1

The (5-15) expression is as such:

$$T^2 = \frac{(n-1)|\hat{\Sigma}_0|}{|\hat{\Sigma}|} - (n-1) = \frac{(n-1)\left|\sum_{j=1}^{n}(x_j-\mu_0)\,(x_j-\mu_0)'\right|}{\left|\sum_{j=1}^{n}(x_j-\bar{x})\,(x_j-\bar{x})'\right|} - (n-1)$$

The dimensions of a matrix is n*p, therefore our matrix is:

```
## [1] 4 2
```

Now, our $\bar{x}$ matrix:

```
##      [,1]
## [1,]    6
## [2,]   10
```

Let's now solve for the numerator. First, the summation with μ_0:

```
##      [,1] [,2]
## [1,]   28   -6
## [2,]   -6   10
```

Now, the rest of the numerator:

```
## [1] 732
```

Now, let's solve for the denominator.

```
## [1] 44
```

Now that we have all the parts, let's plug it all into (5-15), and our answer is:

```
## The T-squared value is 13.63636
```

(b) Use the data in Exercise 5.1 to evaluate Λ in (5-13). Also, evaluate Wilk's lambda.

The (5-13) Λ is:

$$\Lambda = \left(\left|\frac{\hat{\Sigma}}{\hat{\Sigma}_0}\right|\right)^{\frac{n}{2}} = \left(\left|\frac{\Sigma_{j=1}^{n}(x_j-\bar{x})(x_j-\bar{x})'}{\Sigma_{j=1}^{n}(x_j-\mu_0)(x_j-\mu_0)'}\right|\right)^{\frac{n}{2}} < c_\alpha$$

We will need to divide the numerator by 3, but otherwise it's just a matter of flipping the fraction from (a) and setting it to the power of $\frac{n}{2}$.

```
## The value of the Lambda is 0.03251814
```

Now, the Wilk's lambda formula is this: $\Lambda^{\frac{2}{n}}$. Plugging in all the values, we get this:

```
## [1] 0.1803279
```

10. Refer to the bear growth data in Example 1.10 (see Table 1.4). Restrict your attention to the measurements of length

(a) Obtain the 95% T^2 simultaneous confidence intervals for the four population means for length.

The T^2 simultaneous confidence interval formula is:

$$\bar{x_p} \pm \sqrt{\frac{p(n-1)}{n-p} F_{n,n-p}(\alpha) \frac{s_{pp}}{n}}$$

Then, the confidence interval for length2 is:

```
## ( 132.9837 , 153.5877 )
```

For length3, the confidence interval is:

```
## ( 132.9074 , 185.6641 )
```

For length4, the confidence interval is:

```
## ( 162.6495 , 183.6362 )
```

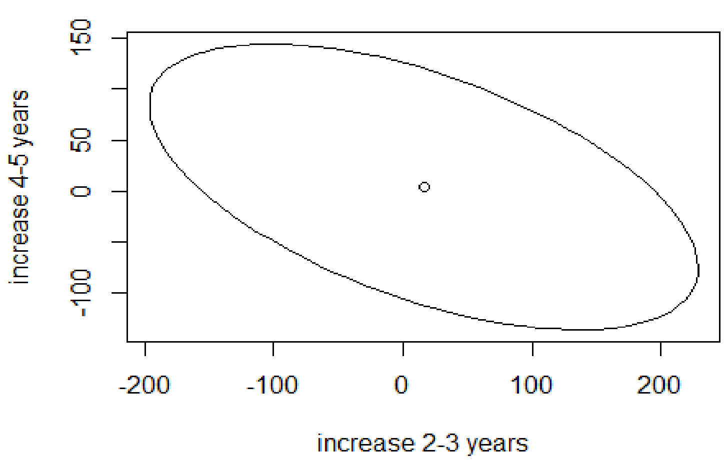
For length5, the confidence interval is:

```
## ( 159.3459 , 194.9399 )
```

(b) Refer to Part a. Obtain the 95% T^2 simultaneous confidence intervals for the three successive yearly increases in mean length

```
## Length2 - Length3: ( -8.283481 , 40.28348 )
## Length3 - Length4: ( -38.05854 , 10.34425 )
## Length4 - Length5: ( -10.37438 , 18.37438 )
```

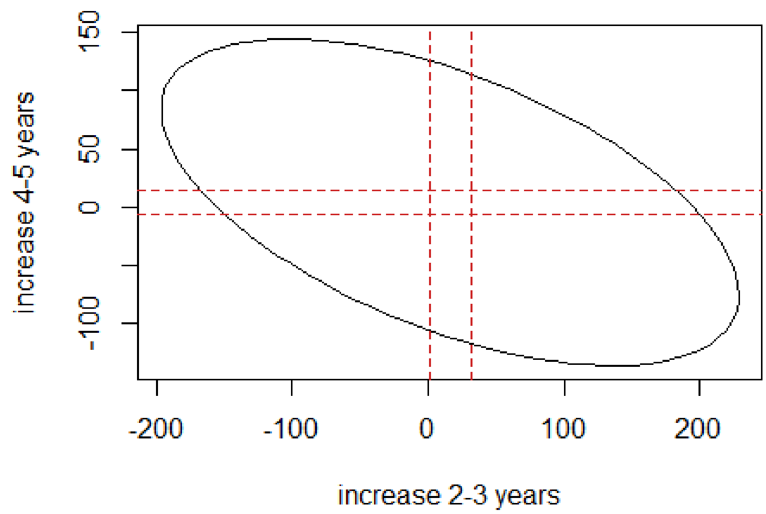
(c) Obtain the 95% T^2 confidence ellipse for the mean increase in length from 2 to 3 years and the mean increase in length from 4 to 5 years.



(d) Refer to Parts a and b. Construct the 95% Bonferroni confidence intervals for the set consisting of four mean lengths and three successive yearly increases in mean length.

```
## Length 2: ( 138.0904 , 148.481 )
## Length 3: ( 145.9831 , 172.5883 )
## Length 4: ( 167.8511 , 178.4347 )
## Length 5: ( 168.1678 , 186.1179 )
## Length 2-3: ( 3.753818 , 28.24618 )
## Length 3-4: ( -26.06193 , -1.652357 )
## Length 4-5: ( -3.249013 , 11.24901 )
```

(e) Refer to Parts c and d. Compare the 95% Bonferroni confidence rectangle for the mean increase in length from 2 to 3 years and the mean increase in length from 4 to 5 years with the confidence ellipse produced by the T^2-procedure.



While I do not believe my 95% Bonferroni confidence rectangle isn't exactly right, I would still say it's much smaller and more informative than the 95% T^2 confidence ellipse. This is simply because from previous results, we learned that T^2 intervals are longer in length compared to Bonferroni ones, therefore the latter would have a more contained area.

12. Given the data

$$X = \begin{bmatrix} 3 & 6 & 0 \\ 4 & 4 & 3 \\ - & 8 & 3 \\ 5 & - & - \end{bmatrix}$$

with missing components, use the prediction-estimation algorithm of Section 5.7 to estimate μ and Σ. Determine the initial estimates, and iterate to find the first revised estimates.

We'll first find the initial sample averages.

```
## The sample averages are: 4 , 6 , and 2
```

Now, we can find $\hat{\Sigma}$.

```
##      [,1] [,2] [,3]
## [1,]  0.5    0  0.5
## [2,]  0.0    2  0.0
## [3,]  0.5    0  1.5
```

Now, based on Example 5.13, we will start doing the prediction steps.

```
## The predicted x31 is 4.333333
## The predicted x31-squared is 19.61111
## The predicted x42 and x43 are 6 3
## The predicted x42 and x43-squared and products are 38 18 18 10
```

Now, we'll jump into the predicted data sufficient statistics.

```
## The T1 matrix is
##          [,1]
## [1,] 16.33333
## [2,] 24.00000
## [3,]  9.00000
##
##  The T2 matrix is
##      [,1]               [,2]  [,3]
## [1,] "68.7777777777778" " "   " "
## [2,] "98.6666666666667" "154" " "
## [3,] "40"               "54"  "28"
```

Now, we can find our first revised estimates of μ and Σ.

```
## The first revised estimate of mu is:
##          [,1]
## [1,] 4.083333
## [2,] 6.000000
## [3,] 2.250000

##  The first revised estimate of Sigma is:
```

```
##            [,1]      [,2]   [,3]
## [1,] 0.5208333 0.1666667 0.8125
## [2,] 0.1666667 2.5000000 0.0000
## [3,] 0.8125000 0.0000000 1.9375
```

15. Let X_{ji} and X_{jk} be the ith and kth components, respectively, of X_j.

(a) Show that $\mu_i = E(X_{ji}) = p_i$ and $\sigma_{ii} = Var(X_{ji}) = p_i(1-p_i)$, $(i = 1, 2, \ldots, p)$.

