Indexing

Lecture 12

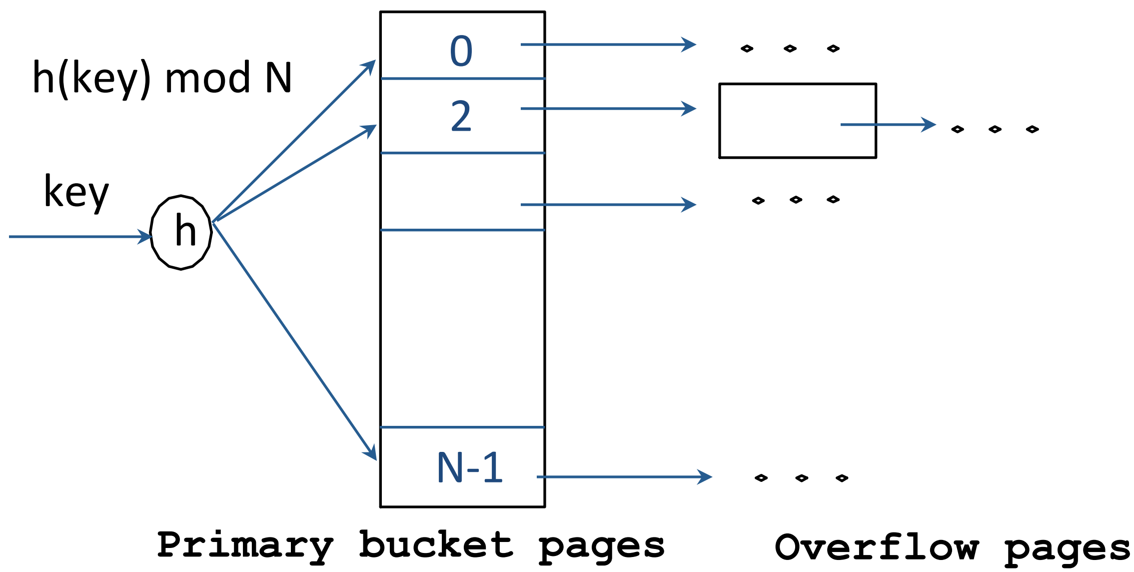
EECS 339

Introduction

- *As for any index, 3 alternatives for data entries* **k***:
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
  - Choice orthogonal to the *indexing technique*
- *Hash-based* indexes are best for *equality selections*. ***Cannot*** support range searches.
- Static and dynamic hashing techniques exist; trade-offs similar to ISAM vs. B+ trees.

Static Hashing

- # primary pages fixed, allocated sequentially, never de-allocated; overflow pages if needed.
- **h**(k) mod M = bucket to which data entry with key k belongs. (M = # of buckets)



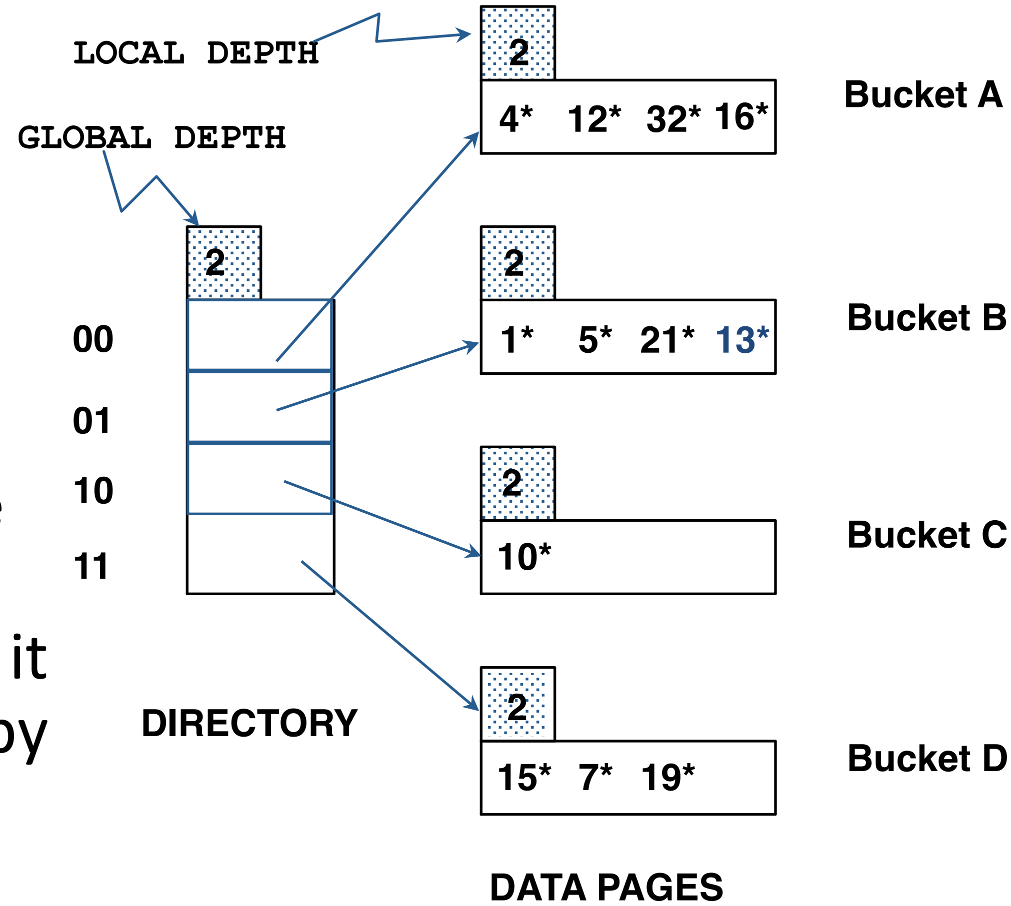
Static Hashing (Contd.)

