

An intelligent ~~trader~~ is carrying three sacks having 30 coconuts each. None of the sacks can carry more than 30 coconuts. Now he has to pass through 30 check points and at each check point he has to give one coconut for each sack he is carrying. How many coconuts will be left?

|   | Sack I<br>check point |    | Sack II<br>check point |    | Sack III<br>check point |                  |
|---|-----------------------|----|------------------------|----|-------------------------|------------------|
|   | 30                    |    | 30                     |    | 30                      |                  |
| 1 | 3                     | 11 | 2                      | 26 | 1                       |                  |
| 2 | 3                     | 12 | 2                      | 27 | 1                       |                  |
| 3 | 3                     | 13 | 2                      | 28 | 1                       |                  |
|   |                       | ↓  |                        | ↓  |                         |                  |
|   |                       |    | 25                     |    |                         |                  |
|   |                       |    |                        | 30 |                         |                  |
|   |                       |    | 2                      |    |                         |                  |
|   |                       |    |                        | 30 |                         |                  |
|   |                       |    |                        |    | 5                       |                  |
|   |                       |    |                        |    |                         | 25 coconuts left |

$$999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7} + 999\frac{5}{7} + 999\frac{6}{7}$$

$$\frac{999}{7} \\ 6993$$

$$= 999 + \frac{1}{7} + 999 + \frac{2}{7} + 999 + \frac{3}{7} + 999 + \frac{4}{7} + 999 + \frac{5}{7} + 999 + \frac{6}{7}$$

$$= 999 \times 6 + \left( \frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7} \right)$$

$$= 5994 + \left( \frac{21}{7} \right) = 5997$$

## TEST OF DIVISIBILITY.

### Multiples :-

what are the multiples of 3.

↓  
3, 6, 9, 12, 15,  
18, 21, 24, 27, 30.

↓  
smallest number is the 3 itself.

### Factors.

#### Factor of 12.



1, 2, 3, 4, 6, 12.

Factor are those numbers which perfectly divide that number.

1 is the smallest.

12 greatest is the number itself.

### # DIVISIBILITY RULE :-

1: All number is divisible by 1.

2: 0/2/4/6/8

3: Sum of the digits must be divisible by 3.

$$\text{eg: } 597324 = 5+9+7+3+2+4 \\ = \underline{30} \text{- divisible by 3.}$$

What will be the remainder of 77777 if divided by 3

$$= 7+7+7+7+7 = 35. \\ = \frac{35}{3} = 11 \frac{2}{3}$$

$$\begin{array}{r} 11 \\ 3 \overline{) 35} \\ \underline{3} \\ 5 \end{array}$$

4: Last two digits must be divisible by 4.

$$\frac{73\textcircled{00}}{4} = 73 \times \textcircled{100} - \underline{\underline{25 \times 4}}$$

5: 0/5

6: if a number is divisible by 2 and 3.

7: divisible by 7.

8: Last 3 digits must be divisible by 8.

16: Last 4 digits divisible by 16.

$$\frac{2^4}{\underline{\underline{1}}} \cdot 2^4 = 16$$

32: Last 5 digits divisible by 32

$$\frac{2^5}{\underline{\underline{1}}} = 32$$

64: Last 6 digits divisible by 64

$$2^6 = 64$$

(62939)

~~125878~~

64  
32

79346125878

= 8 is divisible by 2 by the last 2 digits  
not divisible by 4.

9: Sum of the digits must be divisible by 9.

$$4+5+3+2+1+7+8+6+5 = \underline{\underline{41}}$$

not divisible by 9, remainder

$$5 \cdot 9) \overline{4174}$$
$$\underline{\underline{36}}$$
$$\underline{\underline{5}}$$

co-prime  
3  
6 8  
 $\textcircled{1} \frac{2}{\uparrow}, 3, 6 \quad \textcircled{1} \frac{2}{\uparrow}, 4, 8$

After 1, & if any other number are common, they are not co-prime

14 15  
 $\textcircled{1} \frac{2}{\uparrow}, 7, 14 \quad \textcircled{1} \frac{3}{\uparrow}, 5, 15$

Apart from one, no other number common, they are co-prime.

10: last digit 0.

11: 0 or multiple of 11.

5 4 3 2 7 9 6 8  
| | | | | |  
Odd Even Odd Even

(Adding the odd) and the even)  
then subtracting.

$$(8+9+2+4) - (6+7+3+5)$$

$$23 - 21$$

$$= 2$$

~~4 5 6 8 7 4 1 5 6 2~~ . 4 5 6 8 7 4 1 5 6 2  
| | | | | | | | | |

$$\begin{array}{r} \cancel{4+6+7+1} \\ = 24 \\ - (5+8+4+5+2) \\ = 24 \end{array}$$

Pairing  $= 24 - 24$

$\overbrace{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad}$   $= 0$

$\overbrace{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad}$  8 is not divisible by 11.

odd number not divisible

Pairing  $\overbrace{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad}$  Even number divisible by 11.

12: if a number is divisible by 3 and 4.

$$= 7943152\boxed{68} = 7+9+4+3+1+5+2+6+8 = 45$$

$$\begin{array}{r} 3) \overline{45} (15 \\ 3 \\ \hline 15 \end{array}$$

= Last two digits divisible by 4.

= 45 is divisible by 3.

15 : 3 and 5.

18 : 2 and 9.

20 : 0, 2, 4, 6, 8.

22 : 2 and 11.

24 : 3 and 8.

17395428, <sup>not</sup> divisible by 8.

$$2. 1+7+3+9+5+4+2+8.$$

25 : 25, 50, 75, 80.

55 : 5 and 11.

72 : 8 and 9.

$$\begin{array}{r}
 8 + 7 \\
 \hline
 15
 \end{array}$$

8 0 3 6 4 2  
 1 1 1 1 1 1

$$8+3+4 = 15$$

$$15 - 15$$

$$\begin{array}{r}
 5. 1 \\
 8) \overline{6041} \\
 8) \overline{2891} \\
 \underline{24} \\
 49
 \end{array}$$

$$\begin{array}{r}
 1 1 1 \\
 4 3 7 1 7 \\
 \hline
 1 1 1 1 1 1 \\
 1 4 9 \\
 \hline
 2 4 2 3 \\
 \hline
 1 7
 \end{array}$$

$$1, 2, 3, 5, 6, \frac{23}{19}$$

$$\begin{array}{r}
 882 \\
 6a4 \\
 \hline
 14b6
 \end{array}$$

1 2 3 4 5 6 7 8 9 10 11 12 13 .....

9 digits

111 digits

$$\downarrow 110+1$$

$$2 \times 55$$

$$9+1 = 10$$

$$9+2 = 11$$

$$9+3 = 12$$

$$\downarrow \quad \downarrow$$

$$9+55 = 64$$

Taking the smallest number to be 0.  
as value of a:

$$\begin{array}{r}
 8 \quad 8 \quad 8 \\
 + a \quad + 0 \quad + 1 \\
 \hline
 b \quad 8 \quad 9
 \end{array}
 ]^a_{b=10}$$

$$8) \overline{646}(8$$
  
$$\underline{64}$$
  
$$6$$

10, 11, 12, 13 ..... 63, 64,

119 digits complete only 1 digit is left after 64, we only take 6 of 65.

$$10^{25} - 7 \quad \& \quad 10^{24} - x$$

$$[(10^{25} - 7) - (10^{24} - x)]$$

$$[10^{25} - 7 - 10^{24} - x]$$

$$[10^{25} - 10^{24} - 7 + x]$$

$$10^{24} (10 - 1) - 7 + x$$

$$10^{24} \times 9 - 7 + x$$

$$- 7 + x$$

$$x = 0, -7 + 0 = -7$$

$$= \cancel{-1} - 7 + 1 = -6$$

$$(10^{25} - 7) \& (10^{24} - x)$$

$$10^1 - 7 = 3$$

$$10^2 - 7 = 93$$

$$10^3 - 7 = 993$$

$$x = 0.$$

$$10 - 0 = 10.$$

$$10 - 1 = 9.$$

$$10 - 2 = 8.$$

Induction method.

### FACTORS.

$$18 \rightarrow 1, 2, 3, 6, 9, 18.$$

$$90 \rightarrow$$

$$\begin{array}{c|cc} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \end{array}$$

$$2^1 \times 3^2 \times 5^1$$

Taken for eq.

$$\therefore a^p \times b^q \times c^r$$

$$1 \text{ No. of factors } (n) = (p+1) \times (q+1) \times (r+1)$$

$$= (2+1)(1+1)(2+1)(1+1)$$

$$= 2 \times 3 \times 2$$

$$= 12$$

144, 375, 270.

$$\begin{array}{r} 144 \\ \hline 2 | 72 \\ 2 | 36 \\ 2 | 18 \\ 3 | 9 \\ 3 | 3 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 375 \\ \hline 5 | 75 \\ 5 | 15 \\ 3 | 3 \\ \hline 1 \end{array}$$

$$5^3 \times 3^1$$

$$\begin{array}{r} 270 \\ \hline 2 | 135 \\ 3 | 27 \\ 3 | 9 \\ 3 | 3 \\ \hline 1 \end{array}$$

$$2^1 \times 5^1 \times 3^3$$

$$\begin{array}{r} 240 \\ \hline 2 | 120 \\ 2 | 60 \\ 2 | 30 \\ 3 | 15 \\ 3 | 5 \\ \hline 1 \end{array}$$

$$2^4 \times 3^1 \times 5^1$$

$$a^p \times b^q \times c^r$$

$$(1+4)(1+1)(1+1)$$

$$= 5^4 \times 2^2 \times 2^2$$

$$=$$

$$2^4 \times 3^2$$

$$a^p \times b^q$$

$$a^p \times b^q \times c^r$$

$$a^p \times b^q$$

$$(1+3) \times (1+1)$$

$$4 \times 2$$

$$(1+1)(1+1)(1+3)$$

$$(1+4) \times (1+2)$$

$$= 8$$

$$2 \times 2 \times 4$$

$$= 16$$

$$= 5 \times 3$$

$$= 15.$$

@upsc.riseinfinity

Sum of the factors :

$$\frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)} \dots \dots$$

$$90 \Rightarrow \frac{(2^{1+1}-1)(3^{2+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)}$$

$$= \frac{(2^2-1)(3^3-1)(5^2-1)}{1 \times 2 \times 4}$$

$$\frac{3 \times 26 \times 24}{8}$$

$$= 26 \times 3 \times 3$$

$$= 234.$$

$$\frac{15^2}{24^2} = a^p \times b^q \times c^r$$

$$\frac{(2-1)(3-1)}{(2^2-1)(3^2-1)}$$

$$\frac{4}{2-1} \times \frac{2}{3-1} \times \frac{2}{5-1}$$

$$= \frac{(2-1)(3-1)(5-1)}{15 \times 8 \times 24}$$

$$\frac{15 \times 8 \times 24}{1 \times 2 \times 4}$$

$$\begin{array}{r} 120 \\ 2 \quad | \\ 60 \\ 2 \quad | \\ 30 \\ 3 \quad | \\ 15 \\ 5 \quad | \\ 5 \end{array}$$

$$\frac{15 \times 8 \times 24}{1 \times 2 \times 4}$$

$$= 8$$

144

480

420

$$\begin{array}{r} 144 \\ 2 \quad | \\ 72 \\ 2 \quad | \\ 36 \\ 2 \quad | \\ 18 \\ 3 \quad | \\ 9 \\ 3 \quad | \\ 3 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 480 \\ 2 \quad | \\ 240 \\ 2 \quad | \\ 120 \\ 2 \quad | \\ 60 \\ 2 \quad | \\ 30 \\ 3 \quad | \\ 15 \\ 5 \quad | \\ 5 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 420 \\ 2 \quad | \\ 210 \\ 2 \quad | \\ 105 \\ 3 \quad | \\ 21 \\ 7 \quad | \\ 7 \\ \hline 1 \end{array}$$

$$2^4 \times 3^2 = 403$$

$$2^5 \times 3^1 \times 5^1 = 1512$$

$$a^p \times b^q \times c^r$$

$$a^p \times b^q \times c^r$$

$$\frac{(2^{4+1}-1)(3^{2+1}-1)}{(2-1)(3-1)(5-1)} = \frac{(2^5-1)(3^3-1)}{(2-1)(3-1)(5-1)}$$

$$(2-1)(3-1)$$

$$\frac{(2^5-1)(3^3-1)}{1 \times 2 \times 4}$$

$$= \frac{9 \times 81}{3} = \frac{729}{3} = 243$$

Product of the factors :-

Product of the number

$$= N^{n/2}$$

$$\Rightarrow 90 \rightarrow N$$

$$\text{Factor of } 90 = 12 \rightarrow n$$

$$90^{12/2} = 90^6$$

$$10^{25} - 7 - 10^{24} + n$$

$$10^{25} - 10^{24} - 7 + n$$

$$10^{24}(10-1) - 7 + n$$

$$10^{24} \times 9$$

$$-7 + 3$$

A man decided to visit four temples one by one. As soon as he enters the first temple his money gets doubled. When he comes out he donates rupees 100, then he enters the second temple ..... After the fourth temple, how much money was there with him, if he is left with zero rupees now.

$$x \xrightarrow{\times 2} 2x - 100$$

$$\begin{array}{rcl} 1 & - & 100 \\ 2 & - & 100 \\ 3 & - & 100 \\ 4 & - & 100 \end{array}$$

$$2 \quad 2x - 100 \xrightarrow{\times 2} 4x - 200 - 100$$

$$2 \quad 4x - 300 \xrightarrow{\times 2} 8x - 600 - 100$$

$$2 \quad 8x - 700 \xrightarrow{\times 2} 16x - 1400 - 100$$

$$2 \quad 16x - 1500 = 0$$

$$2 \quad x = \frac{1500}{16}$$

$$0 \xrightarrow{+100} 100 \xrightarrow{\frac{1}{2}} 50 \xrightarrow{+100} 150 \xrightarrow{\frac{1}{2}} 75$$

↓ +100

$\leftarrow \underline{112} \quad 175$

Divisor      7 - Quotient

~~11) 85 (~~

~~77~~

Dividend      8 - Remainder

$$\therefore \text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

The difference b/w 11 - 8 = 3 added to 85 will be greater than Dividend perfectly divisible by 11.

The greatest 3 digit number perfectly divisible by 36.

$$\begin{array}{r} 27 \\ 36) \overline{999} \\ - 72 \\ \hline 279 \\ - 252 \\ \hline 27 \end{array}$$

$$\begin{array}{r} 36 ) \overline{999} \\ - 72 \\ \hline 279 \\ - 252 \\ \hline 108 \\ - 108 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 999 \\ - 27 \\ \hline 972 \end{array}$$

972 — Perfectly divisible by 36.

Again adding +7 will leave a remainder of 5  
not divisible.

The greatest 3 digit number perfectly divisible by  
27 and 42.

$$\begin{array}{r} 42 \\ 4 ) \overline{126} \\ - 12 \\ \hline 68 \\ - 64 \\ \hline 4 \\ 27 ) \overline{274} \\ - 27 \\ \hline 4 \\ - 3 \\ \hline 1 \end{array}$$

999 is perfectly divisible by 42  
by 27.

In order  $999 - 33 = 966$   
perfectly divisible by 42.

$$\begin{array}{r} 37 \\ 27 ) \overline{999} \\ - 81 \\ \hline 189 \\ - 189 \\ \hline 0 \end{array}$$

Find out the smallest four digit number perfectly  
divisible by 32.

$$32 - 8 = 24.$$

$$\begin{array}{r} 32 \\ 16 ) \overline{32} \\ - 32 \\ \hline 0 \end{array}$$

$1000 + 24 = 1024$  is perfectly  
divisible by 24.

$$\begin{array}{r} 32 ) \overline{1000} \\ - 96 \\ \hline 40 \\ - 32 \\ \hline 8 \end{array}$$

Smallest four digit number perfectly divisible by  
40 and 54.

$$54 - 28 = 26$$

$$1000 + 26 = 1026$$

perfectly divisible by 54.

$$54 \overline{)1000} \\ 54 \\ \hline 460 \\ 432 \\ \hline 28$$

$$40 \overline{)1000} \\ 80 \\ \hline 200 \\ 200 \\ \hline 0$$

$$\begin{array}{r} 18 \\ 54 \overline{)1000} \\ 54 \\ \hline 460 \\ 432 \\ \hline 28 \\ 28 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 25 \\ 50 \overline{)1000} \\ 50 \\ \hline 200 \\ 200 \\ \hline 0 \end{array}$$

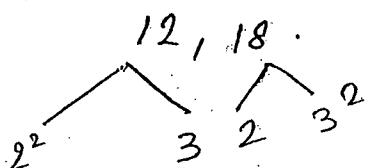
972 + 7

$$= 979$$

is wrong

LCM

The smallest possible number which is perfectly divisible



$$\text{LCM} = 2^2 \times 3^2 \\ = 36$$

$$\begin{array}{r} 2 \overline{)12, 18} \\ 2 \overline{)6, 9} \\ 3 \overline{)3, 9} \\ 3 \overline{)1, 3} \\ 1, 1 \end{array}$$

$$2 \times 2 \times 3 \times 3$$

$$= 36$$

Find out the LCM of 24, 35, 18.

$$3 \times 3 \times 2 \times 2 \times 2 \times 5 \times 7$$

$$= 2520$$

$$\begin{array}{r} 3 \overline{)24, 35, 18} \\ 3 \overline{)8, 35, 6} \\ 2 \overline{)8, 35, 2} \\ 2 \overline{)8, 35, 1} \\ 2 \overline{)2, 35, 1} \\ 5 \overline{)1, 25 } \end{array}$$

LCM of 27, 15, 6, 32

$$3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \\ 2 \quad 4320$$

$$\begin{array}{r} 160 \\ 27 \quad | \\ 20 \quad | \\ 20 \quad | \\ \hline 1320 \end{array}$$

LCM of 12, 15

$$2 \quad 60$$

Perfectly divisible by both 12 & 15

$$60 \times 2 = 120$$

$$60 \times 3 = 180$$

$$60 \times 4 = 240$$

Greatest 3 digit perfectly divisible by 12 and 15.

$$60) \overline{99946}$$

$$\begin{array}{r} 60 \\ \hline 399 \\ 360 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 999 \\ - 39 \\ \hline 960 \end{array}$$

$$\begin{array}{r} 3 \quad | \quad 27, 15, 6, 32 \\ 3 \quad | \quad 9, 5, 6, 32 \\ 3 \quad | \quad 3, 5, 2, 32 \\ 2 \quad | \quad 1, 5, 2, 32 \\ 2 \quad | \quad 1, 5, 1, 16 \\ 2 \quad | \quad 1, 5, 1, 8 \\ 2 \quad | \quad 1, 5, 1, 4 \\ 2 \quad | \quad 1, 5, 1, 2 \\ 5 \quad | \quad 1, 5, 1, 1 \\ \hline 1, 1, 1, 1 \end{array}$$

$$\begin{array}{r} 60 \\ 9 \\ \hline 540 \end{array}$$

Smallest three digit number perfectly divisible by 8, 12, 15.

$$2 \times 2 \times 2 \times 3 \times 5 = 120$$

1080 will be perfectly divisible by 8, 12, 15.

$$120) \overline{1000}$$

$$\begin{array}{r} 2 \\ | \\ 8, 12, 15 \\ | \\ 4, 6, 15 \\ | \\ 2, 3, 15 \\ | \\ 1, 3, 15 \\ | \\ 1, 1, 5 \\ | \\ 1, 1, 1 \end{array}$$

Find the greatest 3 digit number when divided by 12 and 15 leaves remainder 7.

$$\text{LCM} = 12 \times 15 = 60$$

$$60) \overline{999} \quad (\text{16})$$

$$\begin{array}{r} 60 \\ 399 \\ \hline 399 \\ 360 \\ \hline 39 \\ +7 \\ \hline 46 \\ = 967 \end{array}$$

Smallest four digit number which when divided by 11, 12, 15, leaves remainder 3.

$$\text{LCM} = 660$$

$$660 \times 2 = 1320$$

Perfectly divisible by 11, 12, 15.

$$1320 + 3 = 1323$$

$$660) \overline{1000} \quad (\text{1})$$

$$\begin{array}{r} 11 \\ | \\ 11, 12, 15 \\ | \\ 11, 4, 5 \\ | \\ 2, 2, 5 \\ | \\ 220 \\ | \\ 220 \\ | \\ 5 \\ | \\ 660 \\ | \\ 660 \\ | \\ 11 \\ | \\ 11, 1, 5 \\ | \\ 11 \\ | \\ 11, 1, 1 \\ | \\ 1, 1, 1 \end{array}$$

Smallest no, which when divided by 12 leaves remainder 10, and when divided by 15 leaves remainder 13.

$$\text{LCM of } 12, 15 = 60$$

1020 perfectly divisible.  $\therefore 60$

by 12 & 15

$$\begin{array}{r} 1020 \\ 10 \\ \hline 1020 \\ 10 \\ \hline 1030 \\ \hline 1033 \end{array}$$

$$60) \overline{9600} \quad \begin{array}{r} 16 \\ 60 \\ 400 \\ 360 \\ \hline 40 \end{array}$$

In case of common difference.

$$2 - \left( \begin{matrix} 12 \\ \downarrow \\ 10 \end{matrix} \right) \quad 2 - \left( \begin{matrix} 15 \\ \downarrow \\ 13 \end{matrix} \right)$$

i)  $\text{LCM} = 60$ .

ii) 60 is perfectly divisible.

$$60 - 2,$$

$$\begin{array}{r} 4 \\ 12) \overline{58} \\ 48 \\ \hline 10 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ 15) \overline{58} \\ 45 \\ \hline 13 \\ \hline \end{array}$$

$$= 58.$$

In case of greatest 3 digit no.

i)  $\text{LCM} = 60$     ii) 960 perfectly divisible.

Now subtract the common difference.

$$960 - 2 = 958.$$

Find out the greatest 3 digit which when divided by 7, 8, 12, leaves respective remainder of 3, 5, 9.

Common Diff. = 3

$$\text{i) LCM} = 168$$

ii) 840 - perfectly divisible.

$$\begin{aligned}\text{iii)} \quad & 840 - 3 \\ & = 837\end{aligned}$$

$$\begin{array}{r} 2 | 7, 8, 112 \\ 2 | 7, 4, 6 \\ \hline 7, 2, 3 \end{array}$$

$$\begin{array}{r} 168 | 1008 \\ 168 \overline{) 1008} \\ 840 \\ \hline 150 \\ 168 \\ \hline 840 \\ 840 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1018 \\ - 3 \\ \hline 1015 \end{array} \quad \begin{array}{r} 168 \\ - 150 \\ \hline 18 \end{array}$$

$$60 | 999(5)$$

Find out the smallest four digit number which when divided by 6, 7, 8, 9 leaves remainder of 3, 4, 5, 6.

Common Diff - 3.

$$\text{LCM} = 504$$

$504 \times 2 = 1008$  perfectly divisible.

$$1008 - 3$$

$$= \underline{\underline{1005}}$$

$$\begin{array}{r}
 \frac{36}{5} 3 \\
 \frac{11}{5} 2 \times 2 \times 3 \times 3 \times 5 \times 5 \\
 \frac{55}{5} 02 \\
 \frac{55}{10} 0 \\
 \hline
 1000 \\
 -100 \\
 \hline
 900 \\
 \hline
 550) \overline{1000} \\
 \frac{550}{450} \\
 \hline
 550 \\
 -450 \\
 \hline
 100 \\
 \hline
 24 \\
 \hline
 72
 \end{array}$$

$$\begin{array}{r}
 2 \times 2 \times 2 \times 3 \times 3. \\
 2 \quad 72. \\
 3 \quad 9, 12, 18 \\
 3 \quad 3, 4, 6 \\
 2 \quad 1, 4, 2 \\
 1, 2, 1 \\
 \hline
 12 \\
 72 \\
 \hline
 180 \\
 144 \\
 36 \\
 \hline
 64 \\
 \hline
 1036 \\
 7 \\
 \hline
 1093
 \end{array}$$

$$\begin{array}{l}
 I - 36, 72, 108, 144 \\
 II - 48, 96, 144 \\
 III - 72, 144, 216 \\
 IV - 108, 216, 324
 \end{array}$$

$$LCM = 36, 48, 72, 108.$$

$$2 \quad 432$$

$$2 \quad 7 \text{ mins } \frac{12}{4} \text{ sec.}$$

Seven thieves stole a few number of diamonds from a diamond merchant while returning they decided to take rest, while all were sleeping 2 of them got up and decided to divide all of the diamonds equally among them. They divided equally but one was left. So, they woke up the third thief.

$$\text{LCM} = 2, 3, 4, 5, 6 = 60.$$

Common remainder = 1.

$$\begin{array}{r}
 60 + 1 \\
 60 \times 2 = 120 \\
 \quad + 1 \\
 \hline 121
 \end{array}
 \qquad
 \begin{array}{r}
 60 \times 5 = 300 \\
 + 1 \\
 \hline 301
 \end{array}$$

$$\begin{array}{r}
 60 \times 3 = 180 \\
 \quad + 1 \\
 \hline 181
 \end{array}
 \qquad
 \begin{array}{r}
 24212 \\
 \quad 22 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 60 \times 4 = 240 \\
 \quad + 1 \\
 \hline 241
 \end{array}$$

Home Assignments -

$$\frac{40}{120}$$

$$2 \times 2 \times 2 \times 5 \times 3$$

$$2 \mid 120$$

$$\times 2$$

$$\underline{240}$$

$$+ 2$$

$$\underline{\underline{242}}$$

$$38$$

$$- 66$$

$$\underline{82}$$

$$66$$

$$80$$

$$\underline{86}$$

$$2 \mid \boxed{5, 8, 12, 30}$$

$$2 \mid \boxed{5, 4, 6, 15}$$

$$5 \mid \boxed{5, 2, 3, 15}$$

$$3 \mid \boxed{1, 2, 3, 3}$$

$$1, 2, 1, 1$$

$$\begin{array}{r} 880 \\ \times 19 \\ \hline 792 \\ 88 \times \\ \hline 1672 \end{array}$$

$$2 \mid \boxed{38, 66, 80}$$

$$2 \mid \boxed{19, 11, 40}$$

$$2 \mid \boxed{19, 11, 20}$$

$$19, 11, 10$$

$$2 \times 2 \times 2 \times 5 \times 19 \times 11$$

$$2 \mid 16720$$

$$15, 20, 25, 35$$

$$L.C.M = 2100$$

④ We subtract the

common diff.

$$2100 - 10 \quad 2 \quad 2090$$

13.

$$LCM = 5, 6, 8, 9$$

$$2 \mid 360$$

$$\boxed{6, 7, 8, 9, 12}$$

④ Common remainder we

$$add = 360 + 3$$

$$2 \quad 363$$

- $\underline{I}$  - 2, 4, 6, 8 . . . .  
 $\underline{II}$  - 3, 9, 12, 15 . . . .  
 $\underline{III}$  - 4, 8, 12, 16 . . . .  
 $\underline{IV}$  - 5, 10, 15, 20 . . . .  
 $\underline{V}$  - 6, 12, 18, 24 . . . . .

$\boxed{2, 3, 4, 5, 6}$  - LCM  
 260 days. 3 days.

- 1st time they are meeting on the 1st day. ✓
- 2nd time they will be meeting on 61st days. ✓
- 3rd time they will be meeting on 121st days. ✓
- 4th time they will be meeting on 181st days.

Exam approach.

i)  $\text{LCM} = 60 \text{ days.}$

ii)  $\frac{180}{60}^3 \text{ days}$

If it was 181 then it would have been  $\frac{181}{60}^{3.5}$

then would have



## H.C.F.

H C F  
Highest Common Factor.

$$12 : 1, 2, 3, 4, \cancel{6}, 12$$

$$18 : 1, 2, 3, \cancel{6}, 18$$

$$\text{H.C.F.} = 6$$

1.

$$\begin{array}{c} 24, 36 \\ \diagdown \quad \diagup \\ 2^3 \times 3 \quad 2^2 \times 3^2 \end{array}$$

$$\text{H.C.F.} = 2^2 \times 3^1$$

$$= 12.$$

2. 72, 180, 432.

$$\text{H.C.F. of } 180, 432$$

$$2 \boxed{36}$$

$$\begin{array}{r} 2 \\ 180 ) 432 ( \\ \underline{-360} \quad 72 \\ 72 \quad \boxed{180} \\ \underline{-144} \quad 2 \\ 2 \end{array}$$

$$36 ) 72 ( 2$$

$$36 ) 72 ( 2$$

3. 90, 180, 432.

$$\text{H.C.F. } 36.$$

$$36 ) 90 ( 2$$

$$\underline{72}$$

$$18 ) 36 ( 2$$

$$\underline{36}$$

$$\underline{\times}$$

4. 180, 540, 1188

$$540 ) 1188 ( 2$$

$$\underline{1080}$$

$$108 ) 540 ( 8$$

$$\underline{540}$$

$$\underline{\times}$$

$$540$$

$$\frac{1080}{540}$$

$$\frac{1620}{108}$$

$$\frac{108}{54}$$

$$\frac{54}{27}$$

$$\frac{27}{13.5}$$

$$\frac{13.5}{7.2}$$

$$\frac{7.2}{3.6}$$

$$\frac{3.6}{1.8}$$

$$\frac{1.8}{0.9}$$

80, H.C.F. of 90, 180, 432

is 18.

$$108 ) 180 ( 1$$

$$72 ) 108 ( 1$$

$$\underline{72}$$

$$36 ) 72 ( 2$$

$$\underline{72}$$

$$\underline{\times}$$

$$26 ) 26 ( 1$$

$$\frac{26}{13}$$

$$\text{H.C.F.} = 36$$

$$26 ) 26 ( 1$$

$$\frac{26}{13}$$

5. 27, 63, 144, 384

$$27) \overline{63} (2 \\ \underline{-54} \\ \underline{9}) 27 (3 \\ \underline{-27} \\ \underline{x})$$

$$144) \overline{384} (2 \\ \underline{-288} \\ \underline{96}) 144 (1 \\ \underline{-96} \\ \underline{x})$$

$$\begin{array}{r} 96^2 \\ \underline{-4} \\ \underline{96} \\ \underline{-288} \\ \underline{192} \\ \underline{432} \\ \underline{-48} \\ \underline{96} \end{array}$$

$$48) \overline{96} (2 \\ \underline{-96} \\ \underline{x})$$

$$9) \overline{48} (5 \\ \underline{-45} \\ \underline{3})$$

$$3) \overline{9} (3 \\ \underline{-9} \\ \underline{x})$$

$$\text{H.C.F} = \underline{\underline{3}}$$

6. Find out the greatest number that will divide 105 leaving remainder 9 and 175 leaving remainder 7.

$$\begin{array}{r} 90 \\ 105 \\ -9 \\ \hline 96 \end{array}$$

$$\begin{array}{r} 175 \\ -7 \\ \hline 168 \end{array}$$

$$96) \overline{168} (1 \\ \underline{-96} \\ \underline{72}) 96 (1 \\ \underline{-72} \\ \underline{24})$$

$$24) \overline{72} (3 \\ \underline{-72} \\ \underline{x})$$

$$\begin{array}{r} 96^2 \\ \underline{-192} \\ \underline{192} \end{array}$$

$$\begin{array}{r} 72 \\ 144 \\ \underline{-144} \\ \underline{72} \end{array}$$

$$\begin{array}{r} 24 \\ 72 \\ \underline{-72} \\ \underline{24} \end{array}$$

$$\text{Ans} : \underline{\underline{24}}$$

7.  $\begin{array}{r} 81, 99, 180 \\ \text{Difference} \quad \text{Difference} \\ 18 \quad 81 \end{array}$

$18, 81, 99$ .

The H.C.F cannot be greater than  
18

H.C.F of  $18, 81, 99$

= 9.

H.C.F cannot be greater than  
their difference.

### Class Test -

4.  $37$  — leaves remainder  $2$

$$58 \quad \underline{\quad} \quad \text{u} \quad 4 \quad 3$$

$$\begin{array}{r} 37 & 58 \\ - 2 & - 3 \\ \hline 35 & 55 \end{array}$$

$$35) \overline{55} (1$$

$$\underline{35}$$

$$20) \overline{35} (1$$

$$\underline{20}$$

$$15) \overline{20} (1$$

$$\underline{15}$$

$$5) \overline{15} (3$$

$$\underline{15}$$

Ans : 5.

5. 144, 216, 480, 528

$$\frac{72}{2} \quad \frac{144}{2} \\ \underline{288}$$

$$144) \overline{216}(1 \\ \underline{144} \\ 72) \overline{144}(2 \\ \underline{144} \\ \underline{x})$$

$$480) \overline{528}(1 \\ \underline{480} \\ 48) \overline{480}(10 \\ \underline{480} \\ \underline{x})$$

$$\frac{48}{2} \\ \underline{96}$$

$$48) \overline{72}(1 \\ \underline{48} \\ 24) \overline{48}(2 \\ \underline{24} \\ \underline{x})$$

6.

$$786 \quad \underline{\quad\quad\quad} 4 \\ 991 \quad \underline{\quad\quad\quad} 5 \\ 1196 \quad \underline{\quad\quad\quad} 6$$

$$786 \quad 991 \\ - 4 \quad - 5 \\ \underline{782} \quad \underline{986}$$

$$1196 \\ - 6 \\ \underline{1190}$$

$$\frac{172}{2} \quad \frac{986}{2} \\ \underline{340} \quad \underline{1972}$$

$$986) \overline{1190}(1 \\ \underline{986} \\ 204) \overline{986}(4 \\ \underline{816} \\ \underline{170})$$

$$204) \overline{204}(1$$

$$34) \overline{782}(10 \\ \underline{340} \\ \underline{440} \\ \underline{000}$$

$$34) \overline{782}( \\ \underline{68} \\ 102 \\ 102 \\ \underline{x})$$

$$\frac{204}{4} \quad \frac{408}{816} \\ \underline{816} \quad \underline{612} \\ 204 \\ 5 \\ \underline{1020}$$

$$\frac{342}{170}$$

$$34) \overline{102}( \\ \underline{68} \\ 34 \\ 34 \\ \underline{02}$$

Ans : 34

Home Assignment

5.  $36, 42, 105, 135$

$$36) \overline{42} (1$$

$$\begin{array}{r} 36 \\ - 6 \\ \hline 36 \end{array}$$

$$\begin{array}{r} 36 \\ - 36 \\ \hline \alpha \end{array}$$

$$105) \overline{135} (1$$

$$\begin{array}{r} 105 \\ - 30 \\ \hline 90 \end{array}$$

$$\begin{array}{r} 90 \\ - 15 \\ \hline 30 \end{array}$$

$$\begin{array}{r} 30 \\ - 30 \\ \hline \alpha \end{array}$$

$$6) \overline{15} (2$$

$$\begin{array}{r} 12 \\ - 3 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 6 \\ - 6 \\ \hline \alpha \end{array}$$

Aus : 6 3 =

6.  $117, 130, 143$

$$130) \overline{143} (1$$

$$\begin{array}{r} 130 \\ - 13 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 130 \\ - 130 \\ \hline \alpha \end{array}$$

$$13) \overline{117} (9$$

$$\begin{array}{r} 117 \\ - 117 \\ \hline \alpha \end{array}$$

H.C.F 13

$$18) \overline{(117, 130)} 143$$

$$\boxed{\begin{array}{|c|c|c|} \hline 9 & 10 & 143 \\ \hline \end{array}}$$

42

$$2. \quad \overbrace{38, 66, 80}^{28} (2$$

$$\begin{array}{r} 38 \\ - 28 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 66 \\ - 28 \\ \hline 38 \end{array}$$

$$\begin{array}{r} 80 \\ - 56 \\ \hline R=10 \end{array}$$

$$14) \overline{38 (114) 66} 4 (4$$

$$\begin{array}{r} 28 \\ - 10 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 56 \\ - 56 \\ \hline 0 \end{array}$$

$$14) \overline{80} (5$$

$$\begin{array}{r} 70 \\ - 70 \\ \hline 10 \end{array}$$

Aus : 14

$$\begin{array}{r} 105 \\ - 2 \\ \hline 103 \end{array}$$

$$\begin{array}{r} 30 \\ - 9 \\ \hline 21 \end{array}$$

$$\begin{array}{r} 30 \\ - 12 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 130 \\ - 26 \\ \hline 104 \end{array}$$

$$\begin{array}{r} 13 \\ - 11 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 14 \\ - 14 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 14 \\ - 14 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 14 \\ - 14 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 14 \\ - 14 \\ \hline 0 \end{array}$$

17. 1709, 1879, 2399. 6. 117, 130, 143.

$$\begin{array}{r} 1879 \\ - 1709 \\ \hline 170 \end{array} \quad \begin{array}{r} 2399 \\ - 1709 \\ \hline 690 \end{array}$$

$$\text{H.C.F} = 13.$$

$$\frac{117}{13} = 9.$$

$$\frac{130}{13} = 10.$$

$$\frac{143}{13} = 12.$$

$$= 9 + 10 + 13 = \underline{\underline{30}}.$$

↪ LCM of Fractions :  $\frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$

↪ HCF of Fractions :  $\frac{\text{HCF of Numerator}}{\text{LCM of Denominators}}$

$$\frac{3}{5}, \frac{9}{25}, \frac{81}{125}$$

LCM

$$= \frac{\text{LCM of } 3, 9, 81}{\text{HCF of } 5, 25, 125} = \frac{81}{5}$$

$$\text{HCF} = \frac{\text{HCF of } 3, 9, 81}{\text{LCM of } 5, 25, 125} = \frac{3}{125}$$

$$\hookrightarrow axy = \text{LCM} \times \text{HCF}$$

LCM of 24, 36 & 72.

HCF of 24, 36 & 12.

$$\begin{array}{r} 24 \\ \times 362 \\ \hline 144 \\ 72x \\ \hline 864 \end{array}$$

$$24 \times 36 = 864$$

$$72 \times 12 = 864$$

### HOME ASSIGNMENT :

4.

(3, 5, 7, 11)

Suppose, A, B, C, D. are 4 runners running at 3 km/hr.

$$A = \frac{3}{3} = 1 \text{ hr}$$

<sup>2</sup> <sub>1</sub> hr, 2, 3, 4.

$$B = \frac{3}{5} \text{ hrs}$$

$$\text{LCM of } 1, \frac{3}{5}, \frac{3}{7}, \frac{3}{11}$$

$$= \frac{3}{5}, \frac{6}{5}, \frac{9}{5}$$

$$= \frac{3}{1} \text{ hr}$$

$$C = \frac{3}{7}$$

$$= 3 \text{ hrs.}$$

$$= \frac{6}{7}, \frac{9}{7}, \frac{12}{7}$$

They started at

5 am

$$D = \frac{3}{11}$$

then they meet

at 5 + 3

$$= \frac{6}{11}, \frac{9}{11}$$

28 am.

$$8. \text{ H.C.F} = a$$

LCM  $\geq 2900\%$  more than HCF.

$\geq 2900\%$  more than 'a'

$\geq 29$  times more than 'a'

$$\therefore \text{LCM} = 29a + a \\ = 30a$$

$$\text{LCM} + \text{HCF} = 310$$

$$30a + \text{HCF} = 310$$

$$30a + a = 310$$

$$31a = 310$$

$$a = 10$$

$$\text{HCF} = a \rightarrow 10.$$

$$x \times y = \text{LCM} \times \text{HCF}$$

$$20 \times y = \frac{15}{300} \times 10$$

$$= \underline{\underline{150}}$$

$$2) x \times y = \text{LCM} \times \text{HCF}$$

$$21 \times 4641 = x \times y$$

$$b) 231 \times 378$$

$$= 8$$

$$a) 273, 357$$

$$c) 252, 378$$

$$16$$

$$d) 273, 399$$

$$24$$

Factorial  $n!$

$$n! = n \times (n-1) \times (n-2) \times \underline{3 \times 2 \times 1}$$

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$7! = 7 \times 6!$$

$$7! = 7 \times 6 \times 5!$$

What will be the LCM of 8 & 5 Factorial?

