Problem 1

Let D_i represent the disagreement set for the i^{th} element-wise comparison during the most general unification algorithm, Unify(x, y, θ) (figure 9.1 in AIMA). θ is the substitution list built up so far.

Part A

$S_0 = \{P(A, B, B), P(x, y, z)\}$
$\theta = \{x/A, y/B, z/B\}$ is the most general unifier.

Part B

$S_0 = \{Q(y, G(A, B)), Q(G(x, x), y)\}$
$D_0 = \{y, G(x, x)\}$
$\theta = \{y/G(x, x)\}$
$S_1 = \{Q(G(x, x), G(A, B)), Q(G(x, x), G(x, x))\}$
$D_1 = \{A, x\}$
$\theta = \{y/G(x, x), x/A\}$
$S_2 = \{Q(G(A, x), G(A, B)), Q(G(A, x), G(A, X))\}$
$D_2 = \{B, x\}$ No unification possible.

Part C

$S_0 = \{R(x, A, z), R(B, y, z)\}$
$\theta = \{x/B, y/A, z/z\}$ is the most general unifier.

Part D

$S_0 = \{Older(Father(y), y), Older(Father(x), John)\}$
$D_0 = \{y, x\}$
$\theta = \{y/x\}$
$S_1 = \{Older(Father(x), x), Older(Father(x), John)\}$
$D_1 = \{x, John\}$
$\theta = \{y/John, x/John\}$ is the most general unifier.

Part E

$S_0 = \{Knows(Father(y), y), Knows(x, x)\}$
$D_0 = \{Father(y), x\}$
$\theta = \{x/Father(y)\}$
$S_1 = \{Knows(Father(y), y), Knows(Father(y), Father(y))\}$
$D_1 = \{y, Father(y)\}$ No unification possible.

Problem 2

Consider the following sentences:

1. John likes all kinds of food.
2. Apples are food.
3. Chicken is food.
4. Anything anyone eats and isnt killed by is food.
5. If you are killed by something, you are not alive.
6. Bill eats peanuts and is still alive.*
7. Sue eats everything Bill eats.

Part A

Translation of sentences 1-7 into FOL formulae:

$$\forall x[food(x) \Rightarrow likes(John, x)] \tag{S1}$$
$$\forall x[apple(x) \Rightarrow food(x)] \tag{S2}$$
$$\forall x[chicken(x) \Rightarrow food(x)] \tag{S3}$$
$$\forall x[\exists y\ eats(y, x) \wedge \neg killed(x, y) \Rightarrow food(x)] \tag{S4}$$
$$\forall x[\exists y\ killed(y, x) \Rightarrow \neg alive(x)] \tag{S5}$$
$$[eats(Bill, peanuts) \wedge alive(Bill)] \tag{S6}$$
$$\forall x[eats(Bill, x) \Rightarrow eats(Sue, x)] \tag{S7}$$

Part B

Conversion of formulae of part A into CNF (also called clausal form).

$$\forall x[food(x) \Rightarrow likes(John, x)] \quad \text{(S1)}$$
$$\forall x[\neg food(x) \vee likes(John, x)] \quad \text{(S1)}$$
$$[\neg food(x) \vee likes(John, x)] \quad \text{(C1)}$$

$$\forall x[apple(x) \Rightarrow food(x)] \quad \text{(S2)}$$
$$\forall y[\neg apple(y) \vee food(y)] \quad \text{(S2)}$$
$$[\neg apple(y) \vee food(y)] \quad \text{(C2)}$$
$$\equiv food(apples) \quad \text{(C2)}$$

$$\forall x[chicken(x) \Rightarrow food(x)] \quad \text{(S3)}$$
$$\forall z[\neg chicken(z) \vee food(z)] \quad \text{(S3)}$$
$$[\neg chicken(z) \vee food(z)] \quad \text{(C3)}$$
$$\equiv food(chicken) \quad \text{(C3)}$$

$$\forall x[\exists y\ eats(y,x) \land \neg killed(x,y) \Rightarrow food(x)] \tag{S4}$$
$$\forall x[\neg(\exists y\ eats(y,x) \land \neg killed(x,y)) \lor food(x)] \tag{S4}$$
$$\forall x[\forall y \neg(eats(y,x) \land \neg killed(x,y)) \lor food(x)] \tag{S4}$$
$$\forall u[\forall v(\neg eats(v,u) \lor killed(u,v)) \lor food(u)] \tag{S4}$$
$$[\neg eats(v,u) \lor killed(u,v) \lor food(u)] \tag{C4}$$

$$\forall x[\exists y\ killed(y,x) \Rightarrow \neg alive(x)] \tag{S5}$$
$$\forall x[\neg(\exists y\ killed(y,x)) \lor \neg alive(x)] \tag{S5}$$
$$\forall s[\forall t\ \neg killed(t,s) \lor \neg alive(s)] \tag{S5}$$
$$[\neg killed(t,s) \lor \neg alive(s)] \tag{C5}$$