- Buckets contain *data entries*.
- Hash fn works on *search key* field of record *r*. Must distribute values over range 0 ... M-1.
  - **h**(*key*) = (a * *key* + b) usually works well.
  - a and b are constants; lots known about how to tune **h**.
- Long overflow chains can develop and degrade performance.
  - *Extendible* and *Linear Hashing*: Dynamic techniques to fix this problem.

Extendible Hashing

- Situation: Bucket (primary page) becomes full. Why not re-organize file by *doubling* # of buckets?
  - Reading and writing all pages is expensive!
  - *Idea*: Use *directory of pointers to buckets*, double # of buckets by *doubling the directory,* splitting just the bucket that overflowed!
  - Directory much smaller than file, so doubling it is much cheaper. Only one page of data entries is split. *No overflow page*!
  - Trick lies in how hash function is adjusted!

Example

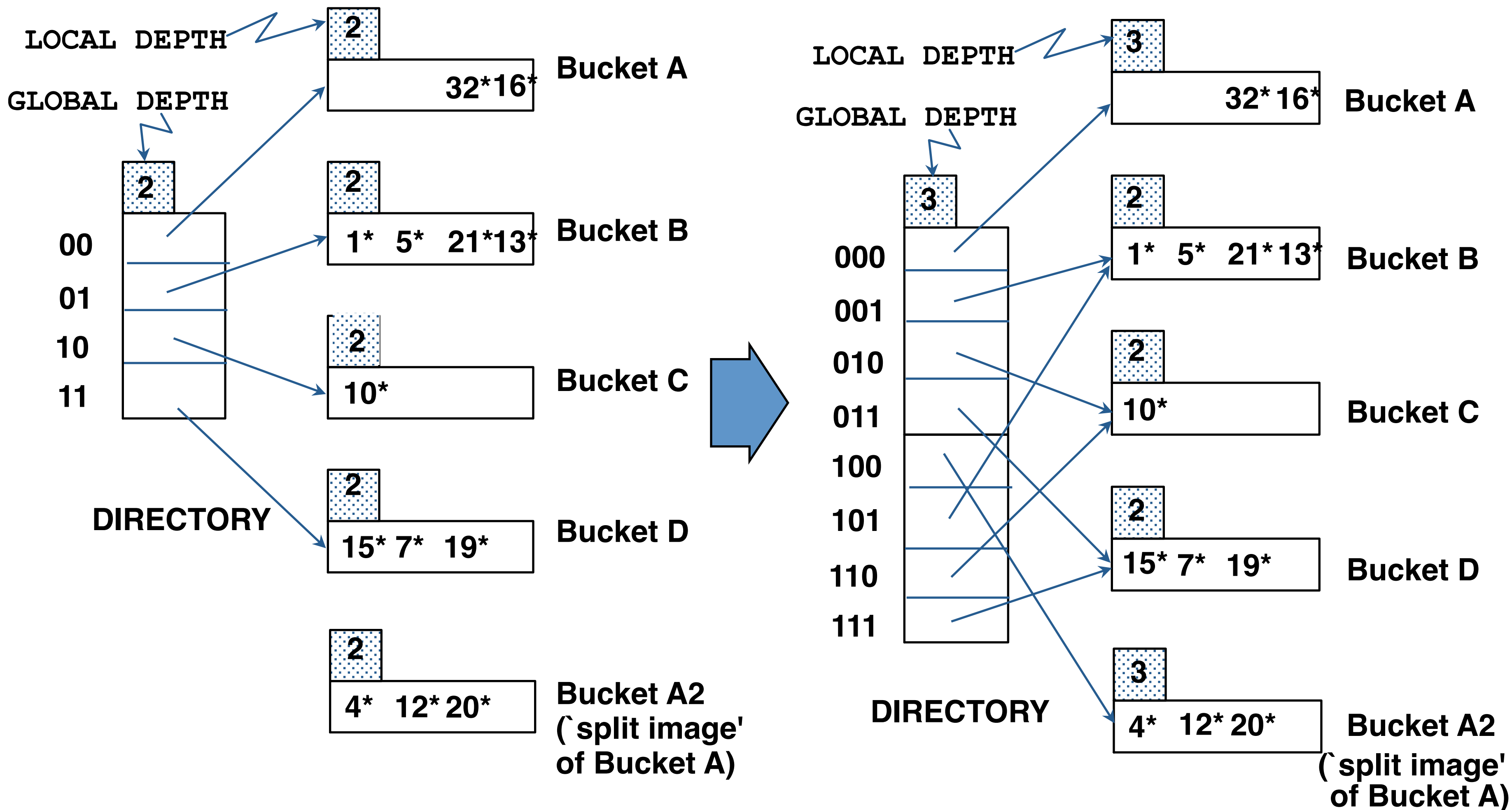


- Directory is array of size 4.
- To find bucket for *r*, take last \`*global depth*' # bits of **h**(*r*); we denote *r* by **h**(*r*).
