Math 760

Chapter 1 HW

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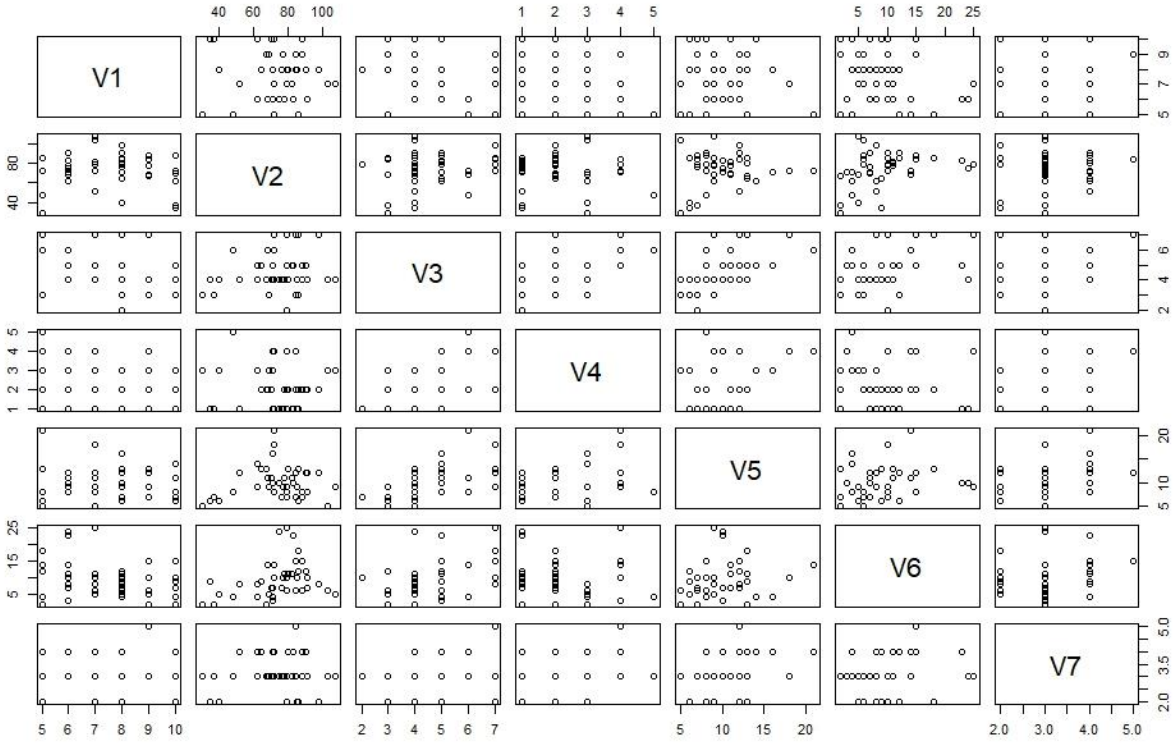
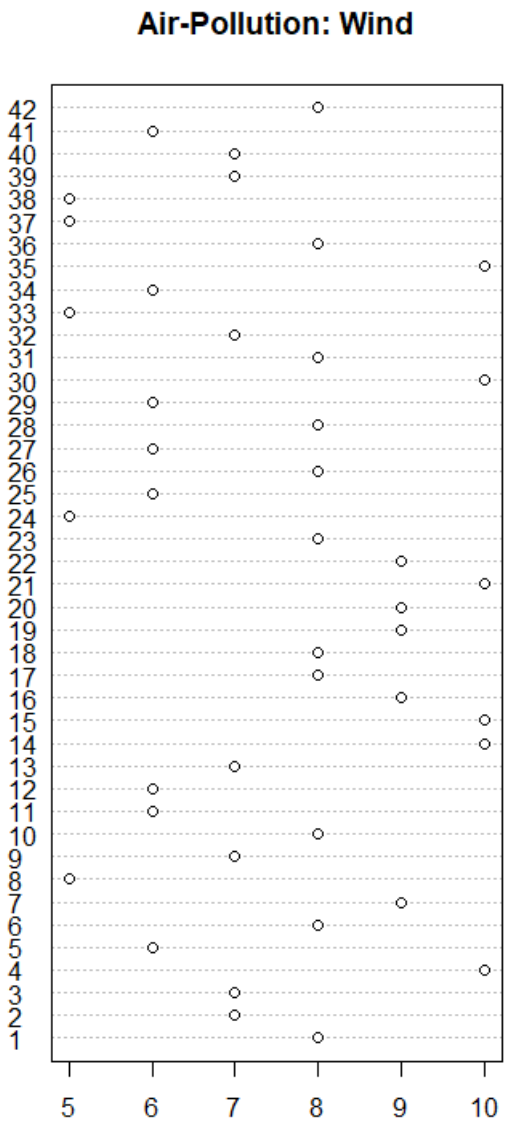
Feb 14, 2024

6. The data in Table 1.5 are 42 measurements on air-pollution variables recorded at 12:00 noon in the LA area on different days.

(a) Plot the mariginal dot diagrams for all the variables.