LCM = 8 - can't be greater than the numbers  
HCF = 5 - smallest one

$$73!, 225!, 786!$$

$$\text{LCM} = 786!$$

$$\text{HCF} = 73!$$

→ Find the maximum power of 3 that can perfectly divide  $10!$

$$2 \frac{10!}{3^n}$$

$$2 \frac{10!}{3^n} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 3}$$

$$3 \overline{)1073} \\ 9 \\ \hline 1$$

∴ find out the multiples of 3 in 10 factorials.

$$\frac{10!}{3^n} = \frac{10}{3} + \frac{10}{3^2}$$

$\Downarrow$        $\Downarrow$

$$3 + 1$$

→ Maximum power of 3 that can perfectly divide  
 $100!$

$$\frac{100!}{3^n} = \frac{100}{3} + \frac{100}{3^2} + \frac{100}{3^3} + \frac{100}{3^4}$$

$\Downarrow$        $\Downarrow$        $\Downarrow$

$$233 + 99 + 3 + 1$$

$2 \quad 3^{48}$

$$\frac{100!}{3^n} = \frac{100}{3} 233 +$$

$$\Rightarrow \frac{33}{3} 211 +$$

$$\Rightarrow \frac{11}{3} 23 +$$

$$\Rightarrow \frac{3}{3} 21$$

48

→ max. power of  $\frac{1000}{11}$  that can perfectly divide  $400!$ , Max. power of  $7 \rightarrow 800!$

$$\frac{400!}{11^2} = \frac{400}{11^2} = 36.$$

$$\begin{array}{r} 36 \\ 47400 \\ -33 \\ \hline 14 \\ -70 \\ \hline 66 \\ -4 \\ \hline \end{array}$$

$$\Rightarrow \frac{36}{11} = 3$$

$$\begin{array}{r} 3 \\ 11 \\ \hline 39. \end{array}$$

$$\Rightarrow \frac{800}{7^2} = 114$$

$$\begin{array}{r} 114 \\ 7) 800 \\ -7 \\ \hline 10 \\ -7 \\ \hline 30 \\ -28 \\ \hline 2. \end{array}$$

+

$$\Rightarrow \frac{114}{7^2} = 16$$

$$\begin{array}{r} 16 \\ 7) 114 \\ -7 \\ \hline 44 \\ -42 \\ \hline 2. \end{array}$$

+

$$\Rightarrow \frac{16}{7^2} = 2$$

$$\begin{array}{r} 132 \\ \hline \end{array}$$

→ Find the max. power of 8 that can divide  
 $200!$

$$\textcircled{8} \quad \frac{200!}{8^n} = \frac{200}{8} = 25.$$

$$2) \frac{25}{8} \quad 2 \quad 3.$$

$$2) \frac{3}{8} \quad 2) \underline{\underline{28}}.$$

$$8) \overline{200} \quad 25 \\ 16 \overline{)40} \\ 40 \overline{)0} \\ \underline{\underline{x}} \\ 8) \overline{25} \quad 3 \\ 24$$

$$\text{multiples of } 8 = 2 \times 2 \times 2$$

as we have 3 multiples of 2

we will ~~multiples!~~ of 8.

$$@upsc.riseinfinity \quad 2) \frac{200!}{2} \quad 2) \frac{100}{2} \quad 2) \frac{100}{2} \quad 2) \frac{197}{2} \quad 2) \frac{65}{2}$$

$$\begin{array}{rcl} \frac{200!}{8^n} & = & 2) \frac{200!}{2} + 2) \frac{100}{2} + 2) \frac{197}{2} + 2) \frac{65}{2} \\ & = & 2) \frac{100}{2} + 2) \frac{50}{2} + 2) \frac{3}{2} + 2) \frac{8}{2} \\ & = & 2) \frac{50}{2} + 2) \frac{25}{2} + 2) \frac{8}{2} \\ & = & 2) \frac{25}{2} + 2) \frac{12}{2} + 2) \frac{8}{2} \\ & = & 2) \frac{12}{2} + 2) \frac{6}{2} + 2) \frac{8}{2} \\ & = & 2) \frac{6}{2} + 2) \frac{3}{2} + 2) \frac{8}{2} \\ & = & 2) \frac{3}{2} + 2) \frac{1}{2} + 2) \frac{8}{2} \end{array}$$

$$2) \frac{50}{2} + 2) \frac{25}{2} + 2) \frac{8}{2} \quad 2) \frac{200!}{8^n} + 2) \frac{197}{2} + 2) \frac{65}{2}$$

$$\Rightarrow 2) \frac{25}{2} + 2) \frac{12}{2} + 2) \frac{8}{2}$$

$$\Rightarrow 2) \frac{12}{2} + 2) \frac{6}{2} + 2) \frac{8}{2}$$

$$\Rightarrow 2) \frac{6}{2} + 2) \frac{3}{2} + 2) \frac{8}{2}$$

$$\Rightarrow 2) \frac{3}{2} + 2) \frac{1}{2} + 2) \frac{8}{2} = 197$$

$$\hookrightarrow \frac{100!}{6^n}$$

$\begin{array}{c} 2 \\ \diagdown \quad \diagup \\ 2 \times 3 \end{array}$

$$2) \frac{100!}{3^n} = 48$$

$$3^{48}$$

$$\frac{4!}{6^n} = \frac{4 \times 3 \times 2 \times 1}{6^3}$$

Suppose I am taking

for  $6^n$

$$2) 6^{48}$$

$$\frac{(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2 \times 2}{(3 \times 3) \times (3 \times 3) \times (3 \times 3)}$$

$$\hookrightarrow \frac{500!}{35}$$

$\begin{array}{c} \diagdown \quad \diagup \\ 5 \times 7 \end{array}$

$$2) \frac{500}{7^n} = 71$$

+

$$2) \frac{71}{7} = 10$$

+

$$2) \frac{10}{7} = 1$$

$$2) \frac{1}{7} - \underline{\quad} = 82$$

$$35^{82}$$

$$\frac{(\mathbb{Q})!}{6} \frac{300!}{18^n}$$

$\begin{array}{c} \diagdown \quad \diagup \\ 2 \times 3^2 \end{array}$

$$2) \frac{300}{3} = 100$$

+

$$2) \frac{100}{3} = 33$$

+

$$\Rightarrow \frac{33}{3} = 11$$

+

$$\Rightarrow \frac{11}{3} = 3$$

+

$$2) \frac{3}{3} = 1$$

—

$$7) \frac{500}{49}$$

$\begin{array}{c} 71 \\ 10 \\ 7 \\ 3 \\ \hline \end{array}$

$$3) \frac{3}{11}$$

$\begin{array}{c} 9 \\ 2 \\ \hline \end{array}$

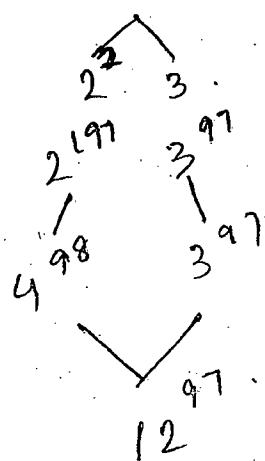
$$2) \frac{74}{148}$$

$\begin{array}{c} 9 \\ 18 \\ 18 \\ \hline \end{array}$

$$= 18 = 9 \sqrt{74}$$

→  $200!$

$$\frac{1}{12^n}$$



$$\frac{200}{2} = 100$$

$$\frac{100}{2} = 50$$

$$\frac{50}{2} = 25$$

$$\frac{25}{2} = 12$$

$$\frac{12}{2} = 6$$

$$\frac{6}{2} = 3$$

$$\frac{3}{2} = 1$$

$$\frac{200}{3} = 66$$

$$\frac{66}{3} = 22$$

$$\frac{22}{3} = 7$$

$$\frac{7}{3} = 2$$

—  
97

197

2 197

→ How many Zeros are there at the end of  $100!$  factorials?

The maximum power of 10 and the number of zeroes at the end will be same.

$$\frac{10}{\underline{\quad}}^{24}$$

→ Find the max. power 10 of  $100!$

$$\begin{array}{r} 100 \\ \overline{10} \\ \quad | \\ 2 \times 5 \end{array} \quad \begin{array}{r} 2 \frac{100}{5} \\ 2 \quad 20 \\ \Rightarrow 20 \quad 20 \quad + \\ \quad \quad \quad \overline{5} \end{array}$$

Multiples of 5  
will be greater  
than  $2^2 \cdot 5^1$ ,  
leave 2.

$$\begin{array}{r} 24 \\ \overline{10} \\ \quad | \\ 2 \quad 4 \end{array} \quad \begin{array}{r} 4 \\ \overline{5} \\ \quad | \\ 24 \end{array}$$

→ 12.

$$H.C.F = a$$

$$L.C.M = 12 \text{ times } \cancel{a}$$

$$L.C.M = \cancel{12a} + 12a + a$$

$$2 \cancel{12a} \cdot 12a$$

$$\frac{L.C.M + H.C.F}{12a} = 403$$

$$\cancel{12a} + a = 403$$

$$12a = 403$$

$$a = \frac{403 - 3}{12}$$

$$\begin{array}{r} 4 \\ 14) 403 \\ 38 \\ \hline 23 \\ 14 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 403 \\ 4 \\ \hline 372 \\ 372 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 372 \\ 93) 372 (4 \\ 372 \\ \hline 0 \end{array}$$

$$H.C.F = a - 3$$

$$a \times b = H.C.F \times L.C.M$$

$$93 \times 4 = 31 \times 372$$

$$= 124$$

20 2:3 30

$$\begin{array}{r} & \downarrow \\ 30 & - \frac{10}{\downarrow} \\ 40 & \end{array}$$

$$\begin{array}{r} \cancel{3} \cancel{1} 6 \\ x \\ \cancel{2} \cancel{1} 4 \end{array}$$

$$\begin{aligned} x &= 30 + 6 \\ &= 36 \end{aligned}$$

$$x = 40 - 4$$

$$2 \quad 36$$

49 7:9 63

$$\begin{array}{r} \downarrow \qquad \downarrow \\ 326 - \frac{64}{\downarrow} \qquad 390 \end{array}$$

$$\begin{array}{r} \cancel{9} \cancel{4} \\ \cancel{3} \cancel{6} \quad x \\ \cancel{1} \cancel{7} \end{array}$$

$$\begin{array}{r} 9 \times 64 \\ \hline 16 \end{array}$$

$$326 + 36$$

$$2 \quad 362$$

$$\begin{array}{ccccccc}
 & 8 & 1 & 9 : 11 & 99 & 11+9 & \\
 & \downarrow & & & \downarrow & & \\
 786 & - & \cancel{100} & & 686 & \frac{11}{20} \times \cancel{100} & \frac{4 \times 24}{10} \\
 & & & & & 255 & 6:4 \\
 & \cancel{11} & & & \cancel{9} & & \\
 & & 55 & x & & & \\
 786 - 55 & = & 731 & & & & \frac{210}{2420} \\
 & & & & & & \frac{4}{2} \\
 & & & & & & 0
 \end{array}$$

36 Articles avg. price is Rs 420.  $x$  articles avg. price Rs 455 and the overall avg. price is Rs 440.

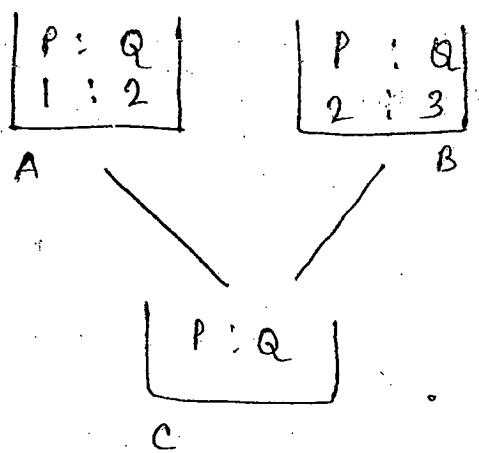
$$\begin{array}{ccccc}
 36 & & x & 420 & 455 \\
 \downarrow & & \downarrow & & \\
 420 & & 455 & 440 - 420 & 220 \\
 & \cancel{20} & \cancel{15} & 455 - 440 & 215 \\
 & & & & \\
 & & & &
 \end{array}$$

The ratio of the diff. = 4:3.

Then inverse it = 3:4.

$$x = \frac{4}{3} \times \frac{12}{36}$$

$$= 48$$



Vol. of vessel A =  $x$ .

$$\text{Quantity of } P = \frac{1}{3}x$$

$$\text{“ “ } Q = \frac{2}{3}x$$

Vol. of vessel B =  $x$

$$\text{Qty of } P = \frac{2}{5}x$$

$$\text{Qty of } Q = \frac{3}{5}x$$

$$\frac{P}{Q} = \frac{x}{3} + \frac{2x}{5}$$

$$\frac{2x}{3} + \frac{3x}{5}$$

$$= \frac{5x + 6x}{15} = \frac{11x}{15}$$

$$10x + 9x$$

$$\frac{15}{15}$$

= 11 : 19.

$$\begin{array}{ccc}
 A & & B \\
 \downarrow & & \downarrow \\
 P : Q & & P : Q \\
 5 \times \underbrace{1 : 2}_{3} & & \underbrace{2 : 3}_{5} \times 3 \\
 & & x 3 \\
 & & 15
 \end{array}$$

LCM of 3 & 5  
= 15

$$\begin{array}{ccc}
 A & & B \\
 \downarrow & & \downarrow \\
 P : Q & & P : Q \\
 5 : 10 & & 6 : 9
 \end{array}$$

Now adding the value of  $P = 5 + 6$   
 $= 11$

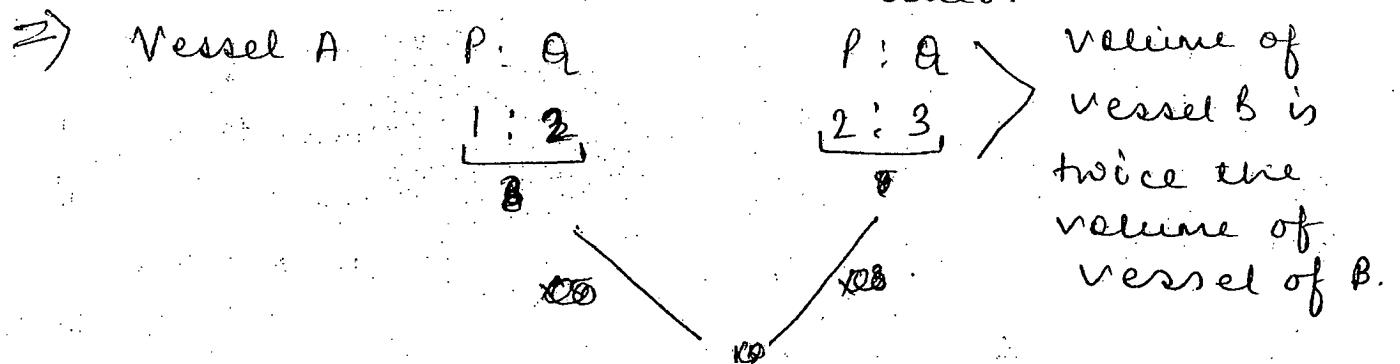
$$\begin{array}{ccc}
 " & & " \text{ of } Q = 10 + 9 \\
 & & = 19 \\
 & & \frac{2}{11} \\
 & & 19
 \end{array}$$

$$\begin{array}{ccc}
 \Rightarrow \text{Vessel A, } P : Q & & P : Q \\
 \times 7 \underbrace{2 : 3}_{5} & & \underbrace{3 : 4}_{7} \times 5 \\
 & & x 7 \\
 & & 35
 \end{array}$$

$$\begin{array}{r}
 14 \\
 + 15 \\
 \hline
 29
 \end{array}$$

$$= 14 : 21 \quad 15 : 20$$

$$\begin{array}{r}
 29 \\
 \hline
 41
 \end{array}$$



Science Class.

$$\text{Vol. of vessel A} = x - \frac{1}{3}x = \frac{2}{3}x$$

$$\text{Vol. of } \text{vessel B} = 2x - \frac{2}{5} \times 2x = \frac{3}{5} \times 2x$$

$$\begin{aligned} \frac{P}{Q} &= \frac{x + 4x}{\frac{2x}{3} + \frac{6x}{5}} \\ &= \frac{5x + 12x}{15} \\ &= \frac{17x}{28x} \end{aligned}$$

|                                  |                                  |
|----------------------------------|----------------------------------|
| $A$                              | $B$                              |
| 1                                | 1                                |
| $P:Q$                            | $P:Q$                            |
| $\boxed{1:2}$                    | $\boxed{2:3}$                    |
| $\times 5$                       | $\times 2$                       |
| <del><math>\times 5</math></del> | <del><math>\times 3</math></del> |

$$\begin{array}{ll} A & B \\ P:Q & P:Q \\ 5:10 & 12:18 \end{array}$$

$$\begin{aligned} &= 5 + 12 = 17 \\ &= 10 + 18 = 28 \end{aligned}$$

$$2 \frac{17}{28}$$

$$\Rightarrow \begin{array}{l} P : Q \\ \times 7 \quad \boxed{2 : 3} \\ \times 4 \quad 5 \end{array} \qquad \begin{array}{l} P : Q \\ \boxed{3 : 4} \times 5 \\ 7 \end{array}$$

$\frac{21}{84}$  volume of vessel is  
4 times that of B.  
so, after mixing them  
find out the  
ratio of P : Q.

~~56~~

$$60 : 84$$

$$15 : 20$$

~~total~~ Balloons. 71 : 104

$$P = 56 + 15 = 71$$

$$Q = 84 + 20 = 104$$

$$\frac{14}{56}$$

$$\frac{84}{104}$$

$$\frac{15}{79}$$

$$\frac{15}{71}$$

$$\frac{17}{33}$$

Vessel B is  
twice that of  
A.

$$\Rightarrow \begin{array}{l} P : Q \\ \cancel{2} \cancel{3} \quad \boxed{5 : 6} \\ 11 \end{array}$$

$$\begin{array}{l} P : Q \\ \boxed{17 : 16} \\ 33 \times 1 \end{array}$$

The ratio is  
already  
twice.

$$\begin{array}{l} 45 : 54 \\ \cancel{6} \cancel{2} : \cancel{6} \cancel{0} \\ 17 : 16 \end{array}$$

$$\frac{P}{Q} = \frac{5+17}{6+16}$$

$$\frac{1}{54} \quad \frac{1}{54} \quad \frac{1}{54}$$

$$\frac{1}{22} \quad 2 : 1 : 1$$

Handout :

$$19. \quad 20 \text{ kg} \left[ \begin{array}{c} A \\ 1 : 5 \\ \hline 6 \end{array} \quad \begin{array}{c} B \\ 1 : 6 \\ \hline 7 \end{array} \right] 25 \text{ kg}$$

$$\text{Alu} = \frac{1}{5}$$

$$= \frac{1}{5} \times 20$$

= 4 kg of

Aluminium

$$Ni = \frac{4}{5} \times 20$$

$$= 16 \text{ kg}$$

$$\frac{1}{6} \times 25$$

$$= \frac{25}{6} \text{ kg}$$

$$\frac{5}{6} \times 25$$

$$= \frac{125}{6} \text{ kg}$$

$$\frac{1}{7} \times 25$$

$$= \frac{25}{7}$$

$$\frac{10}{3} + \frac{25}{7}$$

$$= \frac{10+25}{21} =$$

$$\frac{35}{21} = \frac{5}{3}$$

$$\frac{5}{6} \times 20$$

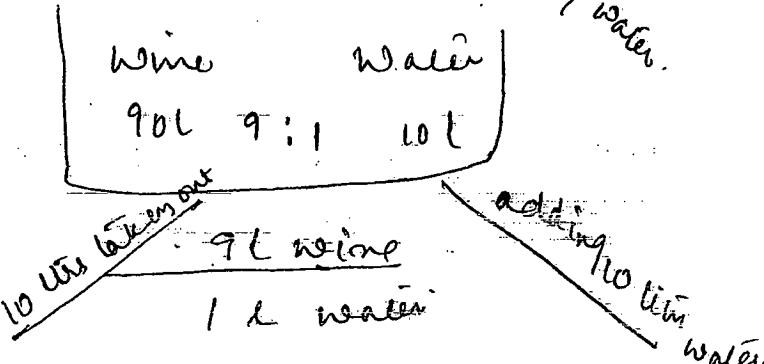
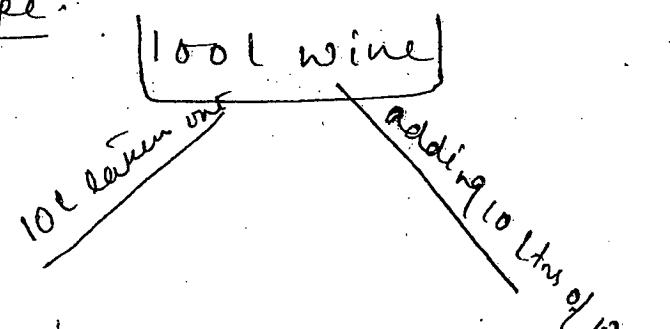
$$\frac{10}{3} + \frac{50}{3} = \frac{60}{3} = 20$$

$$\frac{6}{7} \times 25$$

$$= \frac{150}{7}$$

$$\frac{\text{Alu}}{Ni} = \frac{4 + \frac{25}{6}}{16 + \frac{125}{6}} = \frac{24 + 25}{96 + 125} = \frac{49}{121}$$

Third Type :



|  |  |
|--|--|
| wine   | water                                      |
| ↓  | ↓  |
| $\frac{90}{100} - \frac{9}{100}$<br>= $\frac{81}{100}$ l | $\frac{10+9}{100}$<br>= $\frac{19}{100}$ l |

So, the ratio is 81 : 19.

Ratio of wine in

$$\text{the mixture} = \frac{81}{81+19} \\ = \frac{81}{100}$$

Formula:

If the initial qty of pure liquid

Replaced qty  $\frac{2x}{2x-y}$   
No. of replacement done  $= n$ .

The qty of pure liquid remaining  
after 'n' operations  $= n \left(1 - \frac{y}{x}\right)^n$

Qty of mixture at any point of time  
will be  $x$ .

$$x = 100 \text{ l.}$$

$$y = 10 \text{ l}$$

$$\text{Replacement done} = n = 2.$$

$$\text{neine } 2 \cdot 100 \left(1 - \frac{10}{100}\right)^2$$

$$= 100 \times \frac{9}{10} \times \frac{9}{10}$$

$$= 81 \text{ l.}$$

Handout

$$7. \quad x \left(1 - \frac{4}{x}\right)^3$$

$$100 \left(1 - \frac{10}{100}\right)^3$$

$$= 100 \times \frac{9}{10} \times \frac{9}{10}$$

$$= 81 \text{ liter}$$

$$100 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

$$8. \quad 500 \left(1 - \frac{100}{500}\right)^4$$

$$= 500 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$$

$$= \frac{4^5}{5} \text{ l}$$

$$= \frac{1024}{5} \text{ l}$$

$$= 204.8 \text{ l.}$$

$$2 \frac{729}{10} = 72.9$$

$$9. \quad x = ?$$

$$y = 50$$

$$n = 3$$

Qty of <sup>wine</sup> remaining

$$= x \left(1 - \frac{50}{x}\right)^3$$

$$\text{Mixture} = x$$

Ratio of Wine : Mixture.

$$\frac{\text{wine}}{\text{mixture}} = \frac{x \left(1 - \frac{50}{x}\right)^3}{x}$$

$$= \left(1 - \frac{50}{x}\right)^3$$

$$\frac{\text{wine}}{\text{water}} = \frac{343}{169}$$

$$\frac{\text{wine}}{\text{mixture}} = \frac{343}{343 + 169}$$

$$= \frac{343}{512}$$
$$\frac{(1 - \frac{50}{x})^3}{(1 - \frac{50}{x})^3 + \frac{343}{512}}$$

$$\left(1 - \frac{50}{x}\right)^3 = 2 \left(\frac{7}{8}\right)^3$$

$$2 \cdot 1 - \frac{50}{x} = 2 \cdot \frac{7}{8}$$

$$= 1 - \frac{7}{8} = \frac{50}{x}$$

$$\frac{1}{8} = \frac{50}{x}$$

$$x = 400 \text{ l}$$

28.

$$\text{Profit \%} = \frac{\text{Qty of water}}{\text{Qty of milk}} \times 100 \%$$

$$= \frac{1 \phi}{4 \phi} \times \frac{25}{100}$$

$$= 25 \%$$

The cost price &  
Selling Price is  
same.

$$29. \text{ Ratio of water to milk} = \frac{P \%}{100}$$

$$= \frac{1}{3} : \frac{33}{3} \% = \frac{1}{3} : 1 : 3$$

$\Rightarrow$  The Cost Price of an article = Rs 12000.  
 If  $\frac{1}{3}$  rd of it sold at the loss of 10%.

Then, at what profit % remaining article must be sold to make an overall profit of 20%.

$$\begin{array}{ccc}
 & \text{to make profit} & \\
 12,000 & \xrightarrow{\text{of}} & 20\%
 \\ 
 \swarrow & & \downarrow \\
 \begin{array}{l}
 \frac{1}{3} \text{ sold} \\
 @ 10\% \text{ loss} \\
 \frac{1}{3} \times \frac{4,000}{12,000}
 \end{array} & 
 \begin{array}{l}
 \frac{2}{3} \\
 \text{Profit \% = ?} \\
 \frac{20}{100} \times 12,000 \\
 = 2400
 \end{array} & 
 \begin{array}{l}
 \frac{2}{3} \times \frac{4,000}{12,000} \times \frac{9}{10} \\
 = 8,000 \times \frac{2}{3} \\
 = 1600
 \end{array}
 \end{array}$$

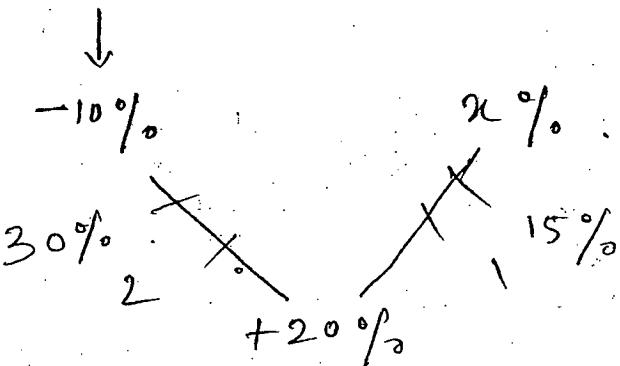
$$\begin{aligned}
 \text{Profit} &= 2400 + 400 \\
 &= 2800
 \end{aligned}$$

$$P\% = \frac{2800}{8000} \times 100$$

$$= 35\%$$

$\frac{1}{3} \quad 1:2 \quad \frac{2}{3}$

$\frac{2}{3}x$



$$-10 + 20 \quad x = 20\% + 15\%$$

$$\therefore 30\% \quad \therefore 35\%$$

8.

|      |       |
|------|-------|
| wine | water |
| 74%  | 26%   |

$$\text{Qty} = x$$

Initial qty of wine = 74 % of  $x$ .

" " " water = 26 % of  $x$ .

taking out 14 ltrs.

$$\text{wine} = 74\% \text{ of } 14.$$

$$\text{Qty of water} = 26\% \text{ of } 14.$$

adding 14 ltrs

$$\text{wine} = 14 \text{ ltrs}$$

$$\text{Qty of wine} = 76 \% \text{ of } x.$$

$$\text{u u water} = 24 \% \text{ of } x.$$

$$74\% \text{ of } x - 74\% \text{ of } 14 + 14 = 76\% \text{ of } x$$

water

$$26\% \text{ of } x - 26\% \text{ of } 14 = 24\% \text{ of } x$$

$$\frac{26}{100} \times x - \frac{26}{100} \times 14 = \frac{24}{100} \times x$$

$$\frac{26}{100} \times x - \frac{24}{100} \times x = \frac{26}{100} \times 14$$

$$\cancel{\frac{2}{100}} \times x = \frac{13}{100} \times 14$$

$$x = 182 \text{ litres}$$

Or,  $26\% - 24\% = 2\% \text{ of } 14$

~~$\frac{2}{100} \times 14$~~

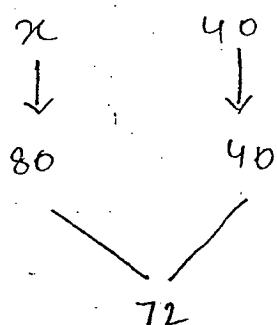
$$1\% = \frac{13}{2} \% \text{ of } 14$$

$$= 13\% \text{ of } 14$$

$$1\% = \frac{13}{100} \times 14 = \frac{13 \times 14}{100}$$

Q. 6, 10, 11, 12, 15, 17, 20, 21, 23, 25, 27

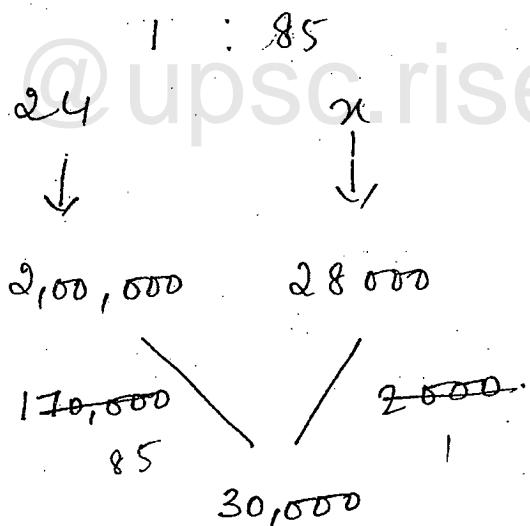
15.



$$72 \rightarrow P\% : 20\% \cdot 2100 + 20 \\ = 120$$

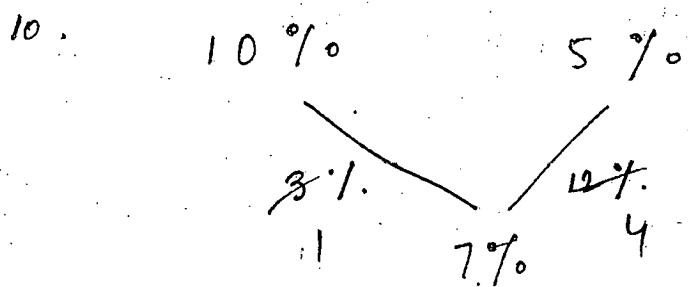
$$\text{Cost price} = \frac{72 \times 100}{120} \\ = 60$$

17.



$$x = 85 \times 24$$

$$= 2040$$



$1:4$

$$= \frac{4}{16}$$

~~60 kg~~ &  $40 \text{ kg}$ . &  $10 \text{ kg}$ .

24.

$$\frac{7}{12}x - \frac{7}{12} \times 9 = \frac{7}{16}x$$

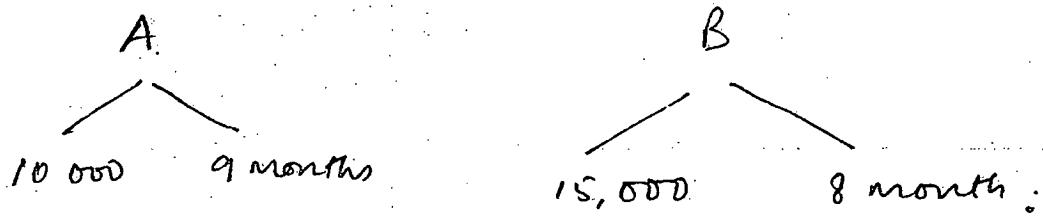
$$7:5 \\ 7+5 \\ 12$$

@  $x = 36$

$$= \frac{7}{12} \times 36$$

= 21.

## PARTNERSHIP



$$\begin{aligned}
 \frac{P_A}{P_B} &= \frac{2}{\frac{10,000 \times 9}{15,000 \times 8}} \\
 &= 3 : 4
 \end{aligned}$$

Profit = 28,000



$$\begin{aligned}
 \frac{3 \times 28000}{7} &= 12,000 \\
 \frac{4 \times 28000}{7} &= 16,000
 \end{aligned}$$

For eg.

A                              B                              C

P<sub>A</sub> :      P<sub>B</sub>      ;      P<sub>C</sub>

$$(I_A \times T_A) : (I_B \times T_B) : (I_C \times T_C)$$

1. A started a business by investing Rs 1,00,000, B joins in 3 months later by investing Rs 70,000. If the total profit earned at the end of the year is Rs 1,22,000, find out the profit share of A & B.

$$\frac{1,00,000 \times 12}{70,000 \times 9}$$

$$\frac{120}{63} : 40 = 40 : 21$$

@upsc.riseinfinity  
1,22,000.

$$40 : 21$$

$$\frac{40}{61} \times \frac{2000}{122,000} : \frac{21}{61} \times \frac{2000}{122,000}$$

$$80,000 : 42,000$$

Handout :-

1. A & B invest in the business  
= 3 : 2.

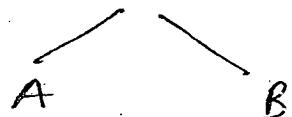
Profit % goes to charity 5%.

$$\text{Remaining} = 100\% - 5\% = 95\%$$

$\begin{array}{r} 1713 \\ \times 95 \\ \hline 855 \\ 1539 \times \\ \hline 16245 \end{array}$ 
  
 $\frac{171}{95} \times \frac{171}{855} = 5$

$$\text{A's share} = 855.$$

95% Profit



$$3 : 2$$

$$\frac{3 \times 95}{5} = 57\%$$

$$\frac{2 \times 95}{5} = 38\%$$

57% of Rs 855

$$100\% = \frac{15}{855 \times 100} = \frac{15}{855}$$

= Rs 1500.

5.

$$P_A : P_B : P_C$$

$$5 : 7 : 8$$

$$(x \times 14) : (y \times 8) : (z \times 7)$$

$$14x : 8y : 7z = 5 : 7 : 8$$

$$\frac{14x}{8y} = \frac{5}{7}$$

$$\frac{x}{4} = \frac{20}{49}$$

$$\frac{8y}{7z} = \frac{7}{8}$$

$$\frac{4}{z} = \frac{49}{64}$$

$x : y : z$

$$= 20 : 49 : 64$$

2.

A

B

C

7400

↓ 5%

Remaining = 95%

7400  
x 95  
-----  
6830

95% of 7400

$$\Rightarrow \frac{95}{100} \times 7400$$

|                            |                            |                             |
|----------------------------|----------------------------|-----------------------------|
| $\frac{13}{6500} \times 6$ | $\frac{14}{8400} \times 5$ | $\frac{10}{10000} \times 3$ |
| <b>A</b>                   | <b>B</b>                   | <b>C</b>                    |

13 : 14 : 10

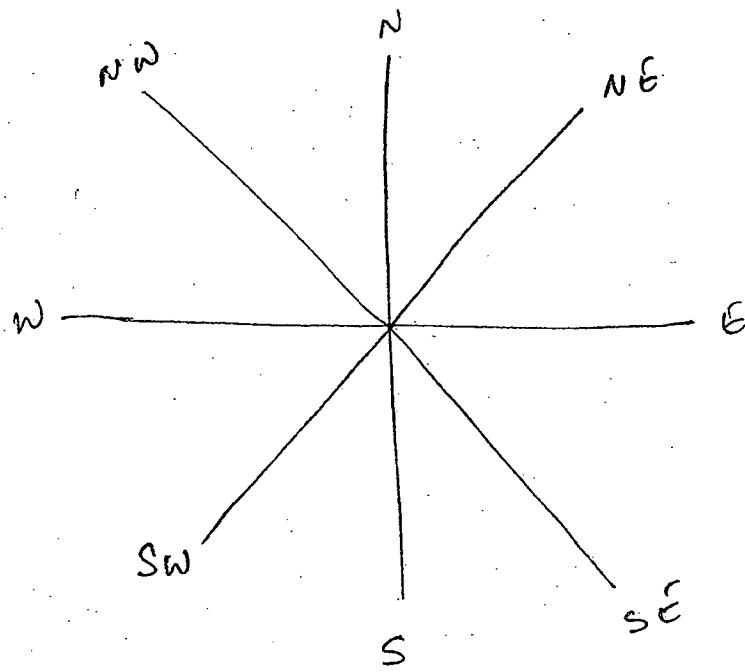
$$\text{B's share} = \frac{14}{37} \times 95\% \text{ of } 7400$$

$$= \frac{14}{37} \times \frac{95}{100} \times 7400^2$$

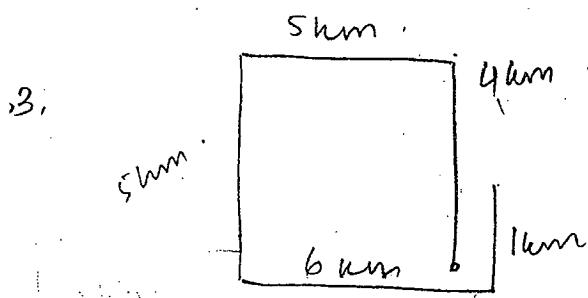
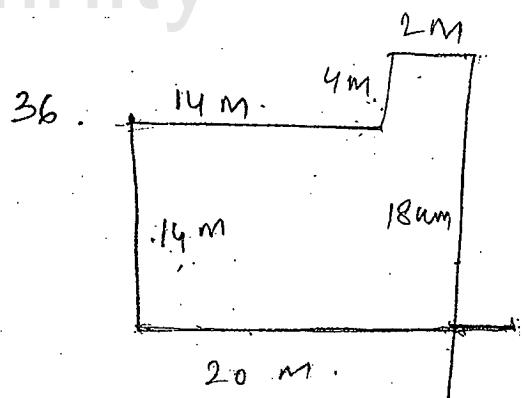
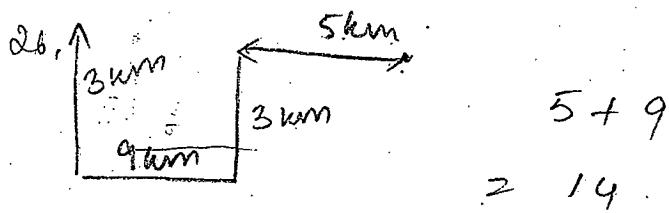
$$= 2660$$

CLOCK

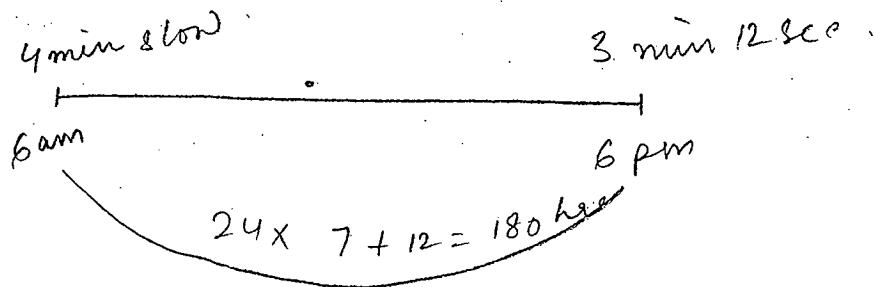
Directions



- i) Turning towards right is clockwise
- ii) Turning towards left is anti-clockwise.



A watch which gains uniformly is 4 mins slow at 6 am Sunday and is 3 mins 12 sec fast the following Sunday <sup>6 pm</sup>. When did this watch show the correct time?



4 mins + 3 mins 12 sec

$\therefore$  7 mins 12 sec gained in 180 hrs.

$$\begin{aligned} & 7 \times 60 + 12 \quad 4 \text{ min} \quad ? \\ & = 432 \text{ sec} \quad \swarrow \quad \swarrow \\ & 4 \times 60 \quad 240 \text{ sec} \end{aligned}$$

$$432 = 180$$

$$\begin{array}{r} 10 \\ 240 \\ \underline{-} 180 \\ 60 \\ \underline{-} 48 \\ 12 \\ \underline{-} 10 \\ 2 \end{array}$$

100 hrs

6 am Sunday

$$\begin{array}{l} \downarrow \\ 100 \text{ hrs} \rightarrow 96 \text{ hrs} + 4 \end{array}$$

Thursday 10 hours  $\therefore$  4 days & 4 hours.