  - If **h**(*r*) = 5 = binary 101, it is in bucket pointed to by 01.

❖ **Insert**: If bucket is full, *split* it (*allocate new page, re-distribute*).

❖ *If necessary*, double the directory. (As we will see, splitting a bucket does not always require doubling; we can tell by comparing *global depth* with *local depth* for the split bucket.)

Insert **h**(r)=20 (Causes Doubling)



Points to Note

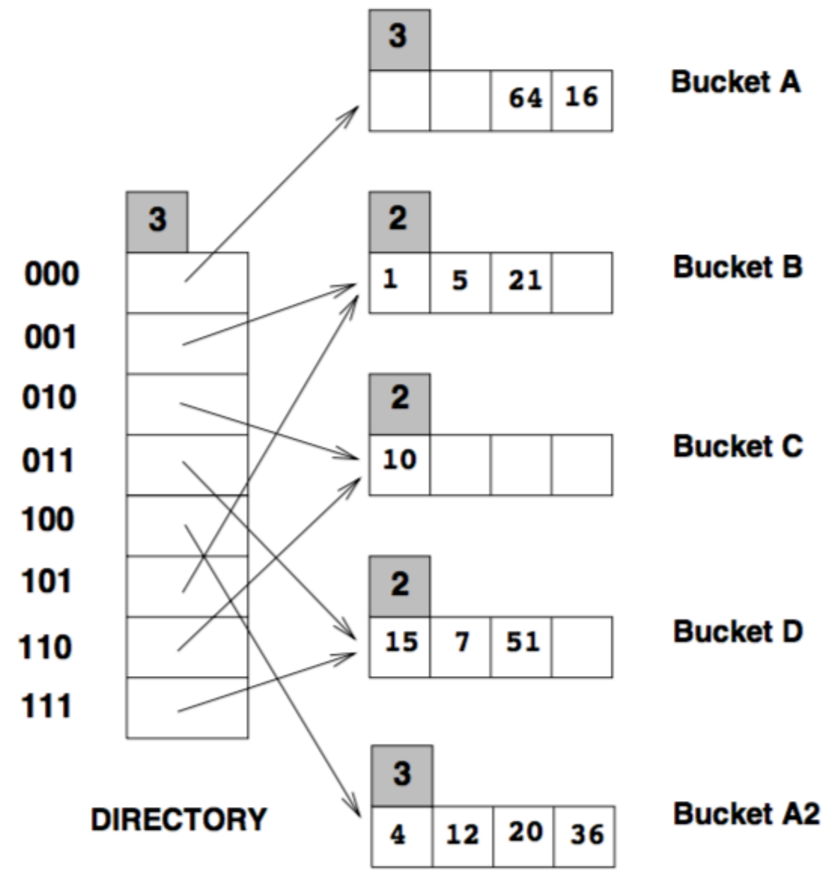
- 20 = binary 10100. Last **2** bits (00) tell us *r* belongs in A or A2. Last **<u>3</u>** bits needed to tell which.
  - *Global depth of directory*: Max # of bits needed to tell which bucket an entry belongs to.
  - *Local depth of a bucket*: # of bits used to determine if an entry belongs to this bucket.
- When does bucket split cause directory doubling?
  - Before insert, *local depth* of bucket = *global depth*. Insert causes *local depth* to become > *global depth*; directory is doubled by *copying it over* and `fixing' pointer to split image page. (Use of least significant bits enables efficient doubling via copying of directory!)