Air-Pollution: Wind
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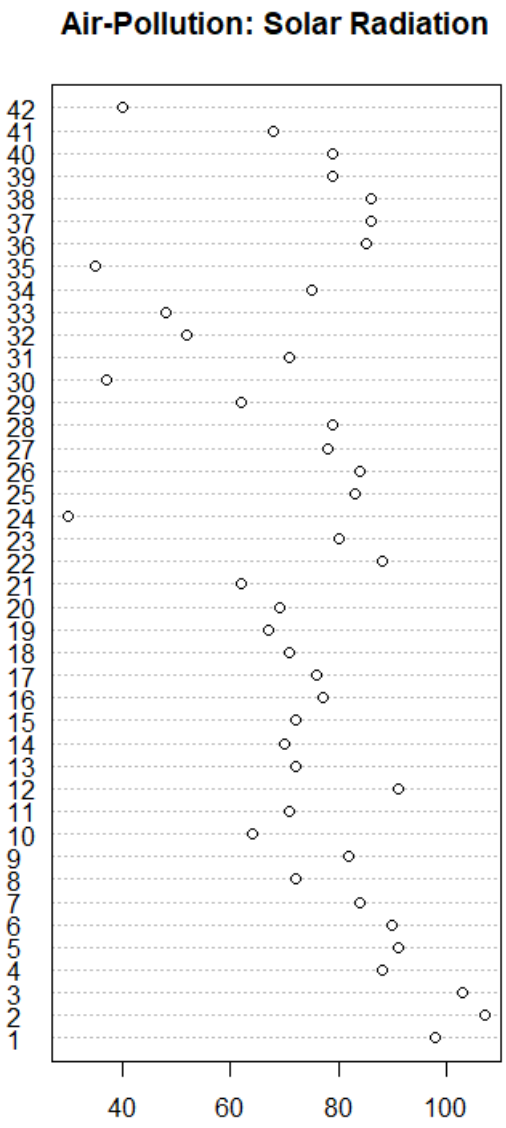
Air-Pollution: Solar Radiation
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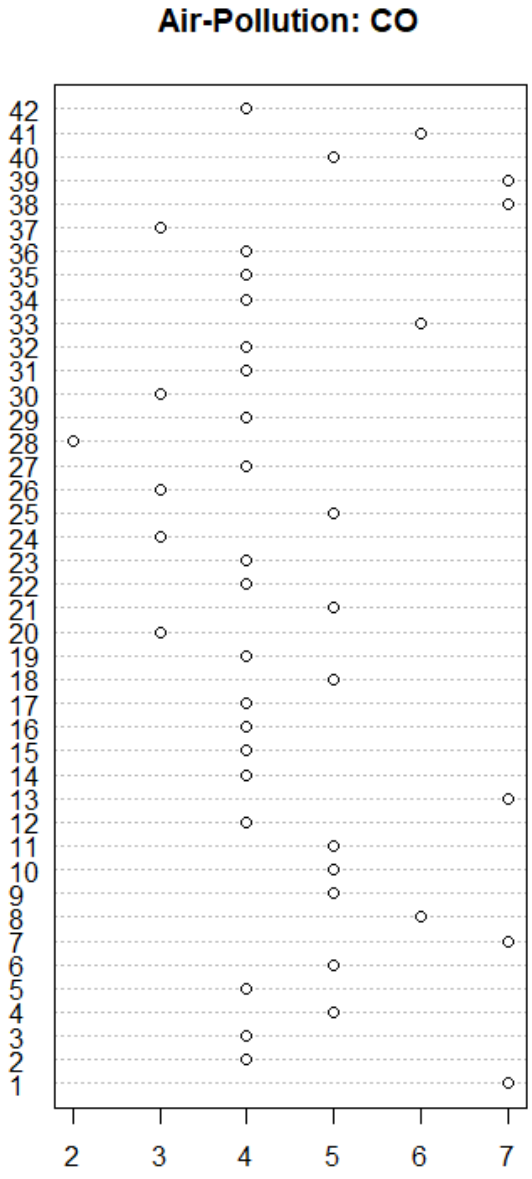
Air-Pollution: CO
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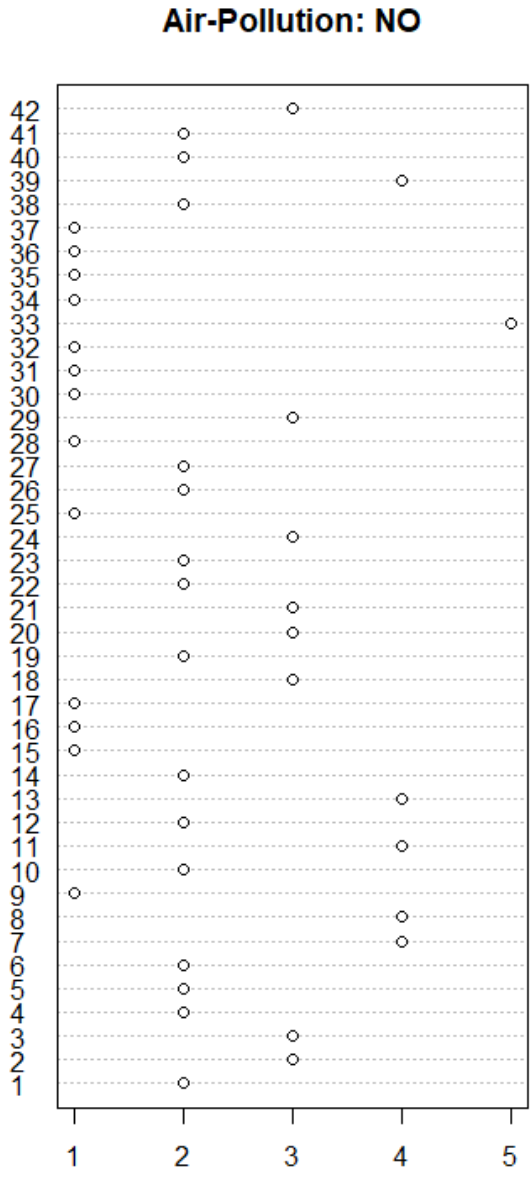
Air-Pollution: NO
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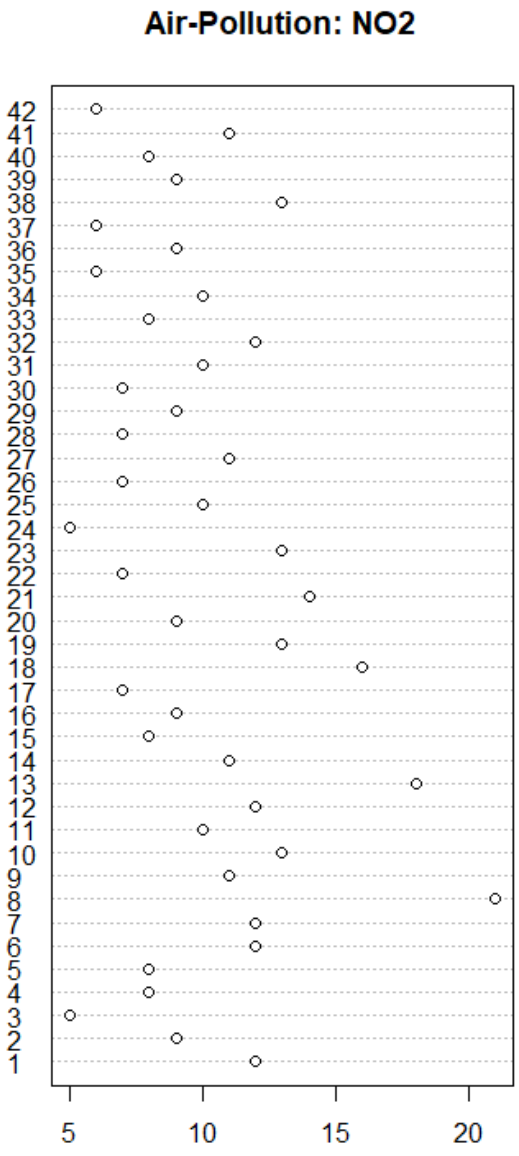
Air-Pollution: NO2
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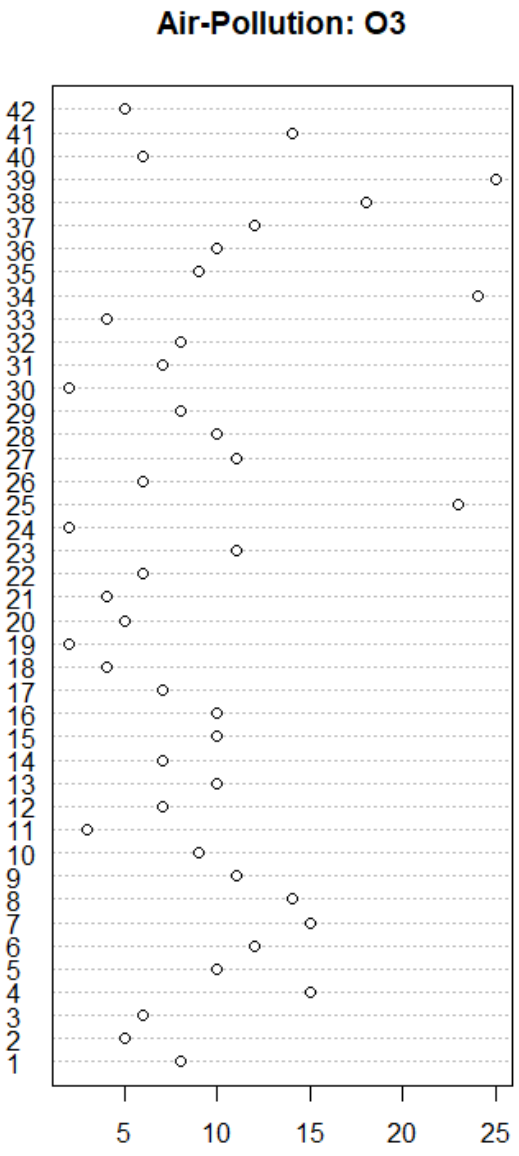
Air-Pollution: O3
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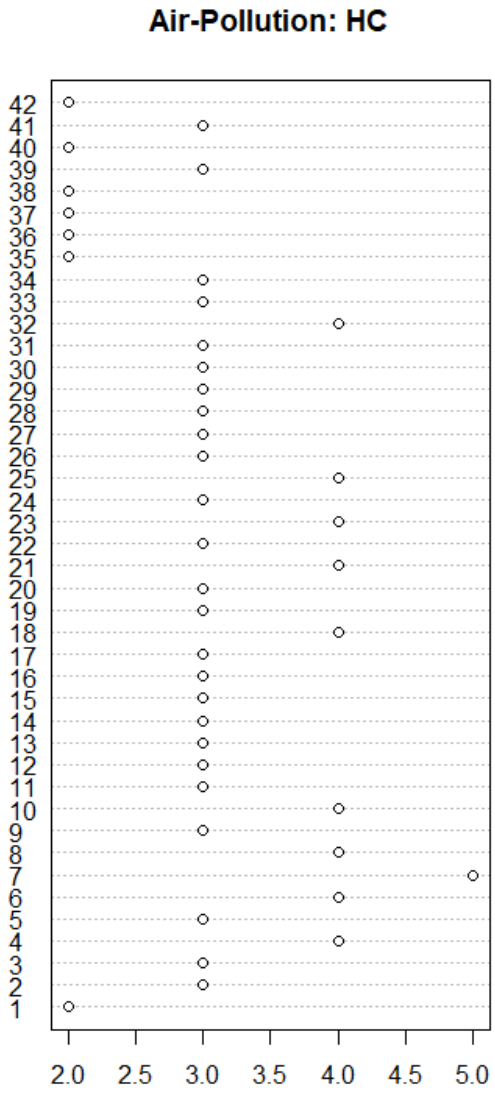