The general expected value formula is: $\sum x_k P_X(x_k)$. We also know generally probability is either p_i or $1-p_i$; and if not given specific values for the x_k's, we assume it's either 1 or 0. Thus, $\mu_i = E(X_{ji}) = 1(p_i) - 0(1-p_i) = p_i$.

The general variance formula is: $Var(X) = E(X^2) - (EX)^2$.
And thus:
$\sigma_{ii} = Var(X_{ji}) = (1-p_i)^2 p_i + (0-p_i)^2(1-p_i) = (1-2p_i+p_i^2)p_i + p_i^2(1-p_i)$
$\sigma_{ii} = p_i - 2p_i^2 + p_i^3 + p_i^2 - p_i^3 = p_i - p_i^2 = p_i(1-p_i)$

(b) Show that $\sigma_{ik} = Cov(X_{ji}, X_{jk}) = -p_i p_k,\ i \neq k$. Why must this covariance necessarily be negative?

The general covariance formula is: $Cov(X,Y) = E[(X-EX)(Y-EY)] = E(XY) - EX(EY)$.
Thus, $\sigma_{ik} = Cov(X_{ji}, X_{jk}) = E(X_{ji}, X_{jk}) - E(X_{ji})E(X_{jk}) = 0 - p_i p_k = -p_i p_k$.
$E(X_{ji}, X_{jk}) = 0$, because we are assuming the x's are independent.
Covariance can be negative, positive, or zero. But in terms of this question, it must be necessarily be negative, because there is an inverse relationship between the variables.

18. Use the college test data in Table 5.2 (See Example 5.5)

(a) Test the null hypothesis $H_0: \mu' = [500, 50, 30]$ vs $H_1: \mu' \neq [500, 50, 30]$ at the $\alpha = 0.05$ level of significance. Suppose [500,50,30]' represent average scores for thousands of college students over the last 10 years. Is there reason to believe that the group of students represented by the scores in Table 5.2 is scoring differently? Explain.

$$H_0: \mu' = [500,50,30] \text{ vs } H_1: \mu' \neq [500,50,30]\ \alpha = 0.05$$

According to (5-7), H_0 would be rejected if $T^2 = n(\bar{x} - \mu_0)' S^{-1} (\bar{x} - \mu_0) > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$

```
## The critical region for this alpha is 8.557973
##
##  Hotelling's one sample T2-test
##
## data:  college
## T.2 = 72.706, df1 = 3, df2 = 84, p-value < 2.2e-16
## alternative hypothesis: true location is not equal to c(500,50,30)
```

We find that the p-value is small and $T^2 = 72.706 > 8.557973$, therefore we reject H_0. This means, the average scores for thousands of college students over the last 10 years do differ significantly from the students in Table 5.2. The scores in Table 5.2 could be scoring differently because of sample size and perhaps because these students are scoring notably higher or lower in the chosen subject.

(b) Determine the lengths and directions for the axes 95% confidence ellipsoid for μ.

The formula for finding the length and direction of the axes of the 95% confidence ellipsoid is: $\sqrt{\lambda_i \left[\frac{p(n-1)}{n(n-p)}\right] F_{p,n-p}(\alpha) e_i}$, where λ_i and e_i are the eigenvalues and eigenvectors from the sample covariance matrix.

We will first find the direction of the 3 axes.

The direction of the social science & history axis is:

```
## [ 0.9939054 0.1034434 0.03809906 ]
```

The direction of the verbal axis is:

```
## [ 0.1037315 -0.9945892 -0.005660238 ]
```

The direction of the science axis is:

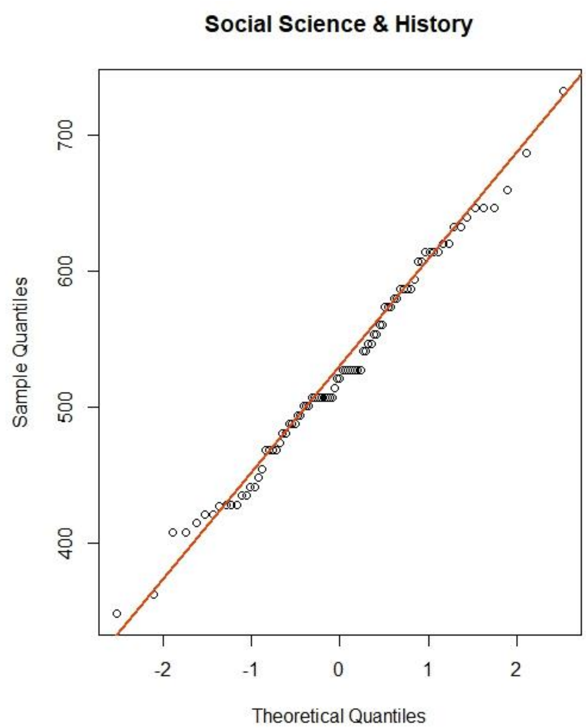
```
## [ -0.0373074 -0.009577815 0.9992579 ]
```

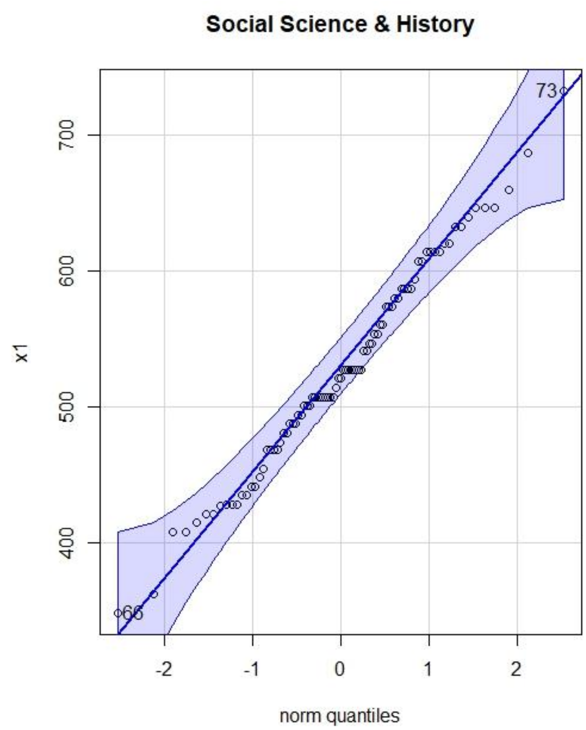
Now, we find the axes' lengths.

```
## The axis length of social science & history is 23.73
## The axis length of verbal is 2.472768
## The axis length of science is 1.1825
```