\* The angle between 12 & 1

$$= 30^\circ$$

between 12 & 2.

$$= 60^\circ$$

Hour hand in 12 hours

$$= 360^\circ$$

$$1 \text{ hour} = \frac{360}{12}$$

$$60 \text{ min} = 30^\circ$$

$$1 \text{ min} = \frac{30}{60} = \frac{1}{2}^\circ$$

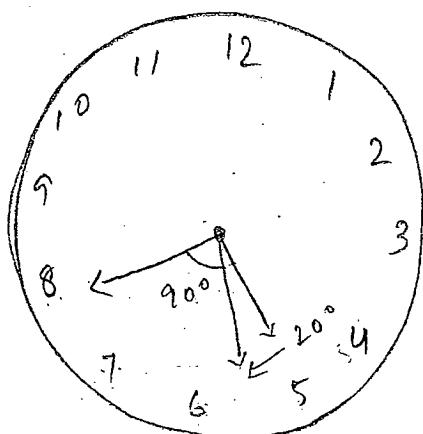
Minute hand in 1 hour

$$1 \text{ hour} = 360^\circ$$

$$60 \text{ min} = 360^\circ$$

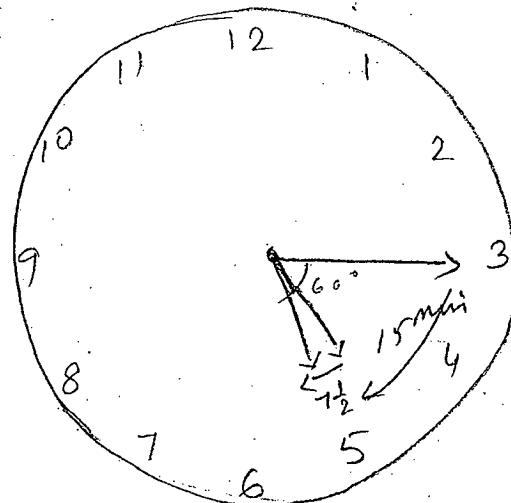
$$1 \text{ min} = \frac{360}{60} = 6^\circ$$

$$= 6^\circ$$



$$90 - 20$$

$$= 70^\circ$$



$$\text{If } 1 \text{ min} = \frac{1}{2}^\circ$$

$$\text{Then } 15 \text{ min} = 15 \times$$

$$= 7\frac{1}{2}^\circ$$

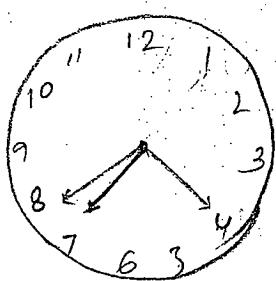
$$2 \text{ Angle} = 67\frac{1}{2}^\circ$$

minute hand  
moves  $5\frac{1}{2}^\circ$

more than  
the hour  
hand.

5 hrs 40 min.

7 : 20



1 : 05

$$2 \times 15^\circ = 2 \times 15^\circ$$

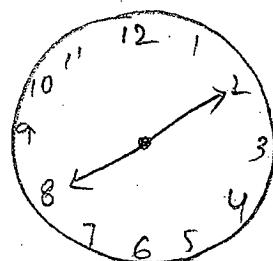
$$20^\circ + 10^\circ$$

$$90^\circ + 10^\circ$$

$$= 100^\circ$$

8 : 10

$$180^\circ$$



$10^\circ > 25^\circ$   
 $= 185^\circ$   
 Reflex angle  
 $> 180^\circ$

11 : 25

$$\frac{210}{2}$$

$$= 21.5^\circ$$

$$2167.5^\circ \text{ or}$$

$$192.5^\circ$$

1 : 45

$$142.5^\circ / 217.5^\circ$$

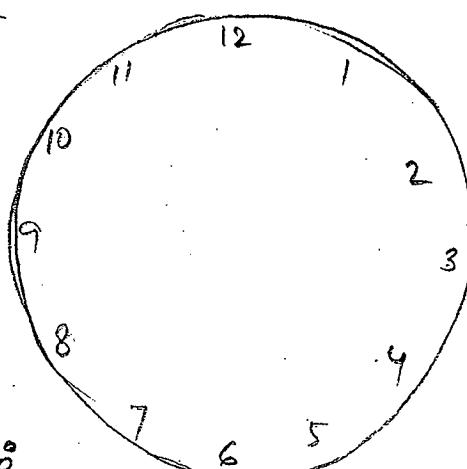
@upsc.risefinity

$$\text{Angle} = \frac{11}{2} M - 30H$$

$$5 : 15 = \frac{11}{2} \times 15 - 30 \times 5$$

$$= 82.5 - 150$$

$$= 67.5^\circ$$



$$60 \text{ min} = 360^\circ$$

$$30 \text{ min} = 180^\circ$$

$$15 \text{ min} = 90^\circ$$

### Hour hand.

$$60 \text{ min} = 5 \text{ min}$$

minute hand

$$60^{\circ} \text{ min} = 60 \text{ min}$$

minute hand

is moving ~~more~~  
 $55^{\circ}$  more in  
60 min.

minute hand gains 55 mins 60 min.

over hour hand in 60 min.

Min. hand gains 55 min. in 60 min.

$$1 \text{ min} = \frac{60}{55}$$

$$= 12 \text{ mins.}$$

$$\boxed{1 \text{ min} = \frac{12}{11} \text{ min}}$$

12:00 the min. hand and hour hand coinciding then again 1:05 it will coincide.

$$1 \text{ min} = \frac{12}{11} \text{ min.}$$

$$5 \text{ min} = \frac{12 \times 5}{11}$$

$$= \frac{60}{11} = 5 \frac{5}{7}$$

$1 \frac{5}{7}$  they will coincide

2 : 10

$$1 \text{ min} = \frac{12}{11}$$

$$10 \text{ min} = \frac{12 \times 10}{11}$$

$$\frac{2 \frac{120}{11}}{11} = 10 \frac{10}{11} \text{ min.}$$

$$\begin{array}{r} 12 \\ \times 15 \\ \hline 60 \\ 12 \\ \hline 180 \\ 11 \\ \hline 16 \end{array}$$

Exactly at  $2: \frac{10}{11}$  it bome & min. hand coinciding the exact time

3 : 15

$$= \frac{12}{11} \times 15$$

$$\frac{2 \frac{180}{11}}{11} = 16 \frac{4}{11}$$

$$3:16 \frac{4}{11} \text{ exact time.}$$

$$\begin{array}{r} 12 \\ \times 35 \\ \hline 60 \\ 35 \\ \hline 95 \\ 7 \\ \hline 35 \\ 2 \\ \hline 35 \end{array}$$

After 7  $\rightarrow$  it will coincide

$$\frac{12 \times 35}{11}$$

$$\frac{420}{11} = 38 \frac{2}{11}$$

$$7:38 \frac{2}{11} \text{ exact time}$$

$$\frac{5}{55} \times \frac{12}{4}$$

$$= 60 \text{ min}$$

at 12:00 they coincide.

\* The hour hand & the min. hand coincides  
in 12 hours  
11 times and in a day  
24 hours 22 times

At what regular intervals hour hand and minute hand coincide?

$$12 : 00 \\ 1 : 5 \frac{5}{11} \text{ coinciding}$$

After every  $1 : 5 \frac{5}{11}$  hour they will coincide.

$$1 \text{ hour } 5 \frac{5}{11} \text{ min.}$$

$\Rightarrow$  12:00 straight line will be at 12:30 forming an angle of  $180^\circ$  and the time difference will be  $30^\circ$  minutes\*. In a straight line but opposite direction.

At 12:00, the min. hand & hr. hand has no difference.

$$\begin{aligned} * 1 \text{ min } 30 \times \frac{12}{11} &= 360 \quad \text{min. hand } 30 \text{ min.} \\ &\quad \text{ahead and min.} \\ &\quad \text{hand gets this} \\ &\quad \text{time at } 32 \frac{8}{11} \end{aligned}$$
$$\therefore = 12 : 32 \frac{8}{11}$$

2 At 1:35 they again need to be <sup>in</sup> a straight line,

min. hand was 5 min behind  
now at 1:35 it is 35 min ahead.

$$= 35 \times \frac{12}{11} = \frac{420}{11}$$

$$= 32 \frac{2}{11}$$
  
$$1 : 32 \frac{2}{11}$$

2 After 5 o' clock exactly at what time.

$$55 \text{ min.} = 55 \times \frac{12}{11} = 60 \text{ min.}$$

At 60' clock they will be at straight line.

2 After 3 o' clock it will be 3:45  
they will be at straight line but opposite direction.

$$\frac{45 \times 12}{11}$$

2 After 8 o' clock at 8:10 they need

be a straight line.

After 7 o' clock it will be in a straight line at 7: 05 i.e.,  $5 \times \frac{12}{11} = \frac{60}{11}$ .

- \* In 12 hours hr. hand & min. hand are in a straight line 11 times and in 24 hours 22 times
- = At what regular interval hour hand & min. hand are in a straight line and opp. direction;

@upsc.risefinity

$$12 : 00 \quad 12 : 32 \frac{8}{11} \\ 1 : 38 \frac{2}{11} ) 1 \text{ hour } 5 \frac{5}{11}$$

$$1 \text{ hr} : 5 \frac{5}{11} \text{ min}$$

- = After 12 o' clock when are they at right angle to each other i.e  $90^\circ$ .

$$12 : 15$$

The time difference of exact 15 min. they will be at right angle =  $15 \times \frac{12}{11}$

Again at 12:45 they will be at right angle.

$$\frac{45 \times 12}{11}$$

1:20 @ right angle. 1:50 @ right angle.

$$\frac{20 \times 12}{11}$$

$$\frac{50 \times 12}{11}$$

After 2 they are @

right angle = 2:25 and again @ 2:55

$$2:25 \times 12$$

$$= \frac{300}{11}$$

$$= 27 \frac{3}{11}$$

$$\frac{55 \times 12}{11}$$

$$= 60 \text{ mins}$$

@ 3 o'clock

they will be

@ right angle.

After 8 o'clock they

@ 9 o'clock they are  
at right angle.

$$\left\{ \begin{array}{l} 12:30 - 1:30 \\ 1:30 - 2:30 \end{array} \right.$$

Twice.

\* In 12 hours they are right angle  
22 times. and 24 hrs 44 times.

At what regular intervals they are at right angle to each other?

22 times — 12 hrs.

$$1 \text{ time} = \frac{12}{22} \times 60^{\circ} = \frac{360}{11} = 32\frac{8}{11} \text{ min}$$

$$\text{Angle} = \frac{11M}{2} - 30H$$

$$\text{Coincidence} = \frac{60n}{11}$$

$$\text{Straight line: } (5n+30) \times \frac{12}{11} + n < 6$$

$$(5n-30) \times \frac{12}{11}, \text{ if } n \geq 6$$

$$\text{Right angle: } (5n+15) \times \frac{12}{11}$$

## Cyclicity:

$$273 \times 374 \times 679 \times 287 \times 4.$$

$$\begin{array}{ccccccc} & 2 & & 8 & & 6 & \\ \curvearrowleft & 1 \underline{\underline{2}} & & 1 \underline{\underline{8}} & & 5 \underline{\underline{6}} & 2 \underline{\underline{4}} \end{array}$$

Product of unit digit.

$$1^n : 1$$

$$2^n : 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = \underline{16}$$

$$2^5 = 2^4 \times 2, 2^6 = 2^5 \times 2, 2^7 = 8, 2^8 = 6 \\ = 6 \times 2 \qquad \qquad \qquad = 2 \times 2$$

$$2^9 = 2, 2^{10} = 4, 2^{11} = 8, 2^{12} = 6.$$

In case of 2 cyclicity is that of 4.

$$\textcircled{3} \quad 2^{83} = 4 \overline{)83} \qquad \qquad \qquad \begin{array}{l} 2 \\ 8 \\ \hline 3 \end{array}$$

leaves a remainder 3. so, unit digit will be  $\underline{\underline{8}}$ .

$$\textcircled{4} \quad 2^8 \div 4 \quad \begin{array}{r} 8 \\ 4 \\ \hline 0 \end{array}$$

when remainder is  $\underline{\underline{0}}$   
unit digit will be 6

\textcircled{1}  $2^{83} \div 4$  leaves a remainder of 2. corresponding unit digit is  $\underline{\underline{2}}$ .

\textcircled{2}  $2^6 \div 4 = 6/4 = \text{remainder } 2$  then unit digit is 4.

To find out the unit digit of  $2^n$ , we divide last two digits of  $n$  by 4 (cyclicity is 4) & then look for remainder.

If,  $R = 1$  Unit Digit = 2.

$$\begin{array}{rcl} & & = 4 \\ & & = 8 \\ 2 & 3 & = 6 \\ & & = 0 \end{array}$$

$3^n$ :  $3^1 = 3$      $3^2 = 9$      $3^3 = 7$      $3^4 = 1$

divide the power by 4 leaves a remainder of 1 = 3.

$3^5 = 3$      $3^6 = 9$      $3^7 = 7$      $3^8 = 1$

$3^9 = 3$      $3^{10} = 9$      $3^{11} = 7$      $3^{12} = 1$

remainder 2 U.D is 9.

| Remainder | Unit Digit |
|-----------|------------|
| 1         | 3          |
| 2         | 9          |
| 3         | 7          |
| 0         | 1          |

$4^n$ :  $4^1 = 4$      $4^2 = 6$     cyclicity is 2 in case

$4^3 = 4$      $4^4 = 6$     of 4.

$4^5 = 4$      $4^6 = 6$

power is odd U.D will be 4, even it will be 6.

$$5^n: 5^1 = 5 \quad 5^2 = 25 \quad 5^3 = 125 \quad \text{cyclicality of 5}$$

$$6^n: 6^1 = 6 \quad 6^2 = 36 \quad 6^3 = 216. \quad \text{and 6 will be 5 and 6.}$$

$$7^n: 7^1 = 7 \quad 7^2 = 49 \quad 7^3 = 343 \quad 7^4 = 2401 \quad \text{Rem. U.D} \\ 1 \quad 7$$

$$7^5 = 7 \quad 7^6 = 49 \quad 7^7 = 343 \quad 7^8 = 2401 \quad 2 \quad 9$$

$$3 \quad 3$$

$$8^n: 8^1 = 8 \quad 8^2 = 64 \quad 8^3 = 512 \quad 8^4 = 4096 \quad \text{Rem. U.D} \\ 0 \quad 1$$

$$8^5 = 32 \quad 8^6 = 256 \quad 8^7 = 2048 \quad 8^8 = 16384 \quad 1 \quad 8 \\ 2 \quad 4$$

$$9^n: 9^1 = 9 \quad 9^2 = 81 \quad 3 \quad 2$$

$$9^3 = 729 \quad 9^4 = 6561 \quad \text{power is odd.} \quad 0 \quad 6.$$

322 @upsc.riseinfinity  $U.D = 9$ , power is even

$$U.D = 1.$$

322

$$\Rightarrow 567 \\ = 7^{322} \cdot 22/4$$

= Remainder 2, Unit Digit is 9.

$$\begin{array}{r} 321 & 486 & 219 \\ 123 & \times 729 & \times 314 \end{array}$$

$$\begin{array}{r} 321/4 & 486 - \text{power even} & 219 - \text{power odd} \\ 123 \downarrow & \times 729 \downarrow & \times 314 \downarrow \\ 3 & \times 1 (\times 4 & \quad U.D = 2. \end{array}$$

$$1. \begin{array}{ccccccc} 333 & 444 & 555 & 777 & 888 & 999 \\ \underline{222} \times 333 & \times 444 & \times 666 & \times 777 & \times 888 & \times \\ 999 & \text{considering 2 and power 33 divided by 4.} & & & & & \end{array}$$

$$\begin{array}{r} 8 \\ 4 ) 33 \\ 32 \\ \hline 1 \\ -1 \\ \hline 4 ) 44 \\ 4 \\ \hline 0 \\ 4 ) 99 \\ 4 \\ \hline 9 \\ -8 \\ \hline 1 \\ 4 ) 19 \\ 16 \\ \hline 3 \\ \hline \end{array}$$

$$2 \times 1 \times 4 \times 6 \times 1 \times 2 \times 9$$

$$\begin{array}{c} 2 \times 24 \\ \underline{8} \quad \underline{6} \quad \underline{4} \\ 16 \quad 54 \\ \hline 24 \end{array}$$

$$2. \begin{array}{cccccc} 333 & 444 & 555 & 777 & 888 \\ 222 + 333 & * 444 & + 666 & + 777 & + \\ 999 & + 999 & & & \\ 888 & + 999 & \text{III} & & & \end{array}$$

$$\begin{array}{r} 2 + 1 + 4 + 6 + 1 + 2 + 9 \\ 3 + 4 + 7 + 6 \\ \hline 213 + 1 \\ \hline = 4 + 2 + 6 + 9 \\ \hline = 15 \end{array}$$

$$3. \begin{array}{cccccc} 444 & 555 & & U.D = 5 \\ 222 \times 333 \times 444 & \times 666 & \times 777 & \times 888 \times 999 & 222 & 44 \\ & \times 444 & \times 777 & \times 888 & \times 999 & \end{array}$$

$$\begin{array}{cccccc} 6 & \times 7 & \times 6 & \times 6 & \times 3 & \times 4 \times 1 \\ & & & & & \hline 24 \end{array}$$

$$4. 6 + 7 + 6 + 6 + 3 + 4 + 2 \ 3.$$

$$\begin{array}{ccccccccc}
 & 111 & 222 & 333 & 444 & 555 & 666 & 777 \\
 & \times \\
 111 & \times & 222 & \times & 333 & \times & 444 & \times & 555 \\
 & 888 & & 999 & & & & & \\
 & \times & & \times & & & & & \\
 & 888 & & 999 & & & & & \\
 & & & & 20 & & & & \\
 \end{array}$$

### Miscellaneous-

Q. Let the no. of days vacation  
be  $x$  days.

They did activity in the morning.

$$x - 24$$

No. of days in the evening they  
did nothing =  $x - 12$ .

$$x - 24 + x - 12 = 22$$

$$2x - 36 = 22$$

$$2x = 58$$

$$x = 29$$

29 days

Q. 35.  $4^{61} + 4^{62} + 4^{63} + 4^{64}$

$$4^{61}(1 + 4 + 4^2 + 4^3)$$

$$= 4^{61}(1 + 4 + 16 + 64)$$

$$= 4^{61} \times 85$$

$$46^1 \times 5 \times 17$$

∴ not divisible by 3.

$$37: 1^2 + 2^2 + 3^2 + \dots + 10^2 = 385.$$

$$2^2 + 4^2 + 6^2 + \dots + 20^2 = ?$$

$$\begin{aligned} & 1 + 4 + 9 + \dots = 385 \\ & 4 + 16 + 36 + \dots = ? \end{aligned}$$

$$4 \times 1^2 + 4 \times 2^2 + 4 \times 3^2 + \dots + 4 \times 10^2$$

$$4 (1^2 + 2^2 + \dots + 10^2)$$

$$4 \times 385 = 1540$$

3.

$\times 2$  aaab or baaa

Divisible by 9  $\Rightarrow$  9, 18, 27, 36, ...

9: 0009  $\times$  not valid number or,  
9000.

$$\begin{array}{ll} 1116 & \text{or } 6111 \\ 2223 & \text{or } 3222 \\ 3330 & \text{or } 0333 \end{array} \left. \right\} 6$$

$$\begin{array}{ll} 18: 3339 & \text{or } 9333 \\ 4446 & \text{or } 6444 \\ 5553 & \text{or } 3553 \end{array} \left. \right\} 7$$

$$27 : \begin{array}{r} 666 \\ 777 \\ 888 \\ 9990 \end{array} \quad \left. \begin{array}{r} 9666 \\ 6777 \\ 3888 \\ 0999 \end{array} \right\} 7$$

$$36 : 9999 \quad \} .1$$

Note: In the question if we remove the word, exactly, then the answer will be 21,

$$24. \quad |11a+7| - 13.$$

$$\text{If } a = 0.$$

$$\begin{aligned} & |11 \times 0 + 7| - 13 \\ &= |0 + 7| - 13 = 7 - 13 = -6. \end{aligned}$$

$$a = 1$$

$$\begin{aligned} & |11 \times 1 + 7| - 13 \\ &= |11 + 7| - 13 = 18 - 13 = 5. \end{aligned}$$

$$a = -1. \quad |-11 + 7| - 13 = 4 - 13 = -9.$$

$$a = -2. \quad |-22 + 7| - 13 = 15 - 13 = 2.$$

$$25. \quad \left( \frac{1}{\sqrt{5}-2} \right) + \left( \frac{1}{\sqrt{5}+2} \right), \quad \text{l.c.m.}$$

$$\begin{aligned} & \frac{\sqrt{5}+2 + \sqrt{5}-2}{2} \\ &= \frac{8-4}{2} = \frac{4}{2} = 2 \end{aligned}$$

$$= a-b \quad a+b, \quad a^2 - b^2$$

$$26. \quad (1 \times 2 \times 3 \times \dots \times 9) \times (11 \times 12 \times 13 \dots \times 19) \times 21 \times \dots \times 29$$

$$(91 \times 92 \times 93 \times \dots \times 99)$$

$$\underline{99!}$$

$$10 \times 20 \times 30 \times 40 \times 50 \times 60 \times 70 \times 80 \times 90$$

$$\underline{99!}$$

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10^9$$

20 one more zero added that becomes  $10^{10}$ .

$$\underline{99!}$$

$$36288 \times 10^{10}$$

$$28. \quad a + \frac{1}{a} = 2 \quad \therefore a + \frac{1}{a} = 3$$

$$2 \quad a^3 + b^3 + 3ab(a+b)$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a} \left(a + \frac{1}{a}\right)$$

$$5^3 = a^3 + \frac{1}{a^3} + 3 \times 5$$

$$a^3 + \frac{1}{a^3} = 125 - 15$$

$$= 110$$

$$32. \quad 5^{a-1} + 5^a + 5^{a+1} = 775.$$

$$a=1 \quad 5^0 + 5^1 + 5^2$$

$$a=3 \quad 5^2 + 5^3 + 5^4$$

$$25 + 125 + 625 = 775$$

$$9. \quad 100, 101, 102, \dots, 198, 199, 200$$

|     |     |     |
|-----|-----|-----|
| 100 | 106 | 112 |
| 101 | 107 | 113 |
| 102 | 108 | 113 |
| 103 | 109 |     |
| 104 | 110 |     |
| 105 | 111 |     |

Hundredths digit place 1.

100 times.

Ten digit place 1.

2 10 times.

110, 111, 112, 118, 119.

Unit digit place 1.

101, 121, 121, 181, 191

2 10 times.

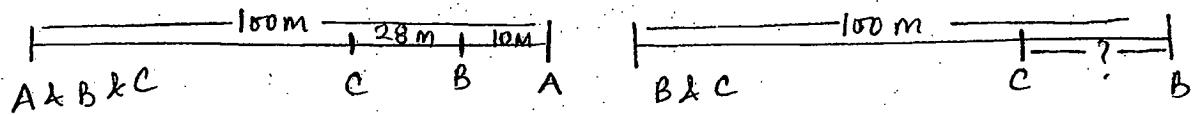
2 120 times.

$$15. \quad S = \frac{n(n+1)(2n+1)}{6}$$

$$2 \quad \frac{50 \times 101 \times 201}{6}$$

$$2 \quad 50 \times 101 \times 67$$

## RACES AND GAMES OF SKILL



$$A = 100 \text{ m}$$

$$B = 90 \text{ m}$$

$$C = 72 \text{ m}$$

$$B - 90 \text{ m} \quad C - 72 \text{ m}$$

$$\rightarrow 1 \text{ m} = \frac{72}{90} \text{ m}$$

$$\rightarrow 100 \text{ m} = \frac{72}{90} \times 100 \text{ m}$$

$$= 80 \text{ m}$$

$$\text{Gap b/w B \& C} = 90 - 72 \\ = 18 \text{ m}$$

$$1 \text{ m} = \frac{18}{90} \text{ m}$$

Ratio of Speed of B &  
C = 90 : 72.

$$\rightarrow 100 \text{ m} = \frac{2}{\frac{9}{8}} \times 100 \text{ m}$$

$$= 20 \text{ m}$$

$$2^5 : 4$$

$$\begin{array}{c} / \quad \backslash \\ 100 \text{ m} \quad \frac{4}{5} \times 100 \\ \downarrow \quad \quad \quad 80 \text{ m} \\ 20 \text{ m} \end{array}$$

b/w A, B & C  
Between a 300 m race, A beats  
B by 80 m and C by 60 m. In a  
600 m race b/w B & C, who will  
win and by what distance.

$$\frac{11}{220} \times \frac{50}{600}$$

$$\frac{24}{240} + 2$$

$$+ 2$$

$$= 550$$

$$A - 300 \text{ m}$$

$$B - 220 \text{ m}$$

$$C - 240 \text{ m}$$

$$\begin{array}{r} 300 \\ - 80 \\ \hline 220 \end{array}$$

$$\begin{array}{r} 300 \\ - 60 \\ \hline 240 \end{array}$$

$$C = 240 \text{ m} \quad B = 220 \text{ m}$$

$$600 \text{ m} - \frac{11}{220} \times 600 \xrightarrow[240]{+2} 50 \text{ m} \quad 550 \text{ m.}$$

$$(Or) \quad C = 240 \quad \overline{C-B} = 20 \text{ m.}$$

$$600 \text{ m} - \frac{10}{20} \times 600 \xrightarrow[240]{+2} 250 \text{ m.}$$

$$(Or) \quad S_B : S_C \quad (\text{Speed of } B : C)$$

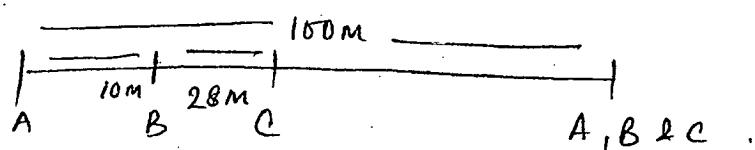
$$220 \text{ m} : 240 \text{ m.}$$

$$\frac{11}{12} : 12$$

$$\frac{11}{12} \times 600 \quad 600$$

$$550 \quad 50 \text{ m.}$$

### Class Assignment.



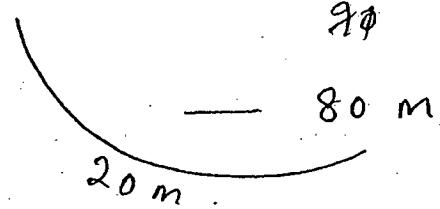
$$A = 100 \text{ M} \quad C = 72 \text{ m.}$$

$$B = 9 \text{ m.}$$

$$B \text{ --- } 90 \text{ --- } C \text{ --- } 72$$

$$1 \text{ m} \text{ --- } \frac{72}{90} \text{ m}$$

$$100 \text{ m} \text{ --- } \frac{72}{90} \times 100 \text{ m}$$



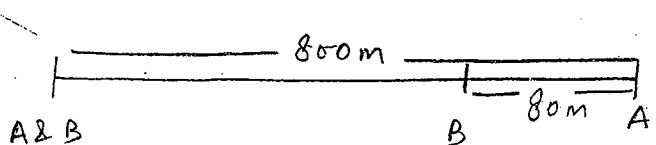
$$\text{Or, } B = 90 \text{ m} \text{ and } B-C = 18 \text{ m.}$$

$$1 \text{ m} \text{ --- } \frac{18}{90}$$

$$= 100 \text{ m} \text{ --- } \frac{18}{90} \times 100$$

$$= 20 \text{ m}$$

B/w 800 m race A & B, A beats B by 80 m or 8 seconds, find in what time A completed the race.



$$- \frac{80}{720}$$

A — 800 m. B covered 80 m in 8 secs.

$$B \text{ --- } 720 \text{ m.}$$

$$B \text{ --- } 800 \text{ m} = \frac{8}{80} \times 80 \text{ s}$$

$$= 80 \text{ seconds}$$

A will finish the race

in  $\approx 80 \text{ seconds} - 8 \text{ seconds}$

$$= \underline{\underline{72 \text{ seconds}}}$$

B — 80 m — 8 seconds

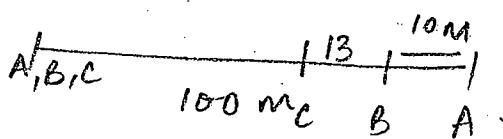
3, 5, 6, 2

$$B - 720 m = \frac{8}{80} \times 72 \phi$$

= 72 seconds,

### Class Assignment

3.



$$\begin{array}{r} 100 \\ - 13 \\ \hline 87 \end{array}$$

A — 100 m.

B — 90 m

C — 87 m

B — 180 m. <sup>B</sup> Will beat C by ?

B — 90      C — 87

$$1 m — \frac{87}{90} m$$

$$180 m — \frac{87}{90} \times \frac{2}{\phi}$$

$$\begin{array}{r} 1 \\ 87 \\ - 180 \\ \hline 174 \end{array}$$

— 174 m  
6 m

5.

Race = 200 m

$$\begin{array}{r}
 200 \\
 -35 \\
 \hline
 165 \\
 -35 \\
 \hline
 0
 \end{array}$$

A — 200 m

B — 165 m

B would cover 35 m — 7 seconds.

$$\begin{array}{r}
 33 \\
 B — 165 m — \frac{7 \times 165}{35} \\
 \hline
 8
 \end{array}$$

= 33 seconds.

or,

B — 35 — 7 seconds.

$$\begin{array}{r}
 40 \\
 B — 200 m — \frac{7 \times 200}{35} \text{ seconds} \\
 \hline
 8
 \end{array}$$

= 40 seconds.

$$T_A = 40 \text{ seconds} - 7 \text{ seconds}$$

= 33 seconds.

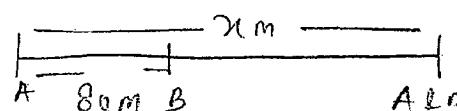
6.

let the length of the race course

be  $x$  m.

$$2\frac{1}{3} = \frac{7}{3}$$

B starts 80 m ahead of A.

A —  $x$  m.B —  $x - 80$  m.

Speed of A =  $\frac{7}{3}$  times of B.

Distance A =  $\frac{7}{3}$  Distance B.

$$x = \frac{7}{3}(x - 80)$$

$$3x = 7x - 560$$

$$4x = 560$$

$$x = \frac{560}{4}$$

$$= 140 \text{ m}$$

\*\* Distance and time  
Speed is directly proportional.

Distance  $\uparrow$   
Time will also  $\uparrow$ .

(Or) If A can give B  $x$  mts start and Speed of A is ' $n$ ' times Speed of B, Length of the race course, so that both reach the end point at the same time

then,

$$L = x \left( \frac{n}{n-1} \right)^m$$

$$x = 80 \text{ m } n = \frac{7}{3}$$

$$L = 80 \left( \frac{\frac{7}{3}}{\frac{7}{3}-1} \right)$$

$$= \frac{80 \times \frac{7}{3}}{\frac{4}{3}} \text{ m}$$

$$= 140 \text{ m}$$

2.

A and B take part in 100 m race.

Speed of A = 5 kmph.

$$= 5 \times \frac{5}{18} \text{ m/s.}$$

$$= \frac{25}{18} \text{ m/s.}$$

7, 9, 12,  
13, 15, 2.

Distance covered by A = 100 m.

$$\text{Time of A} = 100 \div \frac{25}{18}$$

$$= \frac{100}{25} \times \frac{18}{18}$$

$$= 72 \text{ seconds.}$$

Distance covered by B = 100 - 8 m

$$= 92 \text{ m.}$$

Time taken by B = 72 + 8.

$$= 80 \text{ sec}$$

Speed of B =  $\frac{92}{80} \text{ m/s.}$

$$= \frac{92}{80} \times \frac{18}{5} \text{ km/hr.}$$

$$\approx 4.14 \text{ km/hr.}$$

7.

$$\text{Total} = 224 \text{ m}$$

$$A = 28 \text{ s}$$

$$B = 32 \text{ s}$$

$$A - \frac{224}{28}$$

$$B - \frac{224}{32}$$

$$= 8 \text{ m/s.}$$

$$7 \text{ m/s.}$$

$$\begin{array}{r}
 568 \times 4 = 32 \\
 442 \\
 \underline{224} \\
 28 \\
 14 \\
 7 \\
 \hline
 11256 \\
 224 \\
 \hline
 3216 \\
 8 \\
 8 \times
 \end{array}$$

4 min.

$$\begin{array}{r}
 \overline{A-B} = 32 - 28 \\
 = 4 \text{ s.}
 \end{array}$$

$$7 \times 4$$

$$= 28 \text{ m}$$

9.

Let it be  $x$ .

B starts 80 m ahead of A.

$$A = x$$

$$B = x - 80$$

Speed of A =  $\frac{5}{3}$  times of Speed of B.

i.e.,  $\frac{5}{3}$  times

$$\therefore x = \frac{5}{3}(x - 80)$$

$$3x = 5x - 400$$

$$2x = 400$$

$$x = \frac{400}{2} \\ = 200 \text{ m}$$

12. Race = 100 m

A's speed = 2 m/s.

B starts 4 m ahead.

= 96 m.

Distance of A = 100 m

Time of A =  $\frac{100}{2}$   
 $= 50 \text{ sec.}$

Distance of B = 96 m.

Time taken = 50 + 10.

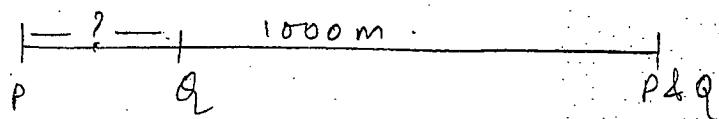
= 60 sec.

Speed of B =  $\frac{96}{60} = 48 \text{ sec.}$

$$15) \overline{241} \\ \underline{15} \\ 90$$

= 1.6 m/sec.

13.



$$P = 180 \text{ sec} \rightarrow 60 \times 3$$

$$Q = 250 \text{ sec} \rightarrow 60 \times 4 + 10$$

$$Q = 250 \text{ sec} \rightarrow 1000 \text{ m}$$

$$180 \text{ s} \rightarrow \frac{1000}{250} \times 180$$

$$= 720 \text{ m}$$

280 m

(a)

$$Q = 70 \text{ sec} \rightarrow \frac{1000}{250} \times 70$$

$$= \frac{100}{250} \times 70$$

$$= 280 \text{ m}$$

15.

$$B = 220 \text{ m} \rightarrow 44 \text{ s.}$$

$$- 190 \text{ m} \rightarrow \frac{42}{44} \times 190$$

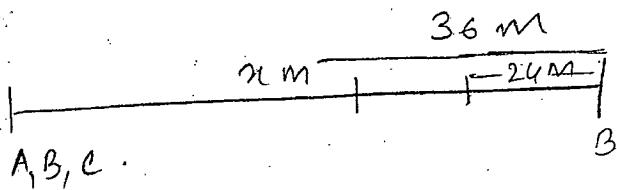
$$= 220$$

$$= 38 \text{ s.}$$

$$\text{A time} = 41 \text{ s} - 38 \text{ s}$$

$$= 3 \text{ sec}$$

28. Let the length of the course  
be  $x$ .



$$\text{Dist. of } B = x \text{ m.}$$

$$\text{Dist. of } C = x \text{ m.}$$

$$\text{Dist. of } C = x - 24 \text{ m.}$$

$$\text{Dist. of } A = x - 16 \text{ m.}$$

$$\text{Dist. of } A = x - 36 \text{ m.}$$

$$\frac{\text{Speed of } C}{\text{Speed of } A} = \frac{x}{x-16}$$

$$\frac{\text{Speed of } C}{\text{Speed of } A} = \frac{x-24}{x-36}$$

$$\therefore \frac{x-24}{x-36} = \frac{x}{x-16}$$

$$\frac{24}{16} = \frac{2}{4}$$

$$\frac{4}{6} = \frac{2}{4}$$

$$\frac{6}{6} = \frac{4}{4}$$

$$\frac{x-24}{x-36} = \frac{x}{x-16}$$

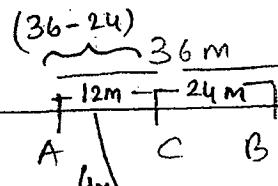
$$x = 96 \text{ m.}$$

\* A gap of 4 mts is created b/w A and C  
as soon as C runs 24 mts.

$$4 \text{ m} = \overline{C-A}, C = 24 \text{ m.}$$

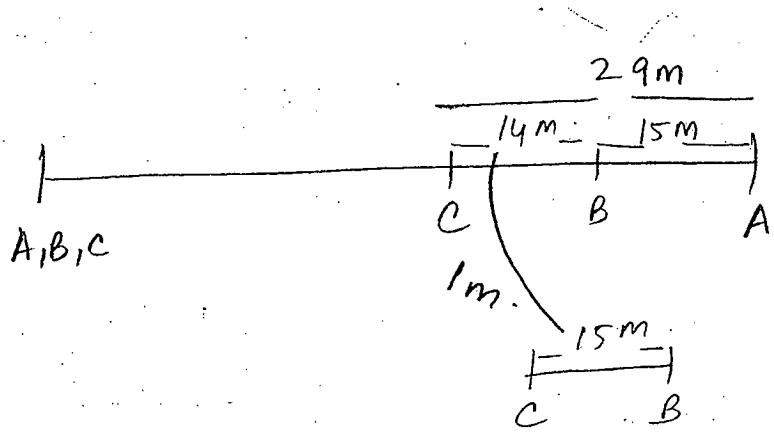
$$16 \text{ m} = \frac{24 \times 16}{x}$$

$$2 \quad 96 \text{ m.}$$



$$16 - 12 \\ = 4 \text{ m.}$$

17.



$$1 \text{ m} \quad B-C \quad B - 15 \text{ m.}$$

$$15 \text{ m} \quad \frac{15 \times 15}{1}$$

$$= 225 \text{ m.}$$

### GAME SKILLS:

100 points

$$\overline{A-B} = 20 \text{ points.} \quad 200 \text{ points:}$$

$$\overline{A-C} = 30 \text{ points.} \quad B - 200 \text{ p.}$$

$$C - ? \quad 175 \text{ p.}$$

$$A = 100 \text{ p.}$$

$$B = 80 \text{ p. } C = 70 \text{ p.}$$

$$B = 80 \text{ p. } C = 70 \text{ p. } \rightarrow 200 \text{ p.} = \frac{70}{80} \times \frac{25}{200}$$

$$25 \text{ p.} \quad 2175 \text{ p.}$$

$$(Or) \quad B = 80 \text{ p. } \overline{B-C} = 10 \text{ p.}$$

$$B = 200 \text{ p.} - \frac{10}{80} \times 200$$

$$= 25 \text{ p.}$$

$$4. \quad A = 60 p$$

$$\overline{A-B} = 15 p$$

$$\overline{A-C} = 20 p$$

$$A = 60$$

$$B = 45$$

$$C = 40$$

$$\begin{array}{r} 60 \\ - 15 \\ \hline 45 \\ - 60 \end{array}$$

$$B = 45 \quad C = 40$$

$$B = 90 \quad C = \frac{8}{40} \times 10$$

$$C = \frac{40}{45} \times 90$$

$$= 80$$

$$10 p.$$

$$B = 90$$

$$B = \frac{9}{8} \times 90$$

$$= \frac{8}{45} \times 10$$

$$C = 80 \text{ points}$$

$$\begin{array}{r} 45 \times 90 \\ \hline 40 \quad 2 \\ \hline 45 \end{array}$$

$$\begin{array}{r} 9 \\ \hline 2 \end{array}$$

$$= 80$$

$$90 - 80$$

$$= 10$$

30. 1. Find out the time taken by each one to complete one round.

$$\text{Time } A = \frac{120}{5} = 24 \text{ s.}$$

$$\text{Time } B = \frac{120}{7} \text{ s.}$$

$$\text{Time } C = \frac{120}{15} = 8 \text{ sec.}$$

2. Find out the L.C.M of  $\frac{24}{1}, \frac{120}{7}, \frac{8}{1}$ .

$$\text{L.C.M} = 24, 120, 8.$$

$$= 120.$$

H.C.F of denominator.

$$1, 7, 1$$

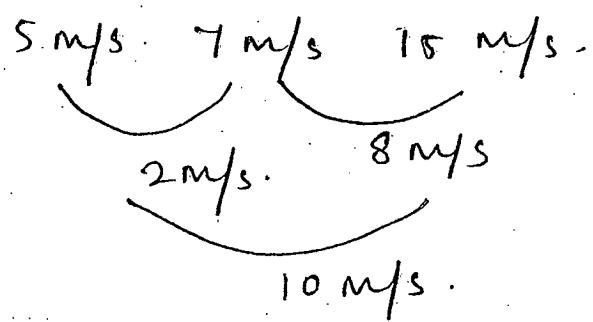
$$= 1.$$

$$\text{L.C.M} = \frac{120}{1} \text{ s,}$$

$$= 120 \text{ s.}$$

For the first time

- 1) Find out the relative speed among different persons.