(b) Construct $\bar{x}$, S_n, and R arrays, and interpret entries in R.

The $\bar{x}$ array is:

```
##          [,1]
## V1  7.500000
## V2 73.857143
## V3  4.547619
## V4  2.190476
## V5 10.047619
## V6  9.404762
## V7  3.095238
```

The S_n array is:

```
##            V1          V2         V3         V4         V5         V6
V7
## V1  2.5000000  -2.7804878 -0.3780488 -0.4634146 -0.5853659 -2.2317073
0.1707317
## V2 -2.7804878 300.5156794  3.9094077 -1.3867596  6.7630662 30.7909408
0.6236934
## V3 -0.3780488   3.9094077  1.5220674  0.6736353  2.3147503  2.8217189
0.1416957
## V4 -0.4634146  -1.3867596  0.6736353  1.1823461  1.0882695 -0.8106852
0.1765389
## V5 -0.5853659   6.7630662  2.3147503  1.0882695 11.3635308  3.1265970
1.0441347
## V6 -2.2317073  30.7909408  2.8217189 -0.8106852  3.1265970 30.9785134
0.5946574
## V7  0.1707317   0.6236934  0.1416957  0.1765389  1.0441347  0.5946574
0.4785134
```

The **R** array is:

```
##            [,1]        [,2]       [,3]        [,4]       [,5]       [,6]
## [1,]  1.0000000 -0.10144191 -0.1938032 -0.26954261 -0.1098249 -0.2535928
## [2,] -0.1014419  1.00000000  0.1827934 -0.07356907  0.1157320  0.3191237
## [3,] -0.1938032  0.18279338  1.0000000  0.50215246  0.5565838  0.4109288
## [4,] -0.2695426 -0.07356907  0.5021525  1.00000000  0.2968981 -0.1339521
## [5,] -0.1098249  0.11573199  0.5565838  0.29689814  1.0000000  0.1666422
## [6,] -0.2535928  0.31912373  0.4109288 -0.13395214  0.1666422  1.0000000
## [7,]  0.1560979  0.05201044  0.1660323  0.23470432  0.4477678  0.1544506
##            [,7]
## [1,] 0.15609793
## [2,] 0.05201044
## [3,] 0.16603235
## [4,] 0.23470432
## [5,] 0.44776780
## [6,] 0.15445056
## [7,] 1.00000000
```

Most of the entries in the correlation array are quite small and have a negative correlation with v_1, wind. But rows v_3 and v_5 generally have a positive correlation with the other variables, besides v_1. Row v_7 is the only one that has a positive correlation with the rest of the variables.

