Both universal forms:
- DNF: Disjunctive Normal Form
- CNF: Conjunctive Normal Form.

Fundamental question about forms: ① are they universal? Can we represent all knowledge we're interested in using this form?

② Is it concise?

CNF

$$\underbrace{(\overset{\text{literal}}{A} \vee \overset{\text{literal}}{\neg \beta})}_{\text{Clause 1}} \wedge \underbrace{(\beta \vee \neg \gamma \vee \neg \Delta)}_{\text{Clause 2}} \wedge \cdots$$

CNF is a conjunction of clauses.

Clause: disjunction of literals

Horn Clause: there is at most one positive literal.
(we gain simplicity, but lose universality.)

DNF

$$\underbrace{(A \wedge \beta)}_{\text{Term 1}} \vee \underbrace{(A \wedge \neg C)}_{\text{Term 2}} \vee \underbrace{(\neg B \wedge \neg D \wedge X)}_{\text{Term 3}}$$

DNF is a disjunction of terms.

term: Conjunction of literals

→ World, → Truth Assignment } $w \models \alpha$ ← α holds at w, w satisfies α.

Example

			Position		Propositional Variables	
Robots		R	Left	L	LR	RR
Dirt		D	Right	R	LD	RD

→ World
→ Truth Assignment

$w \models \alpha$ ← α holds at w, w satisfies α.

Example

			Position		Propositional Variables	
Robots		R	Left	L	LR	RR
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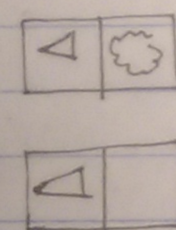
Knowledge Base

Δ

- β_1 $\quad LR \vee RR$
- β_2 $\quad \neg(LR \wedge RR)$
- β_3 $\quad (LR \Rightarrow \neg LD) \wedge (RR \Rightarrow \neg RD)$
- β_4 $\quad LR$
- β_5 $\quad \neg LD$

$\beta_1 \wedge \beta_2 \wedge \beta_3 \wedge \beta_4$ implies β_5

Δ satisfies



From the 2^4 possible worlds, the top two are only satisfied worlds.

Meaning of Sentence α:

$$M(\alpha) = \{w : w \models \alpha\}$$

Note: there may be many knowledge bases (syntax) that correspond to the same meaning (semantics)!

α implies β.

→ $M(\alpha) \subseteq M(\beta)$

→ If $\omega \models \alpha$, then $\omega \models \beta$.

α is equivalent to β.

$M(\alpha) = M(\beta)$.

α, β mutually exclusive

$M(\alpha) \cap M(\beta) = \phi$

α is inconsistent

$M(\alpha) = \phi$

↳ α implies everyt else!

α Vacuous / Invalid

$M(\alpha) = W$ (whole world).

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$M(\alpha \wedge \beta) = M(\alpha) \cap M(\beta)$

$M(\alpha \vee \beta) = M(\alpha) \cup M(\beta)$

$M(\neg A) = M(A)$.