- 2) Find out the H.C.F of relative speed

$$\text{H.C.F of } 2 \text{ m/s, } 8 \text{ m/s, } 10 \text{ m/s.}$$

$$\text{H.C.F} = 2 \text{ m/s.}$$

3. Divide the length of the course by the H.C.F

$$T_2 = \frac{120^{\frac{60}{\text{m}}}}{2 \text{ m/s.}} = 60 \text{ s.}$$

$$\text{Distance A} = 5 \times 60$$

$$= 300 \text{ m}$$

$$= 2R + 60 \text{ m}$$

$$\text{Distance B} = 7 \times 60 \text{ m}$$

$$= 420 \text{ m}$$

$$= 3R + 60 \text{ m}$$

R = Round

$$\text{Distance C} = 15 \times 60$$

$$= 900 \text{ m}$$

$$= 7R + 60 \text{ m}$$

20.

$$48 \text{ ft}$$



$$40 \text{ ft}$$



no. of revolution

5 revolutions more

$$= n$$



$$48 \times n$$

$$= 48n$$



$$n+5$$

$$= 40(n+5)$$

$$48n$$

$$= 40n + 200$$

$$8n = 200$$

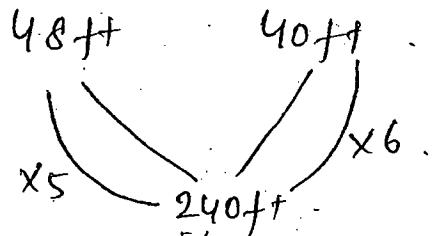
$$n = 25$$

$$\text{Distance} = 48 \times 25$$

$$= 1200 \text{ ft}$$

L.C.M of 48 & 40

Ques.



$$= 240$$

$$\begin{array}{r} 48 \\ \times 4 \\ \hline 240 \end{array}$$

$$\begin{array}{r} 148,40 \\ 2 \overline{) 24,20} \\ 2 \overline{) 12,10} \\ 2 \overline{) 6,5} \\ 3,5 \end{array}$$

$$1 \text{ revolution} = 240 \text{ ft}$$

$$5 \text{ revolution} = 240 \times 5$$

$$= 1200 \text{ ft}$$

Ques.

I      II      III

$$x \quad 2x \quad \frac{2}{3}(x+2x)$$

$$\frac{2}{3} \times \frac{2}{3}x \\ \frac{4}{9}x = 2x$$

$$\text{Distance covered} = x + 2x + 2x$$

$$= 5x = D$$

Dist' travelling at the speed of  
 $x$  for 3 hours  $= 2x \times 3$

$$= 3x = D'$$

$$D - D' = 120$$

$$5x - 3x = 120$$

$$2x = 120$$

$$x = 60$$

Avg. Speed = Total Distance

Total Time

$$= \frac{5x}{3}$$

$$= \frac{5 \times 60}{3} \\ = \underline{20}$$

$$= 100 \text{ km/hr}$$

10. A — 100 points.

A-B — 20 points.

A-C — 28 points.

A — 100.

B — 80 points.

C — ~~72~~ 72 points.

B — 80 C — 72.

$$1 \text{ m} \quad 2 \quad 72 \\ \underline{80}$$

$$100 \quad 2 \quad \frac{72}{80} \times 100 \\ = 90$$

10 Percent 2 90 percent

10 points.

ii.

$$A \text{ --- } 90 \text{ points.}$$

$$\overline{A-B} \text{ --- } 15 \text{ points.}$$

$$\overline{A-C} \text{ --- } 30 \text{ points.}$$

$$\begin{array}{r} 90 \\ 15 \\ \hline 75 \end{array}$$

$$A \text{ --- } 90 \text{ points.}$$

$$B \text{ --- } 75 \text{ points.}$$

$$C \text{ --- } 60 \text{ points.}$$

$$B \text{ --- } 75 \text{ p } C \text{ --- } 60 \text{ p.}$$

$$1 \text{ p } - 60$$

$$\overline{75}$$

$$100 \text{ p } - \frac{20}{60 \times 100} \text{ 4}$$

$$\begin{array}{r} 25 \\ 3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 25 \\ 3 \\ \hline 75 \end{array}$$

$$\begin{array}{c} 20 \\ 75 \\ \hline 20 \end{array}$$

points, 2 80

2 20 points.

## DATA INTERPRETATION.

1. India can spend  $\approx$  3 bn.

$$1. \text{ Rs } 3 \times 10^9$$

• 1 barrel of oil costs 110 Riyal.

$$3.75 \text{ Riyal} \quad 2. \text{ Rs } 48$$

$$1 \text{ Riyal} \quad 2 \frac{\text{Rs } 48}{3.75}$$

$$110 \text{ Riyal} \quad 2 \frac{\text{Rs } 48}{3.75} \times 110$$

$$n = \frac{\text{Rs } 3 \times 10^9}{\frac{\text{Rs } 48}{3.75} \times 110}$$

$$2 \frac{3.75 \times 10^8}{16 \times 110}$$
$$= \frac{3.75 \times 10^8 \times 10^6}{176}$$

$$2 \frac{375}{176} \text{ million}$$

$\approx$  2.1 million

2.

France imported goods of 6 bn.

Germany 15% of import

∴ 15% of 6 bn francs.

$$\Rightarrow 6.4 \text{ franc} = 1.9 \text{ DM}$$

$$1 \text{ franc} = \frac{1.9}{6.4} \text{ DM}$$

$$= \frac{1.9}{64} \text{ DM}$$

$$\Rightarrow 15\% \text{ of } 6 \times 10^9 \times \frac{1.9}{64} \text{ DM}$$

$$= \frac{15}{100} \times 6 \times 10^9 \times \frac{1.9}{64} \text{ DM}$$

$$= 90 \times \frac{1.9}{64} \times 10^7 \text{ DM}$$

$$= 90 \times 10 \times \frac{1.9}{64} \times 10^6 \text{ DM}$$

$$= 900 \times \frac{1.9}{64} \times 10^6 \text{ DM}$$

$$= \frac{900}{3.1} \text{ million DM}$$

$\approx 270$  million DM

4.

$$\boxed{\text{Total Revenue} = \text{S.P} \times \text{Selling Quantity}}$$

\*\*

Total produce quantity =  
Total sold quantity

Total Revenue = Rs. 1.8 lakh

Rice = 35% of Revenue.

$$(\text{i.e}) \quad 35\% \text{ of } 1.8 \text{ lakh} = \text{S.P} \times \frac{90}{360} \times 80 \text{ quintal}$$

$$= \frac{35}{100} \times 1.8 \times \cancel{10^5} = \text{S.P} \times \frac{90}{360} \times \cancel{80} \times \cancel{100}$$

$$\text{S.P} = \text{Rs } 35 \times 0.9 / \text{kg}$$

$$= \text{Rs } 31.5 / \text{kg}$$

~~40  
50  
30~~

5

$$\frac{8}{100} \times 1.8 \times \cancel{10^5} = \text{S.P} \times \frac{3}{150} \times \cancel{80} \times \cancel{100}$$

12.

$$\text{S.P} = 1.8 \times 12 / \text{kg}$$

$$= \text{Rs } 21.6 / \text{kg}$$

Marked price  $\Rightarrow 90\% = \text{Rs } 21.6 / \text{kg}$ .

$$90\% \rightarrow 100 - 10 \\ = 90\%$$

10% discount

$$100\% \Rightarrow \frac{2.4}{A_p} \times 100 / \text{kg}$$

$$= \text{Rs } 24$$

6.

$$\frac{\% \text{ of males in U.P, M.P \& Goa}}{\text{Total population}} \times 100\%$$

$$\frac{\% \text{ of males in U.P, M.P \& Goa} \times 100}{100}$$

\*\* Anything complete  
is 100%

$$\% \text{ of males in U.P} = \frac{3}{5} \times \frac{3}{5} \times 100\% \\ = 9\%$$

$$\% \text{ of males in M.P} = \frac{3}{4} \times \frac{5}{4} \times 100\% \\ = 15\%$$

$$\% \text{ of males in Goa} = \frac{3}{8} \times \frac{3}{2} \times 100\% \\ = 4.5\%$$

$$\Rightarrow 9\% + 15\% + 4.5\% \\ = \underline{\underline{28.5\%}}$$

11.

Market C  $\rightarrow$  children visiting

$$= 20\% \text{ of } 45640$$

Market F  $\rightarrow$  children visiting

$$14\% \text{ of } \cancel{45640} = 59650$$

$$1^b \quad \frac{20\% \text{ of } 45640}{744\% \text{ of } 59650} \times 100\%$$

$$2 \quad \frac{10 \times 4564}{7 \times 5965} \times 100$$

$$2 \quad \frac{10 \times 4564}{7 \times 5965} \times 100 \quad \begin{matrix} 23 \\ 10 \times 4564 \\ 7 \times 5965 \\ 3 \end{matrix}$$

$$2 \quad \frac{23}{21} \times 100\%$$

$$2 \quad 109.30\%$$

$\frac{3.3}{10}$

30. Total Students = 150.

Students in all

$$\begin{aligned} & \text{three disciplines} \\ & = 2 \text{ boys} + 3 \text{ girls} \\ & = 5 \end{aligned}$$

$$2 \quad \frac{5}{150} \times 100\%$$

$$2 \quad 3.33\%$$

37.

2878

1652

1226

Against

1226

796

430

Favone

1425

825

600

Undecided

227

31

196

Total

$$= \begin{array}{r} 1226 \\ + 1425 \\ \hline 2651 \end{array}$$

2878

$$- \begin{array}{r} 2651 \\ \hline 227 \end{array}$$

227

$$- \begin{array}{r} 196 \\ \hline 31 \end{array}$$

$$\begin{array}{r} 796 \\ + 31 \\ \hline 827 \end{array}$$

$$\begin{array}{r} 1652 \\ - 827 \\ \hline 825 \end{array}$$

$$\begin{array}{r} 1226 \\ - 796 \\ \hline 430 \end{array}$$

$$\begin{array}{r} 1425 \\ - 825 \\ \hline 600 \end{array}$$

15.

$$\begin{array}{r} 33 \\ 66 \times 15\% \\ \hline 100 \\ 2 \quad = 99 \end{array}$$

$$\begin{array}{r} 44 \\ 88 \times 18\% \\ \hline 100 \\ 2 \quad = 2132 \end{array}$$

$$\begin{array}{r} 2 \\ 99 \\ 3 \quad 75 \\ 1 \quad 32 \\ 70 \\ 42 \quad 25 \\ 45 \end{array}$$

$$\begin{array}{r} 14 \\ 56 \times 12.5 \\ \hline 100 \\ 1 \quad = 70 \end{array}$$

$$\begin{array}{r} 14 \\ 56 \times 7.5 \\ \hline 100 \\ 1 \quad = 42 \end{array}$$

$$\begin{array}{r} 463 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 96 \\ \times 5\% \\ \hline 100 \\ 1 \quad = 45 \end{array}$$

16.  $\frac{76 \times 100}{100} = 76$

$\frac{88 \times 100}{100} = 88$

$\frac{88 \times 100}{76} = \frac{88 \times 25}{19} = 38\frac{1}{19}$

(19)  $\begin{array}{r} 88 \\ \times 25 \\ \hline 440 \\ 176 \\ \hline 2200 \\ -19 \\ \hline 11 \end{array}$

20. Vehicle A  $= \frac{832}{16}$  } Dist.  
 $= \frac{864}{16}$  } Time  $= x$ ,  
                   } Not equal

Vehicle B  $= \frac{516}{12} = 43$

$= \frac{774}{18} = 43$

} Equal

25.  $\frac{\frac{7}{15} \times 17\%}{\frac{9}{16} \times 16\%} = \frac{7 \times 17}{15 \times 9} = \frac{119}{135}$

44.

$$30\% \text{ of } 40 = 12 \quad 32 + 8$$

240

$$25\% \text{ of } 12 = 3$$

12 - 3

2 9

$$\begin{array}{r} 1 \\ 999 \\ \hline 1998 \end{array}$$

$$\textcircled{1} \quad \left(2 - \frac{1}{3}\right) \left(2 - \frac{3}{5}\right) \left(2 - \frac{5}{7}\right) \cdots \left(2 - \frac{997}{999}\right)$$

$$\textcircled{2} \quad \left(\frac{6-1}{3}\right) \left(\frac{10-3}{5}\right) \left(\frac{14-5}{7}\right) \cdots$$

$$\textcircled{2} \quad \left(\frac{5}{3}\right) \left(\frac{7}{5}\right) \left(\frac{9}{7}\right) \cdots \left(\frac{1998-997}{999}\right)$$

$$\textcircled{2} \quad \frac{2}{3} \times \frac{1}{2} \times \frac{9}{8} \times \cdots \times \frac{1001}{999} = \frac{1001}{3}$$

$$\textcircled{2} \quad \left(1 + \frac{1}{3}\right) \left(1 + \frac{1}{4}\right) \left(1 + \frac{1}{5}\right) \cdots \left(1 + \frac{1}{n}\right)$$

$$\textcircled{2} \quad \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \cdots \times \frac{n+1}{n}$$

$$\textcircled{2} \quad \frac{n+1}{3}$$

$$③ \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$$

$$= \frac{1}{1 \times 2} = \frac{1}{1} - \frac{1}{2} = \frac{2-1}{2 \times 1} = \frac{1}{1 \times 2}$$

$$\frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{2 \times 3} = \frac{1}{2 \times 3}$$

$$\frac{1}{3 \times 4} = \frac{1}{3} - \frac{1}{4}$$

$$\frac{1}{100 \times 101} = \frac{1}{100} - \frac{1}{101}$$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{100} - \frac{1}{101}$$

$$= 1 - \frac{1}{101} = \frac{101 - 100}{101} = \frac{100}{101}$$

④ A number when divided by 663 leaves 53 as remainder, Find the remainder when this number is divided by 39 and 37.

Let the number be  $x$  — dividend.

Quotient =  $y$ :

$$\therefore x = 663y + 53$$

$$\begin{array}{r} x \\ 39 \) \underline{663y + 53} \\ 39 \end{array}$$

$$= \frac{663y}{39} + \frac{53}{39} = \frac{663y}{39} + \frac{53}{39}$$

$\downarrow$   $\downarrow$   
Remainder O.R = 14.

In this case  $\rightarrow$   
denominator  $\rightarrow$   
product of two  
consecutive numbers  
and in numerator  
it is the difference

$$\begin{array}{r} 221 \\ 663 \\ \hline 39 \\ 13 \\ \hline 13 \\ 91 \\ 91 \\ \hline 0 \end{array}$$

$$\Rightarrow \frac{x}{37} = \frac{663y + 53}{37}$$

$$\Rightarrow \frac{663y}{37} + \frac{53}{37}$$

↓

Can't be determined  
as we do not know the value of  $y$ .

Ans. cannot be determined.

5.

$$\begin{array}{r} 777 \\ \underline{- 555} \\ 222 \end{array} = ① \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} - 1 \quad \frac{3}{2} \cdot 1, 1, 1, 1$$

554

6.

$$\begin{array}{r} 777 \\ \underline{- 333} \\ 444 \end{array}$$

@upsc.risefinity

334

7.

$$\begin{array}{r} 786 \\ \underline{- 333} \\ 453 \end{array}$$

334

8.

$$\begin{array}{r} 8090 \\ \underline{- 70} \\ 720 \end{array} = \frac{70}{23}$$

$$\frac{3}{2}^2 R = 1 \quad \frac{3^2}{2} R = 1 \quad \frac{3^3}{2} R = 1 \quad \frac{3^n}{2} R = 1$$

$$\frac{4}{3}^2 R = 1 \quad \frac{4^2}{3} R = 1 \quad \frac{4^3}{3} R = 1 \quad \frac{4^n}{3} R = 1$$

whenever,  $\left(\frac{n+1}{n}\right)^m$  always the remainder is 1

$$\frac{2}{3} \equiv R \equiv 2, \quad \frac{2^2}{3} \equiv R \equiv 1, \quad \frac{2^3}{3} \equiv R \equiv 2, \quad \frac{2^4}{3} \equiv R \equiv 1$$

when power is odd Remainder = 1

when power is even Remainder = 2.

$$\frac{3}{4} \equiv R \equiv 3, \quad \frac{3^2}{4} \equiv R \equiv 1, \quad \frac{3^3}{4} \equiv R \equiv 3, \quad \frac{3^4}{4} \equiv R \equiv 1, \quad \dots, \quad \frac{3^n}{4}$$

whenever power is odd Remainder = 3

whenever power is even Remainder = 1.

$$\left(\frac{n+1}{n}\right)^m \equiv R \equiv 1$$

$$\frac{n^m}{n+1}$$

If, m is odd, R = n.

m is even, R = 1

4. If,  $\frac{x}{y} \Rightarrow R = 1$

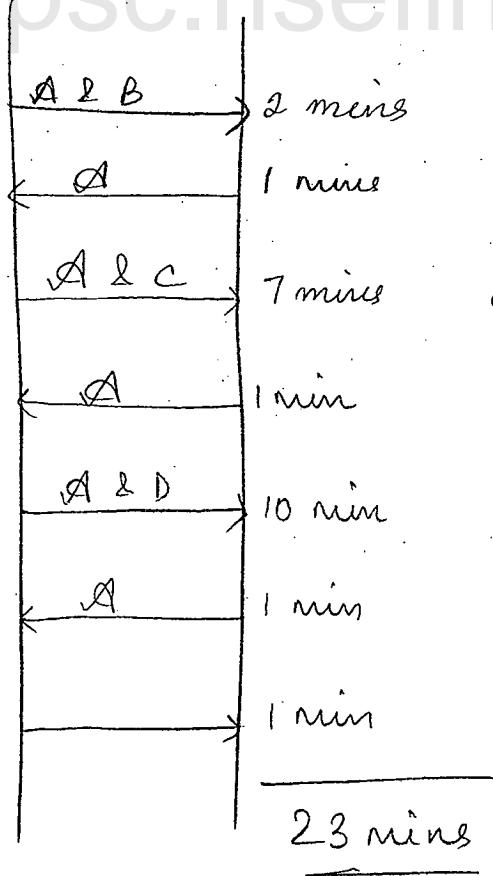
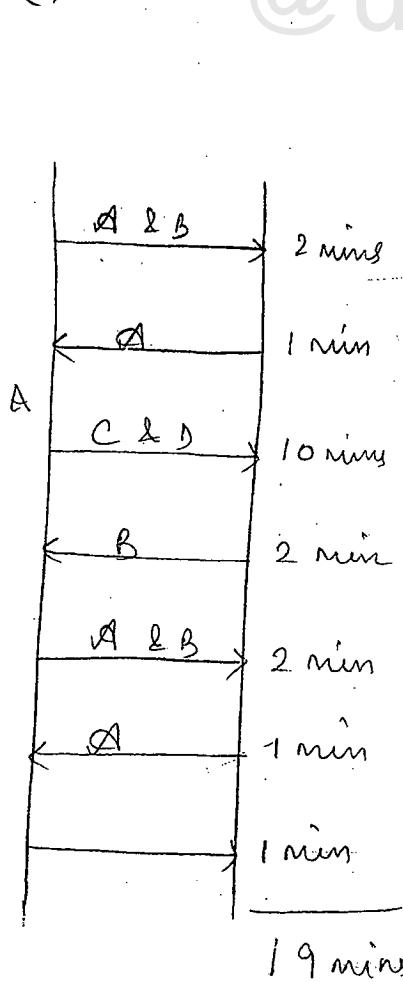
then,  $\frac{x^n}{y} \Rightarrow R = 1$

|        |          |        |          |  |
|--------|----------|--------|----------|--|
| $50^2$ | $= 2500$ | $51^2$ | $- 2601$ | $\begin{array}{r} 4 \\ 49 \\ \hline 49 \\ 49 \\ \hline 13 \\ 196 \\ \hline 2401 \end{array}$ |
| $49^2$ | $= 2401$ | $52^2$ | $- 2704$ |  |
| $48^2$ | $= 2304$ | $53^2$ | $- 2809$ |  |
| $47^2$ | $= 2209$ | $54^2$ | $- 2916$ |  |
| $46^2$ | $= 2116$ | $55^2$ | $- 3025$ |  |
| $45^2$ | $= 2025$ | $56^2$ | $- 3136$ |  |
| $44^2$ | $= 1936$ | $57^2$ | $- 3249$ |  |
| $43^2$ | $= 1849$ | $58^2$ | $- 3364$ |  |
| $42^2$ | $= 1764$ | $59^2$ | $- 3481$ |  |
| $41^2$ | $= 1681$ | $60^2$ | $- 3600$ |  |

### CLASS ASSIGNMENT

1.(a)

b)



Two slow persons  
are always together

$$5. \quad \begin{array}{r} 630 \\ -x \\ \hline \end{array}$$

$$x + x - 3 + x - 6 = 630$$

$$\begin{array}{r} 70 \\ 630 \\ \hline 30 \end{array}$$

$$\underline{x-3}$$

$$3x - 9 = 630$$

$$3x = 639$$

$$\underline{x-6}$$

$$x = \frac{639}{3}$$

$$\underline{x-9}$$

$$x + x - 3 + x - 6 + x - 9 = 630$$

$$\underline{x-12}$$

$$4x - 18 = 630$$

$$4x = 648$$

$$x = \frac{648}{4}$$

$$x + x - 3 + x - 6 + x - 9 + x - 12 = 630$$

$$5x - 30 = 630$$

$$5x = 660$$

$$x = \frac{660}{5}$$

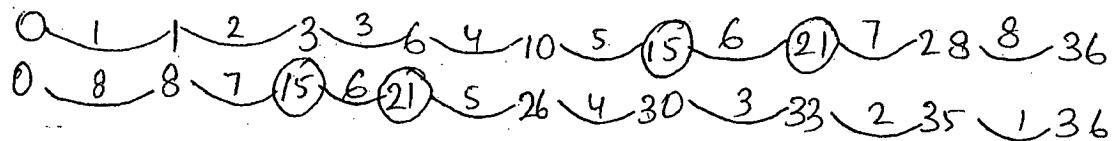
$$x + x - 3 + x - 6 + x - 9 + x - 12 + x - 15 = 630$$

$$6x - 45 = 630$$

$$6x = 675$$

$$x = \frac{675}{6} \text{ X not possible.}$$

6.



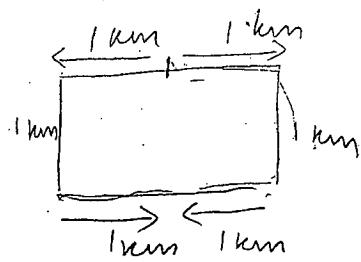
Common points = 2 points

Total points = 14 points

$$\therefore 14 - 2 = 12 \text{ points}$$

2.

After 3 km



3.

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{\dots}}}}$$

$$x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{\dots}}}} = \infty$$

$$x^2 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{\dots}}}} = \infty$$

$$x^2 = 2 + x$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

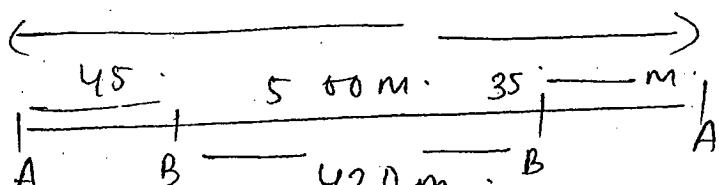
$$(x-2)(x+1) = 0$$

$$x = 2$$

10.

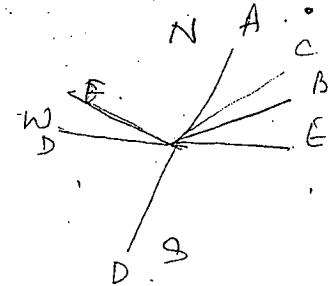
$$\begin{array}{r}
 500 \\
 - 45 \\
 \hline
 455
 \end{array}$$
  

$$\begin{array}{r}
 - 35 \\
 \hline
 420
 \end{array}$$



$$500 : 420$$

$$225 : 21$$



1.  $n$  is the smallest number that has 5 factors. How many factors does  $n-1$  has.
2.  $n$  is the smallest number that has seven factors. How many factors does  $n-3$  has.

|         |             |                   |                |
|---------|-------------|-------------------|----------------|
| 4       | 10          | 12                | 16             |
| ↓       | ↓           | ↓                 | ↓              |
| 1, 2, 4 | 1, 2, 5, 10 | 1, 2, 3, 4, 6, 12 | 1, 2, 4, 8, 16 |

$$N = 16$$

$$N-1 = 15$$



$$1, 3, 5, 15$$

2.  $n-1$  will have 4 factors.

a. i.  $N \rightarrow 'n'$  factors

If ' $n$ ' (total number of factors) is prime

$N \rightarrow (a^P) \rightarrow$  prime factor

$$N \rightarrow a^P \times b^q \times c^r \times \dots$$

$$n = (p+1)(q+1)(r+1) \dots$$

$$\rightarrow n \text{ (total no. of factors)} = p+1$$

$$n \rightarrow 5$$

$$p+1 \rightarrow 5$$

$$p = 2 \text{ or } 4$$

$$n = a^4$$

$$\rightarrow 2^4$$

$$2. \quad n = 87$$

$$p+1 = 7$$

$$p^2 = 6$$

$$N \rightarrow a^6$$

$$= 2^6$$

$\Downarrow$

64.

$$\begin{array}{r} 64 - 3 \\ = 61 \\ \swarrow \quad \searrow \\ 1 \quad 61 \end{array}$$

\* Only applicable in case of prime numbers.  
ex.  $(2^n - 1) 2^{13} \rightarrow 2^{13-1} = 2^{12}$ .

3. Find the smallest possible number by which 5780 must be multiplied or divided to get a perfect square.

4. Find the smallest possible by which 5780 must be multiplied to get a perfect cube.

3.

5780

$$\begin{array}{ccc} & 5780 & \\ \swarrow & & \searrow \\ 578 & & 10 \\ \swarrow & & \searrow \\ 2 \times 289 & & 2 \times 5 \\ \Downarrow & & \\ 17^2 & & \end{array}$$

$$2^2 \times 17^2 \times 5^1$$

power is even

Multiplication by 5 it becomes  $5^2$ , power becomes even.

$$2^2 \times 17^2 \times 5^2$$

when divided by 5 again.

$$\frac{2^2 \times 17^2 \times 5}{5}$$

$$4. \quad 2^2 \times 17^2 \times 5^1 \times (2 \times 17 \times 5^2) \\ = 2^3 \times 17^3 \times 5^3$$

$$\begin{array}{r} 34 \\ \times 25 \\ \hline 170 \\ 68 \times \\ \hline 850 \end{array}$$

5. Find the smallest possible perfect square exactly divisible by 6, 7, 8 and 9.

$$\text{L.C.M.} = 2^3 \times 3^2 \times 7 \\ = 504$$

$$\begin{array}{r} 36 \\ \times 49 \\ \hline 64 \\ 84 \\ \hline 23 \end{array}$$

$$\begin{array}{r} 2 | 6, 7, 8, 9 \\ 2 | 3, 7, 4, 9 \\ 3 | 3, 7, 2, 9 \\ \hline 1, 7, 2, 3 \end{array}$$

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ 7 \\ \hline 504 \end{array}$$

Making power even and to  
make it perfect square.

$$2^3 \times 3^2 \times 7 \times 2 \times 7$$

$$= 2^4 \times 3^2 \times 7^2$$

$$= 7056$$

6. Sq. root of 321489  $\sqrt{567}$

$$\begin{array}{r} 56 \\ \hline 321489 \\ -25 \\ \hline 714 \\ -636 \\ \hline 7889 \\ +6 \\ \hline 112 \end{array}$$

$$\begin{array}{r} 567 \\ \hline 7889 \\ -7889 \\ \hline 0 \end{array}$$

7. Sq. root of 166464.

$$\begin{array}{r}
 408 \\
 \hline
 166464 \\
 -16 \\
 \hline
 6464 \\
 -64 \\
 \hline
 6464 \\
 -64 \\
 \hline
 \alpha
 \end{array}$$

8. Sq. root of 15129.

$$\begin{array}{r}
 123 \\
 \hline
 15129 \\
 -1 \\
 \hline
 51 \\
 -44 \\
 \hline
 729 \\
 -729 \\
 \hline
 \alpha
 \end{array}$$

$$\begin{array}{r}
 808 \\
 \times 86 \\
 \hline
 8464 \\
 \hline
 22 \\
 \hline
 44 \\
 \hline
 23 \\
 \hline
 66 \\
 \hline
 243 \\
 \hline
 729
 \end{array}$$

9. Find the least numbers of 3 digits which must be subtracted from the greatest number of ~~three~~<sup>seven</sup> digit that the remaining number is perfectly divisible by 270.

- a) 9.
- b) 189
- c) 279
- d) Cannot be determined

Find the least number of 2 digits which must be subtracted from the greatest number of ~~three~~<sup>seven</sup> digit that the remaining number is perfectly divisible by 270.

- a. 9
- b. 79
- c. 279
- d. cannot be determined

9.

$$270) \overline{999999}$$

9

$$\begin{array}{r} 8 \\ 899 \\ - 809 \\ \hline 99 \\ - 80 \\ \hline 19 \\ - 18 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 9999999 - 9 \\ 2 \quad 9999990 \\ \quad - 270 \\ \hline 9999720 \\ \quad \quad \quad 9 \end{array}$$

$$\begin{array}{r} 9999999 \\ - 279 \\ \hline 9999720 \end{array}$$

10.

$$\begin{aligned} m+n \div 12 &\Rightarrow R=8 \\ m-n \div 12 &\Rightarrow R=6 \\ mn \div 6 &\Rightarrow R=? \end{aligned}$$

$$\begin{array}{r} m+n = 20(12+8) \\ m-n = 18(12+8) \\ \hline 2m = 38 \\ m = 19 \end{array}$$

116.

$$\begin{aligned} n &= 1 \\ mn &= 19 \times 1 \\ &\frac{6}{\Rightarrow R=1} \end{aligned}$$

11.

$$\begin{array}{r} 93 \times 98 \\ (-7) \quad (-2) \\ 93 \times 98 \\ \hline -2 \quad (-7 \times -2) \\ 2 \quad 93 \quad 2 \quad 91 \\ \hline = 9114 \\ = 9306 \end{array}$$

$$\begin{array}{r} \text{Base} = 100 \\ (-4) \quad (-1) \\ 94 \times 99 \\ \hline \end{array}$$

$$12. \quad 104 \times 107$$

$$\begin{array}{r} & (+4) \\ \times & 104 \\ & 107 \quad (+7) \\ \hline & +4+7=11 \\ & +4 \times 7 = 28 \end{array}$$

$$2 \quad 11128$$

$$\begin{array}{r} (+3) \\ 103 \times 108 \quad (+8) \end{array}$$

$$2 \quad 11124$$

$$13. \quad 94 \times 106 \quad (-6) \quad (+6)$$

$$\begin{array}{r} 94 \quad +6 \\ \times 106 \quad (-6 \times +6) \\ \hline 2 \quad 100 \\ 2 \quad 10000 \quad -36 \end{array}$$

$$91 \times 101 \quad (-9) \quad (+1)$$

$$\begin{array}{r} 2 \quad 92 \quad (-9 \times +1) \\ -9 \\ \hline 2 \quad 9200 \\ 2 \quad 9191 \end{array}$$

$$\begin{array}{r} 10000 \\ -36 \\ \hline 9964 \end{array}$$

$$92 \times 103 \quad (-8) \quad (+3)$$

$$2 \quad 9500 - 24$$

$$14. \quad 85^2$$

$$2 \quad 9476$$

$$2 \quad 7225$$

$$65^2$$

$$2 \quad 4225$$

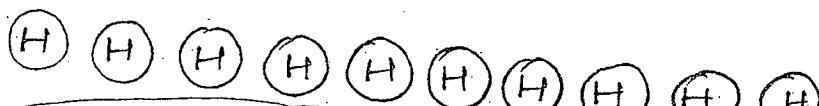
# **PROBABILITY**

@upsc.risefinity

1. There are 10 identical coins and each one of them has 'H' engraved on its one face and 'T' engraved on its other face. These 10 coins are lying on a table and each one of them has 'H' face as the upper face. In one attempt exactly four (neither more nor less) coins can be turned upside down. What is the minimum total number of attempts in which the 'T' face of all the 10 coins can be brought to the upper faces?

\* Probability is certainly.  
It will come.

Possibility → it may or may not happen.



Step-I T T T T H H H H H H

Step-II T T T T H T T T H H H

Step-III T T T T T T T T T T

\* First I turn four coins which is turned upside down and from 'H' → 'T'.

\* Second, I turn three coins of 'H' faced upside down, <sup>to T</sup> and the one from the previously turned one, from 'T' to 'H'

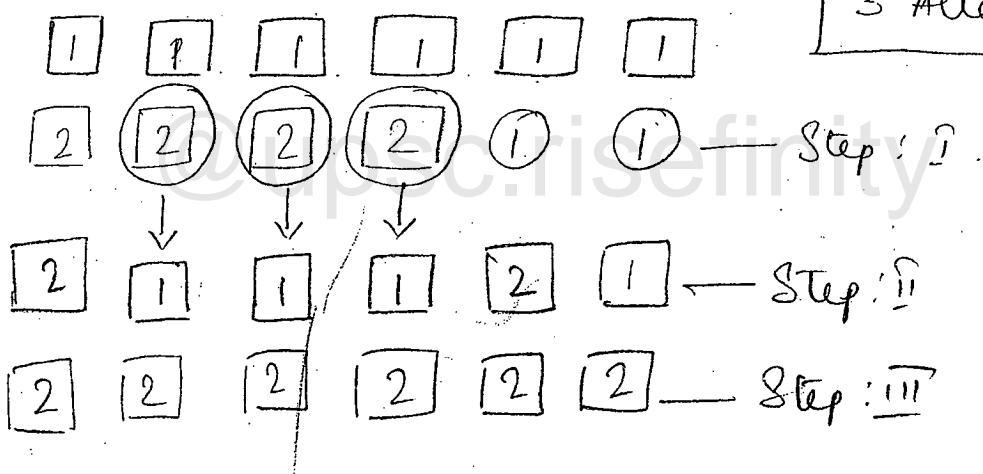
\* Third, the remaining, I am left with will be four coins again. I turn from 'H' to 'T'

2. Six identical cards are placed on the table. Each card has -

- marked on one side
- marked on its other side.

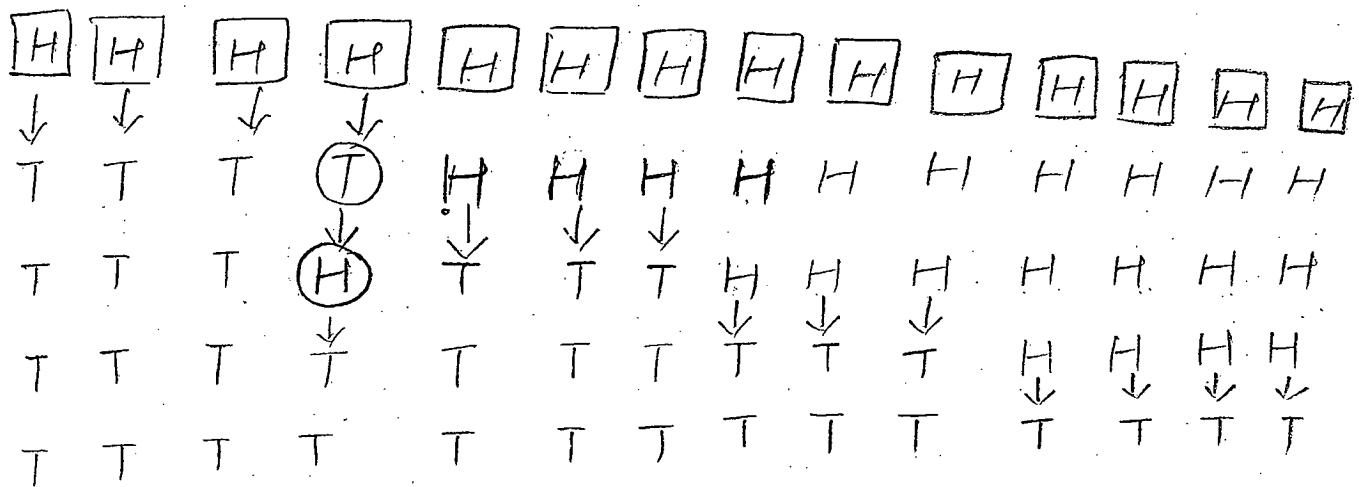
All six cards are placed in such a manner that the number one is on the upper side. In one try exactly four (neither more nor less) cards are turned upside down. In how many number of tries can the cards be turned upside down such that all the six cards shows no. 2 on upper side.

3 Attempts



3. There are 14 identical coins and each one of them has 'H' engraved on its one face and 'T' engraved on its other face. These 14 coins are lying on a table and each one of them has 'H' face as the upper face. In one attempt exactly four (neither more nor less) coins can be turned upside down. What is the minimum

Total number of attempts the 'T' face of all the 14 coins can be brought to the upper faces.

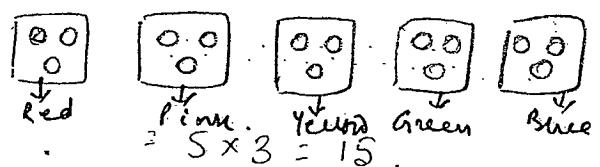


4 Attempts

4. A box contains five sets of balls, while there are three balls in each set. Each of balls has one colour which is different from every other set.

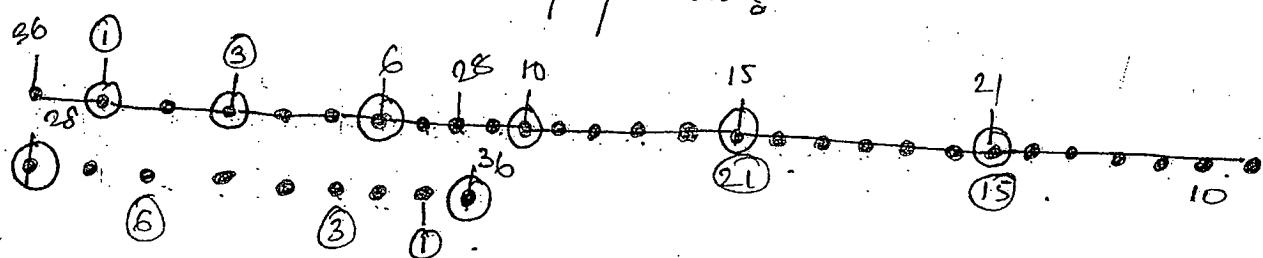
What is the least number of balls removed from the box in order to create certainty. A pair of balls of same colour has been removed.

- a) 2
- b) 3
- c) 5
- d) None of these ✓



R/P/4/G/B. — 6th one  
can be  
repeated  
of the  
previous  
colours.  
= 6 balls.

5. A straight line segment is 36 cm long. Points are to be marked on the line from both the end points. From each end the first point is at a distance of 1 cm from the end, the second point is at a distance of 2 cm from the first point and the third point is at a distance of 3 cm from the second point. and so on..... If points on the ends are not counted and the common points are counted as one what's the number of points?



$\begin{array}{r} 36 \\ 28 \end{array} \Big| \begin{array}{r} 6 \\ 2 \end{array} \Big| \begin{array}{r} 10 \\ . \end{array} \Big| \begin{array}{r} 15 \\ . \end{array} \Big| \begin{array}{r} 21 \\ 6 \end{array} \Big| \begin{array}{r} 28 \\ 3 \end{array} \Big| \begin{array}{r} 36 \\ 1 \end{array}$  Omitting the extreme points, here being  $\underline{\underline{36}}$ . then calculating the points.

