Find answers to the following questions by simulation using SAS (at least 10000 simulations). You must provide a final answer once you have observed the output from SAS and give reasons for your answers. That is, if there is convergence, why and what is the final answer. However, if there is divergence, why.

1. Suppose X_i for i=1, 2, 3… has uniform (0, 1) distribution.

A. Let M = min (n: $X_1 + X_2 + \ldots + X_n > 1$). Find expected value of M; E(M) = Mean of M.

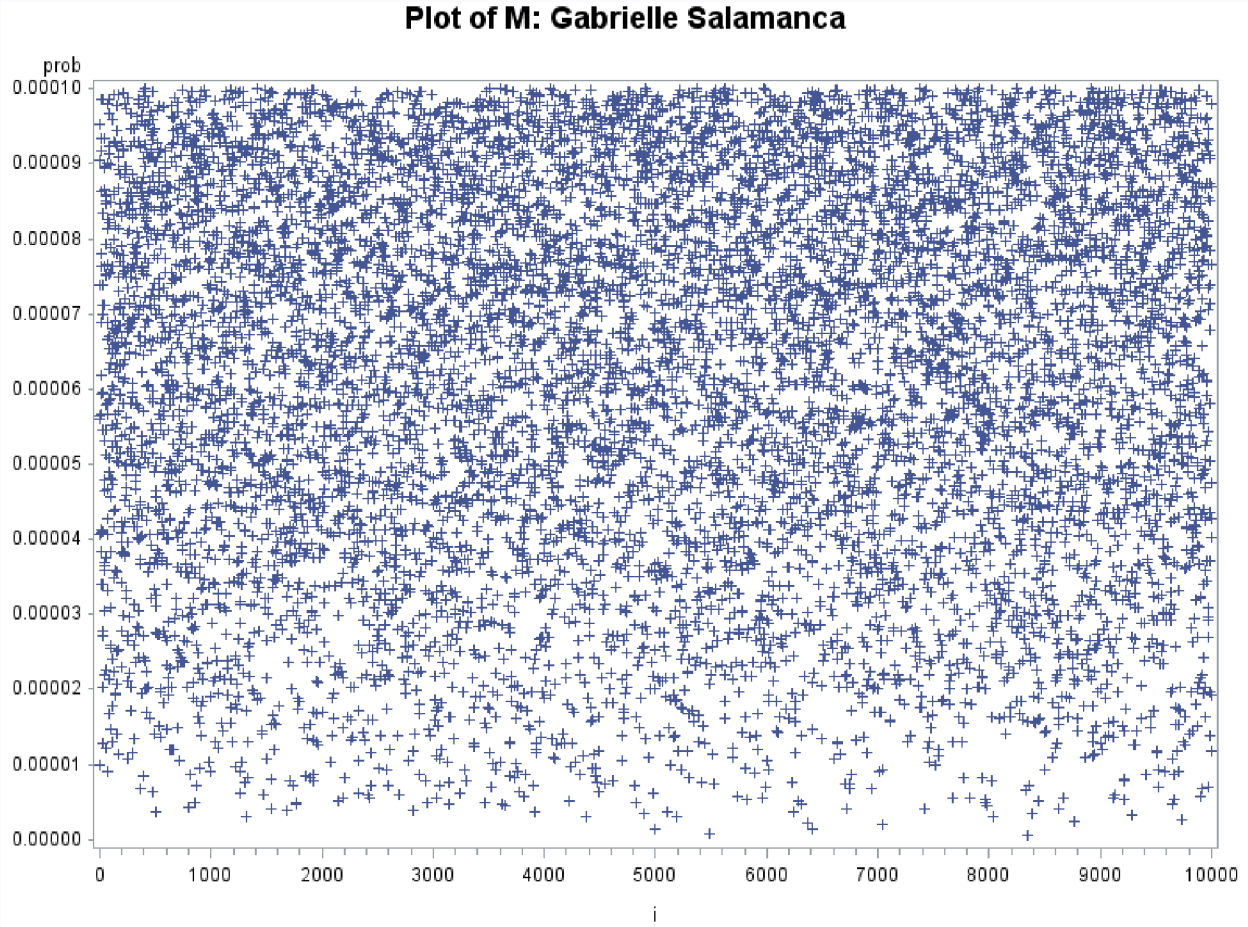
The Mean of M is 0.6369633, or 0.637. Upon observing the graph of E(M) however, it seems to diverge since it doesn't converge to one point.

```
/***** HW3 Q1** Gabrielle Salamanca *****/
options nodate;
data uniform;
call streaminit(0);

a = 10000;

do i = 1 to a;
      sum = 0;
      M=0;
      do until (sum > 1);
            x = rand('uniform', 0, 1);
            sum = sum + x;
            M = M + 1;
            end;
            prob = M / a;
      output;
end;
run;
/***Q3.A: Mean of M***/
proc means data = uniform;
var M;
title 'Mean M: Gabrielle Salamanca';
run;
proc gplot;
plot prob*i;
title 'Plot of M: Gabrielle Salamanca';
run;
```