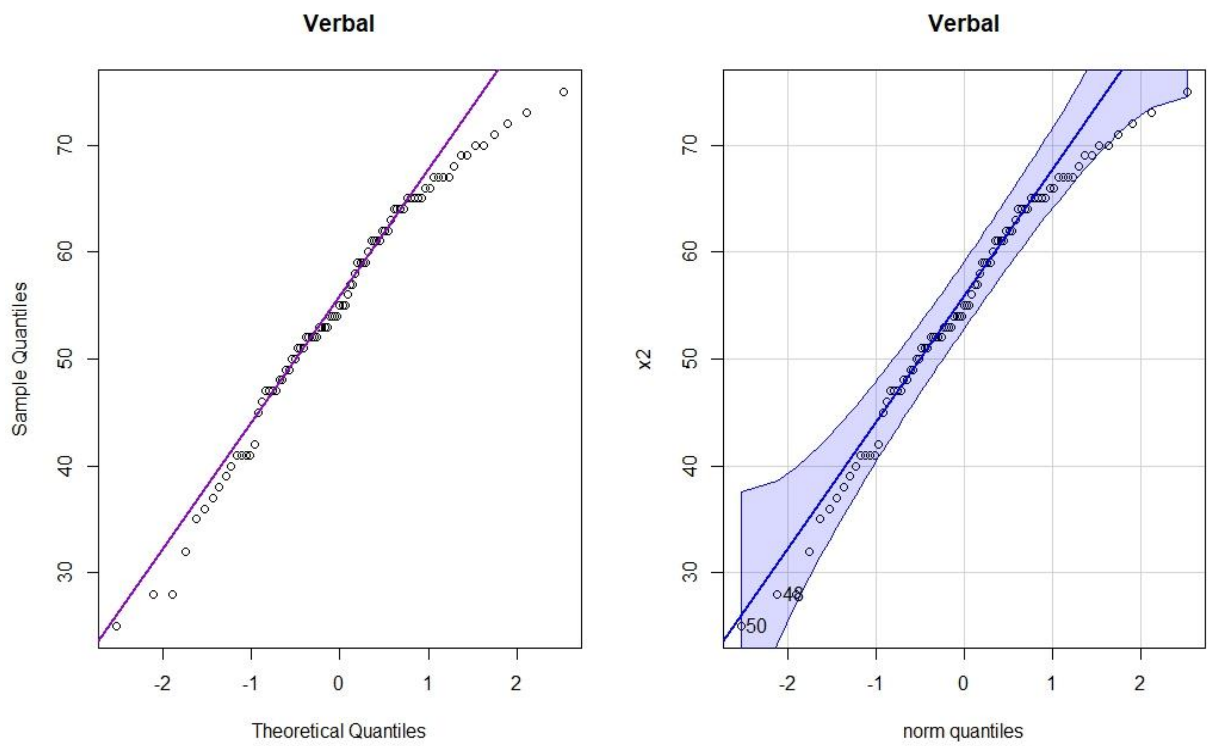
(c) Contstruct Q-Q plots from the marginal distributions of social science and history, verbal, and science scores. Also, construct the three possible scatter diagrams from the pairs of observations on different variables. Do these data appear to be normally distributed? Discuss.

Let's first look at the Q-Q plots of each variable.

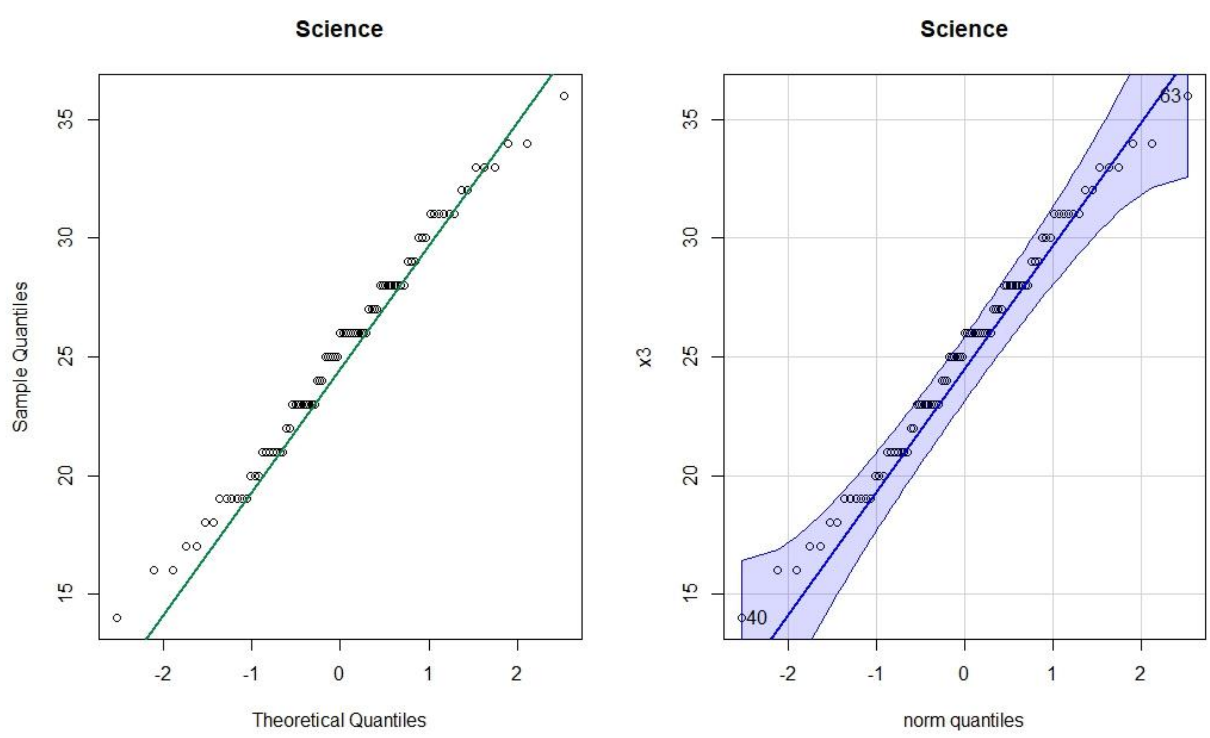





```
## [1] 73 66
```

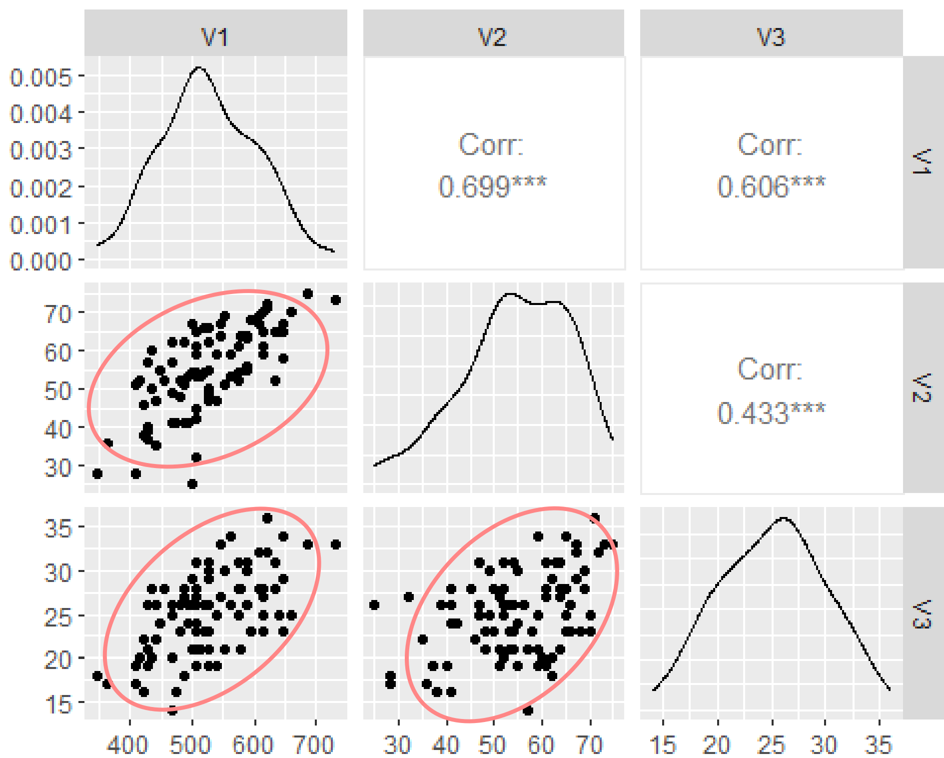

Verbal
Sample Quantiles
Theoretical Quantiles
Verbal
x2
norm quantiles
48
50

[1] 50 48


Science
Sample Quantiles
Theoretical Quantiles
Science
x3
norm quantiles
63
40

[1] 40 63

Overall, each variable the college data seems to follow a normal distribution, if straying just a bit from their respective lines. I would definitely want to investigate the verbal column, because it does seem the points stray farther from their line compared to the other two, and a few do seem to partially step out of the normality region.
Now, let's look over the paired scatter diagrams.