~~36~~ ~~28~~ ~~21~~ ~~15~~ ~~10~~ ~~6~~, ~~3~~, ~~1~~!

$$10 + 2 = 12 \text{ points}$$

## AVERAGE

- Average is the range between the maximum and the minimum.
- Average is the performance of an individual.
- Average comes into picture when the thing is repeated more than once.

Average is  $\frac{\text{sum of all terms}}{\text{total terms}}$

$$\underline{\text{Average speed}} = \frac{\text{distance}}{\text{time}} = \frac{\text{Total distance}}{\text{Total time}}$$

= A man travels from A to B at a speed of 60 km/hr, and from B to A at a speed of 40 km/hr. What is the average speed?



| Time | Distance | Speed |
|------|----------|-------|
| 2    | 60, 40   |       |
| 2    | 30, 20   |       |
| 2    | 15, 10   |       |
| 3    | 15, 5    |       |
| 5    | 5, 5     |       |

$$\text{Average Speed} = \frac{2xy}{x+y} = \frac{120+120}{2+3} = \frac{240}{8} = 48 \text{ km/hr}$$

= First distance  $\frac{1}{3}$  rd covered at 10 km/hr, Second  $\frac{1}{3}$  at a speed of 20 km/hr and the third  $\frac{1}{3}$  at a speed of 30 km/hr. What is the average speed?

L.C.M of 10, 20

& 30

$$\frac{6 \times 10}{3} = \frac{20}{3}$$

$$\frac{60}{30} = \frac{2}{1}$$

$$\text{Average Speed} = \frac{3xyz}{x+y+z}$$

$$\frac{60+60+60}{6+3+2} = \frac{180}{11} \text{ km/hr.}$$

2. A cyclist covered a square field at a speed of 30 km/hr, 20 km/hr, 10 km/hr, 10 km/hr. What is the average speed?

L.C.M of 30, 20,

10, 40.

$$\text{Average Speed} = \frac{120 + 120 + 120 + 120}{12 + 6 + 4 + 3}$$

$$= \frac{480}{25} \text{ km/hr}$$

$$\begin{array}{r} 12 \\ 10 \\ \hline 22 \\ 3 \\ \hline \end{array}$$

$$= 19.2 \text{ km/hr}$$

$$\begin{array}{r} 48 \\ 25 \\ \hline 3 \\ 0 \\ \hline \end{array}$$

Assignment

15.

$$x \text{ kms} \quad v_1 \text{ km/hr.} \quad x/v_1$$

$$y \text{ kms} \quad v_2 \text{ km/hr.} \quad x/v_2$$

$$\frac{x+y}{\frac{x+y}{v_1+v_2}} = \frac{v_1 v_2 (x+y)}{v_2 x + v_1 y}$$

$$\begin{array}{r} 8, 10, 9, 11 \\ 4, 5, 9, 11 \\ \hline 2, 5, 9, 11 \\ 300 \\ 11 \\ \hline 36 \\ 36 \times \\ \hline 3960 \end{array}$$

= Four cars are hired at the rate of Rs 6/km plus the cost of diesel at Rs 40/litre. In this context consider the details given in the following table.

| Car A | Mileage (km/hr) | Hours | Total Payment |
|-------|-----------------|-------|---------------|
| Car B | 10              | 20    | 2120          |
| Car C | 9               | 24    | 1950          |
| Car D | 11              | 22    | 2064          |
|       |                 |       | 1812          |

which car maintained the maximum average speed?

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Suppose total distance as  $a$ .

$$8 \text{ km/hr} = 40$$
$$1 \text{ km/hr} = \frac{40}{8}$$

$$\underline{\text{Car A}}: 6a + 5a = 2120$$

$$11a = 2120$$

$$a = \frac{2120}{11} = \frac{106}{11} = 9\frac{7}{11} \text{ km/hr.}$$

Car B.

$$10 \text{ km/hr} = 40$$

$$1 \text{ km} = \frac{40}{10}$$

car B.

$$\begin{array}{r} 390 \ 78 \\ \hline 1950 \\ - 11 \\ \hline 850 \\ - 25 \\ \hline 500 \\ - 71 \\ \hline 289 \\ - 11 \\ \hline 78 \\ - 71 \\ \hline 7 \end{array}$$
$$\begin{array}{r} 7.09 \\ 11 \ 78 \\ \hline 77 \\ - 77 \\ \hline 0 \end{array}$$
$$\begin{array}{r} 100 \\ 99 \\ \hline 1 \end{array}$$

$$6a + 4a = 1950$$

$$10a = \frac{1950}{10}$$

$$a = \frac{1950}{10} = 195$$
$$= \frac{195}{10} = 19.5$$
$$= 19.5 - 11 = 8.5$$
$$= 8.5 - 7.1 = 1.4$$
$$= 1.4 - 0.7 = 0.7$$
$$= 0.7 - 0.7 = 0$$

$$\text{Average of } 2+4+6 = \frac{12}{3} = 4.$$

$$\text{Add } +1 = 3+5+7 = \frac{15}{3} = 5$$

$$\begin{array}{r} \text{Subtract } -1 \\ \hline 2+4+6 \end{array} = 4$$

$$\text{Multiply } \times 2 \quad 4+8+12 = \frac{24}{3} = 8.$$

$$\text{Divide } \div 2 \quad 2+4+6 = 4.$$

7. Avg. marks of 100 students = 60.

$$\begin{aligned} \text{Total marks of 100 Students} &= 60 \times 100 \\ &= 6000 \end{aligned}$$

$$\begin{array}{r} \text{Correct total} \\ \hline = 6000 - 75 + 65 \\ = 5990. \end{array}$$

$$\begin{array}{r} \text{Average} \\ \hline = \frac{5990}{100} \\ = 59.5 \end{array}$$

12. Average marks of 75 students = 35.

$$\begin{array}{r} \text{Total marks} \\ \hline = 75 \times 35 \end{array}$$

$$= \frac{2625}{75}$$

$$\begin{array}{r} 75-2 \\ 35 \\ \hline 375 \\ 225x \\ \hline 2625 \end{array}$$

The average weight of a class of 30 students is 40 kgs. If however weight of teacher is included, the average becomes 41 kg. What is the weight of the class-teacher?

Avg. weight of 30 students = 40 kgs.

$$\begin{aligned}\text{Total weight of 30 students} &= 40 \times 30 \\ &= 1200 \text{ kgs.}\end{aligned}$$

Avg. weight of 31 persons = 41 kgs.

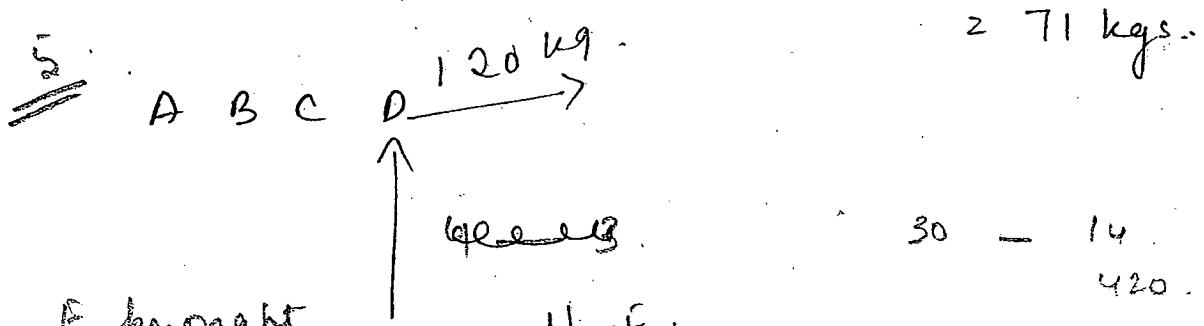
$$\begin{aligned}\text{Total weight of 31 persons} &= 41 \times 31 \\ &= 1271 \text{ kgs.}\end{aligned}$$

$$\begin{aligned}\text{Weight of class teacher} &= 1271 - 1200 \\ &= 71 \text{ kgs.}\end{aligned}$$

Method 2: Avg. is 40 kgs. of 30 students.

Then = now after the class teacher arrives avg increases to 41 kg.

Now the 31 persons =  $40 + 31$



E brought  
120+

replaced by E.

$E = 120$   
if not weight  
replaced.

$$\begin{array}{r} 30 - 14 \\ \hline 420 \end{array} \quad \begin{array}{r} 31 \times 15 \\ \hline 155 \end{array} \quad \begin{array}{r} 31 \times \\ 465 \\ \hline 465 \end{array}$$

$$\begin{array}{r} 465 \\ - 420 \\ \hline 45 \end{array}$$

Class Assignment 08 | Previous class

9. Let per head expenditure =  $Rs x$ .

No. of students = 35

Total expenditure =  $35x$ .

No. of new students =  $35 + 7$   
= 42.

$$\therefore \text{The new expenditure} = \frac{35x + 42}{42}$$

Expenditure diminishes by 1

$$\text{per head} = \frac{35x + 42 - x - 1}{42}$$

$$= 35x + 42 - 42x - 42$$

$$= 7x = 84$$

$$x = 12$$

$$\begin{aligned} \text{Original expenditure} &= 35x = 35 \times 12 \\ &= Rs 420. \end{aligned}$$

10. Let average expenditure of 9 persons =  $x$ .

Total expenditure of 9 persons =  $9x$ .

8 of them pays 12/- each =  $9x = 8 \times 12$ .

$$\Rightarrow 9x = 8 \times 12 + 8 + x$$

$$\downarrow 96 + 8$$

$$\Rightarrow 9x - x = 104$$

$$8x = 104$$

$$x = 13$$

$$\Rightarrow 9x = 9 \times 13$$

11. Avg. of a batsman =  $\frac{\text{Total runs scored}}{\text{Total innings out}}$

Avg. of a bowler =  $\frac{\text{Total runs given up}}{\text{Total wickets taken}}$

Let the avg. till 16 innings =  $x$ .  
Total =  $16x$ .

Total runs scored =  $16x + 87$

17<sup>th</sup> innings  $\Rightarrow \frac{16x + 87}{17} = x + 3$

$$\Rightarrow 16x + 87 = 17x + 51$$
$$x = 36.$$

New avg. =  $36 + 3$   
= 39.

13. Let the number of wickets taken before last match =  $x$ .

Avg. = 12.4

$$\Rightarrow 12.4 = \frac{\text{Total runs}}{x}$$

$$\text{Total runs} = 12.4x$$

Last match given = 26 runs  
 $= 12.4x + 26$

Total wickets taken =  $x + 5$

The avg. decreases by 0.4.

so the new avg. = 12.

$$\Rightarrow \frac{12 \cdot 4x + 26}{x + 5} = 12$$

$$\Rightarrow 12 \cdot 4x + 26 = 12x + 60$$

$$\Rightarrow 0.4x = 340$$

$$4x = 340$$

$$x = \frac{340}{4} \\ = 85$$

21

| No. of students | Avg. marks | Total marks.                |
|-----------------|------------|-----------------------------|
| Cl - I          | $x$        | $m = 35$ $\rightarrow mx$ . |
| Cl - II         | $y$        | $n = 55$ $\rightarrow ny$ . |

Combined Avg.

$$\text{or } \bar{x} = \frac{mx+ny}{x+y}$$

Weighted Avg.

$$\bar{x} = \frac{mx+ny}{x+y}$$

$$\Rightarrow 40 = \frac{35x + 55y}{x+y}$$

$$\Rightarrow 40x + 40y = 35x + 55y$$

$$5x = 15y$$

$$\frac{x}{y} = \frac{15}{5} = \frac{3}{1} = 3:1$$

14. annual income  
Avg. of certain agricultural worker = \$.

Other workers  $\rightarrow$  2 T.

| No. of worker | Avg. | Total |
|---------------|------|-------|
|---------------|------|-------|

|    |   |      |
|----|---|------|
| 11 | S | 11 S |
|----|---|------|

|   |   |   |
|---|---|---|
| 1 | T | T |
|---|---|---|

$$\text{Avg. } \frac{11S + T}{12}$$

\* There is a certain number of cigarettes in a box. They are divided in such a way that the person who gets  $\frac{1}{4}$  of the whole gets twice of what the others get on an average.

Find the no. of people, amongst whom the cigarettes are distributed.

Find the no. of cigarettes.

Let the no. of cigarettes be  $x$ .  
 A,  $12 \times \frac{1}{4} = 3$   $\rightarrow$  just gets 1 each and the no. of persons be  $y$ .  
 which is twice what he has taken.

B, 36 one person take  $\frac{1}{4} = \frac{1}{4}x = \frac{4-1}{4}x$

C, 48 No. of cigarettes left  $= \frac{3}{4}x$ .

Not possible: Each share  $= \frac{3}{4}x$ .

$$\frac{3}{4}x = \left( \frac{\frac{3}{4}x}{y-1} \right) \times 3 = y-1 = 9$$

$$y = 9+1 = 10$$

$$\begin{array}{r} \cancel{2} \\ \cancel{2} \end{array} \quad \begin{array}{r} 45 \times 5 \cancel{00} \\ \hline 100 \\ = 225 \end{array} \quad \begin{array}{r} 45^2 \\ \hline 5 \\ \hline 225 \end{array} \quad \begin{array}{r} 100 \\ \hline 225 \end{array}$$

$$\begin{array}{r} 2140,60 \\ 220,30 \\ 510,15 \\ \hline 2,3 \end{array} \quad \begin{array}{r} 50 \times 40 \\ \hline 150 \\ = 20 \end{array}$$

$$\begin{array}{r} 55 \times 5 \cancel{00} \\ \hline 100 \\ = 275 \end{array} \quad \begin{array}{r} 55^2 \\ \hline 5 \\ \hline 275 \end{array} \quad \begin{array}{r} 2 \\ 120 \\ \hline 120 \\ 60 \end{array} \quad \begin{array}{r} 40+20 \\ = 60 \end{array}$$

$$225 + 275 = 500 \quad + 275 = \begin{array}{r} 225 \\ \hline 500 \end{array}$$

$$\begin{array}{r} 120+120 \\ \hline 3+2 \\ = 240 \end{array}$$

$$= 48.$$

Annual avg. 60 %.

$$45\% + 15 = 60. \quad \underline{3}.$$

$$55/ + 5 = 60.$$

$$60\% + 15 + 5 \\ = 80\%.$$

$$80\% = 500$$

$$\Rightarrow \frac{80}{100} \times 5 \cancel{00}$$

$$= 400.$$

5 consecutive  
odd integers.

$$1, 3, \textcircled{5}, 7, 9 \quad \text{middle} \\ = a + a+2 + \textcircled{a+4} + a+6 \\ + a+8 \\ \hline 5$$

$$= \frac{5a+20}{5} = \underline{\underline{5(a+4)}}$$

a+4 c is the  
avg,

$$\therefore 45 + 55 + x.$$

$$\begin{array}{r} 260 \\ 3 \\ \hline \end{array}$$

80% of 500.

$$100+x = 60 \times 3$$

$$= \frac{80 \times 500}{100}$$

$$100+x = 180.$$

$$= 400$$

$$x = 180 - 100$$

$$= 80.$$

5. Let average till 11 innings be  $x$ .

$$\text{Total} = 11x$$

$$12\text{th innings} = 290$$

$$\text{Total} = 11x + 90$$

$$\text{Avg: } \Rightarrow \frac{11x + 90}{12} = x - 5$$

$$\Rightarrow 11x + 90 = 12x - 60$$

$$12x - 11x = 290 + 60$$

$$x = 150$$

$$\text{Decreases by 5} = 150 - 5$$

$$= \underline{\underline{145}}$$

T - Manya  
Thru - Anni  
Sat - Chanda

\* What can be possible no. of cigarettes

## RATIO AND PROPORTION.

→ Ratio shows relation b/w two numbers.

→ Ratio represents fraction.

$\frac{3}{2}$   
 $\frac{6}{6}$   
20

$$2:4 = \frac{2}{4} = \frac{1}{2}$$

$$8:16 = \frac{8}{16} = \frac{1}{2}$$

→ Proportion is equal

$$2:4 :: 8:16$$

$$\frac{2}{4} = \frac{8}{16}$$

→ Simplest form of Ratio;

$$2:13$$

$$14:15$$

Any ratio which cannot be further reduced is called simplest form of Ratio.

→  $a:b$  is the simplest form of Ratio

Duplicate ratio will be  $a^2:b^2$

Trippleate ratio will be  $a^3:b^3$

Sub-duplicate ratio will be  $\sqrt{a}:\sqrt{b}$

Sub-trippleate ratio will be  $\sqrt[3]{a}:\sqrt[3]{b}$

$$A:B :: 1:2$$

Suppose if I double it.

$$B:C :: 3:4$$

$$A:B = 2:4, 3:6$$

$$\underline{A:B:C :: 3:6:8}$$

Suppose if I double B:C

$$6:8$$

$$\begin{array}{r} a:b :: 1:2 \\ \text{multiply by } 3 \\ b:c :: 3:4 \\ \hline a:b:c :: 3:6:8 \end{array}$$

$$1 \times 3 = 3$$

$$2 \times 3 = 6$$

$$2 \times 4 = 8$$

$$\begin{array}{r} a:b :: 1:5 \\ b:c :: 3:4 \\ \hline a:b:c :: 3:15:20 \end{array}$$

$$\begin{array}{r} a:b:c :: 1:2:3 \\ c:d :: 4:5 \\ \hline a:b:c:d :: 4:8:12:15 \end{array}$$

$$\rightarrow \text{If } a:b :: 1:2$$

$$b:c :: 3:4$$

$$c:d :: 5:6$$

$$\begin{array}{r} a:b:c :: 3:6:8 \\ c:d :: 5:6 \\ \hline a:b:c:d :: 15:30:40:48 \end{array}$$

1. A bag contains 50 paise, rupee 1 and rupee 2 coins in the ratio 2:3:4. If the total amount is Rs 240, what is the total number of coins?

50 paise, ₹ 1 ₹ 2.

2 : 3 : 4

22240

$\times 20$

₹ 1 : ₹ 3 : ₹ 8      1 + 3 + 8 = 12

$20 \times 1$        $3 \times 20$        $20 \times 8$

$\frac{240}{12}$  20.

₹ 20 : ₹ 60 : ₹ 160

How many coins required to make 20 rupees + How many coins reqd. to make ₹ 60 + How many coins reqd. to make ₹ 160  
 $\frac{40}{20}$  +  $\frac{60}{60}$  +  $\frac{80}{80}$  = 180 total coins.

⇒  $a:b::c:d$

mean

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Product of extreme = Product of means

Assignment:

24. Gold Non-Gold.

1 : 3

$x$  :  $3x$ .

$(x+10) : (3x) :: 1:2$

$$2(x+10) = 3x$$

$$2x + 20 = 3x$$

$$x = 20.$$

$$x + 10 + 3x$$

$$4x + 10$$

$$4 \times 20 + 10 = \underline{90}.$$

2. A bag contains rupees 114 in the form of 21, 50 paise, 10 paise in the ratio of 3 : 4 : 10. What is the number of 50 paise coins.

21 50 p 10 p.

3 : 4 : 10.

$$\begin{array}{r} 3+2+1 \\ = 6 \\ \hline 23 : 22 : 21 \end{array}$$

$$\begin{array}{r} 114 \\ 6 \\ \hline 19 \\ 18 \\ \hline 2 \end{array} \quad 219 : 257 : \underline{238} : 219$$

$$57 + 76 + 190$$

$$\underline{76}.$$

3. A bag contains Rs 112 in the form of ₹ 1, 50 paise, and 10 paise coins in the ratio of 3 : 8 : 10. What is the number of 10 paise coins?

₹ 1 . . . 50 p . . . 10 p .

3 : 8 : 10

~~3+4+1 = 8+2+3 = 24~~ : 24 : ₹ 10.

$$8 \overline{)112} (14 \\ \underline{-8} \\ 32 \\ \underline{-32} \\ 0$$

$$112 : \boxed{14} \quad 24 : 256 : 214 \\ \underline{\times 10} \\ 42 + 112 + \underline{140}$$

$$\begin{array}{r} 17 \\ 16 \\ \hline 102 \\ 102 \\ \hline 17 \\ 16 \\ \hline 136 \\ 136 \\ \hline 0 \end{array}$$

$$\boxed{140}$$

4. Kundan has collection of coins of denominations of ₹ 1, 50 paise and 25 paise in the ratio of 12 : 10 : 7. The total worth of coins he has is 75 rupees. Find the number of 25 paise coins.

$$\begin{array}{r}
 & 1^2 \\
 & 5.75 \\
 & 18.75 \\
 \hline
 15 & 19 \times 4 = 76 \\
 & 18.75 \times 4 = 75 \\
 & \hline
 & 7.75 \\
 & 6 \\
 \hline
 & 1.00
 \end{array}
 \quad
 \begin{array}{r}
 21 \quad 50 \text{ p} \quad 25 \text{ p} \\
 12 \quad 10 \quad 7 \\
 21 \quad 25 \\
 \hline
 21
 \end{array}
 \quad
 \begin{array}{r}
 21 \quad 25 \quad 7 \\
 \times 4 \quad \times 4 \\
 \hline
 248 \quad 220 \quad 27
 \end{array}
 \quad
 \begin{array}{r}
 21 \quad 25 \quad 7 \\
 \times 4 \quad \times 4 \\
 \hline
 248 \quad 40 \quad 28
 \end{array}$$

L.C.M of 1, 1, 4

multiplying by

4 for 21 = 4 25p required

$$\text{Then } = 7 \times 4 = 28$$

2. A man willed his  $\frac{1}{3}$  of his estate and the remaining  $\frac{2}{3}$  to his son. Should one be born but in case of a daughter being born,  $\frac{2}{3}$  of the estate will go to the wife and  $\frac{1}{3}$  to the daughter. After the man's death, twins were born, a boy and girl were born. Estate was to be divided as per the wish of the man by maintaining the proportion as desired by the man. What is the ratio in which the property is to be divided among the son, wife and daughter.

Son =  $\frac{2}{3}$       Wife =  $\frac{2}{3}$   
 wife =  $\frac{1}{3}$       Daughter =  $\frac{1}{3}$

Son : wife :: 2 : 1  
 wife : Daughter :: 2 : 1

S : W : D :: 8 : 4 : 2 : 1

Assignment

22.

50 paise, 2 1, 2 2, 2 5. Total 2 50.

$$2 \underbrace{26}_{26} + \underbrace{24}_{24} = 14 + 10$$

$$2x + (x-5)5 = 24.$$

$$2x + 5x - 25 = 24.$$

$$7x = 49.$$

$$x = 7.$$

$$\therefore 7 \times 2 = 14.$$

$$10 + 5 - 5 = 10.$$

$$25. \text{ Total Application} = 120$$

$$= 70$$

Male

$$\text{Female} = 120 - 70$$

$$= 50 \quad : \quad 80 - 50$$

$$\text{No. of those who} \quad : \quad 230$$

$$\text{have driver's license} = 80$$

Minimum no. of males

having drivers' licence : Maximum no. of males  
having drivers' licence

$$30 : 70$$

$$3 : 7$$

\* 30, which is by default male

50 is female.

Among 80 having drivers' licence 50 are  
females then male will be  $80 - 50$

$$= \underline{\underline{30}}$$

$$20. \quad A : B : C$$

$$6 : 19 : 7$$

$$6x \quad 19x \quad 7x \quad : 32x$$

$$\downarrow \quad \downarrow$$

$$6x : 19x + 200 : 7x - 200 \therefore \underline{\underline{3 : 10 : 3}}$$

A & C equal.

$$6x = 7x - 200$$

$$x = 200.$$

$$32x = 32 \times 200$$

$$= \underline{\underline{26400}}$$

$\Rightarrow$  Original value =  $x$ .

Increase by 1%

$$\text{New Value} = \frac{x \times (100 + 1\%)}{100}$$

23. Maths

|                                     | Physics                             | Biology                             |                                    |
|-------------------------------------|-------------------------------------|-------------------------------------|------------------------------------|
| 5                                   | 7                                   | 8                                   | $100 + 40$                         |
| $\frac{24}{5} \times \frac{28}{40}$ | $\frac{3}{7} \times \frac{150}{30}$ | $\frac{4}{8} \times \frac{175}{35}$ | $100 \times \frac{140}{100} = 140$ |
| $\frac{120}{100}$                   | $\frac{150}{100}$                   | $\frac{175}{100}$                   | $100 + 50 = 150$                   |

$$2 : 3 : 4$$

8. Total mixture = 28 litres

Milk : Water :: 5 : 2

$$5+2$$

$$\begin{aligned} \text{Qty of milk} &= \frac{5}{7} \times 28 \\ &= 20 \text{ litres} \end{aligned}$$

$$\begin{aligned} \text{Qty of water} &= \frac{2}{7} \times 28 \\ &= 8 \text{ litres} \end{aligned}$$

$$\text{milk} \quad \text{water} \\ 20 \quad 8+x \quad :: \quad 2:5$$

$$20 \times 5 - 2 = 16 + 2x$$

$$100 = 16 + 2x$$

$$2x^2 - 100 - 16$$

$$2x = 84$$

$$x = 42.$$

Or, milk

Wain

$$\begin{array}{r}
 5 \\
 \downarrow x_4 \\
 20
 \end{array}
 \quad
 \begin{array}{r}
 x_4 \downarrow^2 \\
 8
 \end{array}
 \quad
 \begin{array}{r}
 2 \quad 7 \\
 \downarrow x_4 \\
 28
 \end{array}$$

$$\begin{array}{r} + 42 \\ \hline 20 \\ \downarrow \\ 2 \end{array} \quad \begin{array}{r} - 50 \\ \hline 50 \\ \downarrow \\ 5 \end{array} \quad 50 - 8 \\ 242 \end{array}$$

9. Mille meurt

$$\begin{array}{r}
 2 \\
 \downarrow \\
 \boxed{x} 20 \\
 \hline
 40
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 \downarrow \\
 \boxed{x} 20 \\
 \hline
 20 + 40 \\
 \hline
 80
 \end{array}
 \quad
 \begin{array}{r}
 3 \\
 \downarrow \\
 \boxed{x} 20 \\
 \hline
 60
 \end{array}
 \quad
 \begin{array}{r}
 80 - 20 \\
 \hline
 260
 \end{array}$$

## RATIO & PROPORTION

1. Two glasses of equal volume are respectively half and  $\frac{3}{4}$  th filled with milk. They are then filled to the brim by adding water. Their contents are then pour into another vessel. What is the ratio of milk to water in this vessel.

|      | Milk                          | Water                         |
|------|-------------------------------|-------------------------------|
| V I  | $\frac{1}{2}x$                | $\frac{1}{2}x$                |
| V II | $\frac{3}{4}x$                | $\frac{1}{4}x$                |
|      | <hr/>                         | <hr/>                         |
|      | $\frac{1}{2}x + \frac{3}{4}x$ | $\frac{1}{2}x + \frac{1}{4}x$ |

$$\frac{2x + 3x}{4}$$

$$\frac{2x + 1x}{4}$$

$$5x : 3x$$

$$5 : 3$$

milk      water

$$\frac{1}{2} : \frac{1}{2} = 1 : 1$$

$$\frac{3}{4} : \frac{1}{4} = 3 : 1$$

Or,

|    | Milk         | Water                 |
|----|--------------|-----------------------|
| VI | $1 \times 2$ | $1 \times 2$          |
|    |              | $\overbrace{2}^{x^2}$ |
|    |              | $= 4$                 |

|     | 3     | 1     | $= 4$ |
|-----|-------|-------|-------|
| VII | <hr/> | <hr/> |       |
|     | $5$   | $3$   |       |

Q. Two vessels are full of milk and water in the ratio of 1:3 and 3:5 respectively, if both are mixed in the ratio of 3:2. What is the ratio of milk and water in the new mixture.

$$\begin{array}{r}
 \text{Milk} \quad \text{water} \\
 \hline
 1 \times 24 = 24 \quad 3 \times 24 = 72 \quad 4 \times 8 = 32 \\
 3 \times 8 = 24 \quad 5 \times 8 = 40 \quad 8 \times 4 = 32 \\
 \hline
 48 \quad 112
 \end{array}$$

$$48 : 112$$

@upsc.riceinfinity

$$\begin{array}{r}
 \frac{1 \times 6}{= 6} \quad \frac{3 \times 6 = 18}{\cancel{4 \times 2} \times 3 = 8} \quad \frac{1 \times 6}{= 6} \quad \frac{3 \times 6 = 18}{\cancel{4 \times 2} \times 3 = 8} \quad \frac{2 \times 3}{= 6} \\
 \frac{3 \times 2}{= 6} \quad \frac{5 \times 2 = 10}{\cancel{8 \times 1} \times 2 = 8} \quad \frac{3 \times 2 = 6}{+} \quad \frac{5 \times 2 = 10}{\cancel{8 \times 1} \times 2 = 8} \quad \frac{1 \times 2}{= 2} \\
 \hline
 12 \quad 28 \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}
 \end{array}$$

$$\begin{array}{r}
 \frac{3}{12} : \frac{7}{28} \\
 3 : 7 \\
 \hline
 6 + 6 = 12 \\
 18 + 10 = 28 \\
 \hline
 \frac{3}{12} : \frac{7}{28} \\
 = 3 : 7
 \end{array}$$

or, milk

water

$$\text{VI} \quad \frac{1}{4} \times 3x \quad \frac{3}{4} \times 3x \quad 3x$$

285 115

$$\begin{array}{r} \text{VII} \\ \frac{3}{8} \times 2x \\ \hline \frac{3x}{4} + \frac{3x}{4} \end{array} \quad \begin{array}{r} \frac{5}{8} \times 2x \\ \hline \frac{5x}{4} \end{array} \quad 2x$$
$$\frac{9x}{4} + \frac{5x}{4}$$

$$\frac{6x}{4} : \frac{14x}{4}$$

$$\frac{3}{6} : \frac{7}{14}$$

$$3:7$$

Class Assignment No. 10.

19.

$$\begin{array}{r} 90 \times 2 \\ 180 \end{array} \quad \begin{array}{r} 10 \times 2 \\ 20 \end{array} \quad \begin{array}{r} 100 \times 2 \\ \hline \end{array}$$

$$80 \times 5 \quad 400 \quad 20 \times 5$$

$$70 \times 9 \quad 630 \quad 30 \times 9$$

$$\hline \quad \quad \quad \frac{3}{100} \quad 100 \times 9$$

$$1210 \quad 390$$

$$121 : 39.$$

No. of employees = 9 : 4

Increase their wages = 2 : 5

wage bill = No. of employees  $\frac{9}{X} \times 2 \frac{4}{X} \times 5$

No. of employees =  $\frac{18}{9} : \frac{20}{10}$

= 9 : 10

Original Present

22 : 25

3 : 2

$22 \times 3$

$25 \times 2$

~~66~~

~~33~~

~~50~~

~~25~~

$\cancel{\times 200}$

$\cancel{\times 200}$

~~6600~~

~~5000~~

or, original Present

3 : 2

22 : 25

~~66~~

~~50~~

$66x$

$50x$

$100$

$50x = 500\phi$

$x = 100$

$66 \times 100$

$26600$

$26600$

|                   |                    |
|-------------------|--------------------|
| 300               | 200                |
| 350               | 350                |
| 200               | 600                |
| $\underline{850}$ | $\underline{1150}$ |

$40 \times 5 = 200$   
 $60 \times 5 = 300$   
 $\times 5$

$50 \times 7 = 350$   
 $50 \times 7 = 350$   
 $\times 7$

$75 \times 8 = 600$   
 $25 \times 8 = 200$   
 $\times 8$

$\frac{23}{145\phi} : \frac{17}{85\phi}$

Two candles of the same height are lighted at the same time. The first is consumed in 8 hours and second in 6 hours. Assuming that each candle burns at a constant rate. In how many hours after being lighted, the ratio b/w the first and the second candles becomes 2:1

Let, both the height of the candles be  $h$ .

1st Candle = 8 hours

then in 1 hour =  $\frac{h}{8}$  m/hr.

2nd candle = 6 hours

then in 1 hour =  $\frac{h}{6}$  m/hr.

Then in  $x$  hours it will be :

$$\frac{hx}{8} : \frac{hx}{6} = \left(h - \frac{hx}{8}\right) : \left(h - \frac{hx}{6}\right) :: 2:1$$

$$2 \cdot \frac{hx}{8} = 2 \left(h - \frac{hx}{6}\right)$$

$$2 \cdot h \left(1 - \frac{x}{8}\right) = 2h \left(1 - \frac{x}{6}\right)$$

$$1 - \frac{x}{8} = 2 - \frac{x}{3}$$

$$\frac{x}{3} - \frac{x}{8} = 1$$

$$2 \overline{) 8x - 3x - } \quad 2 \quad |$$

24

$$2 \overline{) 5x \quad 2 \quad 1}$$

24

$$x = \frac{24}{5} = 4\frac{4}{5}$$

or

Two candles

Assignment

$$12 \begin{array}{c|c} 3 & 1 \\ x & x \\ \hline 1 & 3 \end{array}$$

$$3 - x = 2(3 - 3x)$$

$$3 - x = 6 - 6x$$

$$5x = 3$$

$$x = \frac{3}{5}$$

$$2 \quad 1 \text{ hour} = 60 \text{ min}$$

$$\frac{5}{5} = 60$$

$$8 \text{ hours} \quad 6 \text{ hours} \quad 12 \times 3$$

$$\text{LCM of } 8 \text{ & } 6 = 24.$$

$$@upsc.risetinity$$

|  |
|--|
| $\begin{array}{c c} 8 & 6 \\ x & x \\ \hline 3 & 4 \\ 24 & 24 \end{array}$ |
|--|

$$24 - 3x = 2(24 - 4x)$$

$$\Rightarrow 24x - 3x = 48 - 8x$$

$$5x = 24$$

$$x = \frac{24}{5} = 4\frac{4}{5} \text{ hrs.}$$

12.

$$3 \text{ hrs} = 180 \text{ mins}$$

 $\times 1$ 

$$180 - x = 2(180 - 3x)$$

$$180 - x = 360 - 6x$$

$$5x = 180$$

$$x = 36$$

$$1 \text{ hr} = 60 \text{ min}$$

$$\begin{array}{r} \times 3 \\ \hline 180 \end{array}$$

lcm of

123

23

### Class Test.

1. A = 6 ltrs.

B = 10 ltrs

C = 16 ltrs

$$A + B = 6 \text{ ltrs.}$$

$$C = 10 \text{ ltrs.}$$

$$B + C = \underline{\underline{8 \text{ ltrs.}}}$$

2. Boys Girls:

3      2

5

$\times 20$

~~60 x 20~~  
~~120~~

60

40

100

$$60 \times 80 = 48 + 30 = 78$$

~~100~~

$$\text{Scholarship} = 20\% + 80\% + 24\% + 15\% + 25\% + 75\% = \underline{\underline{100}}$$

Old Price

$$= 125$$

Original Price

$$100$$

$$\frac{P \times 1}{Q}$$

$$5$$

$$4$$

New Quantity : Original Qty

$$2 \ 4 : 5$$

$$3 : 4 = 3x & 4x$$

$$\text{Increased by 3} : \frac{3x+3}{4x+3} = \frac{4}{5}$$

$$15x + 15 = 16x + 12$$

$$16x - 15x = 10 \quad 15 - 12$$

$$x = 3$$

History

$$\frac{4 \times 13\phi}{10\phi}$$

Economics

$$\frac{7 \times 12\phi}{10\phi}$$

Mathematics

$$\frac{9 \times 14\phi}{10\phi}$$

$$52 : 84 : 126$$

$$= 26 : 42 : 63$$

6. water Spirit

$$\begin{array}{r} 1 \quad \quad \quad 3 \\ \quad \quad \quad + 1 \\ \hline 1 \quad : \quad 4 \end{array}$$

$\frac{4 \times 251}{100} \\ 4$

8.

$$\frac{4x}{5x-100} = \frac{6}{7}$$

$$28x = 30x - 600$$

$$2x = 600$$
$$x = 300$$

$$B = 4x = 4 \times 300$$
$$= 1200$$

$$G = 5x = 5 \times 300$$
$$= 1500$$

9. Let the first no. be  $x$ .

$$\frac{50}{100}x + y = \frac{4}{3}y$$

$$\frac{x}{2} = \frac{y}{3}$$

$$\frac{x}{y} = \frac{2}{3} \therefore x:y :: 2:3$$

10. Let the no. be

$$5x \quad 4x$$

$$\frac{4p}{8top} \times 8x = 12.6.$$

$$4x = 4 \times 6 \\ = 24$$

$$\frac{5p}{12top} \times 24 = 12 \\ = 12$$

7.  $2A = 3B = 4C$ .

$$A : B : C$$

$$12 : 8 : 6$$

$$\frac{12}{25} \times \frac{100}{2600} = 1200.$$

$$A = B \times C$$

$$B = C \times A$$

$$C = A \times B$$

## AGES & NUMBERS

1. The age of Ramesh is four times the age of Suresh.  
After 10 years age the age of Ramesh will be twice the age of Suresh. Find the present age of Suresh.

| <u>Ramesh</u>       | <u>Suresh</u>      |
|---------------------|--------------------|
| $4x$                | $x$                |
| $4x + 10$           | $x + 10$           |
| $4x + 10$           | $^2 \quad 2(x+10)$ |
| $4x + 10 = 2x + 20$ |                    |

$$4x - 2x = 20 - 10$$

$$2x = 10$$

$$x = 5$$

$$\begin{aligned} \text{Suresh} &= 5 \text{ years.} \\ \text{Ramesh} &= 4 \times 5 \\ &= 20 \text{ years.} \end{aligned}$$

2. The sum of ages of father and son is 50 years.  
Also 5 years ago, the father's age was 7 times the age of the son. The present age of father and son respectively are.

| Father             | Son.         |                           |
|--------------------|--------------|---------------------------|
| $50 - x$           | $x$          | $\text{Son} = 10 - 5.$    |
| $(45 - x)$         | $(x - 5)$    | $^2 5 \text{ years}$      |
| $(45 - x)$         | $= 7(x - 5)$ | $\text{Father} = 50 - 10$ |
| $45 - x = 7x - 35$ |              | $= 40 \text{ yrs.}$       |

$$45 - x = 7x - 35$$

$$8x = 80$$

$$x = 10$$

A father's age is <sup>three times</sup> sum of ages of his two children. 20 years hence his age will be equal to sum of their ages. Then, the father's age is ?

$$\text{Father} = 3(C_1 + C_2)$$

$$\text{Let } C_1 + C_2 = x$$

$$\text{Father} = 3x$$

$$\text{After 20 yrs} = 3x + 20$$

$$\text{And, the children} = x + 20$$

$$x + 20$$

$$3x + 20 = x + 40$$

$$2x = 20$$

$$x = 10$$

$$\text{Father} = 3x$$

$$= 3 \times 10$$

$$= 30 \text{ years}$$

A father said to his son "I was as old as you are at present at the time of your birth. If the father's age is 38 years now, the son's age five years back was ?"

| Father | Son                  |
|--------|----------------------|
| 38     | $x$                  |
| $x$    | $38-x = x-0$         |
| 0      | $2x^2 = 38$          |
|        | $x = 19 \text{ yrs}$ |

At the time of birth  
he was  $0$  years father  
was  $x$  years.

Now father is

Present age of Son  $219 \text{ yrs}$   
five years ago his  
age was  $19-5$   
 $= 14 \text{ yrs}$ .

5. The sum of the ages of 5 members comprising a family three years ago was 80 years. The average age of the family today is the same as it was three years ago, because of an addition of a baby in the intervening period. How old is the baby today?

3 years ago average age of 5 members.

$$\text{The age of family members} = 80 \Rightarrow \frac{80}{5} = 16 \text{ years}$$

increases At present total age of 5 members by three years.

$$(\text{i.e.}) 5 \times 3 = 15 \text{ yrs. } \therefore 80 + 15 = 95$$

At present total age of 6 family members  
 $= 16 \times 6 = 96$

$\therefore$  the age of the baby  $= 96 - 95$

No. 12: 1, 2, 5, 6, 14, 20, 15, 19.

No. 13: 1 to 4.

No. 12: 4, 3.

The age of a mother before two years was 8 times the age of her daughter. After 1 year mother's age will be 5 times the daughter's age. After how many years from now, mother's age will become 3 times the daughter's age?