Mean M: Gabrielle Salamanca

The MEANS Procedure

Analysis Variable : M				
N	Mean	Std Dev	Minimum	Maximum
10000	0.6369633	0.2436196	0.0059316	1.0000000



B. Let N = min (n+1: $X_n > X_{n+1}$). Find expected value of N; E(N) = Mean of N.

The mean of N is 2.7220278, or 2.722. The graph of E(N) converges towards 10000, each probability makes a clear line towards it.

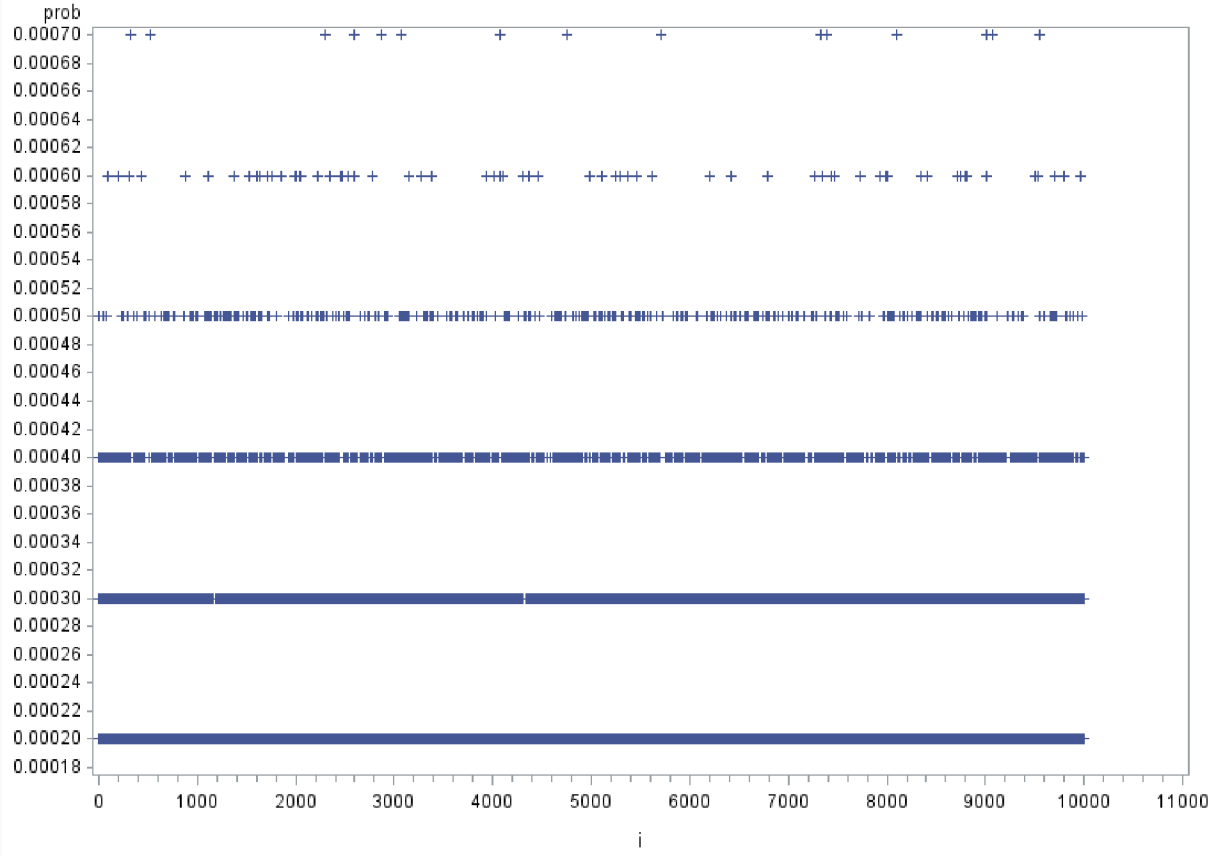
```
/**********************************/
data uni;
call streaminit(0);

b = 10000;

do i = 1 to (b+1);
      sum = 0;
      N = 0;
      do until (sum > 1);
            x = rand('uniform', 0, 1);
            sum = sum + x;
            N = N + 1;
            end;
            prob = N / (b+1);
      output;
end;
run;
/***Q3.B: Mean of N***/
proc means data = uni;
var N;
title 'Mean N: Gabrielle Salamanca';
run;
proc gplot;
plot prob*i;
title 'Plot of N: Gabrielle Salamanca';
run;
/**********************************/
quit;
```