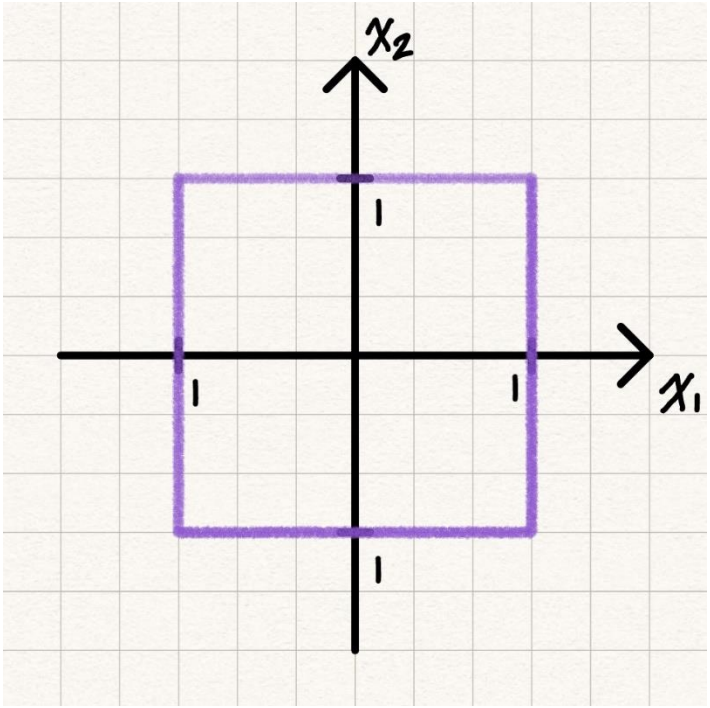
12. Define the distance from the point $P = (x_1, x_2)$ to the origin $O = (0, 0)$ as $d(O, P) = max(|x_1|, |x_2|)$

(a) Compute the distance from $P = (-3, 4)$ to the origin.

$$P = (x_1, x_2) \Rightarrow P = (-3,4)$$
$$d(O, P) = max(|x_1|, |x_2|) \Rightarrow d(O, P) = max(|-3|, |4|)$$

The distance from P = (-3,4) to the origin is 4.

(b) Plot the locus of points whose squared distance from the origin is 1.

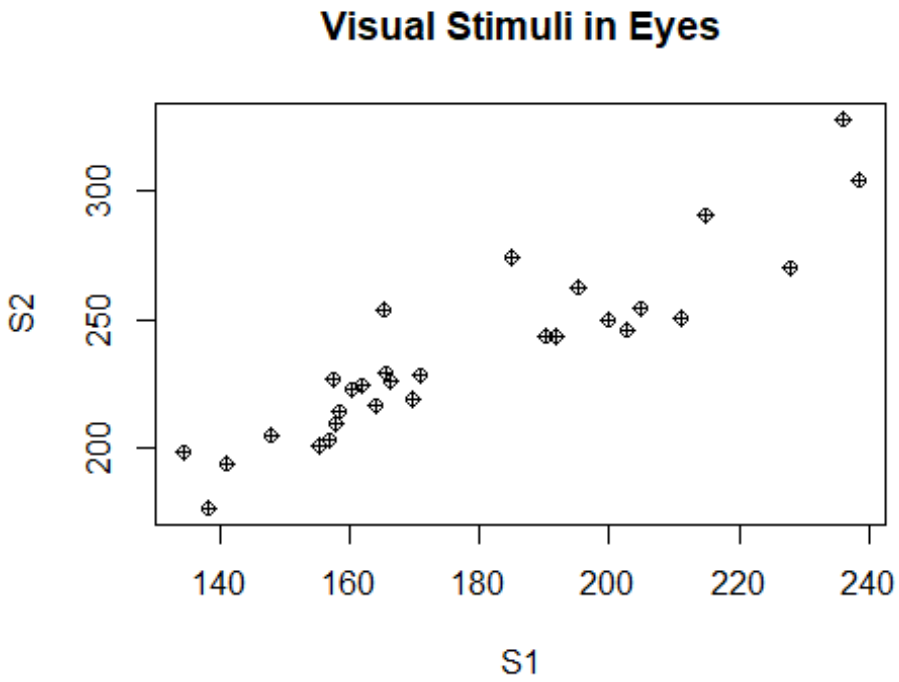


(c) Generalize the foregoing distance expression to the points in p dimensions.

The generalization of the foregoing distance expression in terms of p dimensions is:

$$d(O, P) = max(|x_1|, |x_2|) \Rightarrow d(O, P) = max\left(|x_1|, |x_2|, \ldots, |x_p|\right)$$

14. Table 1.6 contains some of the raw data discussed in Section 1.2. Two different visual stimuli (S1 and S2) produced responses in both the left eye (L) and the right eye (R) of subjects in the study groups. The values recoreded in the table include x_1 (subject's age); x_2 (total response to both eyes to stimulus S1, $|S1L + S1R|$); x_3 (difference between responses of eyes to stimulus S1, $|S1L - S1R|$); and so forth.
(a) Plot the two-dimensional scatter diagram for the variables x_2 and x_4 for multiple-sclerosis group. Comment on the appearance of the diagram.



There is a strong positive correlation according to this scatter plot, and there does not seem to be any outliers.

(b) Compute the $\bar{x}$, S_n, and R for the non-multiple-sclerosis and multiple-sclerosis groups separately.

Let's start with the multiple-sclerosis group.