Ten years ago average age of family of 4 members was 24 years. 3 children having been born the average age of the family today is 22 years. What are the present ages of children, if 2 children are identical twins and differ by 3 years from the younger one?

#### Assignment - 12.

| Present age | Age when she got married |
|-------------|--------------------------|
| $x$         | $x-6$                    |

Today her age is  $x$   $2\frac{1}{4}(x-6)$

$$\Rightarrow x = \frac{5}{4}(x-6)$$

$$\text{Son's age} = \frac{1}{10} \times 30$$

23 yrs.

$$4x = 5x - 30$$

$$x = 30 \text{ years}$$

5. Average age of 5 members = 21 yrs.

$$\begin{aligned} \text{Total age of 5 members} &= 21 \times 5 \\ &= 105. \end{aligned}$$

$$\begin{aligned} 5 \text{ yrs back total age of 5 members} &= 105 - 25 \\ &= 80. \end{aligned}$$

$$\begin{aligned} \text{Avg. age of 4 members} &= \frac{80}{4} \\ &= 20 \text{ yrs.} \end{aligned}$$

6. 3 yrs ago the avg. age of 5

$$\begin{aligned} \text{members} &= 17 \times 5 \\ &= 85 \text{ yrs.} \end{aligned}$$

$$\begin{aligned} \text{At present the total age} &= 5 \times 3 + 85 \\ &= 15 + 85 \\ &= 100 \text{ yrs.} \end{aligned}$$

$$\begin{aligned} \text{At present, avg. age of 6 members} &= 17 \times 6 \\ &= 102 \text{ yrs.} \end{aligned}$$

$$\begin{aligned} \therefore \text{Age of the baby} &= 102 - 100 \\ &= 2 \text{ yrs.} \end{aligned}$$

### Notebook Question.

6. Mother Daughter

$$8x \qquad x$$

$$\text{At present } 8x+2 \qquad x+2.$$

$$\text{After 1 yrs } 8x+3 \qquad x+3.$$

$$\text{After } 8x+3 \qquad 5(x+3)$$

$$8x+3+5(x+3) = 15$$

$$3x = 12$$

$$x = 4$$

$$\text{therefore } 8x = 8 \times 4 \\ = 32 \\ 8x+2 \\ = 34 \\ = 8.$$

Doneghen.

$$3x+y = 3(6+y)$$

$$3x+y = 18+3y$$

$$18 = 2y$$

$$y = 8.$$

10 years ago average age of  
4 family members = 24 years.

$$\text{Total age} = 24 \times 4 \\ = 96 \text{ yrs.}$$

$$\text{Present total age} = 10 \times 4 = 40. \quad \frac{154}{136} \\ \Rightarrow 96 + 40 \\ = 136 \text{ yrs.}$$

Total age of 7 members whose avg. age  
is 22 yrs  $\Rightarrow 22 \times 7 = 154 \text{ yrs.}$

$$\text{The gap is } \frac{154 - 136}{x+x+x-3} = 18 \text{ yrs.}$$

$$3x = 21$$

$$x = 7.$$

$$7, 7, 7-3 = 4$$

7, 7, 4 yrs respectively.

Assignment 112

Aug - of 2 & 4.

3.

$$\begin{array}{r} A \\ \times B \\ \hline 4 \\ ; \quad 3 \\ \hline 3 \quad 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$$

If difference is 1 years.

on adding is 7 years.

$$\text{The difference } 2 \frac{1}{7} \times 6 \frac{9}{7}$$

29 years.

20.9.18:

NUMBERS.

1. A number consists of two digits whose sum is 10. If 18 is subtracted from the number, digits of the number are reversed. What is the product.

Let unit digit be  $x$

Ten's digit be  $y$ .

Original numbers is  $10y + x$ .

After ~~reversing~~<sup>the</sup> digits are reversed

$$10x + y$$

Sum of the 2 digits  $= x + y = 10 \dots ①$

$$10y + x - 18 = 10x + y$$

$$9xy - 9x = 18$$

$$y - x = 2 \quad \text{--- (1)}$$

$$x + y = 10 \quad \text{--- (2)}$$

$$\begin{array}{r} y - x = 2 \\ \hline \end{array} \quad \text{--- (1)}$$

$$2y = 12$$

$$y = 6$$

$$x = 10 - 6 = 4$$

$$x \times y = 6 \times 4$$

$$= 24$$

2. How many numbers of two digits can be formed if the digits of the numbers are interchanged and added to form a perfect square?

Let unit digit =  $x$

Ten's digit =  $y$

Original numbers =  $10y + x$

After the digits are reversed, New numbers

$$= 10x + y$$

$$(10y + x) + (10x + y)$$

$$= 11y + 11x$$

$= 11(y + x)$  should be a perfect square.

$$\rightarrow 11(y+x)$$

|   |   |   |   |    |             |
|---|---|---|---|----|-------------|
| 9 | 2 | 9 | 2 | 29 | } 8 numbers |
| 2 | 9 | 8 | 3 | 38 |             |
| 8 | 3 | 7 | 4 | 47 |             |
| 3 | 8 | 6 | 5 | 56 |             |
| 7 | 4 |   |   |    |             |
| 4 | 7 |   |   |    |             |
| 6 | 5 |   |   |    |             |
| 5 | 6 |   |   |    |             |

121 is a perfect square.

8 two digits number can be formed.

3. How many numbers of 2 digits can be formed, if the digits of numbers are subtracted from the original number a perfect square is formed?

4. How many numbers of 2 digits can be formed, if the digits of numbers are reversed and their differences are taken a perfect cube?

3. Let unit digit be  $x$ .

Ten's digit  $y$ :

$$\text{original numbers} = 10y + x$$

After the digits are

reversed  $= 10x + y$

$$(10y + x) - (10x + y)$$

$$\begin{array}{r} y-x \\ \hline 9x \\ 9x\textcircled{1}=9 \\ 9x\textcircled{9}=81 \\ 9x\textcircled{9}=36 \end{array} \quad 9y - 9x$$

$\Rightarrow 9(y-x)$  is a perfect square.

If difference is 1, if difference is 4, if difference

is 9,

9 8

9 5

8 7

8 4

7 6

7 3

6 5

6 2

5 4

5 1

4 3

4 0

3 2

2 1

1 0

9 0

Ans - 16 numbers when in case of difference  $- 10, 40, 90$  formed will not be counted because it can't be reversed  $\rightarrow$  0 & 1, 4 & 0, 0 & 9, 9 & 0, in case of difference we will leave the 0 digit, in subtraction we will take the numbers.

4.

Let unit digit be  $^2x$

Ten's digit  $^2y$

Original numbers =  $10y + x$

After the digits are reversed,

New numbers =  $10x + y$

Difference

Subtraction

↓  
we ignore gap b/w two  
the sign of numbers  
gap b/w the two  
numbers.  $(10y + x) - (10x + y)$  or  $(10x + y) - (10y + x)$

↓

(-)

$(9y - 9x)$  is a perfect cube.

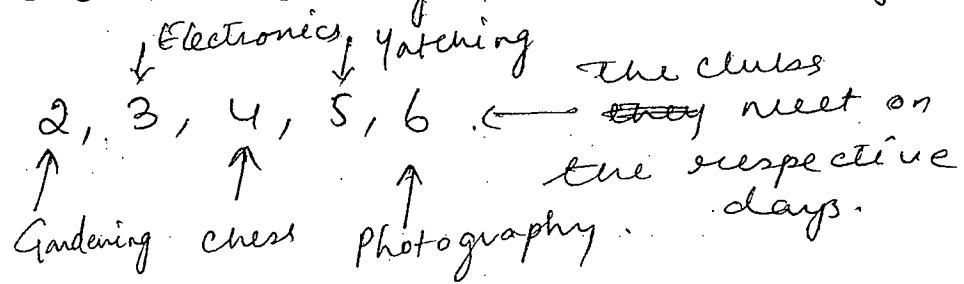
$9(y - x)$  is a perfect cube.

|   |   |    |
|---|---|----|
| 9 | 6 | 69 |
| 8 | 5 | 58 |
| 7 | 4 | 47 |
| 6 | 3 | 36 |
| 5 | 2 | 25 |
| 4 | 1 | 14 |

2 12 numbers.

5. There are five hobby clubs in a college, Photography, yachting, chess, electronics and gardening. The gardening group meets every second day, the electronic group meets every third day, the chess group meets every fourth day, yachting meets every fifth day, photography meets every sixth day.

Ques: How many times all the 5 groups meet on the same day within 180 days.



$$\begin{array}{l} 2 \left( 2, 3, 4, 5, 6 \right) \\ 3 \left( 1, 3, 4, 5, 3 \right) \\ 1, 1, 2, 5, 1 \end{array}$$

$$\begin{array}{c} 3 \\ 180 \\ 60 \end{array}$$

Three times  
Third day they will  
meet within 180 days.

A girl purchased a certain no. of eggs from a shop, while returning to her home, she fell down and some eggs were broken, now she was left with 10 eggs only. She was asked how many eggs she purchased. She could not remember, but she said when it was made 3 <sup>eggs</sup> in a group, 1 egg was left, then she made 4 <sup>eggs</sup> in a group one egg was left, when she made 5 eggs in a group, none was left, how many eggs were broken.

L.C.M. = of 3 & 4

= 12

and remainder = 12 + 1

= 13  $\therefore K = 1, 2, 3, 4, \dots$

$12K + 1$

$= 12 \times 2 + 1$

$= 25$  when divided by

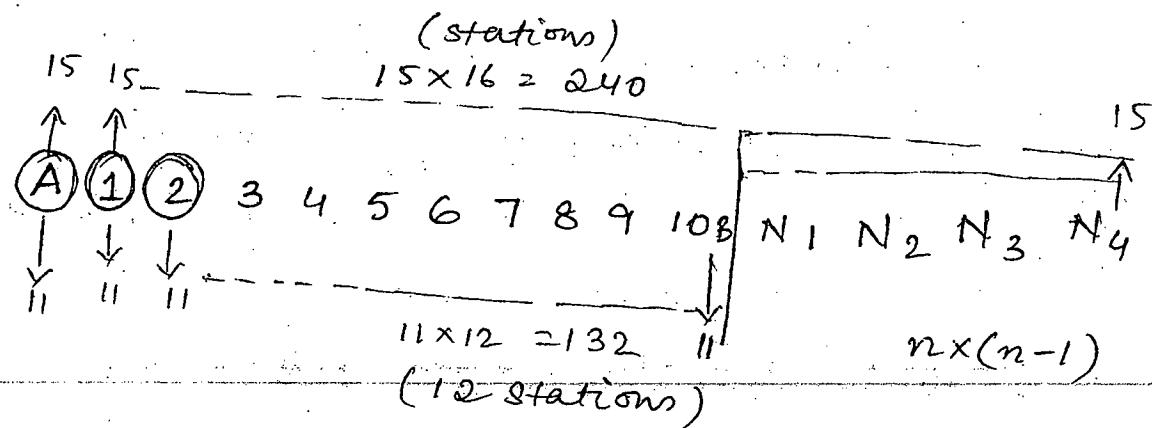
3 & 4 leaves a remainder 1,  
but when divided by 5 perfectly  
divisible.

$25 - 10$

215 eggs broken

Assignment No. 12.

12.



If "n" number of stations are connected to each other and one token is to be issued for one way journey then no. of tokens to be issued will be  $n \times (n-1)$

$$16 \times 15 = 240$$

$$12 \times 11 = 132$$

OR,

|   |       |       |       |       |   |   |   |   |   |    |   |
|---|-------|-------|-------|-------|---|---|---|---|---|----|---|
| A | 1     | 2     | 3     | 4     | 5 | 6 | 7 | 8 | 9 | 10 | B |
|   | $N_1$ | $N_2$ | $N_3$ | $N_4$ |   |   |   |   |   |    |   |
|   | ↓     | ↓     | ↓     | ↓     |   |   |   |   |   |    |   |

$\begin{array}{l} 12_s \quad 13_s \quad 14_s \quad 15_s \\ (12 \times 2) \quad (13 \times 2) \quad (14 \times 2) \quad (15 \times 2) \\ 24 \quad 26 \quad 28 \quad 30 \\ 24 + 26 + 28 + 30 \\ = 108 \end{array}$

\* 12 stations  $\times 2 = 24$ .

## SIMPLE INTEREST

→ Any sum on which interest is charged is Principal

Simple Interest - Bookish Chapter (Theoretical)

Compound Interest - Practical Chapter

$$(R \% \text{ on } P) \times A = S.I.$$

$$(10 \% \text{ on } 100) \times 5 = S.I.$$

$$S.I. = \frac{P \times T \times R}{100}$$

$$\text{Amount} = P + I$$

$$P + \frac{P t r}{100} = P \left(1 + \frac{t r}{100}\right)$$

$$= P \left(\frac{100 + tr}{100}\right)$$

$$P = \frac{100A}{100+tr}$$

|           |                  |           |                  |
|-----------|------------------|-----------|------------------|
| Principal | $\approx$ Rs 100 | Principal | $\approx$ Rs 100 |
| Rate      | $\approx$ 10%    | Rate      | $\approx$ 10%    |
| Time      | $\approx$ 1 yr   | Time      | $\approx$ 2 yrs  |

$$S.I. = \frac{P \times T \times R}{100}$$

$$= \frac{100 \times 10 \times 1}{100}$$

$$\underline{\text{Rs } 10}$$

$$S.I. = \frac{P \times T \times R}{100}$$

$$= \frac{100 \times 10 \times 2}{100}$$

$$\underline{\text{Rs } 20}$$

$$\text{Principal} = \text{Rs } 100$$

$$\text{Rate} = 10\%$$

$$\text{Time} = 3 \text{ yrs.}$$

$$S.I. = \frac{P \times T \times R}{100}$$

$$= \frac{100 \times 10 \times 3}{100} \underline{\text{Rs } 30.}$$

→ S.I. is directly proportional to time if principal and rate are constant.

→ S.I. is equal for every year.

→ In case of S.I. amount grows in the pattern

of arithmetic progression

$$P = \text{Rs } 100.$$

$$R = 10\%$$

$$T = 1 \text{ yr.}$$

$$P = \text{Rs } 100$$

$$R = 20\%$$

$$T = 1 \text{ yrs.}$$

$$S.I = \frac{100 \times 10 \times 1}{100} = \text{Rs } 10$$

$$S.I = \frac{100 \times 20 \times 1}{100} = \text{Rs } 20.$$

S.I is directly proportional to Rate if Principal and time are constant.

S.I is directly proportional to Principal if Time and Rate are constant.

### Assignment -

$$\text{Principal} = \text{Rs } 1200$$

$$\text{Amount} = \text{Rs } 1632$$

$$\text{Interest} = A - P$$

$$= 1632 - 1200$$

$$= 432$$

$$\text{Time} = 4 \text{ yrs.}$$

$$S.I = \frac{P \times T \times R}{100}$$

$$\Rightarrow 432 = \frac{1200 \times 4 \times R}{100}$$

$$R = 9\%$$

If rate increased by  $1\%$  it becomes  $\underline{\underline{10\%}}$ .

$$\text{New Rate} = 10\%$$

$$P = \text{Rs } 1200$$

$$T = 4 \frac{1}{2} \text{ yrs}$$

$$A = P \left( \frac{100 + rt}{100} \right)$$

$$\Rightarrow \frac{1200 \times (100 + 10 \times 4)}{100}$$

$$= 12 \times 140$$

$$= 1680$$

Or,  $1\% \text{ of } 1200 = \text{Rs } 12$ .

Then in 4 yrs it will be

$$= \text{Rs } 12 \times 4$$

$$= \text{Rs } 48.$$

S. I proportionate to Rate %.

$$= \text{Rs } 1632 + 48$$

27

$$= \text{Rs } 1680.$$

$$\text{S.I.} = \frac{P \times 6 \times \frac{12}{100}}{100} + \frac{P \times 9 \times \frac{3}{100}}{100} + \frac{P \times 14 \times \frac{8}{100}}{100}$$

$$11,400 = \frac{P \times 12 + P \times 27 + P \times 56}{100}$$

$$11,400 = \frac{95P}{100}$$

$$\frac{600}{11,400 \times 100} \times 20 \text{ p.} \quad \text{First yr - 2 yrs.}$$

$$= 95\% \quad \text{Rate} = 6\%$$

$$\Rightarrow \text{S.I.B.} = 11,400.$$

$$\text{Second yr} = 3 \text{ yrs.}$$

$$\text{Rate} = 9\%$$

$$\text{S.I.} = 11,400.$$

|  |
|--|
| $\text{Time} = 9 - 2 + 3$<br>$= 4 \text{ yrs}$ |
|--|

For the rest  
of at the  
end = 4 yrs.  
Rate = 14%.

$$\text{S.I.} = 11,400.$$

14. Let Ashok borrowed = Rs.  $x$ .

$$\frac{x \times 2 \times 6}{100} + \frac{x \times 3 \times 9}{100} + \frac{x \times 4 \times 14}{100} = 11,400$$

$$\Rightarrow \frac{12x}{100} + \frac{27x}{100} + \frac{56x}{100} = 11,400$$

$$\frac{95x}{100} = 11,400 - 600$$

$$x = 600 \times 20$$

$$\Rightarrow 12,000$$

or, Let Principal be Rs 100.

$$\begin{aligned} \text{Time Interest} &= 2 \times 2 + 3 \times 9 + 14 \times 4 \\ &= 12 + 27 + 56 = 95. \end{aligned}$$

$$95 = 100.$$

$$11,400 = \frac{100 \times 12}{95} \times 11,400.$$
$$= 12,000.$$

3. Rs 40 on 4 yrs.  
1 = Rs 10.

$$\frac{10}{2000} \times 100 = \frac{1}{2} \%$$

$$\frac{\frac{1}{9} \times 2 \times \frac{1}{9} \times 2 \times 2}{100 \times 25}.$$

r & t  
are equal.

$$r^2 = 25.$$

so it is  
taken r and s.

$$r = 5.$$

$$6. \quad \text{Rs } 100 : \frac{\text{Rs } 30\%}{\text{Rs } 1\%} \text{ Rs } 400.$$

$\therefore r = 10\%$ .

$\therefore t = 30 \text{ yrs.}$

$$\text{Or, } P = x.$$

$$A = 4x.$$

$$I = A - P.$$

$$\therefore 4x - x$$

$$\therefore 3x.$$

$$\text{Rate} = 10\%.$$

$$S. I = \frac{P \times t \times r}{100}$$

$$3x = \frac{x \times t \times 10}{100}$$

$$t = \underline{30 \text{ yrs.}}$$

$$7. \quad \text{Rs } 40 \left( \frac{\text{Rs } 460}{\text{Rs } 500} - \frac{3 \text{ yrs.}}{5 \text{ yrs.}} \right) 2 \text{ yrs.}$$

$$2 \text{ yrs.} = 40\%$$

$$1 \text{ yr.} = \frac{40}{2} = \text{Rs } 20$$

$$3 \text{ yrs.} = \text{Rs } 60.$$

$$P + 60 = 460$$

$$P = 400.$$

↑  $460 - 60$ .

$$R = \frac{20}{2400} \times \frac{5}{100}$$

$$= 5\%$$

11. S.I. in 4 yrs = Rs 150. S.I.  $\propto P$

S.I. in next 4 yrs =  $Rs 150 \times 5$   
Increases 5 times = Rs 750.

S.I. in 8 yrs =  $750 + 150$   
= 900.

15.  $\bar{x}_1 = \frac{28\%}{+21} \bar{x}_2$  S.I.  $\propto R\%$

$$\bar{x}_1 = \frac{1}{4} \bar{x}_2$$

$$x_2 = 8\%$$

17.  $\bar{x}_1 = +4$   $\bar{x}_5 = +4$   $\bar{x}_9$

$$4 + 4 = 8 \text{ yrs.}$$

2 yrs ————— Rs 720. Deposited = Rs P

1 yrs ————— Rs 1020.

—————  
5 yrs ————— Rs 300.

1 yrs —————  $\frac{300}{5}$   
= Rs 60.

2 yrs —————  $60 \times 2$   
= Rs 120.

$$P + 120 = 720.$$

$$P = 600$$

$$R \% = \frac{60}{600} \times 100$$

$$= 10\%$$

4. Let another amount be  $\bar{x}$ .

$$\text{On } \frac{12000 \times 10 \times 1}{100} + \frac{\bar{x} \times 20 \times 1}{100} = \frac{(12000 + \bar{x}) \times 14}{100}.$$

$$120000 + 20\bar{x} = 168000 + 14\bar{x}.$$

$$6\bar{x} = 48000$$

$$\bar{x} = 8000.$$

$$\therefore \text{Total amount} = \text{Rs } 12000 + \text{Rs } 8000 \\ = \text{Rs } 20,000.$$

23. Let the sum lent at 6% p.a be  $x$ .

So,  $\frac{PxRxt}{100}$

$$2 > \frac{x \times 6 \times 5}{100} + \frac{(7000-x) \times 4 \times 5}{100} = 2160.$$

$$30x + 140000 - 20x = 16000 \\ 10x = 20,000 \\ x = 2,000.$$

4.

I.

II.

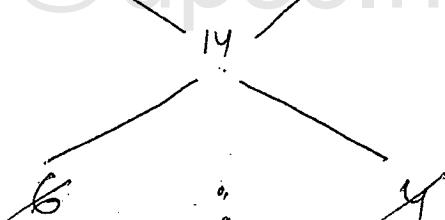
10

20

14

$20 - 14$

26



$$10 - 14 = 6$$

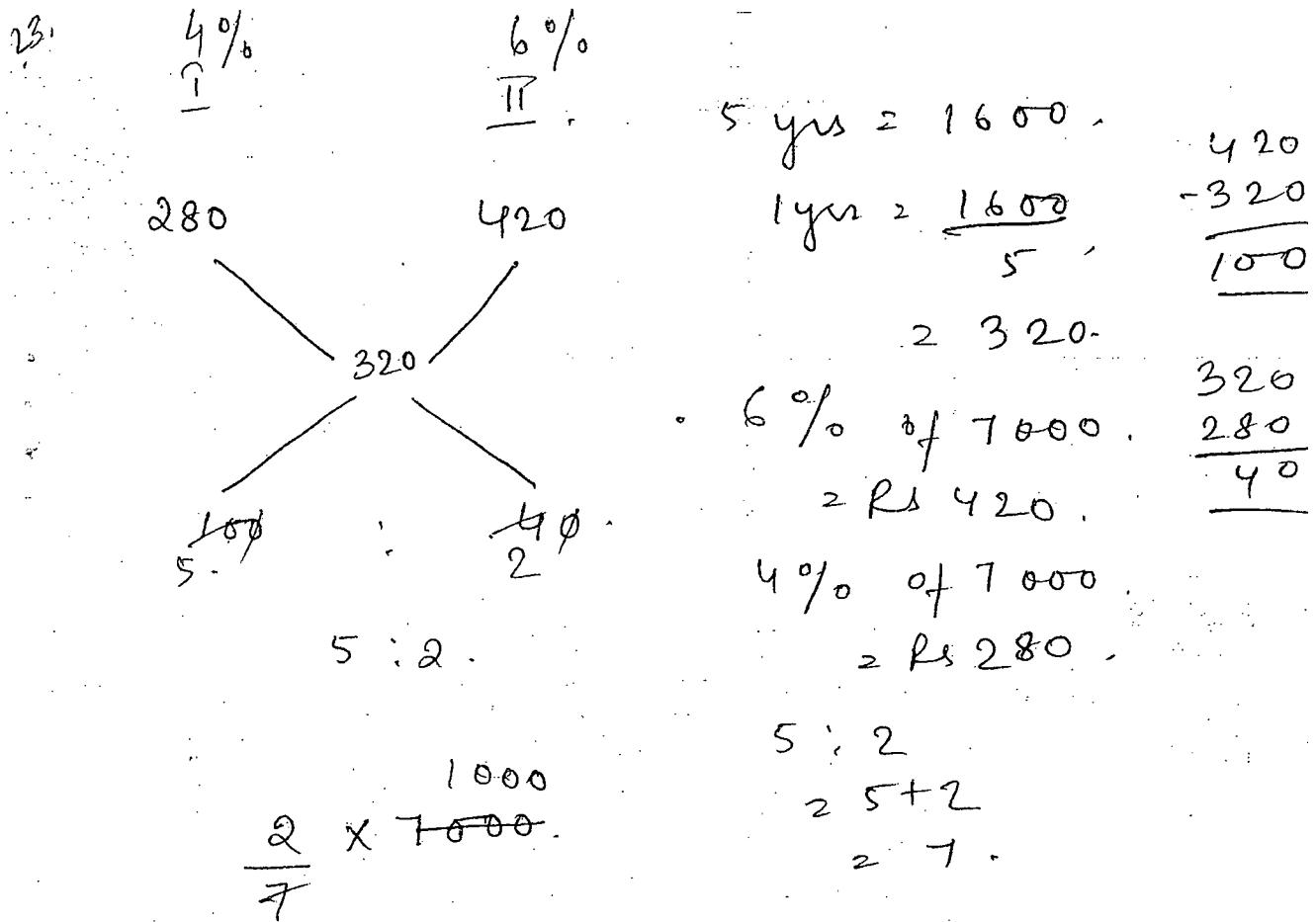
3 : 2

$$3x = 12000.$$

$$x = 4000.$$

$$5x = 5 \times 4000$$

$$= 20,000.$$



Let capital be  $2x$ .      Remainder =  $x - \frac{1}{3}x - \frac{1}{4}x = \frac{12x - 3x - 4x}{12} = \frac{5x}{12}$

$$\frac{1}{3}x \times \frac{1}{100} \times \frac{1}{4}x \times \frac{8+5x}{100/12} \times 10 = 561$$

$$\frac{7x}{300} + \frac{8x}{400} + \frac{50x}{1200} = 561$$

$$\frac{28x + 24x + 50x}{1200} = 561$$

$$12x = 561$$

$$\frac{102x}{600} = \frac{11}{561}$$

$$x = 6600.$$

$$\text{Or, } \frac{1}{3} + \frac{1}{4} = \frac{5}{12} \quad C = 12000.$$

$$7\% \quad 8\% \quad 10\% \quad 102 - 1200$$

$$400 \quad 300 \quad 500$$

$$28 + 24 + 50 = 102$$

$$561 - \frac{600}{1200 \times 561} \times 102$$

$$= 6600$$

## COMPOUND INTEREST

### Simple Interest

14

$$\begin{array}{c} 1440 \\ | \\ x \quad y \quad z \\ \downarrow \\ \frac{x \times x \times 3}{100} \quad = \quad \frac{y \times 3 \times 4}{100} \quad = \quad \frac{z \times 4 \times 5}{100} \end{array}$$

$$\begin{array}{r} 3 \mid 3, 6, 10 \\ 2 \mid 1, 2, 10 \\ \quad \quad \quad 1, 1, 5 \end{array}$$

$$3x = 6y = 10z$$

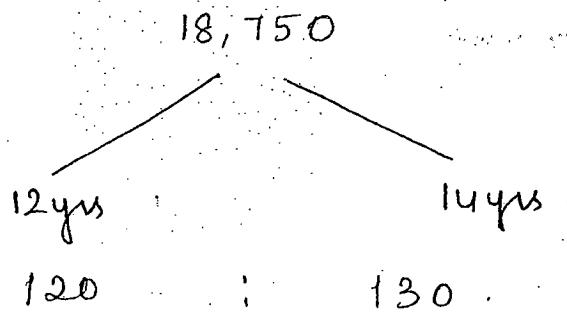
$$x : y : z$$

$$60 : 30 : 18$$

$$10 : 5 : 3$$

$$\therefore \frac{7}{18} \times \frac{80}{1440}$$

$$= 560$$



maturity @ 18 yrs. i.e.,

$$12 + 6 = 18.$$

$$\text{@ } 5\% \text{ interest} = 6 \times 5 \\ = 30.$$

$$\text{Taken as} = 100 \\ = 100 + 30.$$

$$14 + 4 = 18$$

$$\text{@ } 5\% = 4 \times 5$$

$$= 20.$$

$$100 + 20$$

$$= 120.$$

$$12\phi : 13\phi$$

$$750.$$

$$= 12 : 13 \quad 2 \quad \frac{12}{25} \times 18750$$

$$\frac{13}{25} \times \frac{750}{18750} = 9000 \\ = 9750.$$

2.

Suppose

| 2018  | 2019          | 2020          | 2021          | 2022          |       |
|-------|---------------|---------------|---------------|---------------|-------|
| ₹ 100 | ₹ 10<br>₹ 100 | ₹ 10<br>₹ 100 | ₹ 10<br>₹ 100 | ₹ 10<br>₹ 100 | ₹ 140 |
|       |               |               |               |               | ₹ 130 |
|       |               |               |               |               | ₹ 120 |
|       |               |               |               |               | ₹ 110 |
|       |               |               |               |               | ₹ 100 |

If for ₹ 2600 clearing a debt of ₹ 100.  
 $\frac{2600}{100} = 26$

$$\text{₹ 4200} \quad \frac{100 \times 4200}{600}$$

$$= ₹ 700$$

@upco.risefinity  
Class Test Assignment No. 17.

8.

$$\text{Total} = ₹ 25,000.$$

$$\text{first part} = x @ 15\%.$$

$$\text{Second part} = ₹ 25,000 - x @ 18\%.$$

$$\frac{x \times 15 \times 1}{100} + \frac{(25000 - x) \times 18 \times 1}{100} = 4,050.$$

$$\frac{15x}{100} + \frac{25000 - x \times 18}{100} = 4050$$

$$\frac{15x}{100} + \frac{4,50,000}{100} = 4050$$

$$15x + 4,50,000 - 18x = 4,05,000$$

$$3x = 450,000 - 405,000$$

$$3x = 45,000$$

$$x = 15,000$$

$$\begin{aligned}\text{Second part} &= 25,000 - x \\ &= 25,000 - 15,000 \\ &= 10,000.\end{aligned}$$

$$\text{S.I. in 10 yrs} = 600.$$

$$\text{S.I. in 5 yrs} = 300.$$

$$\begin{aligned}\text{S.I. in next 5 yrs} &= 300 \times 3 \\ &= 900.\end{aligned}$$

$$\begin{aligned}900 \text{ next } 5 \text{ yrs} &= 900 + 300 + 900 \\ &= 1200\end{aligned}$$

X

$$= \frac{y \times t \times \%}{100}$$

y

$$\frac{z \times t \times \%}{100} = \frac{x}{y} = \frac{y}{z}$$

$$y^2 = xz$$

$$\frac{1}{3} @ 3\% . \quad \frac{1}{2} @ 8\% .$$

$$\frac{1}{6} @ 6\% .$$

$$\text{Annual income} = \text{Rs } 300.$$

$$\therefore \text{Original sum} = ?$$

Let, the sum be 600

L.C.M of

$$\frac{1}{3} \text{ of } 600 = \frac{1}{3}$$

$$200$$

$$3\%$$

$$\frac{3\% \text{ of } 200}{6}$$

$$6$$

$$\frac{1}{6} = \frac{1}{6} \text{ of } 600$$

$$100$$

$$6\%$$

$$\frac{6\% \text{ of } 100}{6}$$

$$6$$

$$\frac{1}{2} = \frac{1}{2} \text{ of } 600$$

$$300$$

$$8\%$$

$$\frac{8\% \text{ of } 300}{24}$$

$$24 \quad 36.$$

3, 6, 2 and  
since in %  
it is assumed  
as 100.

$$6 \times 100$$

$$= 600.$$

$$\begin{array}{r} 36 \\ \hline 300 \\ \hline 366 \end{array} \quad \begin{array}{r} 600 \\ 100 \quad 50 \\ \hline 600 \times 300 \\ \hline 366 \\ \hline 2 \quad 5000. \end{array}$$

7.

$$\frac{x \times 9 \times 3}{100} - \frac{1150 \times 6 \times 3}{100} = 274.95$$

$$= 1785$$

### Compound Interest

\* Compound interest more closer to % age.

When paying interest simple interest is what we go for.

When taking changing interest

\* Principal changes with every unit of time.

In case of compound interest, principal will change with the change in the next month.

Jan will be common for

Simple and compound after

changing that ~~comes~~, in the month

of Feb. it will change to

compound interest principal amount as well.  
what we go

w.

Suppose, Rs. 10,000  $\times \left( \frac{100+10}{100} \right) + \left( \frac{100+10}{100} \right)$  ... tnos of  
100 years.

$$10,000 \left( \frac{100+10}{100} \right) \left( \frac{100+10}{100} \right) \dots$$
$$10,000 \left( \frac{100+10}{100} \right)^t \quad \downarrow \text{tnos. of 4 years}$$

$$= 10,000 \left( 1 + \frac{10}{100} \right)^t$$
$$F.V = P.V \left( 1 + \frac{r}{100} \right)^t$$
$$A = P \left( 1 + \frac{r}{100} \right)^t$$

Suppose  $r = p = 100$ .

$r = 10\%$  A<sub>1</sub> A<sub>2</sub> A<sub>3</sub> A<sub>4</sub> ..

$$100 \left( 1 + \frac{10}{100} \right)^1 \left( 100 \left( 1 + \frac{10}{100} \right)^2 \right) 100 \left( 1 + \frac{10}{100} \right)^3 100 \left( 1 + \frac{10}{100} \right)^4$$

In case of compound interest, amount grows in the pattern of Geometric Progression.

## COMPOUND INTEREST.

### Class Assignment -

$$4. \quad P = x \quad P = x \\ A = ? 3x \quad A = ? 9x \\ T = ? 3 \text{ yrs.} \quad T = ? \text{ yrs.}$$

$$A = P \left(1 + \frac{r}{100}\right)^t$$

$$3x = x \left(1 + \frac{r}{100}\right)^3 \quad 9x = x \left(1 + \frac{r}{100}\right)^t$$

$$3^2 = \left(1 + \frac{r}{100}\right)^3 \quad \text{squaring both sides.}$$

$$(3)^2 = \left[\left(1 + \frac{r}{100}\right)^3\right]^2 \quad 9 = \left(1 + \frac{r}{100}\right)^t = 6.$$

Or,

Let,  $\frac{21}{\text{in } 3 \text{ yrs}} \frac{23}{\text{in } 3 \text{ yrs.}} \frac{29}{}$

$$80, 3+3$$

$$\underline{\underline{2 \text{ yrs.}}}$$

Compound interest goes in the pattern of geometric progression.

$$A = P \left(1 + \frac{r}{100}\right)^t$$

$$7520 = P \left(1 + \frac{r}{100}\right)^{15}$$

\* Base is same,

difference of

power will be

the answer.

So,

$$\frac{7896}{7520} = P \left(1 + \frac{r}{100}\right)^{16}$$

$$\frac{7896}{7520} = P \left(1 + \frac{r}{100}\right)^{15} =$$

$$\frac{1 + \frac{r}{100}}{7520} = \frac{7896}{7520}$$

$$\frac{r}{100} = \frac{7896}{7520} - 1$$

$$\frac{r}{100} = \frac{376}{7520}$$

$$r = 5\%$$

Or, 15<sup>th</sup> yr. ————— 7520/- interest paid  
 on Rs 7520.  
 16<sup>th</sup> yr. ————— 7896/-  
 ————— 376

In 1 yrs it grew

So,  $\frac{376}{7520}$ , then on 100 it will be,  
 $\frac{376}{7520} \times 100$ .

Rate = 5%  $\neq \frac{376}{7520}$

13.

$$1^{\text{st}} \quad \underline{650}$$

$$2^{\text{nd}} \quad \underline{676}$$

$$\underline{26}$$

$$R = \frac{\frac{2}{26} \times \frac{2}{650}}{5} = 4\%$$

4% is out of 100.

After 1 yrs I will get

$$= 100 + 4$$

$$= 104.$$

$$104 - 100$$

$$650 - \frac{25}{100} \times \frac{80}{650} \cdot 25$$

~~104~~ ~~8~~ yr

$$= \text{Rs } 6.25$$

$$P = \text{Rs } 100$$

| No. of yrs | Bank Simple Interest |      | Bank Compound Interest |      | Difference |
|------------|----------------------|------|------------------------|------|------------|
|            | I                    | II   | III                    | IV   |            |
| R = 10%    | ₹ 10                 | ₹ 10 | ₹ 10                   | ₹ 10 | 0          |

Difference b/w S.I  
and C.I in the  
first year is 0.

100 old P, 10

last year interest

$$= 100 + 10$$

$$\begin{array}{r} 650 \\ 26 \\ \hline 624 \end{array}$$

6, 7, 8, 9, 10 - Assignment

4, 9, 10 - Class Test

$$\frac{5}{100} \times 160$$

1st yrs = \$ \% @ 160.

$$= \frac{5}{100} \times 160$$

$$160 + 8 = 168 . C.I.$$

$$S.I = 160 .$$

$$\text{Difference} = 168 - 160 \\ = 8 .$$

Q. S.I = Rs 80 for

2 yrs i.e,

$$40 + 40 .$$

C.I

$$= 240 + 4\% \text{ of } 40$$

$$= 40 + 1.6$$

$$= 81.60 .$$

$$S.I = 240$$

in 2 yrs = i.e,

$$20 + 20 .$$

$$C.I = 40.80 .$$

in 2 yrs = i.e.

$$(20) + 20 + 0.80 .$$

(1st yr same)

$$R = \frac{8}{20} \times 100 \\ = 4\%$$

$$\frac{4P}{100} = 20$$

$$P = 500$$

8.

$$S.I \quad 160 = 80 + 80$$

$$C.I \quad 160 = 80 + 5\% \text{ of } 80 = 4 \\ \text{i.e., } 80 + 4$$

$$= 164$$

$$164 - 160 \\ = \text{Rs } 4$$

9.

$$\text{Let sum} = \text{Rs } 100$$

$$R = 5\% \text{ p.a.}$$

$$S.I = 5 + 5 = 10$$

$$C.I = 5 + 5\% \text{ of } 5 + 5 = 10.25$$

$$5 + 5.25$$

$\frac{1}{4}$   
 $\frac{1}{4}$   
 $\frac{1}{4}$   
 $\frac{1}{4}$

$$0.25 \quad \underline{\quad} \quad 100 \quad 4$$

$$3 \quad \underline{\quad} \quad \underline{\frac{100}{4}} \times 300$$

$$\cancel{25}$$

$$= 1200$$

Q. Let sum = Rs. 100

$$R = 5\%$$

$$S.I = 5+5 = 10$$

$$C.I = 5+5.25 = 10.25$$

$$\frac{C.I}{S.I} = \frac{10.25}{1000} = \frac{205}{200} = \frac{41}{40} = 41:40$$

Q. Let sum = Rs 100

$$R = 8\%$$

$$S.I = 8+8 = 16$$

$$C.I = 8+8.64 = 16.64$$

$$\begin{array}{rcl} 100 & \xrightarrow{\hspace{1cm}} & 64 \\ 625 & \xrightarrow{\hspace{1cm}} & \frac{64}{100} \times \frac{625}{100} = 400 \\ & & 4 \quad 4 \\ & & \text{Rs } 4 \end{array}$$

### Class- Test

$$P = 100$$

R = 8% p.a payable half yearly

Annual rate = 4% per <sup>6</sup> months

$$100 + 100 \times 6$$

$$2 \times 4 + 4.16$$

$$= 8.16\%$$

10. If  $P = 100$ .

$$4\% = S.I = 4 + 4 = 8.$$

$$C.I = 4 + 4 \cdot 16 = 8 \cdot 16.$$

$$8 \cdot 16 \quad \underline{\quad} \quad 8$$

$$2448 \quad \underline{\quad} \quad \frac{8}{8} \times 2448$$

$$= 2400.$$

### COMPOUND INTEREST.

#### ASSIGNMENT.

1. Difference b/w C.I and S.I for 3 yrs = ~~122~~

$$\frac{P r^2}{(100)^2} \left( \frac{r}{100} + 3 \right)$$

$$122 = \frac{P \times (5)^2}{(100)^2} \left[ \frac{8}{100} + 3 \right]$$

$$2 \frac{122}{4} = \frac{2 \times P}{100 \times 100} \left[ \frac{61}{20} \right]$$

$$2 P = 16000$$

16.  $\frac{64}{8} \times \frac{9}{8} \times \frac{9}{8} = 81$

17. Difference b/w C.I & S.I.

6 : 5

$$\frac{1}{5} \times \frac{20}{100} = 20\%$$

23. Principal  $\equiv P$   
 Rate  $\equiv R\%$  Pa payable half-yearly/  
 k times

Actual rate  $\equiv r/2\%$  here it is  
 $r/k$ .