Comments on Extendible Hashing

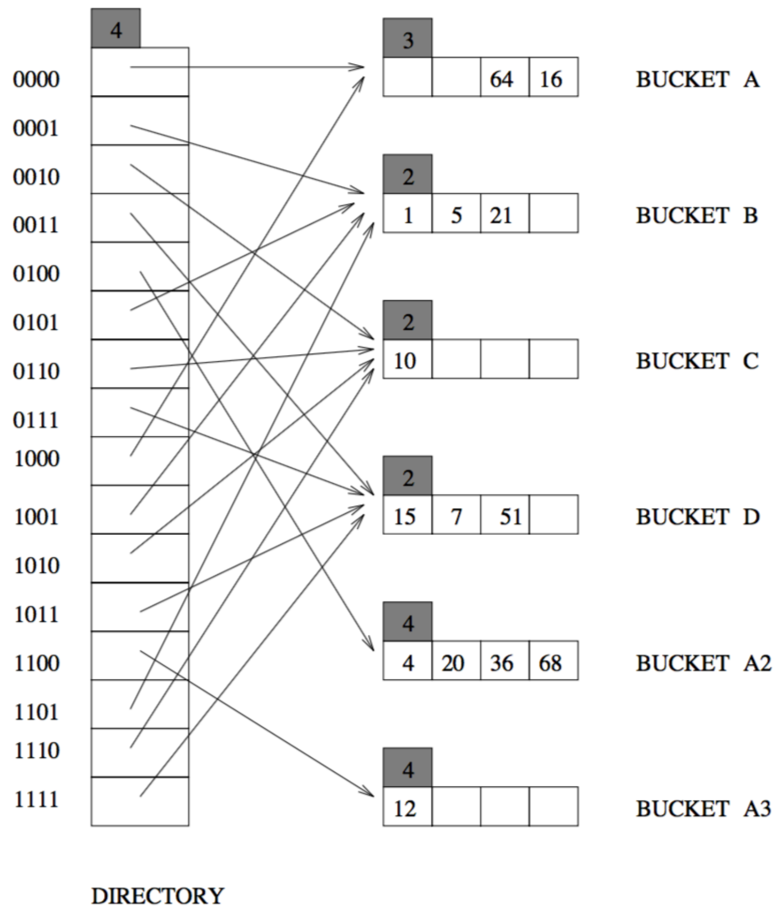
- If directory fits in memory, equality search answered with one disk access; else two.
  - 100MB file, 100 bytes/rec, 4K pages contains 1,000,000 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.
  - Directory grows in spurts, and, if the distribution *of hash values* is skewed, directory can grow large.
  - Multiple entries with same hash value cause problems!
- **Delete**: If removal of data entry makes bucket empty, can be merged with `split image'. If each directory element points to same bucket as its split image, can halve directory.

Study Break: Extendible Hashing

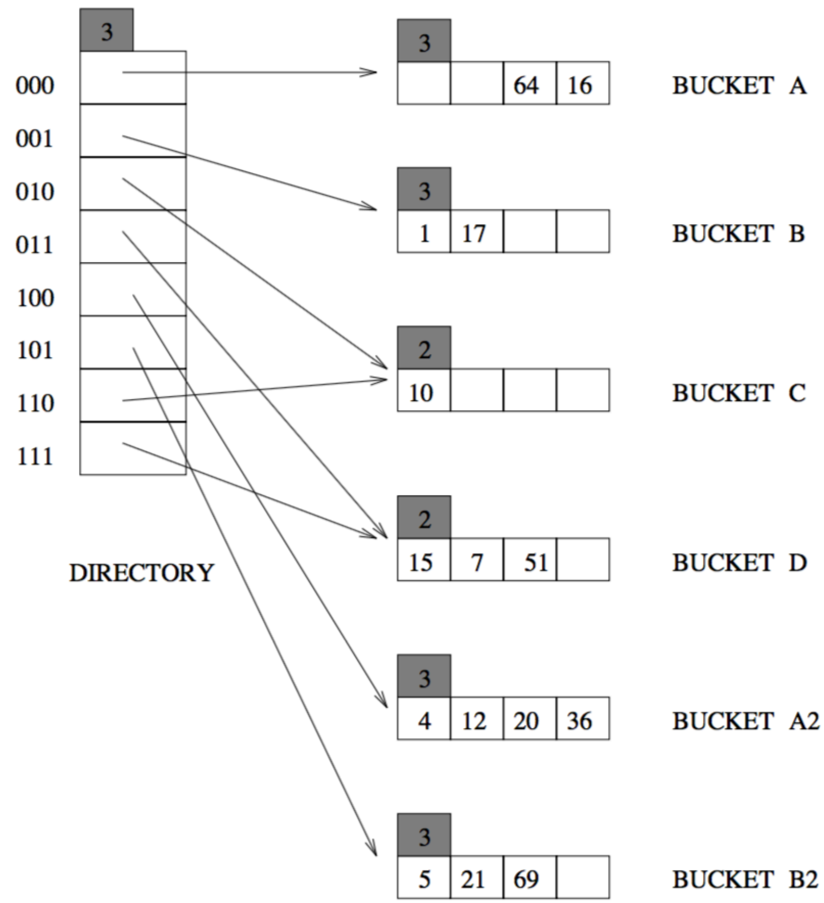
- Consider the index on the right, show it after:
  - inserting a hash value of 68
  - Inserting HV of 17 and 69 (into original tree)
  - Deleting a HV of 21 (from original tree)
  - Deleting an entry of 10 (from original tree) Does this prompt a merge?