The $\bar{x}$ array is:

```
##         [,1]
## V1  42.06897
## V2 178.26897
## V3  12.27586
## V4 236.93103
## V5  13.08276
```

The S_n array is:

```
##           V1        V2       V3         V4        V5
## V1 121.13793  52.79507 -20.2197   68.13350 -29.82020
## V2  52.79507 844.68079 244.4632  912.41493 106.76409
## V3 -20.21970 244.46315 317.2640  232.36542 297.31921
## V4  68.13350 912.41493 232.3654 1180.03222  81.09734
## V5 -29.82020 106.76409 297.3192   81.09734 351.04719
```

The **R** array is:

```
##            [,1]      [,2]       [,3]      [,4]       [,5]
## [1,]  1.0000000 0.1650468 -0.1031393 0.1802078 -0.1446065
## [2,]  0.1650468 1.0000000  0.4722334 0.9139010  0.1960632
## [3,] -0.1031393 0.4722334  1.0000000 0.3797643  0.8909017
## [4,]  0.1802078 0.9139010  0.3797643 1.0000000  0.1260019
## [5,] -0.1446065 0.1960632  0.8909017 0.1260019  1.0000000
```

Now, let's compute for the non-multiple-sclerosis group.

The $\bar{x}$ array is:

```
##          [,1]
## V1  37.985507
## V2 147.289855
## V3   1.562319
## V4 195.602899
## V5   1.620290
```

The S_n array is:

```
##            V1         V2       V3         V4       V5
## V1 277.632140  95.398380 5.361211 103.723572 3.241475
## V2  95.398380 112.294749 1.766377 106.785030 2.042268
## V3   5.361211   1.766377 1.805030   2.234817 0.501364
## V4 103.723572 106.785030 2.234817 185.228815 2.351117
## V5   3.241475   2.042268 0.501364   2.351117 2.355465
```

The **R** array is:

```
##           [,1]      [,2]      [,3]      [,4]      [,5]
## [1,] 1.0000000 0.5402894 0.2394891 0.4573917 0.1267563
## [2,] 0.5402894 1.0000000 0.1240685 0.7404170 0.1255725
## [3,] 0.2394891 0.1240685 1.0000000 0.1222209 0.2431491
## [4,] 0.4573917 0.7404170 0.1222209 1.0000000 0.1125594
## [5,] 0.1267563 0.1255725 0.2431491 0.1125594 1.0000000
```

18. Convert the national track records for women in Table 1.9 to speeds measured in meters per second. For example, the record speed for the 1100-m dash for Argentinian women is 100 m/11.57 sec ≈ 8.643 m/sec. Notice that the records for the 800-m, 1500-m, 3000-m, and marathon runs are measured in minutes. The marathon is 26.2 miles, or 42,195 meters long. Compute $\bar{x}$, S_n, and R arrays. Notice the magnitudes of the correlation coefficients as you go from the shorter (100-meter) to the longer (marathon) running distances. Interpret these pairwise correlations.

Columns 5 through 8 will be converted from minute to second to match the first 4 columns before diving into the arrays.

The $\bar{x}$ array is:

```
##          [,1]
## V2   11.35778
## V3   23.11852
## V4   51.98907
## V5  121.34444
## V6  251.36667
## V7  544.84444
## V8 9217.15556
```

The S_n array is:

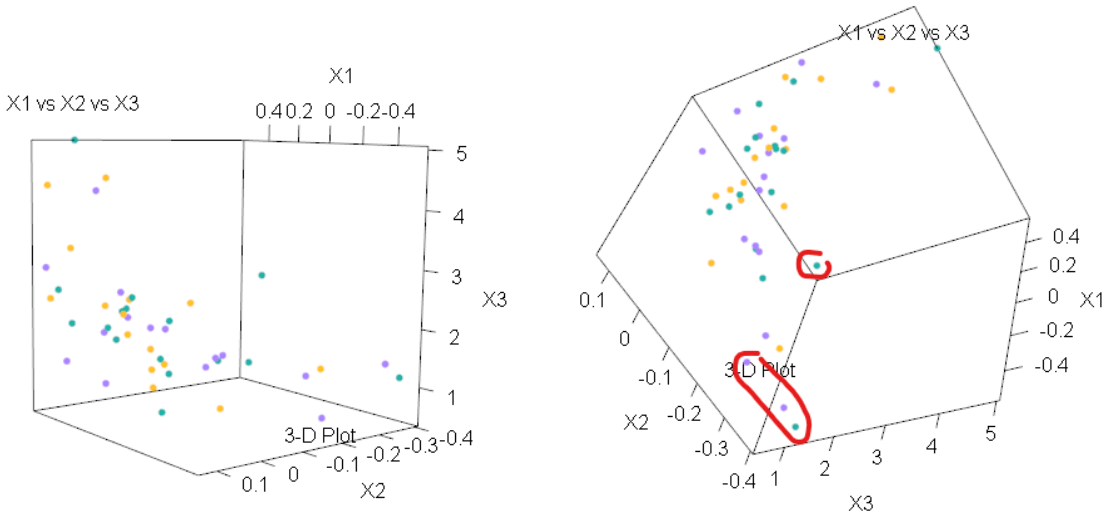
```
##              V2          V3          V4          V5           V6
V7
## V2   0.1553157   0.3445608    0.891296    1.662214     5.033472
14.03297
## V3   0.3445608   0.8630883    2.192836    3.969954    12.165799
33.26101
## V4   0.8912960   2.1928363    6.745458   10.908476    30.550610
85.60895
## V5   1.6622138   3.9699539   10.908476   27.168931    77.092453
220.96553
## V6   5.0334717  12.1657987   30.550610   77.092453   267.057736
778.15849
## V7  14.0329686  33.2610105   85.608948  220.965535   778.158491
2393.12855
## V8 260.0506541 623.0992537 1734.223883 4390.756730 12743.414340
38541.92805
##              V8
## V2    260.0507
## V3    623.0993
## V4   1734.2239
## V5   4390.7567
## V6  12743.4143
## V7  38541.9281
## V8 972972.5414
```

The **R** array is:

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 1.0000000 0.9410886 0.8707802 0.8091758 0.7815510 0.7278784 0.6689597
## [2,] 0.9410886 1.0000000 0.9088096 0.8198258 0.8013282 0.7318546 0.6799537
## [3,] 0.8707802 0.9088096 1.0000000 0.8057904 0.7197996 0.6737991 0.6769384
## [4,] 0.8091758 0.8198258 0.8057904 1.0000000 0.9050509 0.8665732 0.8539900
## [5,] 0.7815510 0.8013282 0.7197996 0.9050509 1.0000000 0.9733801 0.7905565
## [6,] 0.7278784 0.7318546 0.6737991 0.8665732 0.9733801 1.0000000 0.7987302
## [7,] 0.6689597 0.6799537 0.6769384 0.8539900 0.7905565 0.7987302 1.0000000
```

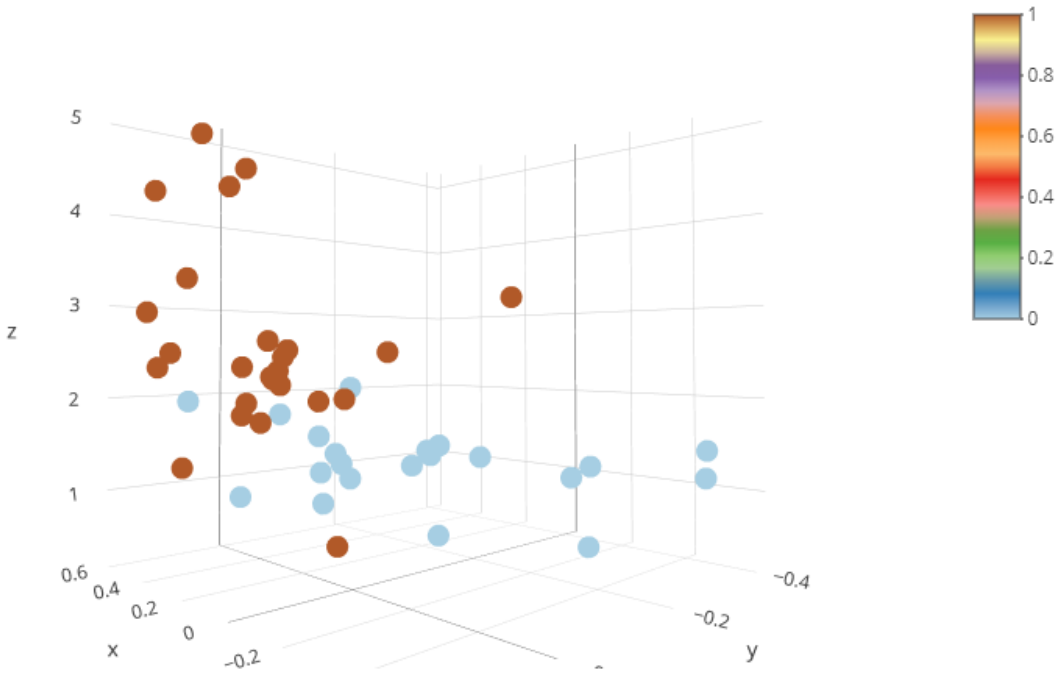
All the correlations are positive through each pair, decreasing in value as the running distances between pairs increase.

20. Refer to the bankruptcy data in Table 11.4, page 657, and on the folllowing website www.prenhall.com/statistics. Using appropriate computer software,
(a) View the entire data set in x_1, x_2, x_3 space. Rotate the coordinate axes in various directions. Check for unusual observations.



The 3D plot shows us an exponential, whether it goes up or down depends on how you look at it. There could be a few unusual observations, ones that stray a bit far from curve. They have been circled.

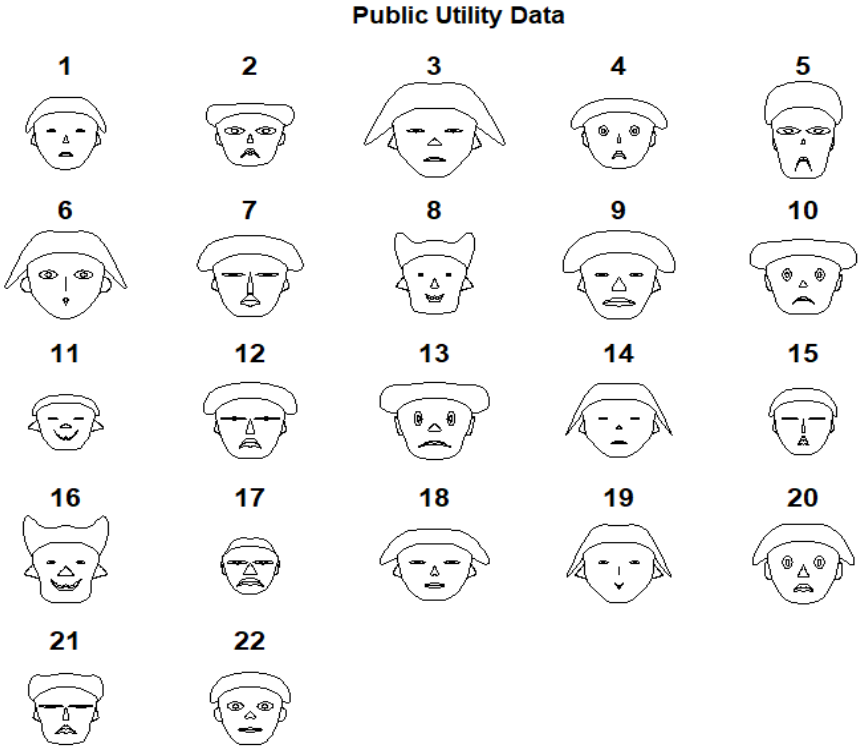
(b) Highlight the set of points corresponding to the bankrupt firms. Examine various 3D perspectives. Are there some orientations of 3D space for which bankrupt firms can be distinguished from the nonbankrupt firms? Are there observations in each of the two groups that are likely to have a significant impact on any rule developed to classify firms based on the sample means, variances, and covariances calculated from these data (See Exercise 11.24).



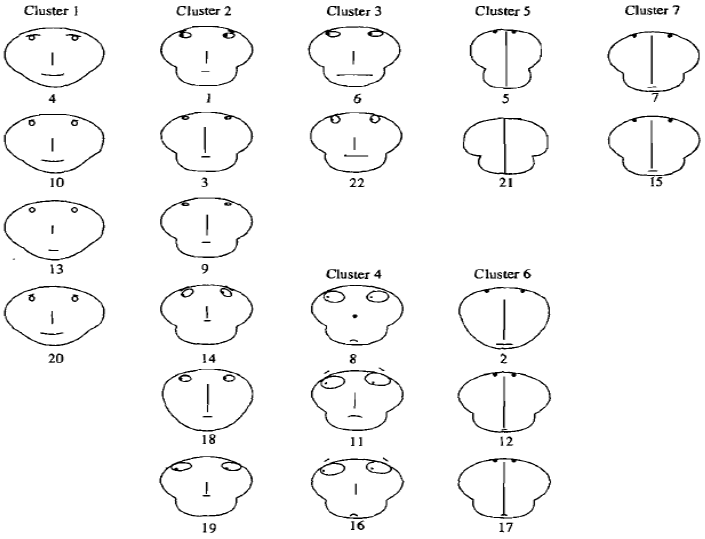