Time  $\equiv 1 \text{ yrs} \equiv n \text{ yrs}$

Actual time  $\equiv k \times n$   
 $= nk$ .

$$\text{Amount} = P \left( 1 + \frac{r}{k \times 100} \right)^{nk}$$

24. Population =  $x$ .

Increase 20% = 120.

Decrease 20% = 80

beginning of third year population = 5184.

$$x \times \frac{120}{100} \times \frac{80}{100} = 5184$$

$$x \times \frac{120}{100} \times \frac{80}{100} = 5184.$$

### SPEED, TIME AND DISTANCE.

$$\underline{\text{Speed}} \quad \underline{\text{Time}} \quad \text{Distance} = S \times T$$

$$10 \text{ km/hr} \quad 1 \text{ hr} \quad \text{Speed} = \frac{D}{T}$$

$$20 \text{ km/hr} \quad \frac{1}{2} \text{ hr} \quad \text{Time} = D/S$$

$$5 \text{ km/hr} \quad 2 \text{ hr}$$

$$\text{Speed} = \frac{\text{Constant}}{\text{Time}}$$

D = km or m

T = hrs or secs

S = km/hr or m/sec

$$S \propto \frac{1}{T}$$

1 km  $\approx$  1000 m

1 hr  $\approx$  3600 Secs.

$$1 \text{ km/hr} \approx \frac{1000}{3600}$$

$$\approx \frac{5}{18} \text{ m/s.}$$

man walks at  $\frac{3}{4}$  of his normal speed and reaches destination 13 minutes late. What is the normal time taken by man, if he walks at his normal speed.

Let normal time taken  
be  $t$  times.

$$\frac{\frac{3}{4} S}{4} \sim \frac{S}{3} t$$

$$\frac{4}{3} t - t = 13$$

$$t = 39 \text{ minutes}$$

man drives his car at 125% of his normal speed and reaches the destination 15 minutes early. What is the normal time taken by the man to cover the same destination if he drives at a normal speed.

$$125\% \text{ of } S = \frac{125}{100} S = \frac{5}{4} S \sim \frac{4}{5} t$$

$$t - \frac{4}{5} t = 15$$

$$t = 75 \text{ minutes}$$

Example:

H ————— V.R.

Student starting  
at x km/hr.

↓  
increased the speed  
by y km/hr.

t<sub>1</sub> minutes, the class  
already started, that is  
t<sub>1</sub> minutes late.  
↓ now the student is  
t<sub>2</sub> minutes earlier.

Let distance be D km and normal time  
taken be t hours.

$$\frac{D}{x} = t + \frac{t_1}{60} \quad \textcircled{1}$$

$$\frac{D}{y} = t - \frac{t_2}{60} \quad \textcircled{2}$$

$$\frac{D}{x} - \frac{D}{y} = \frac{t_1}{60} + \frac{t_2}{60}$$

7 am he took to reach 1 hr 15 mins.

$$\frac{5}{4} = 1 + \frac{1}{4}$$

$$D \left( \frac{1}{x} - \frac{1}{y} \right) = \frac{t_1 + t_2}{60}$$

$\frac{5}{6} = 1 - \frac{1}{6}$  next day  
he leaves home early

$$D \left( \frac{y-x}{xy} \right) = \frac{t_1 + t_2}{60}$$

\*\* The change in signs,  
if he is late both  
the days.

$$D \left( \frac{xy}{y-x} \right) \left( \frac{t_1 + t_2}{60} \right)$$

Required distance is =  $\frac{\text{Product of Speeds}}{\text{Difference of Speeds}} \times \text{Time Gap}$

Rakesh sets out to cycle from Delhi to Mathura and at the same time Suresh starts from Mathura to Delhi. After passing each other, they complete the journey in 9 and 16 hrs respectively. At what speed does Suresh cycle if Rakesh cycles at 16 km/hr.

$$\begin{array}{ccc}
 \text{Delhi} & 16 \text{ hrs} / \frac{16x}{16x} & \xleftarrow{*} \quad \text{Mathura} \\
 \text{Rakesh} & \xrightarrow{16 \text{ km/hr}} & 9 \text{ hrs} \\
 & & = 9 \times 16 \\
 & & = 144 \\
 & & \xrightarrow{\text{D} = \text{S} \times \text{T}}
 \end{array}$$

$$\begin{array}{c}
 \text{Suresh speed} \Rightarrow \frac{\text{Dist.}}{\text{Time}} = \frac{16x}{16} = \frac{144}{x} \\
 \text{Rakesh Dist.} \quad 16
 \end{array}$$

$$x^2 = 144$$

$$x = 12 \text{ km/hr}$$

### Class Assignment

18.

$$\begin{array}{c}
 \text{1500 km} \\
 \hline
 x & & \frac{1}{2} \\
 \cancel{\frac{4}{3}x} & & \text{on time} \\
 \hline
 \cancel{1500} & = & \cancel{x} \times \cancel{\frac{4}{3}x} \\
 750 & & \cancel{3} \\
 \hline
 \cancel{2x/3} & & \times \frac{1}{2}
 \end{array}$$

$$x = 750 \text{ km/hr}$$

$$\begin{aligned}
 & \therefore \frac{4}{3}x = \frac{4}{3} \times 750 \\
 & \qquad \qquad \qquad = 1000 \text{ km/hr}
 \end{aligned}$$

(or)  $S \propto \frac{1}{T}$

$$t - \frac{3}{4} t = \frac{1}{2}$$

$$t = 2 \text{ hrs}$$

$$S = \frac{1500}{1.5} = 1000 \text{ km/hr}$$

Relative Speed = Speed of one objects w.r.t. others.

Case I → when objects are moving in same direction

Relative Speed = difference of their speeds

Case II → when objects are moving in opposite direction

Relative Speed = sum of their speeds.

### Class Assignment

- II. Policeman goes after a thief who has 100 mts start.

$$Sp = \frac{1 \text{ km}}{8 \text{ mins}} = \frac{1000}{8} = 125 \text{ m/mins}$$

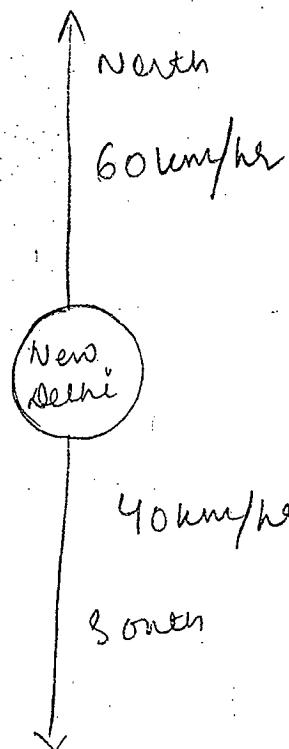
$$St = \frac{1 \text{ km}}{10 \text{ mins}} = \frac{1000}{10} = 100 \text{ m/mins}$$

$$\begin{aligned} \text{Relative Speed} &= 125 - 100 \\ &= 25 \text{ m/mins.} \end{aligned}$$

$$\begin{aligned} \text{Time} &= \frac{D}{R. \text{Speed}} \\ &= \frac{100}{25} \\ &= 4 \text{ mins.} \end{aligned}$$

$$\begin{aligned} \text{Distance covered by thief} &= 24 \times 100 \\ &= 400 \text{ mts.} \end{aligned}$$

12.



$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}}$$

$$= \frac{150}{60+40} = \frac{150}{100}$$

$$= \frac{3}{2}$$

10.

100 mts start

$$\frac{100}{1000} = 0.1 \text{ km}$$

$$S_p = 10 \text{ kmph}$$

$$S_T = 8 \text{ kmph}$$

$$T = \frac{0.1}{20} \text{ hr } 3 \text{ minutes}$$

15.

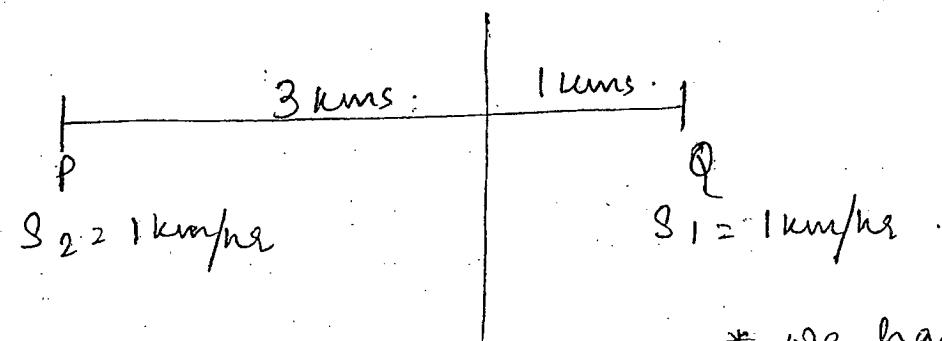
Anjan Speed = 40 kmph.

Starts ahead of Ranee = 5 mins.

$$40 \times 5 \\ = 200 \text{ m}$$

$$T = \frac{200}{50-40} = 20 \text{ mins.}$$

Distance covered by dog =  $20 \times 60$   
= 1200 mts.



1st person started 1 hr earlier.

\* we have taken 3 kms because they cover a distance in 3 hrs.

$$T = \frac{2}{1+1} = \frac{2}{2} = 1 \text{ hrs. } (3-1=2)$$

Time taken = 2 hrs.

Every minutes tree grows jumps = 3 ft.

Slips back = 2 ft.

= 1 ft.

Cliff = 60 ft.

that is = 60 - 3

= 57 ft.

57 ft in 57 mins + 1 min

= 58 mins.

## TRAINS, BOATS AND STREAMS

Time taken to cross insignificant object  
=  $\frac{\text{length of Train}}{\text{Speed of Train}}$

When train has to cover length of the significant object of the time taken will be =  $\frac{\text{length of train} + \text{length of objects}}{\text{Speed of Train}}$

$$\begin{array}{r} 150 \\ 200 \\ 250 \\ \hline 350 \\ 350 \\ 25 \end{array}$$

Moving in same direction - Relative Speed is the difference of their speed

Moving in different direction - Relative Speed is sum of their speed

Time taken by a train to cross a man sitting on another train =  $\frac{\text{length of the train on which the man is not sitting}}{\text{Relative Speed}}$

\* You cannot find out the length on which the person is sitting.

Eg. ~~80~~ km/hr

|     | m/sec |
|-----|-------|
| 9   | 2.5   |
| 18  | 5     |
| 27  | 7.5   |
| 36  | 10    |
| 45  | 12.5  |
| 54  | 15    |
| 63  | 17.5  |
| 72  | 20    |
| 81  | 22.5  |
| 90  | 25    |
| 108 | 30    |
| 126 | 35    |
| 144 | 40    |
| 162 | 45    |

Length of Shatabdi =

LS  
Difference of R.S

$$= \frac{LS}{142-106} = 45$$

$$= \frac{LS}{10} = 45$$

converting it  $\leftarrow 36$   
into m/s.

$$= \frac{LS}{10} = 45$$

$$= L.S = 450 \text{ mts}$$

Length of the Train =

L.S  
Sum of R.S

$$= \frac{L.S}{24+12} = 10$$

$$= \frac{L.S}{36} = 10$$

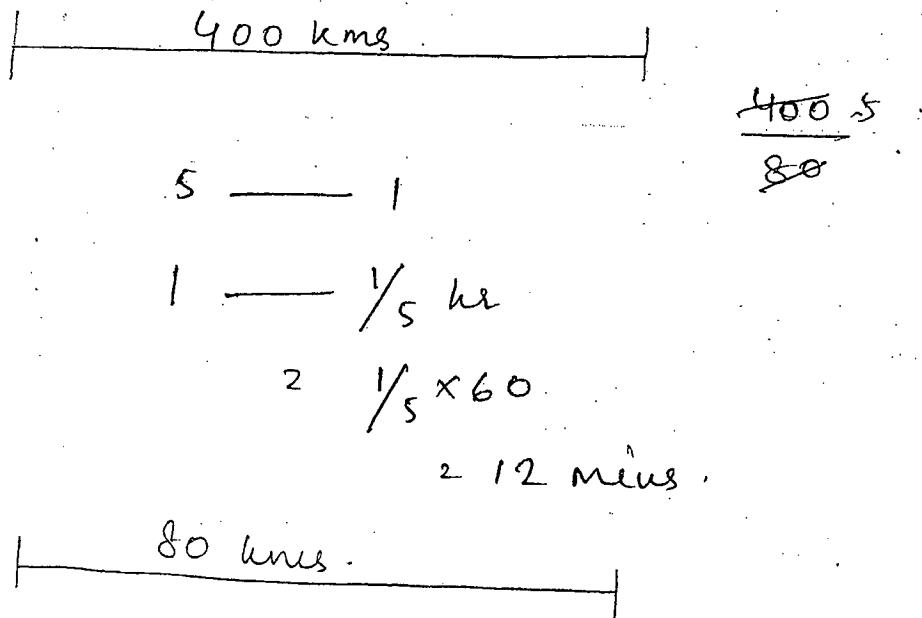
$$= \frac{L.S}{10} = 10$$

$$= \text{Length of Train} = 100 \text{ mts.}$$

A train completes a journey with a few stoppages in between at an average speed of 80 km/hr. If the train had not stopped anywhere it would have completed the journey at an average speed of 100 km/hr. On an average how many minutes per

hour the train stops during the journey.

- \* Assuming a distance that is divisible by 80 and 100 kmph hr.



$$1 = \frac{1}{5} \text{ hr}$$

$$2 = \frac{1}{5} \times 60$$

2 12 mins.

80 kms.

$$\text{time} = \frac{80}{100}$$

$$\begin{array}{r} 6 \times 60 \\ \hline 5 & 272 \\ - & 25 \\ \hline & 20 \\ & - 20 \\ \hline & 0 \end{array}$$

$$2 = \frac{4}{5} \text{ hrs.} \approx \frac{4 \times 60}{5}$$

$$\begin{array}{r} 60 \\ - 48 \\ \hline 12 \end{array}$$

2 48 mins.

$$2 = 60 \text{ min} - 48 \text{ min}$$

2 12 mins.

2.

$$\begin{array}{r} 55 \\ - 35 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 10 \\ 60 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 120 + 80 \\ \hline 55 + 35 \end{array}$$

1, 2, 3, 5, 6, 7, 8,  
10, 11, 12, 13, 14, 15,  
16, 17, 18

$$\begin{array}{r} 200 \\ \hline 90 \end{array}$$

$$\begin{array}{r} 200 \\ \hline 90 \end{array}$$

$$\begin{array}{r} 11 \\ \hline m/s \end{array}$$

$$\begin{array}{r} 200 \\ \hline 40 \end{array}$$

$$\begin{array}{r} 200 \\ \hline 40 \end{array}$$

2 8 seconds.

$$\text{Speed} = \frac{300}{15} \text{ m/sec}$$

$$= 20 \text{ m/sec}$$

converting it to km/hr = 72 km/hr.

$$\frac{192}{32} = 6$$

$$= 6 \text{ kmph}$$

$\frac{2}{2}$  hrs late started after the first train  
and reaches  $\frac{1}{2}$  hr early  $= \frac{2}{2} + \frac{1}{2}$   
 $= 3$  hrs.

$$6 \text{ hrs} - 3 \text{ hrs}$$

$$\text{Train B} = \frac{192}{3} = 64 \text{ km/hr}$$

$$\text{Ratio} = \frac{300}{64} = 32 : 64$$

$$= 1 : 2 \quad 60 + 72$$

192 km

Mumbai

Pune

$$2 \overline{) 72,60}$$

$$2 \overline{) 36,30}$$

$$3 \overline{) 18,15}$$

$$3 \overline{) 6,5}$$

$$2 \overline{) 2,5}$$

$$72 \overline{) 72}$$

$$\frac{36}{6} \text{ p.}$$

$$\frac{6}{2} \text{ p.}$$

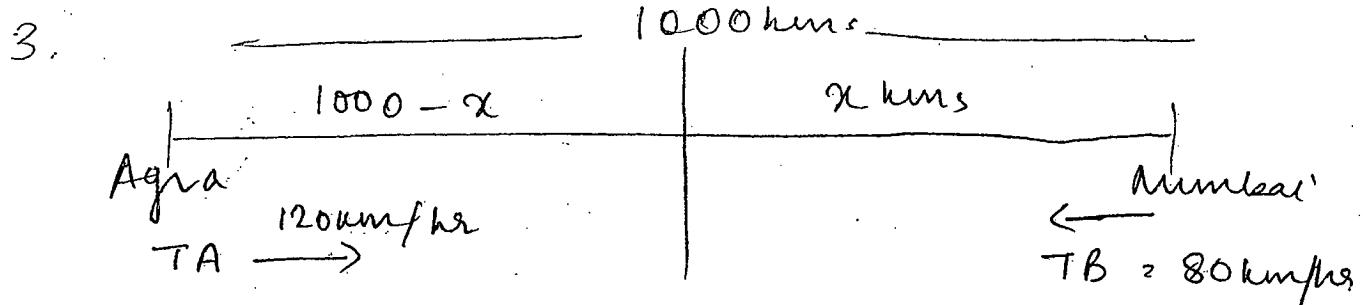
$$\frac{36}{72}$$

3

$$\frac{11}{7}$$

$$\frac{11}{7}$$

$$\frac{2+2+2+2+2+1}{4+6+8+10+12+1}$$



$$\text{Train A} = \frac{1000+x}{120} \quad \text{Train B} = \frac{1000+1000-x}{80}$$

$$2000 + 2x = 3000 + 3000 - 3x$$

$$2000 + 2x = 6000 - 3x$$

$$5x = 4000$$

$$x = 800 \text{ kms}$$

6.

Mumbai Central  
Train A  
= 11 am

Speed = 60 kmph

Train B = 2 pm

Speed = 72 kmph

Alone Train A covered

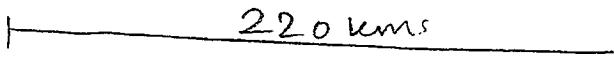
$$2 \frac{60 \times 3}{72} = 11 - 2 \text{ pm}$$

$$2 \frac{180}{72} \text{ hrs}$$

$$T = \frac{\text{Distance}}{\text{R.S.}} = \frac{180}{72-60}$$

$$2 \frac{180}{12} = 15 \text{ hrs}$$

7.  $^2$  5 AM next day.



Train A  
= a kmph

Train B  
= b kmph

$$\frac{220}{a-b} = 11 \Rightarrow a-b = 20 \quad \text{①}$$

220 kms

$$T_a = a \text{ km/hr} \quad T_b = b \text{ km/hr} \quad \frac{220}{a+b} = a+b = 220 \quad \text{--- (1)}$$

$$\frac{a-b}{a+b} = 20 \quad \text{--- (2)}$$

$$\frac{a+b}{a-b} = \frac{220}{20} = 11$$

$$2b = 200$$

$$b = 100 \text{ km/hr}$$

Goods Train → length of goods train +  
Passenger's Train → length of passenger's train  
 $\frac{\text{length of goods train} + \text{length of passenger's train}}{\text{Relative Speed}} = 20$

length of Good Train = 20.

R.S

~~$$\frac{L_G + L_P}{R.S} = 30$$~~

$$\frac{L_G + L_P}{L_G} = \frac{3}{2}$$

~~$$\frac{L_G}{R.S} = 20 \quad 2L_G + 2L_P = 3L_G$$~~

$$3L_G - 2L_G = L_G \quad L_G : L_P = 2 : 1$$

$$2L_G = L_G / L_P = 2 : 1$$

$$= L_G : L_P = 2 : 1$$

10.

$$\frac{L_t + L_p}{S_t} = 10 \text{ sec}$$

$$\frac{L_t}{S_t} = 6 \text{ sec}$$

$$S = \frac{200}{4} = 50 \text{ m/sec.}$$

$$= 180 \text{ km/hr.}$$

Length of Train

$$= \frac{L}{50} \times 6$$

$$= 50 \times 6$$

$$= 300 \text{ mts.}$$

14.

Train A — 5 sec.

Train B — 6 sec.

L.C.M of 5 &amp; 6 = 30 m.

$$\frac{30 \text{ m.}}{5} = 6 \text{ m/s.}$$

$$\frac{30}{6} = 5 \text{ m/s.}$$

$$\frac{30+30}{6-5} = \frac{60}{1}$$

= 60 sec.

Speed of a man in still water =  $x$  km/hr

Speed of water =  $y$  km/hr

Speed in downstream ( $v$ ) =  $(x+y)$  km/hr.

Speed in upstream ( $u$ ) =  $(x-y)$  km/hr.

$$x + y = v$$

$$\begin{array}{r} x - y = u \\ (-) \quad (+) \end{array}$$

$$2y = v - u$$

$$y = \frac{v - u}{2}$$

$$x + y = v$$

$$\begin{array}{r} x - y = u \\ (-) \quad (+) \end{array}$$

$$2x = v + u$$

$$x = \frac{v + u}{2}$$

Speed of man = Speed down stream + Speed up stream

2.

20.

$$25 = 20 + x$$

20, 21, 22, 23

$$20 + x = 25$$

$$21 \cdot \frac{14+5}{2}$$

$$\begin{array}{r} x = 25 - 20 \\ = 5 \end{array}$$

$$2 \cdot \frac{19}{2} =$$

9.5

$$23. \quad \frac{26+11}{2} = \frac{37}{2} \quad 22 \quad \frac{11-7}{2} = \frac{4}{2}$$

= 2 km/hr

If a man takes twice as much time to row upto a certain than to row down. Then speed of man is 3 times that speed of water.

$$\frac{D}{V} = t.$$

$$\frac{D}{u} = 2t \Rightarrow t = \frac{D}{2u}.$$

~~$$\frac{D}{V} = \frac{D}{2u}$$~~

$$= 2u = v$$

$$= 2(x-y) = x+y$$

$$= 2x - 2y = x+y$$

$$= x = 3y.$$

$$\rightarrow 3m = 3 \times \text{Speed water}$$

If a man takes thrice as much time to row upto a certain than to row down. Then speed of man is twice that speed of water.

## Quantitative Aptitude

- Number System
- Percentage
- Profit & Loss
- Time & Work
- Work & Wages
- Pipe & Cistern

6th - 8th Std NCERT

- Number System
- Percentage
- Ratio & Proportion
- Average
- CSAT 2011 - 2018

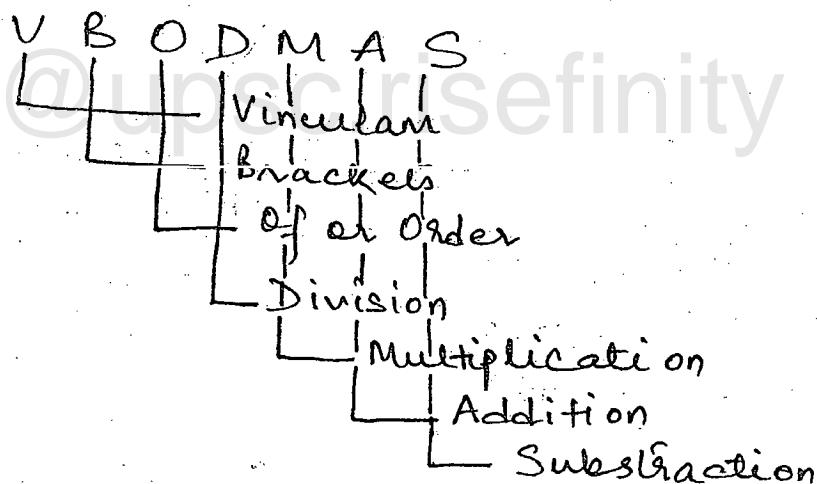
## Reasoning

- Series
- Coding-Decoding
- Character Puzzle
- Number Ranking
- Blood Relations
- Cube & Dice
- Calendar

## Data Interpretation

- Non-verbal reasoning
- General Mental Ability

Rule for



$$2 + 7 \times 5 - 3 \div 6 = (2 + 7 \times 5 - 3 \div 6)$$

$$= 5 + 2 \times 7 + 10 \div 2 + 12 \div 6 + 6$$

$$= 5 + 2 \times 7 + 10 \div 2 + 12 \div 12$$

$$= 5 + 2 \times 7 + 5 + 1$$

$$= 5 + 14 + 5 + 1 = 25$$

$$\frac{80 \times (80+1)}{2} \quad \dots \quad \textcircled{1}$$

$$= 40 \times 81 = 3240$$

$$= \frac{20 \times (20+1)}{2} \quad (11)$$

$$2 \quad 10 \times 21 = 210$$

1-1 3240-210

2 3030

19.7.18.

## Vertical Movement

① 11 Pen = Rs 15.

$$\begin{array}{r} \times 3 \\ \hline 33 \end{array} \quad \begin{array}{r} \text{Pen} \\ \hline 2 \end{array} \quad \begin{array}{r} 15 \times 3 \\ \hline 245 \end{array}$$

② 9 Pencils, 20

$$x 4 \downarrow \quad : \quad \downarrow \\ 36 \text{ Pencils} = 20 \times 4 \\ = 80$$

$$\textcircled{3} \quad 35 \text{ marker} = 60$$

$$\begin{array}{r} \div 5 \\ \downarrow \\ 7 \end{array} \quad \text{Marker?} \quad \begin{array}{r} \downarrow \div 5 \\ \text{P} 60 \div 5 \end{array}$$

④. 4 Dusters = Rs 25

$$12 \text{ Dusters} = 25 \times 3$$

2 75

## Horizontal Movement

⑤. 11  $\xrightarrow{\text{Pen } 2}$  Rs 33.

$$+ 5 \xrightarrow{\times 3} \text{Pens} = 15 \times 3$$

$$\textcircled{6} \quad 7 \xrightarrow{x \rightarrow} \text{Pencils} = \$35$$

$$\begin{array}{r} \xrightarrow{\times 5} \\ 20 \text{ pencils} = 20 \times 8 \end{array}$$

$$\textcircled{7} \quad 12 \xrightarrow{\times 4} \text{markers} = \text{Rs } 48.$$

$$19 \xrightarrow{\times 4} \text{markers} = 19 \times 4 \\ = 76.$$

$$\textcircled{8} \quad 1.150 \text{ kg} \\ @ 15/\text{kg}$$

$$\begin{array}{r} 1 \text{ kg} \\ 100 \text{ gm.} \\ 50 \text{ gm.} \end{array} \left. \begin{array}{l} \text{Breaking} \\ \text{down in} \\ 1.150 \text{ kg} \end{array} \right.$$

$$\begin{array}{r} 15.00 \\ 1.50 \\ .75 \\ \hline 17.25 \end{array}$$

$$\textcircled{9} \quad 1.850 \text{ kg} \\ @ 15/\text{kg}$$

$$\begin{array}{r} 2 \text{ kg} = \text{Rs } 30.00 \\ - 100 \text{ gm.} = 1.50 \\ - .75 \\ 50 \text{ gm.} = \end{array}$$

Rs 32.25

$$\textcircled{10} \quad 49 \times 19$$

$$\begin{aligned} &= (50-1) \times 19 \\ &= 950 - 19 \\ &= 931 \end{aligned}$$

$$\textcircled{11} \quad 11 \% \text{ of } 880$$

$$\begin{array}{r} 10\% = 8.8 \\ 1\% = 8.8 \\ \hline 96.8 \end{array}$$

$$\textcircled{12} \quad 19 \% \text{ of } 760$$

$$\begin{array}{r} 20\% = 152 \\ - 1\% = 7.6 \end{array}$$

13. 12. 5% of 816. 14. 11.11% of 909. 15. 09.09% of 1100.

$$2 \frac{1}{8} \times \frac{102}{8} =$$

$$2 \frac{1}{9} \times \frac{101}{9} =$$

$$2 \frac{1}{11} \times \frac{100}{11} =$$

$$2 \frac{102}{8} =$$

$$2 \frac{101}{9} =$$

$$2 \frac{100}{11} =$$

$$\frac{1}{1} = 100\% = 1.00 \quad \frac{1}{2} = 50\% = 0.50$$

$$\frac{2}{1} = 200\% = 2.00 \quad \frac{2}{2} = 100\% = 1.00$$

$$\frac{3}{1} = 300\% = 3.00 \quad \frac{3}{2} = 150\% = 1.50$$

$$\frac{1}{3} = 33.33\% = 0.33 \quad \frac{1}{4} = 25\% = 0.25$$

$$\frac{2}{3} = 66.66\% = 0.66 \quad \frac{2}{4} = 50\% = 0.50$$

$$\frac{3}{3} = 100\% = 1.00 \quad \frac{3}{4} = 75\% = 0.75$$

$$\frac{4}{4} = 100\% = 1.00$$

$$\frac{1}{5} = 20\% = 0.20 \quad \frac{1}{6} = 16.66\% = 0.166$$

$$\frac{2}{5} = 40\% = 0.40 \quad \frac{2}{6} = 33.33\% = 33.33\%$$

$$\frac{3}{5} = 60\% = 0.60 \quad \frac{3}{6} = 50\% = 0.50$$

$$\frac{4}{5} = 80\% = 0.80 \quad \frac{4}{6} = 66.66\%$$

$$\frac{5}{5} = 100\% = 1.00 \quad \frac{5}{6} = 83.33\%$$

|                             |                        |                         |
|-----------------------------|------------------------|-------------------------|
| $\frac{6}{6} = 100\%$       | $\frac{1}{8} = 12.5\%$ | $\frac{1}{12} = 8.33\%$ |
| $\frac{1}{7} = 14.28\%$     | $\frac{2}{8} = 25\%$   | $\frac{1}{13} = 7.7\%$  |
| $\frac{2}{7} = 28.57\%$     | $\frac{3}{8} = 37.5\%$ | $\frac{1}{14} = 7.14\%$ |
| $\frac{3}{7} = 42.85\%$     | $\frac{4}{8} = 50\%$   | $\frac{1}{15} = 6.66\%$ |
| $\frac{4}{7} = 57.14\%$     | $\frac{5}{8} = 62.5\%$ |                         |
| $\frac{1}{9} = 11.11\%$     | $\frac{6}{8} = 75\%$   | $\frac{1}{16} = 6.25\%$ |
| $\frac{2}{9} = 22.22\%$     | $\frac{7}{8} = 87.5\%$ | $\frac{1}{19} = 5.25\%$ |
| $\frac{3}{9} = 33.33\%$     | $\frac{8}{8} = 100\%$  | $\frac{1}{20} = 5\%$    |
| $\frac{4}{9} = 44.44\%$     | $\frac{1}{10} = 10\%$  | $\frac{1}{21} = 4.75\%$ |
| $\frac{7}{9} = 77.77\%$     | $\frac{2}{10} = 20\%$  |                         |
| $\frac{1}{11} = 0.09, 09\%$ | $\frac{3}{10} = 30\%$  | $\frac{1}{25} = 25\%$   |
| $\frac{2}{11} = 18.18\%$    | $\frac{4}{10} = 40\%$  |                         |
| $\frac{3}{11} = 27.27\%$    | Multiples of 9.        |                         |
| $\frac{4}{11} = 36.36\%$    |                        |                         |
| $\frac{7}{11} = 63.63\%$    |                        |                         |

$$16. 112.5\%$$

$$= 150\% + 12.5\%$$

$$= \frac{1}{1} + \frac{1}{8}$$

$$= \frac{8+1}{8} = \frac{9}{8}$$

$$17. 111.11\%$$

$$= 100 + 11.11\%$$

$$= \frac{1}{1} + \frac{1}{9}$$

$$= \frac{9+1}{9} = \frac{10}{9}$$

$$18. 93.75\%$$

$$= 100\% - 6.25\%$$

$$= \frac{1}{1} - \frac{1}{16}$$

$$= \frac{16-1}{16} = \frac{15}{16}$$

$$19. 125\%$$

$$= 100\% + 25\%$$

$$= \frac{1}{1} + \frac{2}{8}$$

$$= \frac{8+2}{8} = \frac{10}{8} = \frac{5}{4}$$

or,

$$\frac{125}{100} = \frac{5}{4}$$

$$20. x \times 120\% \times 120\% \times 120\% = 4$$

$$x \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} = 4$$

$$x = \frac{216}{125} = 4$$

### ① Type 1. Practice Questions.

$$2D \quad \text{Rectangle} = L \times B = \textcircled{A}$$

$$\text{Square} = D_1 \times D_2 = \textcircled{A}$$

$$\text{Product} \quad P \times C = \textcircled{B}$$

$$\text{Speed} \times \text{Time} = \text{Distance}$$

$$M \times D = W$$

### Percentage Change Graph (P. & Q.)

$$100\% \text{ increase} \xrightarrow{+100\% \uparrow} 200 \uparrow \\ +100$$

$$\text{decrease } 200 \xrightarrow{-100} \frac{200}{-100} = \frac{1}{2}$$

$$= \frac{1}{2} \times 100 \\ = 50\% \downarrow$$

$$100 \xrightarrow{+50\%} \frac{100}{+50} = \uparrow 150\%$$

$$\text{decrease} = \frac{50}{150} = \frac{1}{3} = 33.33\%$$

$$100 + 33.33\% = 133.33 \uparrow \\ + 33.33$$

$$133.33 \xrightarrow{33.33} \frac{133.33}{33.33} = \frac{1}{4} \downarrow \\ 33.33$$

$$100 \xrightarrow{+25\%} \frac{100}{+25} = 125$$

$$100\% \uparrow = \frac{1}{1} \quad 50\% \downarrow = \frac{1}{2}$$

33.33

$$50\% \uparrow = \frac{1}{2} \quad 33.33\% \downarrow = \frac{1}{3}$$

$$33.33\% \uparrow = \frac{1}{3} \quad 25\% \downarrow = \frac{1}{4}$$

$$25\% \uparrow = \frac{1}{4} \quad 20\% \downarrow = \frac{1}{5}$$

= If the length of the rectangle is increased by 40%. Then by what percentage breadth should be decreased so that overall area remains the same.

Method 1:

$$\begin{array}{rcl} 100 & \xrightarrow{-40\%} & 100 - 40 \\ & + 40 \% & \xrightarrow{-40\%} 100 \end{array}$$

$$40\% \text{ of } 100 = 40\%$$

$$100 - 240 = 760$$

$$-40$$

$$= \frac{2}{7} = 28.56\% \downarrow \text{decrease}$$

Method 2:

$$\frac{n}{D} \uparrow \quad \frac{n}{D+n} \downarrow$$

$$\frac{n}{D} = \frac{1}{2} \uparrow \quad \frac{1}{3} \downarrow \frac{1}{2+1} \downarrow \frac{n}{D+n} \downarrow$$

$$\frac{n}{D} \frac{1}{3} \uparrow \quad \frac{1}{4} \downarrow \frac{1}{3+1} \downarrow \frac{n}{D+n} \downarrow$$

$$40\% : \frac{2}{5} \uparrow = \frac{2}{5+2} \downarrow$$

$$= \frac{2}{7} = 28.56 \downarrow$$

If the petrol price is increased by 75%, then by what percentage consumption will be reduced so, that overall budget remains the same.

$$100 \xrightarrow[+75\%]{75\%} 175 \xrightarrow[-75\%]{175} 100$$

$$\quad \quad \quad - 75 \downarrow$$

$$\quad \quad \quad 2.42.85\%$$

If length of the rectangle is decreased by 40%, then by what percentage breadth should be increased so that overall area remains the same.

$$100 \xrightarrow{-40\%} 60 \xrightarrow[+40]{\frac{2}{3} \text{ up}} 100$$

$$\frac{2}{3} = 66.66\% \uparrow \text{ increase}$$

$$\frac{n}{D} \downarrow \quad \frac{n}{D-n} \uparrow$$

40%

$$2 \cdot \frac{2}{5} \downarrow \quad \frac{2}{5-2} \uparrow$$

$$2 \cdot \frac{2}{3} \uparrow = 66.66\%$$

$$\frac{n}{D} \cdot \frac{1}{3} \downarrow$$

$$\frac{1}{2} \uparrow \quad \frac{1}{3-1} \uparrow \frac{n}{D-n} \uparrow$$

Type 2

Type 1 - Product same

Type 2 - Product different.

Change in 1st dimension -  $\pm x$ .

u u 2nd dimension  $\pm y$ .

$$\frac{\pm x \pm y + (\pm x)(\pm y)}{100} \%$$

5.

$$\frac{+10 - 10 + (+10)(-10)}{100} \%$$

$$= \frac{100}{100} \% = 1\%$$

1% decrease.

Method 1

$$100 \xrightarrow[+10]{+10} \frac{110}{-11} = 99$$

2  $100 - 99$

2 1% decrease in the area.

3  $\frac{\pm x \pm y + (\pm x)(\pm y)}{100}$

$$\frac{+10 + 10 + (+10)(-10)}{100}$$

20  $\frac{+100}{100} = 21\%$

Method 2

$$100 \xrightarrow[+10]{+10} 110 \xrightarrow[+11]{+10} 121\%$$

$$= 100 - 121$$

$$= 21\%$$

8

$$-10 - 10 + \frac{(-10)(-10)}{100} \%$$

$$-20 + \frac{100}{100} \% = -19\%$$

$$100 \quad \frac{-10\%}{-10} = 90 \quad \frac{\frac{90\%}{10}}{-10} \% / . \quad 81 \quad \frac{\frac{100}{-81}}{19} \quad \frac{\frac{90}{9}}{-81}$$

2 19%

$$\textcircled{9} \quad \textcircled{9} - + 12 - 12 + \frac{(+12)(-12)}{100} \% / .$$

$$\frac{-144}{100} \% / . - 1.44\% / .$$

$$\textcircled{10} \quad 100 \quad \frac{+10\%}{10} = 110 \quad \frac{-10\%}{-11} = 99$$

2 1%

$$\begin{aligned} & \pm x \pm y \pm z + \frac{(\pm x)(\pm y) + (\pm y)(\pm z) + (\pm z)(\pm x)}{100} \\ & + \frac{(\pm x)(\pm y)(\pm z)}{100 \times 100} \end{aligned}$$

Length increased by 10%.

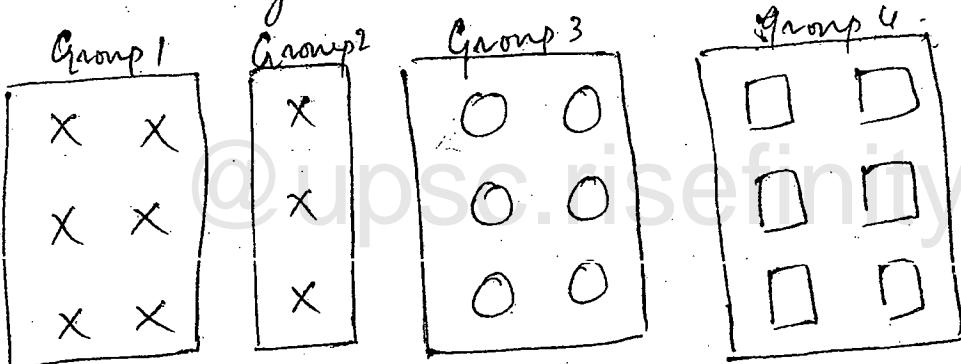
Breadth reduced by 20%.

Height increased by 10%.

Find the % change in the volume of cuboid.

$$\frac{150 + 10\%}{+ 10} - 110 = \frac{-20\%}{- 22} \quad 8 \quad 8 \quad \frac{+10\%}{8 \cdot 8} = 96.8$$

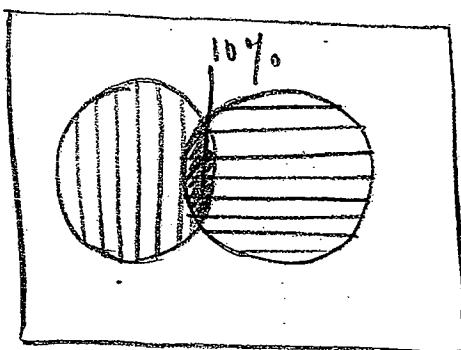
Type 3 Venn Diagram



Failed in one subject =  $G_1 + G_3$ .

Failed in 2 subjects =  $G_2$ .

Passed exactly in one subject =  $G_1 + G_3$ .