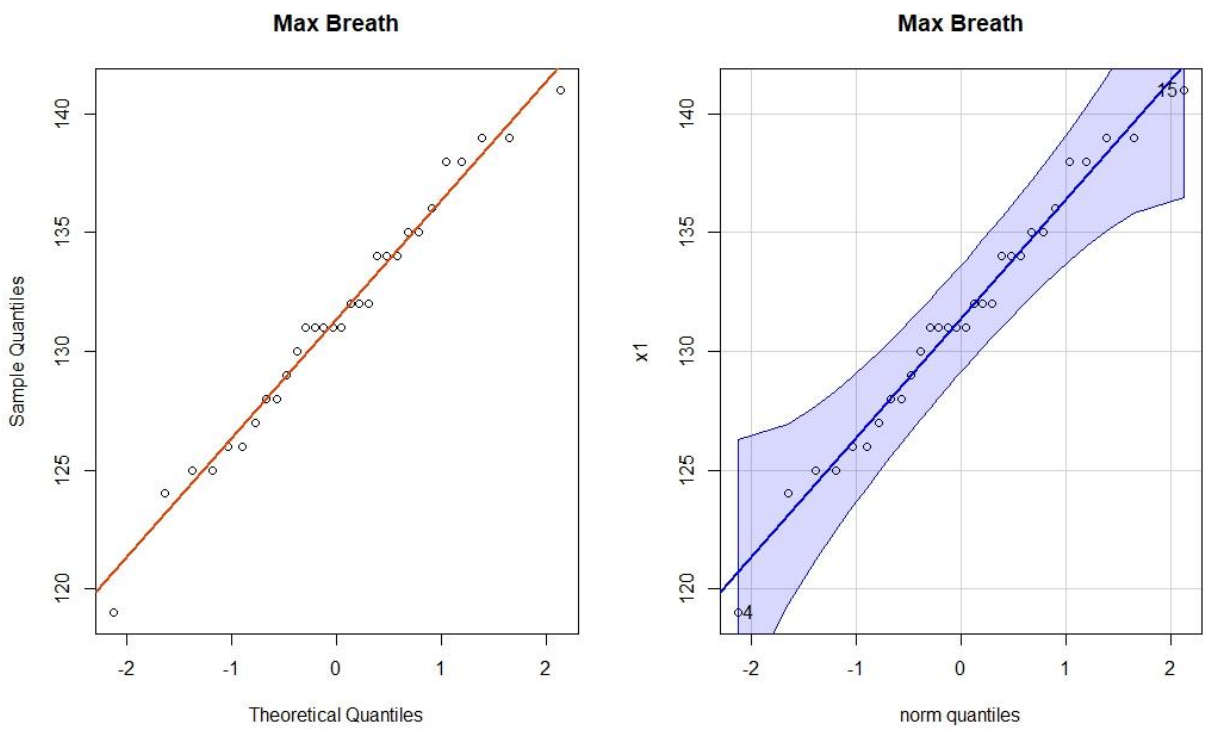


If we were to draw an ellipse over each paired scatter plot, it would at least seem that a majority of the points fall within it. And we know from Chapter 4, if roughly 50% of the points fall within the ellipse, we may assume normality.

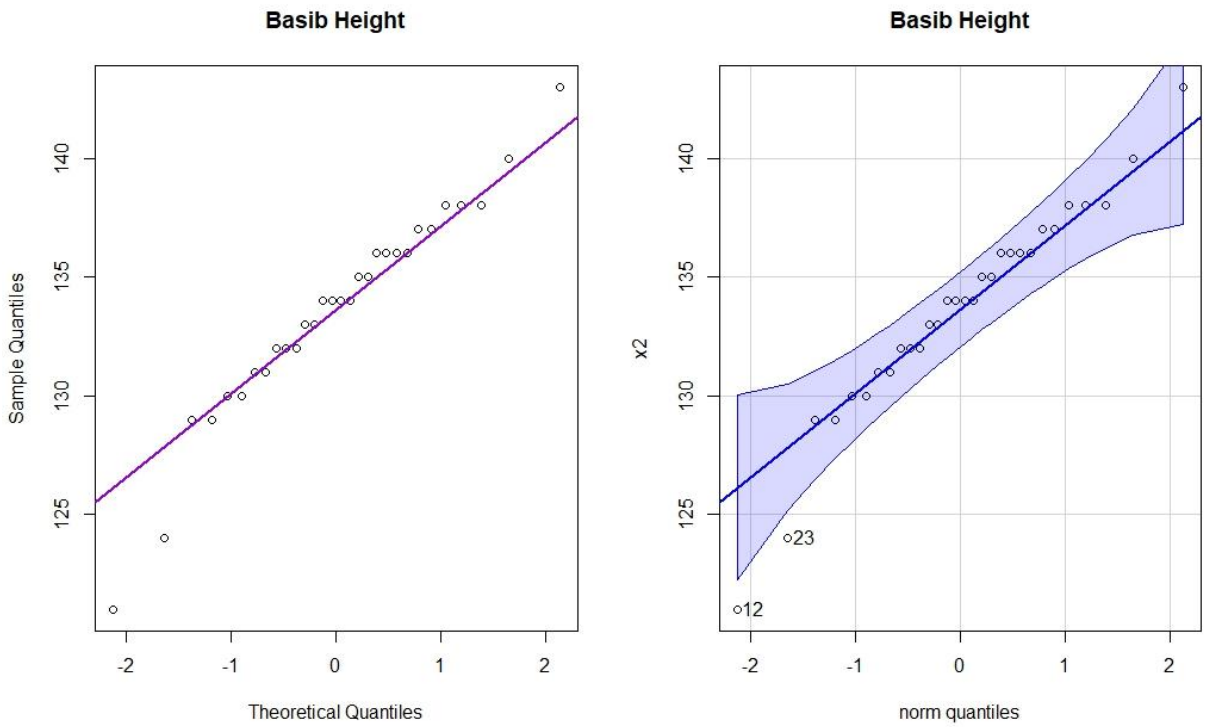
23. Consider the 30 observations on male Egyptian skulls for the first time period given in Table 6.13 on page 349.

(a) Construct Q-Q plots of the marginal distributions of the maxbreath, basheight, baslength, and nasheight variables. Also, construct a chi-square plot of the multivariate observations. Do these data appear to be normally distributed? Explain.

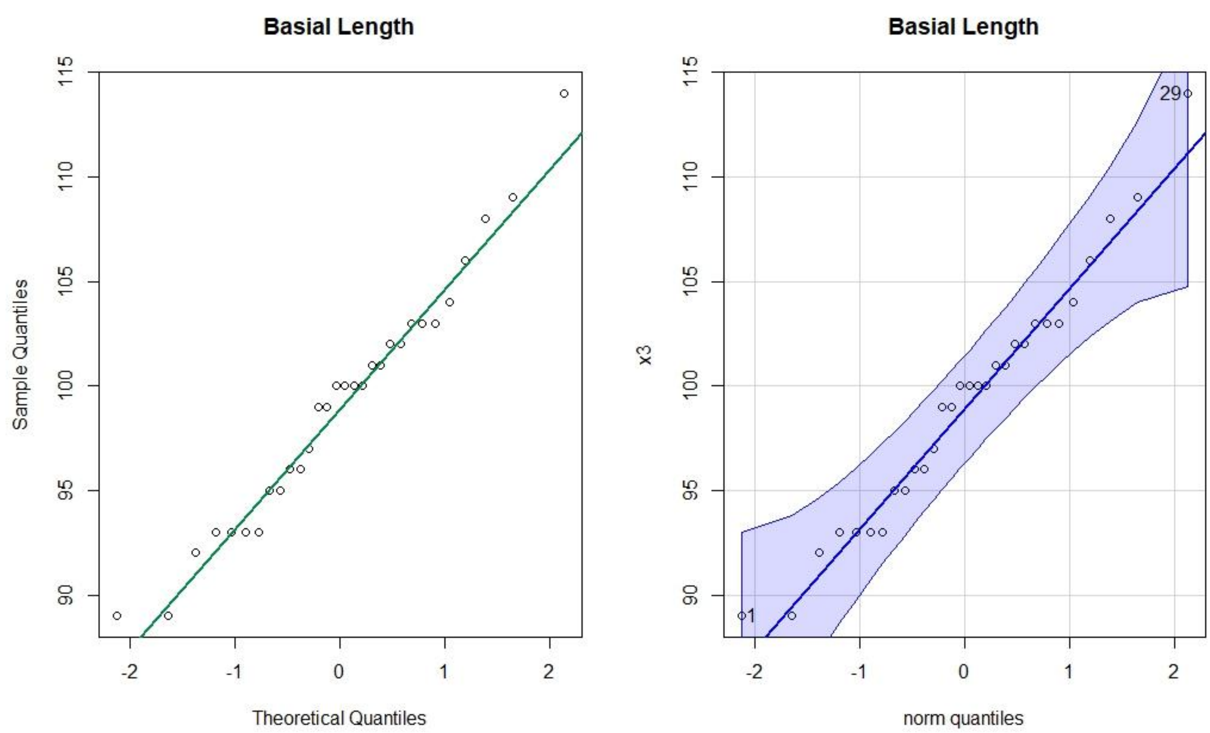
Let's first look at the Q-Q plots of each variable.