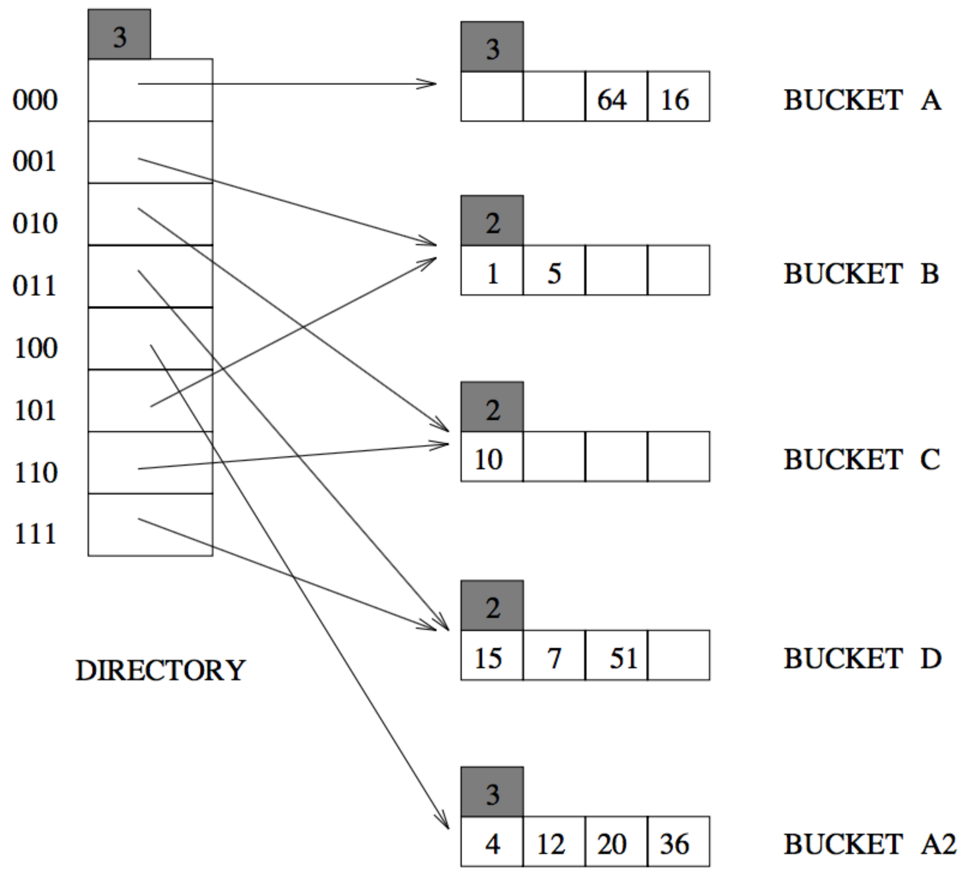
Study Break Solution



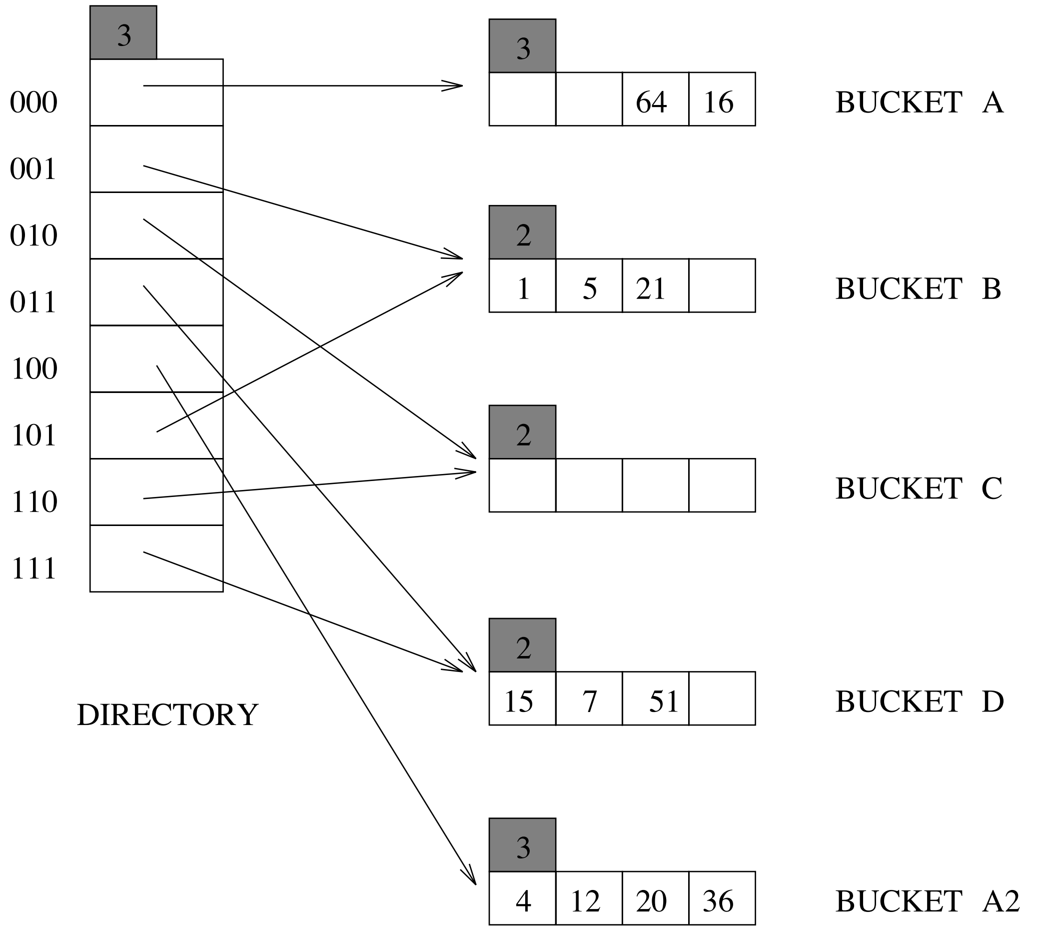
Study Break Solution



Study Break Solution



Study Break Solution



The deletion of the data entry 10 which is the only data entry in bucket C doesn't trigger a merge because bucket C is a primary page and it is left as a place holder. Right now, directory element 010 and its split image 110 already point to the same bucket C. We can't do a further merge.

Linear Hashing

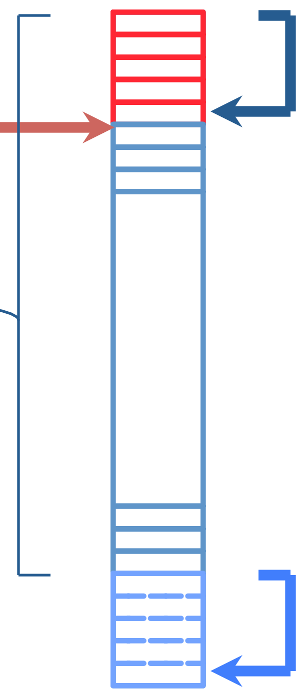
- This is another dynamic hashing scheme, an alternative to Extendible Hashing.
- LH handles the problem of long overflow chains without using a directory, and handles duplicates.
- <u>*Idea*</u>: Use a family of hash functions $\mathbf{h}_0$, $\mathbf{h}_1$, $\mathbf{h}_2$, ...
  - $\mathbf{h}_i(key) = \mathbf{h}(key) \bmod(2^i N)$; N = initial # buckets
  - **h** is some hash function (range is *not* 0 to N-1)
  - If $N = 2^{d0}$, for some *d0*, $\mathbf{h}_i$ consists of applying **h** and looking at the last *di* bits, where $di = d0 + i$.
  - $\mathbf{h}_{i+1}$ doubles the range of $\mathbf{h}_i$ (similar to directory doubling)

Linear Hashing (Contd.)

- Directory avoided in LH by using overflow pages, and choosing bucket to split round-robin.
  - Splitting proceeds in `rounds'. Round ends when all N_R initial (for round R) buckets are split. Buckets 0 to *Next-1* have been split; *Next* to N_R yet to be split.
  - Current round number is *Level*.
  - **Search:** To find bucket for data entry r, find $\mathbf{h}_{Level}(r)$:
    - If $\mathbf{h}_{Level}(r)$ in range `*Next* to N_R', r belongs here.
    - Else, r could belong to bucket $\mathbf{h}_{Level}(r)$ or bucket $\mathbf{h}_{Level}(r) + N_R$; must apply $\mathbf{h}_{Level+1}(r)$ to find out.

Overview of LH File

- In the middle of a round.