Yes, there are orientations of 3D space for which bankrupt firms can be distinguished from the non-bankrupt firms. One is shown above. They have also been color-coded to make it easier to distinguish them: the bankrupt observations are colored blue, while the non-bankrupt ones are colored brown. There are observations that are likely to have a significant impact on any rule developed to classify firms based on the sample means, variances, and covariances calculated from this data. There are a few or so observations that overlap into the group that they aren't part of.

24. Using the utility data in Table 12.4, page 688, and on the web at www.prenhall.com/statistics. represent the public utility companies as Chernoff faces with assignments of variables to facial characteristics different from those considered in Example 1.12. Compare your faces with the faces in Figure 1.17. Are different groupings indicated?

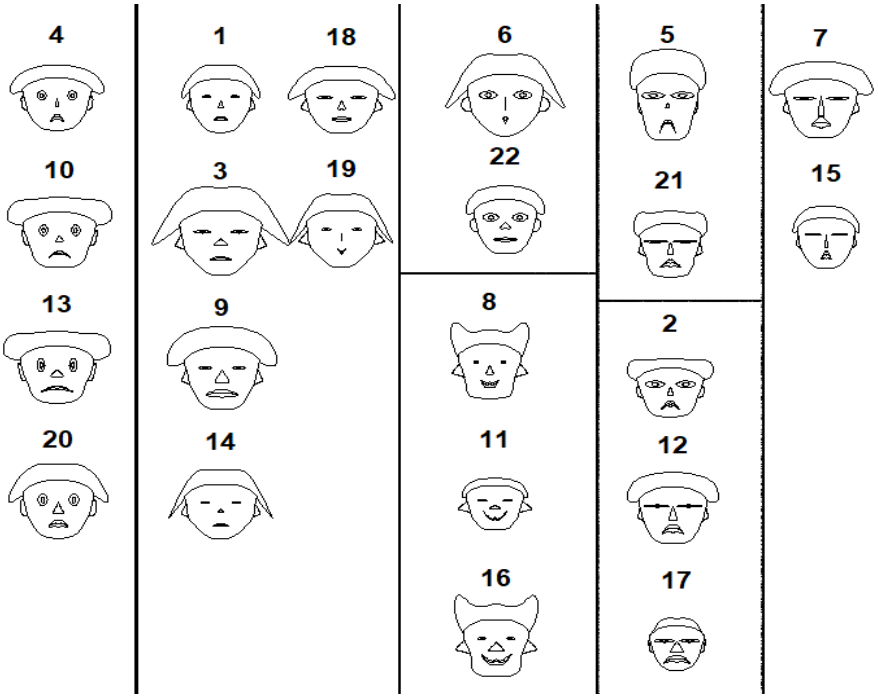
We'll first try the Chernoff faces with the default functions.



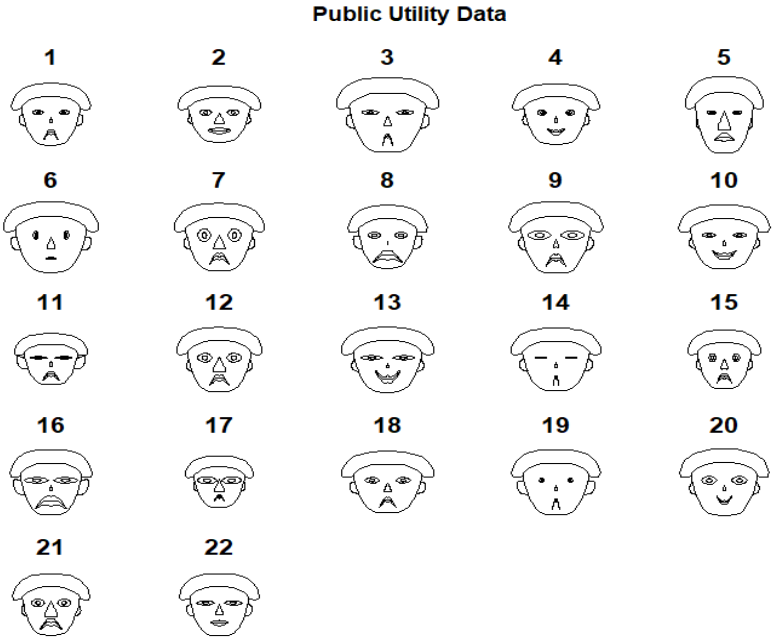
Our faces are quite notably different with the ones in Example 1.12 and Figure 1.17. There's hair and ears on ours, while they're lacking in Figure 1.17 below.



If we grouped our faces like in Example 1.13, I could nearly say the clusters are near similarly.



Let's try assigning each variable with one feature, a few may double up. For the features that didn't receive a variable, they will be assigned a constant.



Based on these new faces, the clusters could be kept the same, but I would make a few changes. I could easily group Face 11 and Face 14, both sharing closed eyes, deep frown, and small nose. Face 2 doesn't quite fit along with Face 12 and Face 17 anymore.

Code