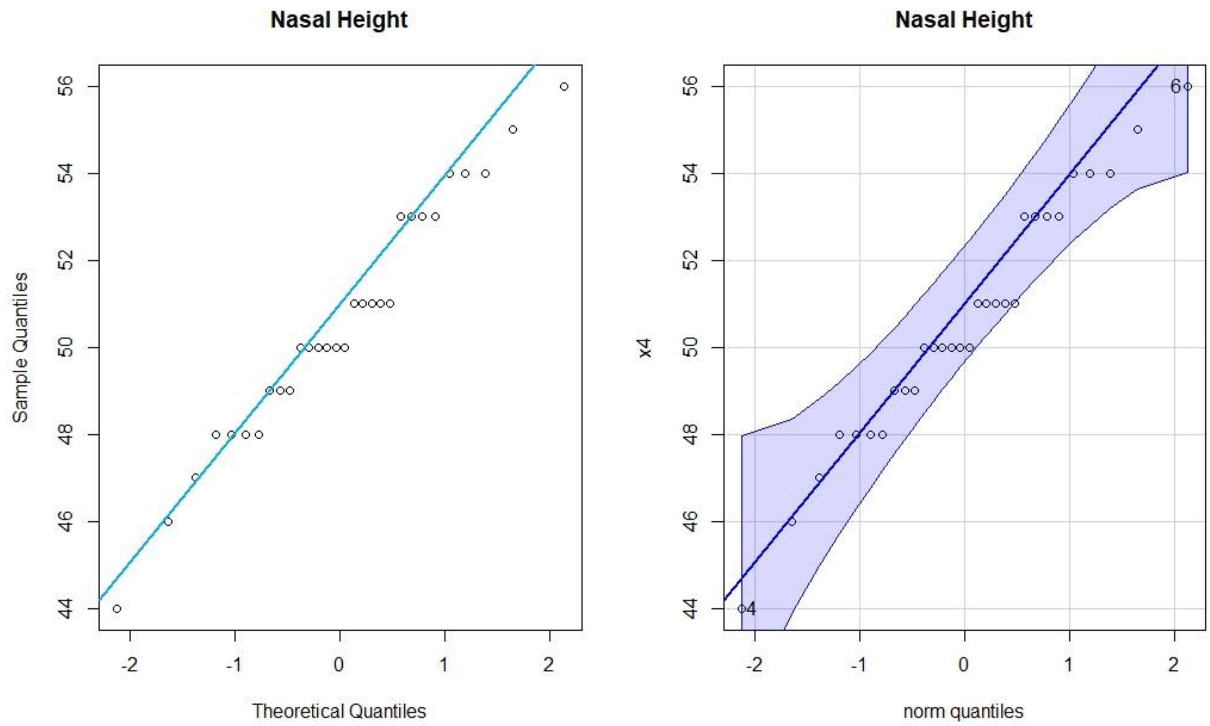

```
## [1]  4 15
```


Basib Height
Sample Quantiles
Theoretical Quantiles
Basib Height
x2
norm quantiles
23
12

```
## [1] 12 23
```


Basial Length
Sample Quantiles
Theoretical Quantiles
Basial Length
x3
norm quantiles
29
1

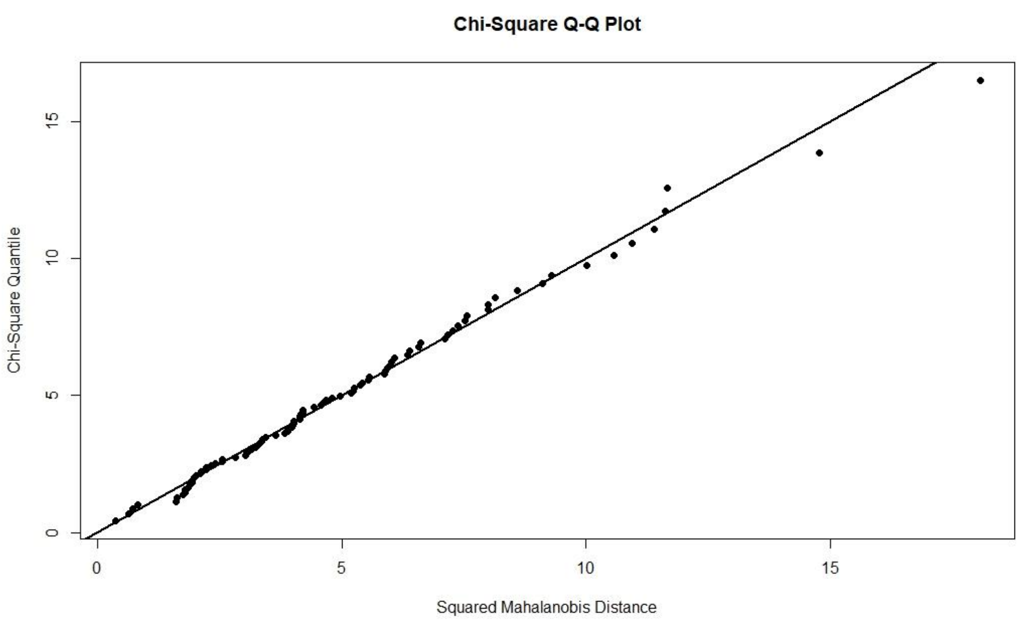
```
## [1] 29  1
```



[1] 4 6

Overall, each variable looks normal when looking at both versions of their plot.

Let’s look at the χ^2 Q-Q plot.



With this plot, I would say the skull data follows a multivariate normal.

(b) Construct 95% Bonferroni intervals for the individual skull dimension variables. Also, find the 95% T^2-intervals. Compare the two sets of intervals.

The simultaneous Bonferroni intervals for each skull variable are:

```
## Max Breadth: ( 128.8727 , 133.8607 )
## Basibregmatic Height: ( 131.427 , 135.773 )
## Basialveolar Length: ( 96.30548 , 102.0279 )
## Nasal Height: ( 49.18965 , 51.87702 )
```

The 95% T^2 intervals for each skull variable are:

```
## Max Breadth: ( 126.6252 , 136.1081 )
## Basibregmatic Height: ( 129.4688 , 137.7312 )
## Basialveolar Length: ( 93.72711 , 104.6062 )
## Nasal Height: ( 47.97878 , 53.08789 )
```

If we subtracted each of these intervals,

```
##      Bonferroni        T2
## [1,]   4.987998  9.482954
## [2,]   4.345980  8.262379
## [3,]   5.722376 10.879120
## [4,]   2.687371  5.109107
```

We find that the Bonferroni inetervals are roughly half the length of the T^2 intervals.

30. Refer to the data on energy consumption in Exercise 3.18.

(a) Obtain the large sample 95% Bonferroni confidence intervals for the mean consumption of each of the four types, the total of the four, and the differnce, petroleum minus natural gas

The Bonferroni confidence interval formula is: $\bar{x}_i \pm t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{ii}}{n}}$.

Therefore, each of the 95% Bonferroni confidence intervals for each type is:

```
## Petroleum: ( 0.4520686 , 1.079931 )
## Natural Gas: ( 0.2996904 , 0.7163096 )
## Coal: ( 0.3752871 , 0.5007129 )
## Nuclear: ( 0.1452301 , 0.1767699 )
## Total: ( 1.272276 , 2.473724 )
## Petroleum - Gas: ( 0.1523782 , 0.3636218 )
```

(b) Obtain the large sample 95% simultaneous T^2 intervals for the mean consumption of each of the four tyopes, the total of the four, and the difference, petroleum minus natural gas. Compare your results for Part a.

The T^2 confidence interval formula is: $\overline{x_\iota} \pm \sqrt{\chi_p^2(\alpha)}\sqrt{\frac{s_{pp}}{n}}$.