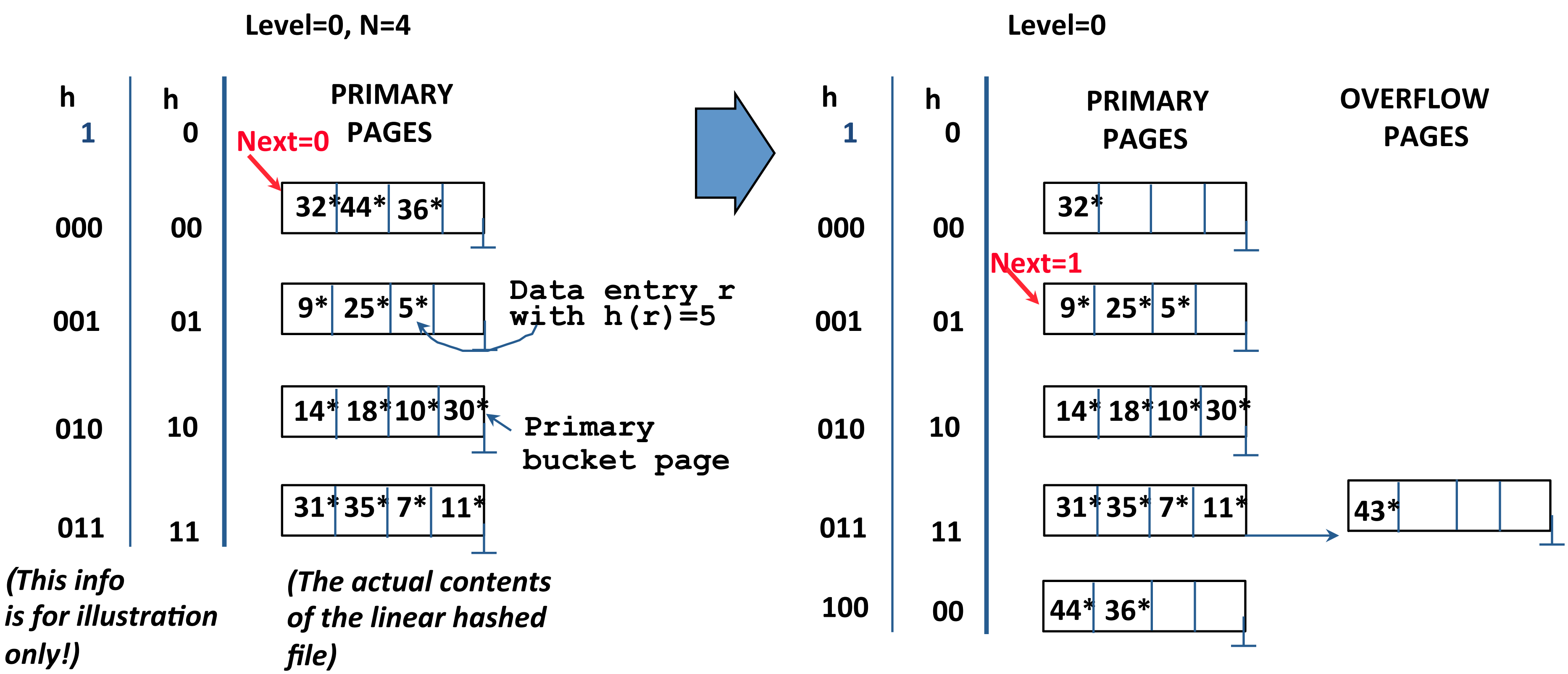


Linear Hashing (Contd.)