Mean N: Gabrielle Salamanca

The MEANS Procedure

Analysis Variable : N				
N	Mean	Std Dev	Minimum	Maximum
10001	2.7220278	0.8785919	2.0000000	7.0000000

Plot of N: Gabrielle Salamanca



2. A one unit stick is broken randomly into two pieces.

```
/***HW3.2***Gabrielle Salamanca***/
options nodate;
data stick;

l=0;
s=0;
n=10000;

do i = 1 to n;
    x = rand('uniform',0,1);

    if x > 0.5 then l = l + x;
    else s = s + x;

    if x < 0.5 then s = s + x;
    else l = l + x;

    if x > 0.5 then h = x;
    if x < 0.5 then d = x;

    output;
      tlc = l/s;
      qlc = s/l;

end;

run;
proc means data = stick;
var h d tlc qlc;
run;
proc print;
var h d tlc qlc;
run;
/*****************************/
quit;
```

The SAS System

The MEANS Procedure

Variable	N	Mean	Std Dev	Minimum	Maximum
h	10000	0.7496146	0.1440399	0.5000047	0.9999150
d	9997	0.2522606	0.1426150	0.000436250	0.4999986
tlc	9996	3.0698961	0.2464217	2.2364618	17.5962413
qlc	9999	0.3265878	0.0156302	0	0.4471348

The mean of the long piece is 0.7496146, or roughly 0.75; while the mean of the short piece is 0.2522606, or roughly 0.25. Both converge, because both graphs reach towards those points.

A. Find E (the short piece divided by the long piece).

When using proc means, the expected value of the short piece divided by the long piece is 0.3265878, or 0.33. If we divide the expected value of the short piece by the expected value of the long piece in a calculator, it will equal to 0.3365203933 or 0.333 when using the rounded version.

B. Find E (the long piece divided by the short piece).

When using proc means, the expected value of the short piece divided by the long piece is 3.0698961, or 3.07. If we divide the expected value of the short piece by the expected value of the long piece in a calculator, it will equal to 2.971588112 or exactly 3 when using the rounded version.

The graphs of E (the short piece divided by the long piece) and E (the long piece divided by the short piece) show that both converge towards each point, 0.333 for the former and 3 for the latter.