Therefore, each of the 95% T^2 confidence intervals for each type is:

```
## Petroleum: ( 0.3931193 , 1.138881 )
## Natural Gas: ( 0.2605745 , 0.7554255 )
## Coal: ( 0.363511 , 0.512489 )
## Nuclear: ( 0.1422688 , 0.1797312 )
## Total: ( 1.159474 , 2.586526 )
## Petroleum - Gas: ( 0.1325448 , 0.3834552 )
```

We know from Problem 23, T^2 intervals are roughly twice the length of Bonferroni intervals. Also, its critical region (shown below) is larger than Bonferroni's, so T^2 intervals will be wider.

```
## The Bonferroni crit region is 2.59326
## The T2 crit region is 3.080216
```

Appendix

```
knitr::opts_chunk$set(echo = FALSE)
library(car)
library(DescTools)
library(ellipse)
library(GGally)
library(ggplot2)
library(graphics)
library(gridExtra)
library(investr)
library(matlib)
library(MVN)
library(robustbase)
library(SIBER)
xMat <- c(2,12,8,9,6,9,8,10)
x <- matrix(xMat, nrow = 4, ncol = 2, byrow = TRUE)
muMat <- c(7,11)
mu <- matrix(muMat, nrow = 2, ncol = 1, byrow = TRUE)
dim(x)
n <- dim(x)[1]
one <- as.matrix(rep(1, dim(x)[1]))
xbar <- 1/n*t(x)%*%one
xbar
sum1 <- 0
# mu for Loop
for (i in 1:nrow(x)) {
  subtract <- x[i,] - mu
  multi <- subtract %*% t(subtract)
  sum1 <- sum1 + multi
}
print(sum1)
mutrix <- (28*10) - (-6)*(-6)
num <- (n-1)*mutrix
num
sum2 <- 0
# xbar for Loop
for (i in 1:nrow(x)) {
  subtract <- x[i,] - xbar
  multi <- subtract %*% t(subtract)
  sum2 <- sum2 + multi
}
denom <- (24*6) - (-10)*(-10)
print(denom)
t2 <- (num/denom) - (n-1)
cat("The T-squared value is", t2)
lambda <- (denom/(num/3))^(n/2)
cat("The value of the Lambda is", lambda)
lambda^(2/n)
```

```r
bear <- read.table("D:/Coding/R Storage/T1-4.dat", header = FALSE, sep = " ")
# new data
length <- bear[,5:8]
n <- dim(length)[1]
p <- dim(length)[2]
x2 <- length$V5 #Length2
x3 <- length$V6 #Length3
x4 <- length$V7 #Length4
x5 <- length$V8 #Length5
length <- as.matrix(length)
# xbar
one <- as.matrix(rep(1,dim(length)[1]))
n <- dim(length)[1]
xbar <- 1/n * t(length)%*%one
# covariance
mean_matrix <- matrix(data = 1, nrow = n)%*%cbind(xbar[[1]], xbar[[2]],
                                                  xbar[[3]], xbar[[4]])
xstar <- length - mean_matrix
covar <- 1/(n-1)*t(xstar)%*%xstar
# vars
crit <- qf(0.05, df1 = n, df2 = p, lower.tail = FALSE)
frac <- (p*(n-1))/(n-p)
right <- crit*frac
neg2 <- xbar[1] - sqrt(frac*crit*(covar[1,1]/n))
pos2 <- xbar[1] + sqrt(frac*crit*(covar[1,1]/n))
cat("(", neg2, ",", pos2, ")")
neg3 <- xbar[2] - sqrt(frac*crit*(covar[2,2]/n))
pos3 <- xbar[2] + sqrt(frac*crit*(covar[2,2]/n))
cat("(", neg3, ",", pos3, ")")
neg4 <- xbar[3] - sqrt(frac*crit*(covar[3,3]/n))
pos4 <- xbar[3] + sqrt(frac*crit*(covar[3,3]/n))
cat("(", neg4, ",", pos4, ")")
neg5 <- xbar[4] - sqrt(frac*crit*(covar[4,4]/n))
pos5 <- xbar[4] + sqrt(frac*crit*(covar[4,4]/n))
cat("(", neg5, ",", pos5, ")")
neg23 <- (xbar[2]-xbar[1]) - sqrt(frac*crit*((covar[2,2]-covar[1,1])/n))
pos23 <- (xbar[2]-xbar[1]) + sqrt(frac*crit*((covar[2,2]-covar[1,1])/n))
cat("Length2 - Length3: (", neg23, ",", pos23, ") \n")
# 3-4
neg34 <- (xbar[2]-xbar[3]) - sqrt(frac*crit*((covar[2,2]-covar[3,3])/n))
pos34 <- (xbar[2]-xbar[3]) + sqrt(frac*crit*((covar[2,2]-covar[3,3])/n))
cat("Length3 - Length4: (", neg34, ",", pos34, ") \n")
# 4-5
neg45 <- (xbar[4]-xbar[3]) - sqrt(frac*crit*((covar[4,4]-covar[3,3])/n))
pos45 <- (xbar[4]-xbar[3]) + sqrt(frac*crit*((covar[4,4]-covar[3,3])/n))
cat("Length4 - Length5: (", neg45, ",", pos45, ")")
a <- matrix(c(-1,1,0,0), ncol = 1)
b <- matrix(c(0,0,-1,1), ncol = 1)
# T2
meandiff <- c(t(a)%*%xbar, t(b)%*%xbar)
```

```r
Sdiff <- matrix(c(t(a)%*%covar%*%a, t(a)%*%covar%*%b, t(b)%*%covar%*%a,
                  t(b)%*%covar%*%b), 2, 2)
plot(ellipse(Sdiff, centre = meandiff, t = right/sqrt(n)), type = "l",
     xlab = "increase 2-3 years", yla = "increase 4-5 years")
points(meandiff[1], meandiff[2])
crit1 <- qt(0.05/(2*p), df = n-1, lower.tail = FALSE)
# L2
bneg1 <- xbar[1] - crit1*sqrt((covar[1,1]/n))
bpos1 <- xbar[1] + crit1*sqrt((covar[1,1]/n))
cat("Length 2: (", bneg1, ",", bpos1, ") \n")
# L3 height
bneg2 <- xbar[2] - crit1*sqrt((covar[2,2]/n))
bpos2 <- xbar[2] + crit1*sqrt((covar[2,2]/n))
cat("Length 3: (", bneg2, ",", bpos2, ") \n")
# L4
bneg3 <- xbar[3] - crit1*sqrt((covar[3,3]/n))
bpos3 <- xbar[3] + crit1*sqrt((covar[3,3]/n))
cat("Length 4: (", bneg3, ",", bpos3, ") \n")
# L5
bneg4 <- xbar[4] - crit1*sqrt((covar[4,4]/n))
bpos4 <- xbar[4] + crit1*sqrt((covar[4,4]/n))
cat("Length 5: (", bneg4, ",", bpos4, ") \n")
# L2-3
bneg23 <- (xbar[2]-xbar[1]) - crit1*sqrt((covar[2,2]-covar[1,1])/n)
bpos23 <- (xbar[2]-xbar[1]) + crit1*sqrt((covar[2,2]-covar[1,1])/n)
cat("Length 2-3: (", bneg23, ",", bpos23, ") \n")
# L3-4
bneg34 <- (xbar[2]-xbar[3]) - crit1*sqrt((covar[2,2]-covar[3,3])/n)
bpos34 <- (xbar[2]-xbar[3]) + crit1*sqrt((covar[2,2]-covar[3,3])/n)
cat("Length 3-4: (", bneg34, ",", bpos34, ") \n")
# L4-5
bneg45 <- (xbar[4]-xbar[3]) - crit1*sqrt((covar[4,4]-covar[3,3])/n)
bpos45 <- (xbar[4]-xbar[3]) + crit1*sqrt((covar[4,4]-covar[3,3])/n)
cat("Length 4-5: (", bneg45, ",", bpos45, ") \n")
a <- matrix(c(-1,1,0,0), ncol = 1)
b <- matrix(c(0,0,-1,1), ncol = 1)
# T2
meandiff <- c(t(a)%*%xbar, t(b)%*%xbar)