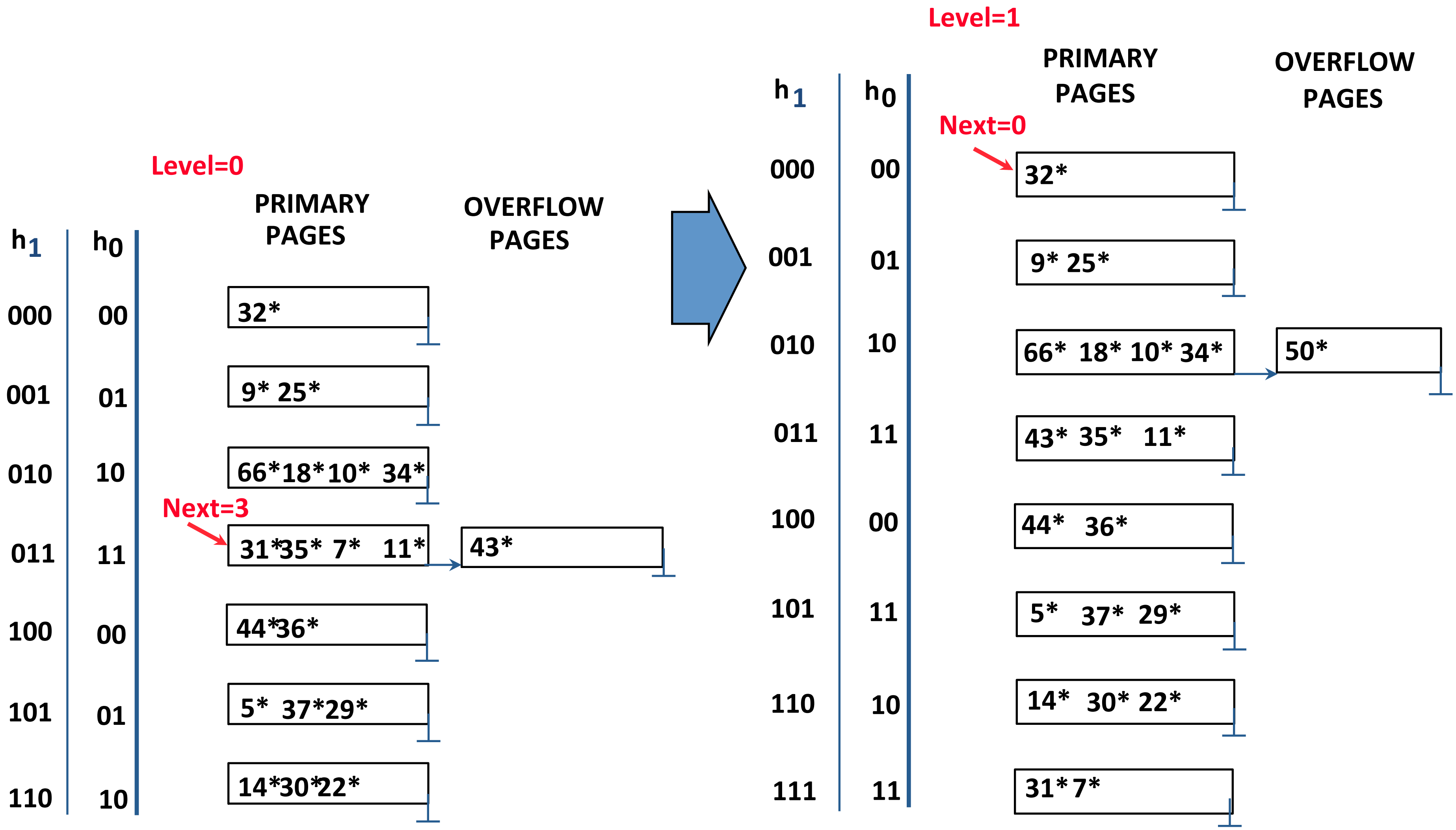
- **Insert**: Find bucket by applying $\mathbf{h}_{Level}$ / $\mathbf{h}_{Level+1}$:
  - If bucket to insert into is full:
    - Add overflow page and insert data entry.
    - (*Maybe*) Split *Next* bucket and increment *Next*.
- Can choose any criterion to `trigger' split.
- Since buckets are split round-robin, long overflow chains don't develop!
- Doubling of directory in Extendible Hashing is similar; switching of hash functions is *implicit* in how the # of bits examined is increased.

Example of Linear Hashing

- On split, $\mathbf{h}_{Level+1}$ is used to re-distribute entries.



Example: End of a Round



LH Described as a Variant of EH

- The two schemes are actually quite similar:
  - Begin with an EH index where directory has *N* elements.
  - Use overflow pages, split buckets round-robin.
  - First split is at bucket 0. (Imagine directory being doubled at this point.) But elements <1,*N*+1>, <2,*N*+2>, ... are the same. So, need only create directory element *N*, which differs from 0, now.
    - When bucket 1 splits, create directory element *N*+1, etc.
- So, directory can double gradually. Also, primary bucket pages are created in order. If they are *allocated* in sequence too (so that finding i' th is easy), we actually don' t need a directory! Voila, LH.

Bitmap Index

Color	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
Purple	*	*	*	*		*	*		*	*
White					*					
Red								*		

1 map per distinct value
1 bit per tuple

I/O Costs for Indexing

	Heap File	Bitmap	Hash Index	B+Tree
Insert	O(1)	O(1)	O(1)	$O(\log_B n)$
Delete	O(P)	O(1)	O(1)	$O(\log_B n)$
Range Scan	O(P)	-- / O(P)	-- / O(P)	$O(\log_B n + R)$
Lookup	O(P)	O(C)	O(1)	$O(\log_B n)$

n : number of tuples
P : number of pages in file
B : branching factor of B-Tree (keys / node)
R : number of pages in range
C: cardinality (#) of unique values on key

Summary

- Hash-based indexes: best for equality searches, cannot support range searches.
- Static Hashing can lead to long overflow chains.
- Extendible Hashing avoids overflow pages by splitting a full bucket when a new data entry is to be added to it. (*Duplicates may require overflow pages.*)
  - Directory to keep track of buckets, doubles periodically.
  - Can get large with skewed data; additional I/O if this does not fit in main memory.

Summary (Contd.)

- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages.
  - Overflow pages not likely to be long.
  - Duplicates handled easily.
  - Space utilization could be lower than Extendible Hashing, since splits not concentrated on `dense' data areas.
    - Can tune criterion for triggering splits to trade-off slightly longer chains for better space utilization.
- For hash-based indexes, a *skewed* data distribution is one in which the *hash values* of data entries are not uniformly distributed!