$$[eats(Bill, peanuts) \land alive(Bill)] \tag{S6}$$
$$[eats(Bill, peanuts)] \tag{C6}$$
$$[alive(Bill)] \tag{C7}$$

$$\forall x[eats(Bill,x) \Rightarrow eats(Sue,x)] \tag{S7}$$
$$\forall q[\neg eats(Bill,q) \lor eats(Sue,q)] \tag{S7}$$
$$[\neg eats(Bill,q) \lor eats(Sue,q)] \tag{C8}$$

Part C

Prove that John likes peanuts using resolution.
Let $\alpha = likes(John, peanuts)$.
Then, $\neg\alpha = \neg likes(John, peanuts)$.
KB $\models \alpha \Leftrightarrow$ KB $\wedge\neg\alpha$ is unsatisfiable.
KB:

$$[\neg food(x) \vee likes(John, x)] \quad \text{(C1)}$$
$$[food(apples)] \quad \text{(C2)}$$
$$[food(chicken)] \quad \text{(C3)}$$
$$[\neg eats(v, u) \vee killed(u, v) \vee food(u)] \quad \text{(C4)}$$
$$[\neg killed(t, s) \vee \neg alive(s)] \quad \text{(C5)}$$
$$[eats(Bill, peanuts)] \quad \text{(C6)}$$
$$[alive(Bill)] \quad \text{(C7)}$$
$$[\neg eats(Bill, q) \vee eats(Sue, q)] \quad \text{(C8)}$$
$$\neg\alpha :$$
$$\neg likes(John, peanuts) \quad \text{(C9)}$$

$$[\neg likes(John, peanuts)], [\neg food(x) \vee likes(John, x)] \quad \text{(C9 and C1)}$$
$$\theta = \{x/peanuts\}$$
$$[\neg food(peanuts)] \quad \text{(C10)}$$

$$[eats(Bill, peanuts)], [\neg eats(v, u) \vee killed(u, v) \vee food(u)] \quad \text{(C6 and C4)}$$
$$\theta = \{v/Bill, u/peanuts\}$$
$$[killed(peanuts, Bill) \vee food(peanuts)] \quad \text{(C11)}$$

$$[\neg killed(t, s) \vee \neg alive(s)], [killed(peanuts, Bill) \vee food(peanuts)] \quad \text{(C5 and C11)}$$
$$\theta = \{t/peanuts, s/Bill\}$$
$$[\neg alive(Bill \vee food(peanuts)] \quad \text{(C12)}$$

$$[alive(Bill)], [\neg alive(Bill) \vee food(peanuts)] \quad \text{(C7 and C12)}$$
$$[food(peanuts)] \quad \text{(C13)}$$

$$[\neg food(peanuts)], [food(peanuts)] \quad \text{(C10 and C11)}$$
$$\varnothing \quad \text{(C13)}$$

Thus, KB $\wedge\neg\alpha$ is unsatisfiable by resolution, and we can conclude that John likes peanuts. ■

Part D

Let $\alpha = \exists x[food(x) \wedge eats(Sue, x)]$.
Then, $\neg\alpha = \neg\exists x[food(x) \wedge eats(Sue, x)] \equiv \forall x \neg[food(x) \wedge eats(Sue, x)] \equiv \forall m[\neg food(m) \vee \neg eats(Sue, m)]$.
KB:

$[\neg food(x) \vee likes(John, x)]$ (C1)

$[food(apples)]$ (C2)

$[food(chicken)]$ (C3)

$[\neg eats(v, u) \vee killed(u, v) \vee food(u)]$ (C4)

$[\neg killed(t, s) \vee \neg alive(s)]$ (C5)

$[eats(Bill, peanuts)]$ (C6)

$[alive(Bill)]$ (C7)

$[\neg eats(Bill, q) \vee eats(Sue, q)]$ (C8)

$\neg\alpha$:

$[\neg food(m) \vee \neg eats(Sue, m)]$ (C9)

$[eats(Bill, peanuts)], [\neg eats(v, u) \vee killed(u, v) \vee food(u)]$ (C6 and C4)

$\theta = \{v/Bill, u/peanuts\}$

$[killed(peanuts, Bill) \vee food(peanuts)]$ (C11)

$[\neg killed(t, s) \vee \neg alive(s)], [killed(peanuts, Bill) \vee food(peanuts)]$ (C5 and C11)

$\theta = \{t/peanuts, s/Bill\}$

$[\neg alive(Bill \vee food(peanuts)]$ (C12)

$[alive(Bill)], [\neg alive(Bill) \vee food(peanuts)]$ (C7 and C12)