Sdiff <- matrix(c(t(a)%*%covar%*%a, t(a)%*%covar%*%b, t(b)%*%covar%*%a,
                  t(b)%*%covar%*%b), 2, 2)
plot(ellipse(Sdiff, centre = meandiff, t = right/sqrt(n)), type = "l",
     xlab = "increase 2-3 years", yla = "increase 4-5 years")
#points(meandiff[1], meandiff[2])
# Bonferroni
amu.L=t(a)%*%xbar-crit1*sqrt(t(a)%*%covar%*%a/n)
amu.U=t(a)%*%xbar+crit1*sqrt(t(a)%*%covar%*%a/n)
bmu.L=t(b)%*%xbar-crit1*sqrt(t(b)%*%covar%*%b/n)
bmu.U=t(b)%*%xbar+crit1*sqrt(t(b)%*%covar%*%b/n)
abline(v = amu.L, lty = 2, col = "firebrick3")
abline(v = amu.U, lty = 2, col = "firebrick3")
```

```r
abline(h = bmu.L, lty = 2, col = "firebrick3")
abline(h = bmu.U, lty = 2, col = "firebrick3")
xMat <- c(3,6,0,4,4,3,"-",8,3,5,"-","-")
x <- matrix(xMat, nrow = 4, ncol = 3, byrow = TRUE)
m1 <- (3+4+5)/3
m2 <- (6+4+8)/3
m3 <- (0+3+3)/3
muMat <- c(m1,m2,m3)
mu <- matrix(muMat, nrow = 3, ncol = 1, byrow = TRUE)
cat("The sample averages are:", m1,",", m2,", and", m3)
# row 1
s11 <- ((3-m1)^2 + (4-m1)^2 + (m1-m1)^2 + (5-m1)^2)/4
s12 <- ((3-m1)*(6-m2) + (4-m1)*(4-m2) + (m1-m1)*(8-m2) + (5-m1)*(m2-m2))/4
s13 <- ((3-m1)*(0-m3) + (4-m1)*(3-m3) + (m1-m1)*(3-m3) + (5-m1)*(m3-m3))/4
# row 2
s21 <- ((6-m2)*(3-m1) + (4-m2)*(4-m1) + (8-m2)*(m1-m1) + (m2-m2)*(5-m1))/4
s22 <- ((6-m2)^2 + (4-m2)^2 + (8-m2)^2 + (m2-m2)^2)/4
s23 <- ((6-m2)*(0-m3) + (4-m2)*(3-m3) + (8-m2)*(3-m3) + (m2-m2)*(m3-m3))/4
# row 3
s31 <- ((0-m3)*(3-m1) + (3-m3)*(4-m1) + (3-m3)*(m1-m1) + (m3-m3)*(5-m1))/4
s32 <- ((0-m3)*(6-m2) + (3-m3)*(4-m2) + (3-m3)*(8-m2) + (m3-m3)*(m2-m2))/4
s33 <- ((0-m3)^2 + (3-m3)^2 + (3-m3)^2 + (m3-m3)^2)/4
# matrix
sigMat <- c(s11, s12, s13, s21, s22, s23, s31, s32, s33)
sigma <- matrix(sigMat, nrow = 3, ncol = 3, byrow = TRUE)
sigma
# predict 31
sig22Mat <- c(0,0.5)
sig22.1 <- matrix(sig22Mat, nrow = 1, ncol = 2, byrow = TRUE)
sig12Mat <- c(2,0,0,1.5)
sig12.1 <- matrix(sig12Mat, nrow = 2, ncol = 2, byrow = TRUE)
muxMat <- c(8 - m2, 3 - m3)
mux <- matrix(muxMat, nrow = 2, ncol = 1, byrow = TRUE)
x31 <- 4 + sig22.1 %*% solve(sig12.1) %*% mux
sig31 <- 0.5 + sig22.1 %*% solve(sig12.1) %*% mux + x31^2
x3Mat <- c(8,3)
x3 <- matrix(x3Mat, nrow = 1, ncol = 2, byrow = TRUE)
row3 <- x31%*%x3
# predict 42-43
mat4 <- matrix(c(m2,m3), nrow = 2, ncol = 1, byrow = TRUE)
sig12 <- matrix(c(0,0.5), nrow = 2, ncol = 1, byrow = TRUE)
sig22 <- sigma[3,1]
xmu <- matrix(5-m1, nrow = 1, ncol = , byrow = TRUE)
x42.3 <- mat4 + sig12 %*% solve(sig22) %*% xmu
sig4 <- sigma[2:3, 2:3] - (sig12 %*% solve(sig22) %*% sig22.1) + (x42.3 %*%
t(x42.3))
row4 <- x42.3 %*% 5
# cat
cat("The predicted x31 is", x31, "\n")
cat("The predicted x31-squared is", sig31, "\n")
```

```r
cat("The predicted x42 and x43 are", x42.3, "\n")
cat("The predicted x42 and x43-squared and products are", sig4)
T1mat <- c(3+4+x31+5, 6+4+8+6, 0+3+3+3)
T1 <- matrix(T1mat, nrow = 3, ncol = 1, byrow = TRUE)
T2mat <- c(3^2+4^2+x31^2+5^2, " ", " ",
           (3*6)+(4*4)+(x31*8)+(5*6), 6^2+4^2+8^2+38, " ",
           (3*0)+(4*3)+(x31*3)+(5*3),(6*0)+(4*3)+(8*3)+(6*3),0^2+3^2+3^2+10)
T2 <- matrix(T2mat, nrow = 3, ncol = 3, byrow = TRUE)
# print
cat("The T1 matrix is \n")
T1
cat("\n The T2 matrix is \n")
T2
n <- dim(x)[1]
newMu <- 0.25 %*% t(T1)
cat("The first revised estimate of mu is: \n")
t(newMu)
# sigma
newT2 <- c(68.7777777777778, 98.6666666666667, 40,
          98.666666666666, 154, 54,
          40, 54, 28)
T2 <- matrix(newT2, nrow = 3, ncol = 3, byrow = TRUE)
newSig <- (0.25*T2) - t(newMu)%*%newMu
cat("\n The first revised estimate of Sigma is: \n")
newSig
college <- read.table("D:/Coding/R Storage/T5-2.dat", header = FALSE)
# vars
x1 <- college$V1 #social science & history
x2 <- college$V2 #verbal
x3 <- college$V3 #science
# vars
n <- 87
p <- 3
muMat <- c(500,50,30)
mu <- matrix(muMat, nrow = 3, ncol = 1, byrow = TRUE)
# testing
crit <- qf(0.05, df1 = n, df2 = p, lower.tail = FALSE)
cat("The critical region for this alpha is", crit, "\n")
HotellingsT2Test(college, mu = mu)
score <- as.matrix(college)
# xbar
one <- as.matrix(rep(1,dim(score)[1]))
n <- dim(score)[1]
xbar <- 1/n * t(score)%*%one
# covariance
mean_matrix <- matrix(data = 1, nrow = n)%*%cbind(xbar[[1]], xbar[[2]],
xbar[[3]])
xstar <- score - mean_matrix
covar <- 1/(n-1)*t(xstar)%*%xstar
# inverse
```

```r
inv <- solve(covar)
# eigen
eigen <- eigen(covar)
eigenval <- eigen$values
eigenvec <- eigen$vectors
cat("[",eigenvec[,1],"]")
cat("[",eigenvec[,2],"]")
cat("[",eigenvec[,3],"]")
# function
axis_length <- function(lambda, n, p, alpha = .95) {
return(sqrt(lambda*(p*(n-1))/(n*(n-p))*qf(alpha, p, n-p)))
}
# axis length
ax1 <- axis_length(eigenval[1], n, p)
ax2 <- axis_length(eigenval[2], n, p)
ax3 <- axis_length(eigenval[3], n, p)
# cat
cat("The axis length of social science & history is", ax1, "\n")
cat("The axis length of verbal is", ax2, "\n")
cat("The axis length of science is", ax3)
par(mfrow = c(1,2))
# x1
qqnorm(x1, main = "Social Science & History")
qqline(x1, col = "orangered2", lwd = 2)
qqPlot(x1, main = "Social Science & History")
# x2
qqnorm(x2, main = "Verbal")
qqline(x2, col = "darkviolet", lwd = 2)
qqPlot(x2, main = "Verbal")
# x3
qqnorm(x3, main = "Science")
qqline(x3, col = "springgreen4", lwd = 2)