```
knitr::opts_chunk$set(echo = FALSE)

library(aplpack)
library(cowplot)
library(ggplot2)
library(ggExtra)
library(ggpubr)
library(plotly)
library(rgl)
library(rglwidget)
library(webshot)
library(webshot2)
pollute <- read.table("D:/Coding/R Storage/T1-5.dat", header = FALSE)

# vars
x1 <- pollute$V1 # wind
x2 <- pollute$V2 # solar radiation
x3 <- pollute$V3 # CO
x4 <- pollute$V4 # NO
x5 <- pollute$V5 # NO2
x6 <- pollute$V6 # O3
x7 <- pollute$V7 # HC
pairs(pollute)

# dot plot
par(mfrow = c(1,2))
dotchart(x1, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
Wind")
dotchart(x2, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
Solar Radiation")
dotchart(x3, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
CO")
dotchart(x4, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
NO")
dotchart(x5, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
NO2")
dotchart(x6, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
O3")
dotchart(x7, labels = row.names(pollute), cex = 1, main = "Air-Pollution:
HC")
pollMat <- as.matrix(pollute)
ID <- as.matrix(rep(1, dim(pollMat)[1]))
n <- dim(pollMat)[1]
xbar <- 1/n*t(pollMat)%*%ID
print(xbar)
meanMat <- matrix(data = 1, nrow = n)%*%cbind(xbar[[1]], xbar[[2]],
xbar[[3]], xbar[[4]], xbar[[5]], xbar[[6]], xbar[[7]])
```

```r
poll <- pollMat - meanMat
coVar <- 1/(n-1)*t(poll)%*%poll
print(coVar)
D <- diag(diag(coVar)^(-1/2))
corr <- D%*%coVar%*%D
print(corr)
square <- ggplot() + geom_rect(aes(xmin = -1, xmax = 1, ymin = -1, ymax = 1))
+ theme_minimal_grid(12)
square + coord_equal()
multiScler <- read.table("D:/Coding/R Storage/T1-6.dat", header = FALSE)

# splitting the data by v6
non <- subset(multiScler, V6 == 0)
ms <- subset(multiScler, V6 == 1)

# removing V6 from both sets
non <- non[,-6]
ms <- ms[,-6]
# vars
x2 <- ms$V2 #S1R + S1L
x4 <- ms$V4 #S2R + S2L

# plot
plot(x2, x4, main = "Visual Stimuli in Eyes", xlab = "S1", ylab = "S2", pch =
10)
msMat <- as.matrix(ms)
ID <- as.matrix(rep(1, dim(msMat)[1]))
n <- dim(msMat)[1]
xbar <- 1/n*t(msMat)%*%ID
print(xbar)
meanMat <- matrix(data = 1, nrow = n)%*%cbind(xbar[[1]], xbar[[2]],
xbar[[3]], xbar[[4]], xbar[[5]])
multi <- msMat - meanMat
coVar <- 1/(n-1)*t(multi)%*%multi
print(coVar)
D <- diag(diag(coVar)^(-1/2))
corr <- D%*%coVar%*%D
print(corr)
nonMat <- as.matrix(non)
ID <- as.matrix(rep(1, dim(nonMat)[1]))
n <- dim(nonMat)[1]
non.xbar <- 1/n*t(nonMat)%*%ID
print(non.xbar)
non.meanMat <- matrix(data = 1, nrow = n)%*%cbind(non.xbar[[1]],
non.xbar[[2]], non.xbar[[3]], non.xbar[[4]], non.xbar[[5]])
nons <- nonMat - non.meanMat
non.coVar <- 1/(n-1)*t(nons)%*%nons
print(non.coVar)
nonD <- diag(diag(non.coVar)^(-1/2))
nonCorr <- nonD%*%non.coVar%*%nonD
```

```r
print(nonCorr)
track <- read.table("D:/Coding/R Storage/T1-9.dat", header = FALSE, sep =
"\t")

# vars
x1 <- track$V1 # country
x2 <- track$V2 # 100m/s
x3 <- track$V3 # 200m/s
x4 <- track$V4 # 400m/s
x5 <- track$V5# 800m/min
x6 <- track$V6 # 1500m/min
x7 <- track$V7 # 3000m/min
x8 <- track$V8 # marathon/min

# per second
OG <- track[,1:4]
second <- track[,5:8]*60
second$V1 <- track$V1

record <- merge(OG, second, by = "V1")
record <- record[,-1]
recordMat <- as.matrix(record)
ID <- as.matrix(rep(1, dim(recordMat)[1]))
n <- dim(recordMat)[1]
xbar <- 1/n*t(recordMat)%*%ID
print(xbar)
meanMat <- matrix(data = 1, nrow = n)%*%cbind(xbar[[1]], xbar[[2]],
xbar[[3]], xbar[[4]], xbar[[5]], xbar[[6]], xbar[[7]])
meter <- recordMat - meanMat
coVar <- 1/(n-1)*t(meter)%*%meter
print(coVar)
D <- diag(diag(coVar)^(-1/2))
corr <- D%*%coVar%*%D
print(corr)
bank <- read.table("D:/Coding/R Storage/T11-4.dat", header = FALSE)

# vars
x1 <- bank$V1 # CF/TD
x2 <- bank$V2 # NI/TA
x3 <- bank$V3 # CA/CL
x4 <- bank$V4 # CA/NS
x5 <- bank$V5 # pop, i = 1,2
plot3d(x1, x2, x3, type = "p", size = 6, lit = FALSE, box = FALSE, col =
c("lightseagreen","mediumpurple1","goldenrod1"),expand = 1, main = "X1 vs X2
vs X3", sub = "3-D Plot", xlab = "X1", ylab = "X2", zlab = "X3")
plot_ly(x = x1, y = x2, z = x3, type = "scatter3d", mode = "markers", color =
x5, colors = "Paired")
utility <- read.table("D:/Coding/R Storage/T12-4.dat", header = FALSE)
```

```r
# vars
x1 <- utility$V1 # fixed-charge coverage ratio (income/debt)
x2 <- utility$V2 # rate of return on capital
x3 <- utility$V3 # cost per KW capacity in place
x4 <- utility$V4 # annual load factor
x5 <- utility$V5 # peak kWh demand growth from 1974 and 1975
x6 <- utility$V6 # sales (kWh use/year)
x7 <- utility$V7 # % nuclear
x8 <- utility$V8 # total fuel costs (cents/kWh)
x9 <- utility$V9 # company
faces(utility[,1:8], face.type = 0, main = "Public Utility Data")
uti <- matrix(1, nrow = 22, ncol = 15)

uti[,1] <- x1
uti[,2] <- x2
uti[, c(4,5)] <- x3
uti[,7] <- x4
uti[,8] <- x5
uti[, c(14,15)] <- x6
uti[,6] <- x7
uti[, c(12,13)] <- x8

faces(uti, face.type = 0, main = "Public Utility Data")
```