$[food(peanuts)]$ (C13)

$[eats(Bill, peanuts)], [\neg eats(Bill, q) \vee eats(Sue, q)]$ (C6 and C8)

$\theta = \{q/peanuts\}$

$[eats(Sue, peanuts)]$ (C14)

$[eats(Sue, peanuts)], [\neg food(m) \vee \neg eats(Sue, m)]$ (C14 and C9)

$\theta = \{m/peanuts\}$

$[\neg food(peanuts)]$ (C15)

$[food(peanuts)], [\neg food(peanuts)]$ (C13 and C15)

$\varnothing$ (C16)

We reach a contradiction when $\theta = \{m/peanuts\}$, and KB $\models \alpha$ when $\theta = \{m/peanuts\}$. Thus, peanuts are food and Sue eats peanuts. ■

Part E

"If you dont eat, you die," becomes $\forall x[\forall y \neg eats(x,y) \Rightarrow dies(x)]$
"If you die, you are not alive," becomes $\forall x[dies(x) \Rightarrow \neg alive(x)]$
"Bill is alive," becomes $alive(Bill)$.

$$\forall x[\forall y \neg eats(x,y) \Rightarrow dies(x)] \tag{S6}$$
$$\forall x[\forall y eats(x,y) \vee dies(x)] \tag{S6}$$
$$[eats(a,b) \vee dies(a)] \tag{C6}$$

$$\forall x[dies(x) \Rightarrow \neg alive(x)] \tag{S7}$$
$$\forall x[\neg dies(x) \vee \neg alive(x)] \tag{S7}$$
$$[\neg dies(c) \vee \neg alive(c)] \tag{C7}$$

$$[alive(Bill)] \tag{C8}$$

Then, we have KB:

$$[\neg food(x) \vee likes(John,x)] \tag{C1}$$
$$[food(apples)] \tag{C2}$$
$$[food(chicken)] \tag{C3}$$
$$[\neg eats(v,u) \vee killed(u,v) \vee food(u)] \tag{C4}$$
$$[\neg killed(t,s) \vee \neg alive(s)] \tag{C5}$$
$$[eats(a,b) \vee dies(a)] \tag{C6}$$
$$[\neg dies(c) \vee \neg alive(c)] \tag{C7}$$
$$[alive(Bill)] \tag{C8}$$
$$[\neg eats(Bill,q) \vee eats(Sue,q)] \tag{C9}$$
$\neg\alpha$:
$$[\neg food(m) \vee \neg eats(Sue,m)] \tag{C10}$$

$$[\neg eats(v,u) \vee killed(u,v) \vee food(u)], [\neg killed(t,s) \vee \neg alive(s)] \tag{C4 and C5}$$
$\theta = \{v/s, u/t\}$
$$[\neg eats(s,t) \vee \neg alive(s) \vee food(t)] \tag{C11}$$

$$[\neg eats(s,t) \vee \neg alive(s) \vee food(t)], [eats(a,b) \vee dies(a)] \tag{C11 and C6}$$
$\theta = \{s/a, t/b\}$
$$[\neg alive(a) \vee food(b) \vee dies(a)] \tag{C12}$$

$$[\neg alive(a) \vee food(b) \vee dies(a)], [\neg dies(c) \vee \neg alive(c)] \tag{C12 and C7}$$
$\theta = \{a/c\}$
$$[\neg alive(c) \vee food(b)] \tag{C13}$$

$[\neg eats(Bill, q) \lor eats(Sue, q)], [\neg food(m) \lor \neg eats(Sue, m)]$ (C9 and C10)

$\theta = \{q/m\}$

$[\neg eats(Bill, m) \lor \neg food(m)]$ (C14)

$[\neg eats(Bill, m) \lor \neg food(m)], [eats(a, b) \lor dies(a)]$ (C14 and C6)

$\theta = \{a/Bill, m/b\}$

$[dies(Bill) \lor \neg food(b)]$ (C15)

$[\neg dies(c) \lor \neg alive(c)], [dies(Bill) \lor \neg food(b)]$ (C7 and C15)

$\theta = \{c/Bill\}$

$[\neg alive(Bill) \lor \neg food(b)]$ (C16)

$[\neg alive(Bill) \lor \neg food(b)], [\neg alive(c) \lor food(b)]$ (C16 and C13)

$\theta = \{c/Bill\}$

$[\neg alive(Bill)]$ (C17)

$[\neg alive(c)], [alive(Bill)]$ (C17 and C8)

$\theta = \{c/Bill\}$

$\varnothing$ (C18)

We reach a contradiction when $\theta = \{c/Bill\}$. KB $\models \alpha$ when $\theta = \{c/Bill\}$. Thus, we can conclude that there is some food that Sue eats. ■