qqPlot(x3, main = "Science")
ggpairs(college)
skull <- read.table("D:/Coding/R Storage/T6-13.dat", header = FALSE)
skullDim <- skull[1:30,1:4]
# vars
x1 <- skullDim$V1 #max breadth
x2 <- skullDim$V2 #basibregmatic height
x3 <- skullDim$V3 #basialveolar length
x4 <- skullDim$V4 #nasal height
par(mfrow = c(1,2))
# x1
qqnorm(x1, main = "Max Breath")
qqline(x1, col = "orangered2", lwd = 2)
qqPlot(x1, main = "Max Breath")
# x2
qqnorm(x2, main = "Basib Height")
qqline(x2, col = "darkviolet", lwd = 2)
qqPlot(x2, main = "Basib Height")
```

```
# x3
qqnorm(x3, main = "Basial Length")
qqline(x3, col = "springgreen4", lwd = 2)
qqPlot(x3, main = "Basial Length")
# x4
qqnorm(x4, main = "Nasal Height")
qqline(x4, col = "deepskyblue2", lwd = 2)
qqPlot(x4, main = "Nasal Height")
mvn(skull, multivariatePlot = "qq")
skullDim <- as.matrix(skullDim)
n <- 30
p <- 4
# xbar
one <- as.matrix(rep(1,dim(skullDim)[1]))
xbar <- 1/n * t(skullDim)%*%one
# covariance
mean_matrix <- matrix(data = 1, nrow = n)%*%cbind(xbar[[1]], xbar[[2]],
xbar[[3]], xbar[[4]])
xstar <- skullDim - mean_matrix
S <- 1/(n-1)*t(xstar)%*%xstar
# vars
crit1 <- -qt(0.05/8, df = n-1)
# max breadth
bneg1 <- xbar[1] - crit1*sqrt((S[1,1]/n))
bpos1 <- xbar[1] + crit1*sqrt((S[1,1]/n))
cat("Max Breadth: (", bneg1, ",", bpos1, ") \n")
# basibregmatic height
bneg2 <- xbar[2] - crit1*sqrt((S[2,2]/n))
bpos2 <- xbar[2] + crit1*sqrt((S[2,2]/n))
cat("Basibregmatic Height: (", bneg2, ",", bpos2, ") \n")
# basialveolar length
bneg3 <- xbar[3] - crit1*sqrt((S[3,3]/n))
bpos3 <- xbar[3] + crit1*sqrt((S[3,3]/n))
cat("Basialveolar Length: (", bneg3, ",", bpos3, ") \n")
# nasal height
bneg4 <- xbar[4] - crit1*sqrt((S[4,4]/n))
bpos4 <- xbar[4] + crit1*sqrt((S[4,4]/n))
cat("Nasal Height: (", bneg4, ",", bpos4, ") \n")
# crit
crit2 <- qf(0.05, df1 = n, df2 = p, lower.tail = FALSE)
frac <- (p*(n-1))/(n-p)
# max breadth
tneg1 <- xbar[1] - sqrt(frac*crit2*(S[1,1]/n))
tpos1 <- xbar[1] + sqrt(frac*crit2*(S[1,1]/n))
cat("Max Breadth: (", tneg1, ",", tpos1, ") \n")
# basibregmatic height
tneg2 <- xbar[2] - sqrt(frac*crit2*(S[2,2]/n))
tpos2 <- xbar[2] + sqrt(frac*crit2*(S[2,2]/n))
cat("Basibregmatic Height: (", tneg2, ",", tpos2, ") \n")
# basialveolar length
```

```r
tneg3 <- xbar[3] - sqrt(frac*crit2*(S[3,3]/n))
tpos3 <- xbar[3] + sqrt(frac*crit2*(S[3,3]/n))
cat("Basialveolar Length: (", tneg3, ",", tpos3, ") \n")
# nasal height
tneg4 <- xbar[4] - sqrt(frac*crit2*(S[4,4]/n))
tpos4 <- xbar[4] + sqrt(frac*crit2*(S[4,4]/n))
cat("Nasal Height: (", tneg4, ",", tpos4, ") \n")
Bonferroni <- c(bpos1-bneg1, bpos2-bneg2, bpos3-bneg3, bpos4-bneg4)
T2 <- c(tpos1-tneg1, tpos2-tneg2, tpos3-tneg3, tpos4-tneg4)
cbind(Bonferroni, T2)
# xbar
xbarMat <- c(0.766, 0.508, 0.438, 0.161)
xbar <- matrix(xbarMat, nrow = 4, ncol = 1, byrow = TRUE)
# sample variance
sMat <- c(0.856, 0.635, 0.173, 0.096,
          0.635, 0.568, 0.128, 0.067,
          0.173, 0.127, 0.171, 0.039,
          0.096, 0.067, 0.039, 0.043)
S <- matrix(sMat, nrow = 4, ncol = 4, byrow = TRUE)
# sample size
n <- 50
p <- 4
crit1 <- -qt(0.05/(2*p), df = n-1)
#petrol
neg1 <- xbar[1] - crit1*(S[1,1]/sqrt(n))
pos1 <- xbar[1] + crit1*(S[1,1]/sqrt(n))
cat("Petroleum: (", neg1, ",", pos1, ") \n")
# gas
neg2 <- xbar[2] - crit1*(S[2,2]/sqrt(n))
pos2 <- xbar[2] + crit1*(S[2,2]/sqrt(n))
cat("Natural Gas: (", neg2, ",", pos2, ") \n")
# coal
neg3 <- xbar[3] - crit1*(S[3,3]/sqrt(n))
pos3 <- xbar[3] + crit1*(S[3,3]/sqrt(n))
cat("Coal: (", neg3, ",", pos3, ") \n")
# nuclear
neg4 <- xbar[4] - crit1*(S[4,4]/sqrt(n))
pos4 <- xbar[4] + crit1*(S[4,4]/sqrt(n))
cat("Nuclear: (", neg4, ",", pos4, ") \n")
# total
neg5 <- (xbar[1] + xbar[2] + xbar[3] + + xbar[4]) -
crit1*((S[1,1]+S[2,2]+S[3,3]+S[4,4])/sqrt(n))
pos5 <- (xbar[1] + xbar[2] + xbar[3] + + xbar[4]) +
crit1*((S[1,1]+S[2,2]+S[3,3]+S[4,4])/sqrt(n))
cat("Total: (", neg5, ",", pos5, ") \n")
# petrol - gas
neg6 <- (xbar[1] - xbar[2]) - crit1*((S[1,1]-S[2,2])/sqrt(n))
pos6 <- (xbar[1] - xbar[2]) + crit1*((S[1,1]-S[2,2])/sqrt(n))
cat("Petroleum - Gas: (", neg6, ",", pos6, ")")
crit2 <- sqrt(qchisq(0.05, df = p, lower.tail = FALSE))
```

```r
# petrol
neg1 <- xbar[1] - (crit2*(S[1,1]/sqrt(n)))
pos1 <- xbar[1] + (crit2*(S[1,1]/sqrt(n)))
cat("Petroleum: (", neg1, ",", pos1, ") \n")
# gas
neg2 <- xbar[2] - (crit2*(S[2,2]/sqrt(n)))
pos2 <- xbar[2] + (crit2*(S[2,2]/sqrt(n)))
cat("Natural Gas: (", neg2, ",", pos2, ") \n")
# coal
neg3 <- xbar[3] - (crit2*(S[3,3]/sqrt(n)))
pos3 <- xbar[3] + (crit2*(S[3,3]/sqrt(n)))
cat("Coal: (", neg3, ",", pos3, ") \n")
# nuclear
neg4 <- xbar[4] - (crit2*(S[4,4]/sqrt(n)))
pos4 <- xbar[4] + (crit2*(S[4,4]/sqrt(n)))
cat("Nuclear: (", neg4, ",", pos4, ") \n")
# total
neg5 <- (xbar[1] + xbar[2] + xbar[3] + + xbar[4]) -
(crit2*((S[1,1]+S[2,2]+S[3,3]+S[4,4])/sqrt(n)))
pos5 <- (xbar[1] + xbar[2] + xbar[3] + + xbar[4]) +
(crit2*((S[1,1]+S[2,2]+S[3,3]+S[4,4])/sqrt(n)))
cat("Total: (", neg5, ",", pos5, ") \n")
# petrol - gas
neg6 <- (xbar[1] - xbar[2]) - (crit2*((S[1,1]-S[2,2])/sqrt(n)))
pos6 <- (xbar[1] - xbar[2]) + (crit2*((S[1,1]-S[2,2])/sqrt(n)))
cat("Petroleum - Gas: (", neg6, ",", pos6, ")")
cat("The Bonferroni crit region is", crit1, "\n")
cat("The T2 crit region is", crit2)
```