2 40%.

2 40% - 10%.

2 30%.

1 failed only Math.

$$\begin{array}{r} 20 \\ - 15 \\ \hline 5 \end{array}$$
  
$$\begin{array}{r} 50 \\ - 45 \\ \hline 5 \end{array}$$
  
$$\begin{array}{r} 96 \\ - 75 \\ \hline 21 \end{array}$$
  
$$\begin{array}{r} 100 \\ - 75 \\ \hline 25 \end{array}$$

$$30\% = 30 - 10\%$$

$$2 \cdot 20\%$$

↳ failed in only Engg.

$$10\% - \text{failed in both}$$

$$\text{So, students failed} = 30\% + 10\% + 20\% \\ = 60\%$$

$$\text{Students passed} = 100 - 60$$

$$= 40\%$$

---

$$x\% \text{ of } y = y\% \text{ of }$$

$$\frac{x}{100} \times y = \frac{y}{100} \times \underline{\quad}$$

$$\frac{xy}{100} = \frac{yx}{100}$$

$$12\% \text{ of } 75 = 75\% \text{ of } 12$$

$$= \frac{3}{4} \times 12^3$$

$$= 9$$

$$96\% \text{ of } \$20 = \frac{20}{100} \times 96$$

$x\% \text{ of } y\% z = z\% \text{ of } y\% \text{ of } -$

$$\frac{x}{100} \times \frac{y}{100} \times \frac{z}{100} = \frac{z}{100} \times \frac{y}{100} \times \frac{x}{100}$$

$$\frac{xyz}{100 \times 100} = \frac{xyz}{100 \times 100}$$

~~12.5% of 20% of 40% of x.~~

method 1: 100,000 students

40% - Prelims 40,000

12.5% - Mains  $12.5 \text{ of } 40,000 = \frac{1}{8} \times 40,000 = 5,000$

20% - Interview  $20\% \text{ of } 5,000$

$$\frac{1}{5} \times \frac{1000}{5000}$$

= 1000 quality.

method 2:

20% of 12.5% of 40% of 1,00,000

$$\frac{1}{5} \times \frac{1}{8} \times \frac{2}{5} \times \frac{20,000}{40,000} \times 1,00,000$$

$$= \underline{\underline{1000}}$$

$$\Rightarrow \text{Max salary} = 1,00,000$$

Party = 20% of 20,000 party exp.

Snacks = 25% = 25% of 20,000

$$\frac{1}{4}, \cancel{20,000} 5000$$

Method 2:

25% of 20% of 1,00,000

$$\frac{1}{4} \times \frac{1}{8} \times \cancel{1,00,000} 5000$$

@upsc.riseinfinity

$\geq$  100 Rupees increase 100%, now if it is

Difference  $\times 100$

Reference

$$\frac{200 - 100}{100} \times 100$$

300

$$= \frac{100}{100} 100\%$$

$$\frac{300 - 100}{100} \times 100$$

100

$$= \frac{200 \times 100}{100}$$

= 200%

2018

1,50,000

2019-

4,50,000

$$\begin{array}{r} 3,00,000 \\ \times 100 \\ \hline 1,50,000 \end{array} \quad 2 \text{ } 200\%$$

Types

Double - 100% ↑

Triple - 200% ↑

4 + times - 300% ↑

N times -  $(N-1) \times 100\% \uparrow$

Type 4:

Salary ↑ - 2.5 times. | Salary ↑ 2.25 times

$$(2.5-1) \times 100$$

$$(2.5-1) \times 100$$

$$1.5 \times 100$$

$$= 150 \uparrow$$

$$(2.25-1) \times 100$$

$$1.25 \times 100$$

$$2 \text{ } 125\% \uparrow$$

2.2

Handout:

1. Two numbers = 20% and 50%.

1<sup>st</sup>      2<sup>nd</sup>      3<sup>rd</sup>.

Let take it as



1st

$$100 + 20$$

$$2120$$

2nd

$$100 + 50$$

$$2150$$

3rd

$$100$$

$$100$$

what % is the first of the second.

$$\frac{120}{150} \times \frac{2}{3}$$

$$= 80\%$$

$$\frac{120}{150} = \frac{4}{5}$$

$$= 80\%$$

what % is the third of second.

$$\frac{100}{300} \times \frac{2}{3}$$

$$\frac{200}{300}$$

$$\frac{100}{300} = \frac{2}{3}$$

$$66.66\%$$

2.

1st

$$20$$

2nd

$$80$$

3rd

$$100$$

$$100 + 20$$

$$100 + 80$$

$$2120$$

$$2180$$

$$100$$

what % of first of the second.

$$\frac{120}{180} = \frac{2}{3}$$

$$= 66.66\%$$

| 3. | 1st                        | 2nd                          | Third |
|----|----------------------------|------------------------------|-------|
|    | 100 - 20                   | 100 + 60                     | 100   |
|    | , 80                       | 160                          | 100   |
|    | $\frac{2}{160} \times 100$ | $\frac{2}{80} = \frac{2}{1}$ |       |
|    | = 200 %                    | = 200 %                      |       |

Type 5

Let the number be 100

Now, dividing it by 5 =  $150 \div 5$

instead of multiplying =  $100 \times 5$   
 $\underline{2} \quad 500$

Required correct value.

% of the result of the required  
correct value - 20% of 500.

$$\left| \begin{array}{l} \begin{array}{r} \frac{4}{20} \times 100 \\ \hline 500 \\ 4\% \end{array} & \begin{array}{r} \frac{120}{25500} = \frac{1}{25} \\ 2 \cdot 4\% \end{array} \end{array} \right.$$

If Error is to be calculated  
upon 100

$$80\% - 20\%$$

$$90\% - 10\%$$

$$20\% - 80\%$$

$$95\% - 5\%$$

So, in this case the error is

$$100\% - 4\%$$

$$= 96\%$$

5. Let the number be 100

Now, dividing it =  $\frac{100}{2}$

$$= 50$$

Multiplying =  $100 \times 2$   
 $= 200$

50% of 200

$$\frac{1}{4} \frac{8\%}{20\%} = \frac{1}{4} = 25\%$$

$$\text{Error} = 100 - 25$$

$$= 75\%$$

## Type 6: Successive Percentage

Method 1

6. Let assume 100 boxes of books in the library.

$$20\% \text{ in Hindi} = 20$$

$$\text{So } 100 - 20 = 80\%$$

50% in English = % in successive  
value

is 50% ~~mean~~ of 80%

$$= 40\%$$

30% off in French.

$$\begin{array}{r} 30 \times 40 \\ \hline 120 \end{array}$$

$$= 12$$

$$= 40 - 12$$

$$= 28 \text{ boxes}$$

The left over books = 6300.

If, 28 boxes contain = 6300

$$1 \text{ box} = \frac{6300}{28}$$

$$100 \text{ boxes contain} = \frac{\frac{6300}{28} \times 100}{4}$$

$$= 22,500$$

Method 2

$$100 - \frac{20\%}{= 20} 80 - \frac{50\%}{= 40\%} 40 - \frac{30\%}{= 12} = 28\%$$

• 28% Regional Languages.

$$28\% = 6300$$

$$1\% = \frac{6300}{28}$$

$$100\% = \frac{\cancel{6300}}{28} \times 100 \\ = 225$$

@upsc.ricefinity

$$\frac{28}{5} \times 72$$

7.

$$100 - \frac{10}{= 10} 90 - \frac{20\%}{= 18\%} \text{ Reqd. } 72 - \frac{20\%}{= 14.4}$$

$$\frac{20}{100} \times 99 \\ = 19$$

$$5) 92 \\ \underline{- 5} \\ 42$$

$$\underline{- 40} \\ 20$$

$$\underline{- 20} \\ 0$$

$$57.6 \quad \frac{10\%}{= 5.76} \quad 51.84\%$$

$$\begin{array}{r} 72.0 \\ 14.4 \\ \hline 57.6 \end{array}$$

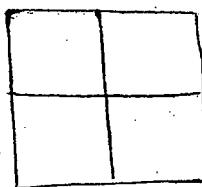
$$51.84\% = 15,552$$

$$\downarrow \quad 1\% = \frac{15552}{51.84} \quad \text{now taken then} \\ 50\% = \sqrt{15,552} \quad \sqrt{51.84} \\ = 15,552$$

$$100\% = 30,000$$

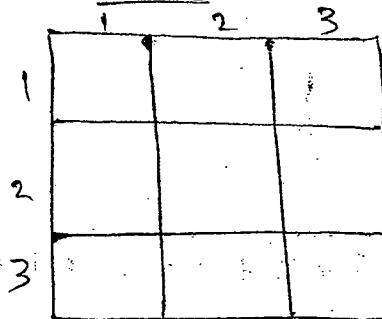
cannot be determined — only if we cannot get answer. None of these option when answer is different from the other values option.

Square →



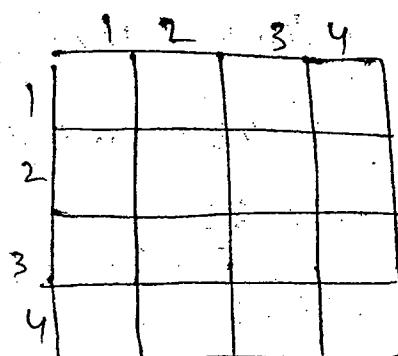
$2 \times 2$

GMA.



$3 \times 3$

$N \times N$



$4 \times 4$

$$N \times N = N \times N + (N-1) \times (N-1) + (N-2) \times (N-2) + \dots$$

$$\begin{aligned} \Rightarrow 3 \times 3 &\Rightarrow 3 \times 3 + 2 \times 2 + 1 \times 1 + 0 \times 0 \\ &9 + 4 + 1 + 0 \\ &= 14 \end{aligned}$$

$$2 \times 2 + 1 \times 1 + 0 \times 0$$

$$4 + 1 + 0$$

$$= 5$$

$$4 \times 4 + 3 \times 3 + 2 \times 2 + 1 + 1 + 0 \times 0$$

$$2 \quad 16 + 9 + 4 + 1 + 0$$

$$2 \quad \underline{\underline{30}}$$

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 |   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |
| 5 |   |   |   |   |   |

$$= 5 \times 5 + 4 \times 4 + 3 \times 3 + 2 \times 2 + 1 \times 1 + 0 \times 0$$

$$= 25 + 16 + 9 + 4 + 1 + 0$$

$$= \underline{\underline{55}}$$

|   | 1 | 2 | 3 |
|---|---|---|---|
| 1 |   |   |   |
| 2 |   |   |   |
| 3 |   |   |   |
| 4 |   |   |   |
| 5 |   |   |   |

No. of column =  $n$

No. of row =  $m$ .

$$5 \times 3.$$

$$n \times m =$$

$$n \times m + (n-1) \times (m-1) + (n-2)(m-2) \dots$$

$$= 5 \times 3 + 4 \times 2 + 3 \times 1 + 2 \times 0$$

$$= 15 + 8 + 3 + 0$$

$$= \underline{\underline{26}}$$

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
|   |   |   |
|   |   |   |
|   |   |   |
|   |   |   |

$$4 \times 3 =$$

$$\begin{aligned} 4 \times 3 + 3 \times 2 + 2 \times 1 \\ 12 + 6 + 2 \\ = 20. \end{aligned}$$

|   |   |   |   |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
|   |   |   |   |
|   |   |   |   |
|   |   |   |   |
|   |   |   |   |

$$5 \times 4$$

2

$$\begin{aligned} 5 \times 4 + 4 \times 3 + 3 \times 2 + 2 \times 1 + 1 \times 0 \\ = 20 + 12 + 6 + 2 + 0 \\ = 40 \end{aligned}$$

## Rectangle

$$N \times N = (1+2+3+\dots+n) \times (1+2+3+\dots+n)$$

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
|   |   |   |
|   |   |   |
|   |   |   |
|   |   |   |

$$3 \times 3 = (1+2+3) \times (1+2+3)$$

$$2 \quad 6 \times 6$$

$$2 \quad 36.$$

|  |  |
|--|--|
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

$$\begin{aligned} 2 \times 2 &= (1+2) \times (1+2) \\ &= 3 \times 3 \end{aligned}$$

$$2 \quad 9.$$

|   |   |   |   |   |
|---|---|---|---|---|
|   | 1 | 2 | 3 | 4 |
| 1 |   |   |   |   |
| 2 |   |   |   |   |
| 3 |   |   |   |   |
| 4 |   |   |   |   |

$4 \times 4 \rightarrow$

$$(1+2+3+4) \times (1+2+3+4)$$

$$10 \times 10$$

$$= 100$$

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 1 | 2 | 3 | 4 | 5 |
| 1 |   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |
| 5 |   |   |   |   |   |

$5 \times 5$

$$(1+2+3+4+5) \times (1+2+3+4+5)$$

$$15 \times 15$$

$$= 225$$

|   |   |   |   |
|---|---|---|---|
|   | 1 | 2 | 3 |
| 1 |   |   |   |
| 2 |   |   |   |
| 3 |   |   |   |
| 4 |   |   |   |
| 5 |   |   |   |

$n \times m$

$$\rightarrow (1+2+3+\dots+n) \times (1+2+3+\dots+m)$$

$5 \times 3$

$$(1+2+3+4+5) \times (1+2+3)$$

$$15 \times 6$$

$$= 90$$

|   |   |   |   |
|---|---|---|---|
|   | 1 | 2 | 3 |
| 1 |   |   |   |
| 2 |   |   |   |
| 3 |   |   |   |
| 4 |   |   |   |

$$4 \times 3$$

$$(1+2+3+4) \times (1+2+3)$$

$$\begin{aligned} & 10 \times 6 \\ & = 60 \end{aligned}$$

|   |   |   |   |   |
|---|---|---|---|---|
|   | 1 | 2 | 3 | 4 |
| 1 |   |   |   |   |
| 2 |   |   |   |   |
| 3 |   |   |   |   |
| 4 |   |   |   |   |
| 5 |   |   |   |   |

$$(1+2+3+4+5) \times (1+2+3+4)$$

$$\begin{aligned} & 15 \times 10 \\ & = 150 \end{aligned}$$

@upsc.risefinity

There are five vertical and five horizontal lines.  
Find what are the maximum no. of squares possible.

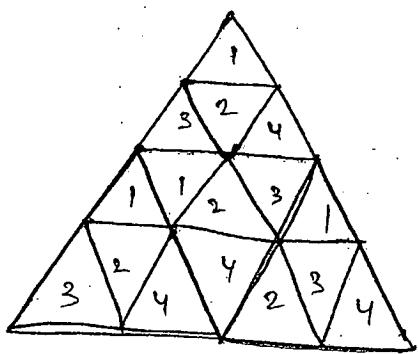
(5x5) →

$$(4 \times 4) + (3 \times 3) + (2 \times 2) + (1 \times 1) \times (0 \times 0)$$

$$16 + 9 + 4 + 1 + 0$$

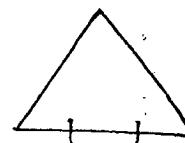
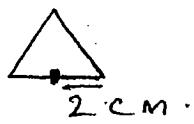
∴ 30

|   |   |   |   |   |
|---|---|---|---|---|
|   | 1 | 2 | 3 | 4 |
| 1 |   |   |   |   |
| 2 |   |   |   |   |
| 3 |   |   |   |   |
| 4 |   |   |   |   |

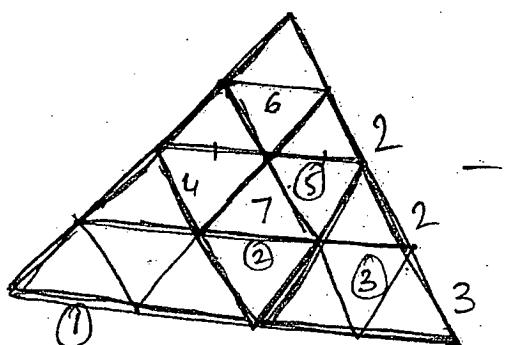


Total = 21.

$$\Delta = 16.$$



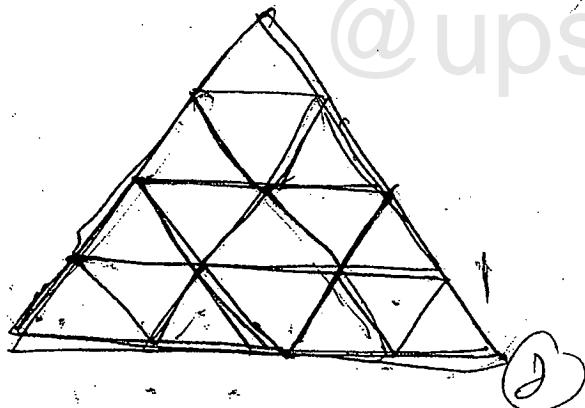
3 Triple base.



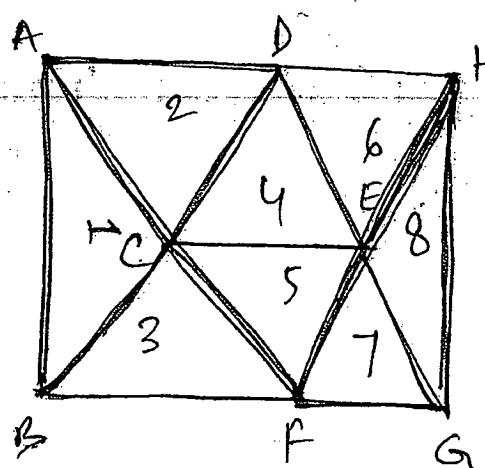
~~$$2 \times 0$$~~

$$2+2+3,$$

2 7 Double base.



$$2+1 \\ 2 \underline{+} 3.$$



Individual 8 triangle.

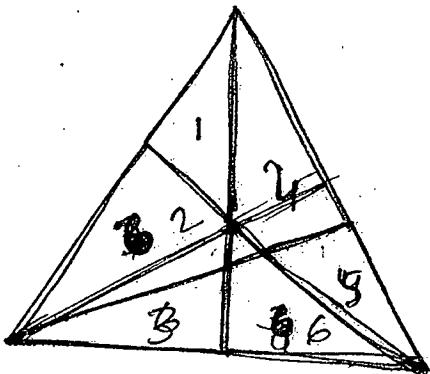
$$\text{Combined} = (1+2)(1+3)$$

$$(6+8)(7+8) = 4 \text{ possibility}$$

4 at a time

$$= (2+4+5+6)(3+4+5+7) \\ (2)$$

$$2 8+4+2+11$$



Individual - 6.

$$\text{Combined } (1+7+6)(2+3+4) \\ = 2$$

$$(2+4)(4+7)$$

$$(1+2)(3+6)(4+5)$$

$$= 3$$

A  $(1+2+3)(4+5+6)$

B  $(2+1+3)(3+6+5)$

C  $(2+3+6)(1+4+5) = 6$

Big Triangle = 1

@upsc.risefinity

1. A has Rs 20.

40% of Total bus fare.

$$\frac{2}{5} \text{ of } 20 =$$

$$20 \times \frac{5}{2} = 50$$

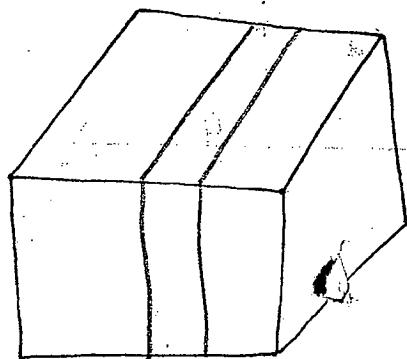
B handed over Rs 30 to A.

$$20 + 30$$

$$= 50$$

## Cube And Dice

1. Cutting of cube
2. Colouring of cube
3. Problem based on dice.



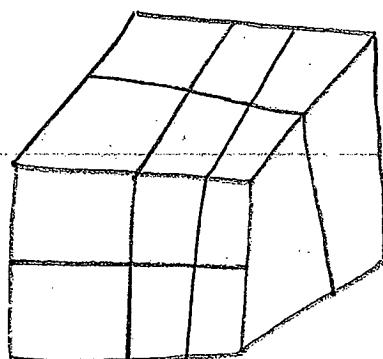
### Properties :

- Every cube has 8 vertices
- Every cube has 12 edges
- Every cube has 6 faces
- Every vertex in a cube is having 3 faces adjacent to it.
- Every edge in a cube is having two faces adjacent to it.
- Every cube is having 3 pairs of opposite faces.

### Cut

### Pieces

|       |         |
|-------|---------|
| 1     | 2       |
| 2     | 3       |
| 3     | 4       |
| 4     | 5       |
| $n$   | $(n+1)$ |
| $1+1$ | 4       |
| $1+2$ | 6       |



$$1+3$$

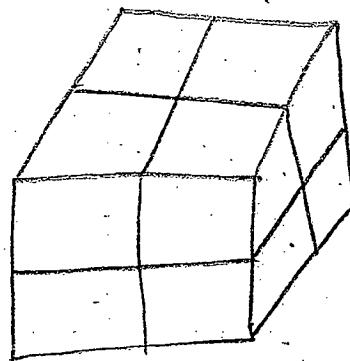
$$1+4$$

$$1+1+1$$

$$8$$

$$10$$

$$8$$



Cut

pieces

$$\times \text{ } 4 \text{ } z$$

$$n + m + p$$

$$\times \text{ } 4 \text{ } z$$

$$(n+1) \times (m+1) \times (p+1)$$

Total number of  
cut we add

Total number of  
pieces we multiply

$$(1+1) \longrightarrow (1+1) \times (1+1) \\ = 4$$

$$(1+2) \longrightarrow (1+1) \times (2+1) \\ @ = 2 \times 3 \\ = 6$$

$$1+3 \longrightarrow (1+1)(3+1) \\ = 2 \times 4 \\ = 8$$

$$1+1+1 \\ = (1+1) \times (1+1) \times (1+1) \\ = 2 \times 2 \times 2 \\ = 8$$

$$6$$

$$7$$

$$5+1$$

$$(5+1) \times (1+1) \quad 4+2 = (4+1) \times (2+1) \\ = 6 \times 2 \quad = 5 \times 3 \\ = 12 \quad = 15$$

$3+3$

$$(3+1) \times (3+1)$$

$$4 \times 4$$

$= 16$  — max. pieces possible

$4+1+1$

$$= (4+1) \times (1+1) \times (1+1)$$

$$= 5 \times 2 \times 2$$

$$= 20$$

$3+2+1$

$$= (3+1) \times (2+1) \times (1+1)$$

$$= 4 \times 3 \times 2$$

$$= 24$$

$$4 \times 4 \times 2$$

$3+3+1$

$2+2+2$

$$= (2+1) \times (2+1) \times (2+1)$$

$$= 3 \times 3 \times 3$$

$= 27$  — max. pieces possible

$$2 \times 2 \times 2$$

$$3 \times 3 \times 3$$

$$= 27$$

$$\begin{array}{r} 25 \\ \underline{-6} \\ 20 \\ \underline{-15} \\ 5 \end{array}$$

Cut ~~8~~

Min. no. of pieces      Max. Pieces

3  $(1+1+1)$

4

$$2 \times 2 \times 2 = 8$$

6  $(2+2+2)$

7

$$3 \times 3 \times 3 = 27$$

$$\begin{array}{r} 36 \\ \underline{-6} \\ 30 \\ \underline{-15} \\ 15 \end{array}$$

9  $(3+3+3)$

10

$$4 \times 4 \times 4 = 64$$

12  $(4+4+4)$

13

$$5 \times 5 \times 5 = 125$$

15  $(5+5+5)$

16

$$6 \times 6 \times 6 = \frac{216}{400}$$

18  $(6+6+6)$

19

$$7 \times 7 \times 7 = 343$$

Cut Min - Max

$$10 \rightarrow 4+4+2 \\ \searrow 3+3+4$$

$$5 \times 5 \times 3 \\ = 75$$

$$4 \times 4 \times 5 \quad \text{nearest} \\ = 80 \quad \text{multiple of 3}$$

$$14 \rightarrow 5+5+4$$

$$6 \times 6 \times 5 \\ = 180$$

$$11 \rightarrow 4+4+3$$

$$= 5 \times 5 \times 4 \quad \begin{array}{l} \text{no. of pieces we} \\ \text{multiply} \\ \text{no. of cuts we} \\ \text{add} \end{array} \\ = 100$$

$$7 \rightarrow \cancel{3} \times \cancel{3} \times 2$$

$$2+2+3$$

$$\cancel{2} \times \cancel{2} \times 3 \\ = 3 \times 3 \times 4 \\ = 36$$

$$8 \rightarrow 2 \ 3+3+2$$

$$4 \times 4 \times 3 \\ = 48$$

| <u>Cut</u>   | <u>Pieces</u> | <u>Cuts</u> | <u>Pieces</u>         | <u>Cuts</u>               |
|--------------|---------------|-------------|-----------------------|---------------------------|
| 10           |               |             | $\times 4 \cancel{2}$ | $\cancel{X} 4 \cancel{2}$ |
| $5 \times 2$ | $4+1$         | $= 2$       | $n \ m \ p$           | $(n-1)(m-1)(p-1)$         |
|              |               |             | $n \times m \times p$ | $(n-1)*(m-1)*(p-1)$       |

$$20 \quad 19 - \text{max.}$$

$$2 \ 10 \times 2 = 9+1 = 10$$

$$5 \times 4 = 4+3 = 7$$

$$5 \times 2 \times 2 = 4+1+1 = 6 - \text{min.}$$

$$24 \quad \textcircled{3} \quad 12 \times 2$$

$$\quad \quad \quad 8 \times 3$$

$$11 \cancel{\times} 1 = 12 \quad \text{clustering we look for lesser multiple}$$

$$7 \cancel{\times} 2 = 9$$

$$= 2 \times 2 \times 2 \times 3$$

$$= 4 \times 2 \times 3$$

$$= 3 \times 1 \times 2$$

$$= 3+1+2 = 6$$

$$100 = 4 \times 25$$

$$= 2 \times 2 \times 5 \times 5$$

$$= 2 \times 2 \times 5 \times 5$$

$$= 4 \times 5 \times 5$$

$$= 3+4+4$$

$$= 11$$

$$\begin{array}{r} 2 \longdiv{36} \\ 2 \longdiv{18} \\ 3 \longdiv{9} \\ \hline 3 \end{array}$$

$$36 = 2 \times 2 \times 3 \times 3$$

@upsc.riseinfinity

$$= 4 \times 3 \times 3$$

$$= 3+2+2$$

$$= 7$$

$$\begin{array}{r} 2 \longdiv{48} \\ 2 \longdiv{24} \\ 2 \longdiv{12} \\ 2 \longdiv{6} \\ 3 \longdiv{3} \\ \hline 1 \end{array}$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$= 4 \times 4 \times 3$$

$$= 3+3+2$$

$$= 8$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$= 2 \cancel{\times} 6 \times 5 = 4 \times 3 \times 5$$

$$= 2 \cancel{+} 5 \cancel{+} 4 = 3+2+4$$

$$= 1$$

$$= 9$$

$$\begin{array}{r} 2 \longdiv{60} \\ 2 \longdiv{30} \\ 3 \longdiv{15} \\ \hline 5 \end{array}$$

72

1000 2

$$\begin{array}{r} 2/72 \\ 2 \cancel{3}6 \\ 2/18 \\ 3 \cancel{9} \\ 3 \end{array}$$

$$= 2 \times 2 \times 2 \times 3 \times 3$$

$$= 10 \times 10 \times 10$$

$$= 4 \times 2 \times 3 \times 3$$

$$= 9 + 9 + 9$$

$$= 4 \times 6 \times 3$$

$$= 27$$

$$= \cancel{8}0 \quad 3 + 5 + 2$$

$$= 10$$

216

$$= 6 \times 6 \times 6$$

$$= 5 + 5 + 5$$

$$= 15$$

512

$$= 8 \times 8 \times 8$$

$$= 7 + 7 + 7$$

$$= 21$$

| <u>Pieces</u> | <u>Max. cut</u> | <u>Min. cut</u> |
|---------------|-----------------|-----------------|
|---------------|-----------------|-----------------|

5

4

4

For prime number of pieces max. and min. number of pieces are always same.

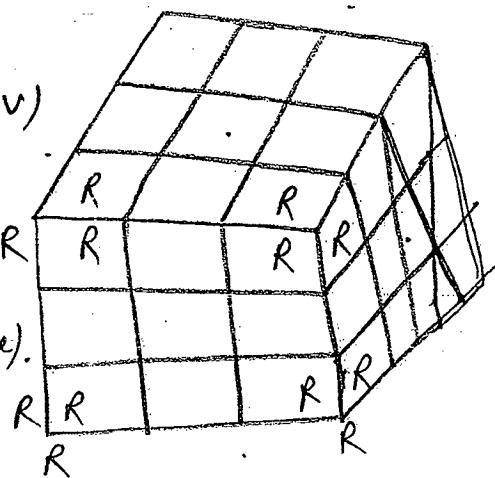
## Colouring of cube

3 faces colour red -  $1 \times 8$  (v)

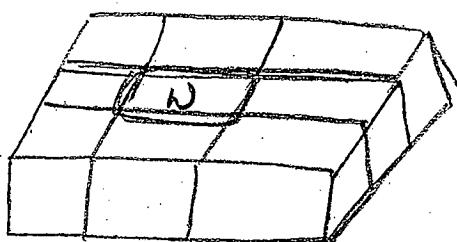
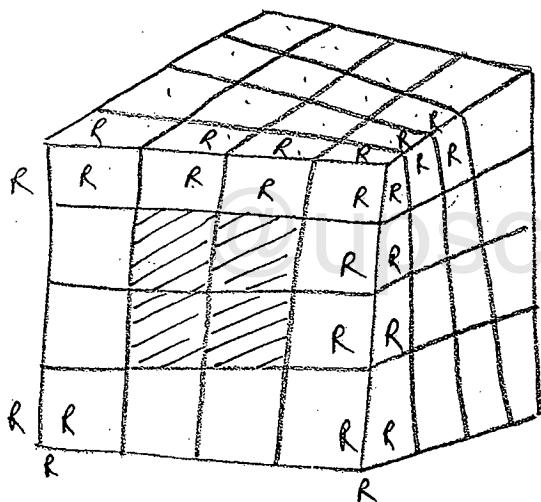
2 faces colour red -  $(1 \times 12)$  (e) R

1 face colour red -  $(1 \times 6)$  (face).

0 face colour red 1.



27



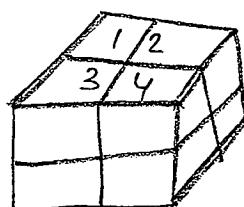
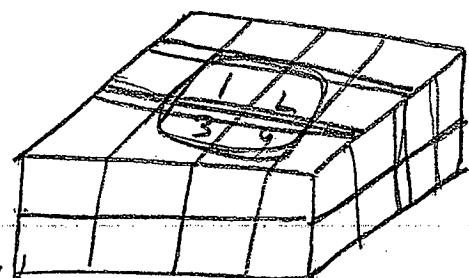
$$16 = \underline{\underline{64}}$$

3 faces colour red -  $1 \times 8$  (v)

2 faces colour red -  $2 \times 12$  (e)

1 face colour red -  $4 \times 6$  (face)

0 face colour red = 8.



$$\frac{125}{\underline{\underline{125}}} \quad \frac{216}{\underline{\underline{216}}} \quad \frac{(5^3)}{(6)^3} \quad \frac{343}{\underline{\underline{343}}} \quad \frac{(7)^3}{N^3}$$

faces colour red

$$8 \quad 8 \quad 8 \quad 8$$

faces colour red

$$(3 \times 12) \quad (4 \times 12) \quad (5 \times 12) \quad (N-2) \times 12$$

face colour

~~$$6000$$~~

$$3^2 \times 6 \quad 4^2 \times 6 \quad 5^2 \times 6 \quad (N-2)^2 \times 6$$

0 face colour red

$$3^3 \quad 4^3 \quad 5^3 \quad (N-2)^3$$

All the six faces of a cube painted in green colour. After that the cube is cut into 1000 small equal cubes. How many are

3 faces colour ~~and~~ green 8

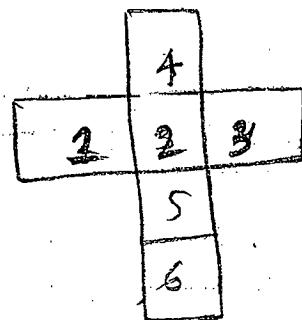
2 faces colour green  $(10-2) \times 12 = 96$

1 face "  $(10-2)^2 \times 6 = 8^2 \times 6 = 384$

0 face "  $(10-2)^3 = 8^3 = 512$

## Construction of Boxes

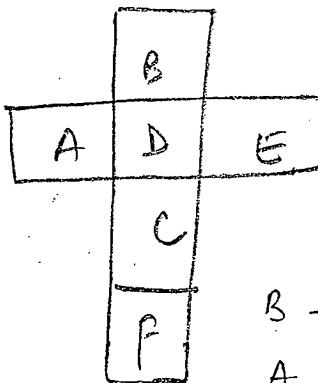
Type 1: When in a column or row alternate faces are always opposite.



Mon — Wed      1 — 3

4 — 5

2 — 6



1 — 3

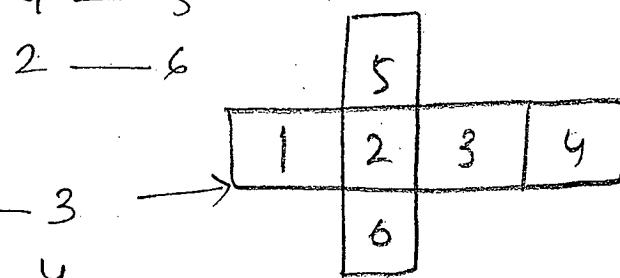
2 — 4

5 — 6

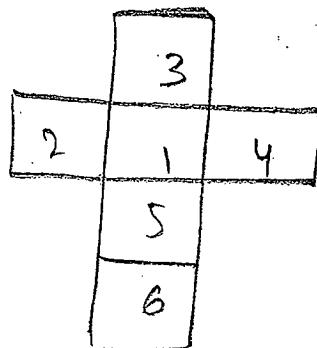
B — C

A — E

D — F



Type 2:

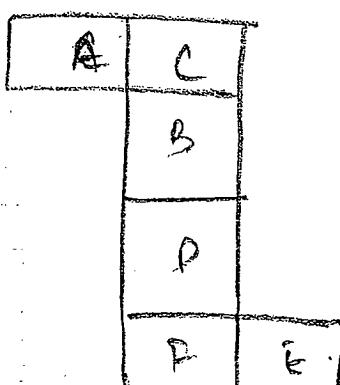


Left hand face opposite to right hand face, no matter where placed

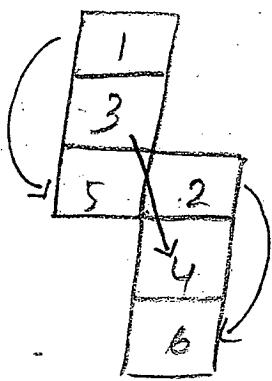
2 — 4

The upper face will be opposite to lower face.

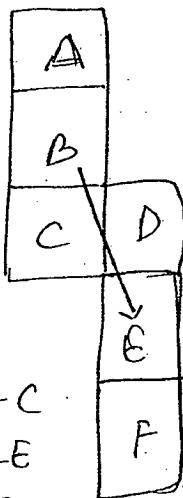
A — D



Type 3:



$$\begin{array}{l} 1 - 5 \\ 2 - 6 \\ 3 - 4 \end{array}$$

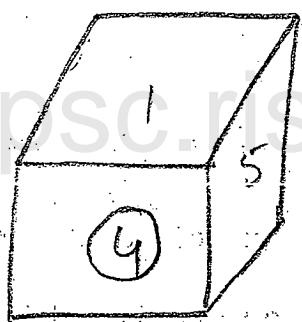
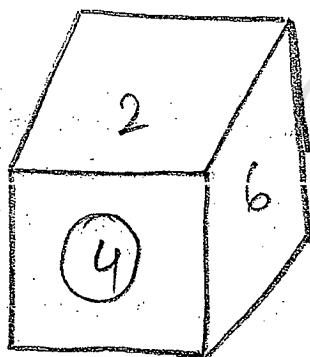


$$\begin{array}{l} A - C \\ A - E \\ D - F \end{array}$$

8.18.

Type : 1

When only one face is common.



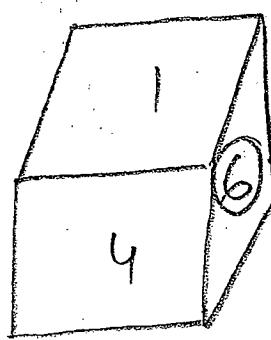
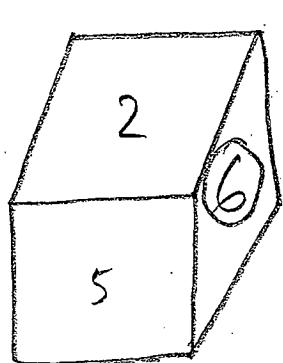
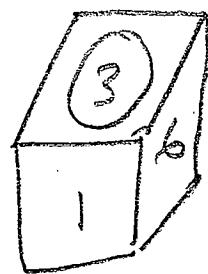
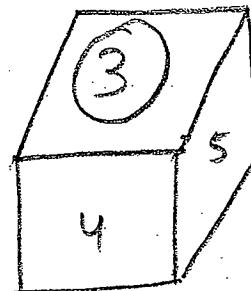
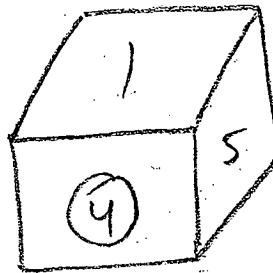
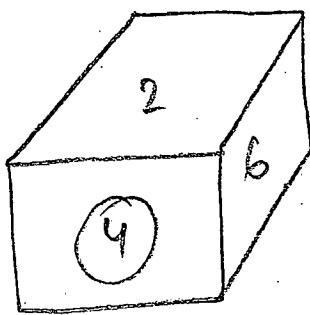
Adjacent's  
Taking numbers  
1 to 6.

$$\begin{array}{l} 4 - 6 \\ 4 - 2 \\ 1 \\ 5 \end{array}$$

\* 4 is common. opposite will be  
3 then.

Properties

Every face in a cube is having four faces adjacent to it and remaining one will always be opposite of it.



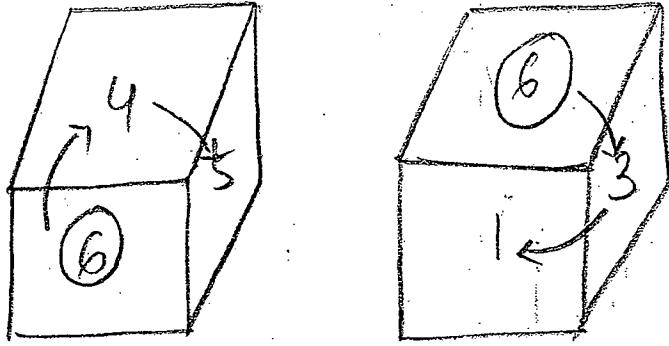
right face  
right face

### Property : 8.

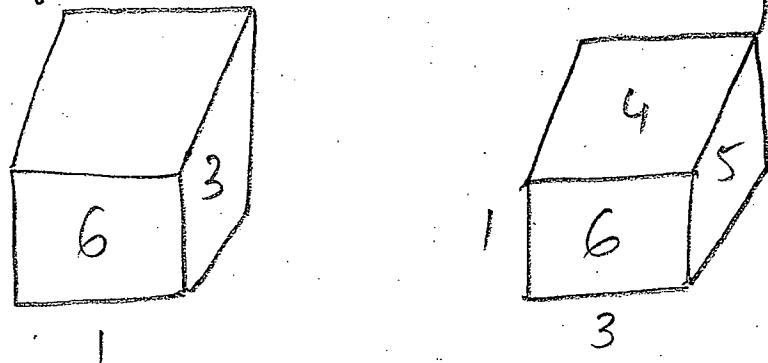
- If the common face is having same position in two different ~~views~~ <sup>views</sup> of same dice, then in the respective remaining positions will always be opposite to each other. (Top face opposite top, right face opp. to right face, side face opp. to side face depending on question)

### Property : 9.

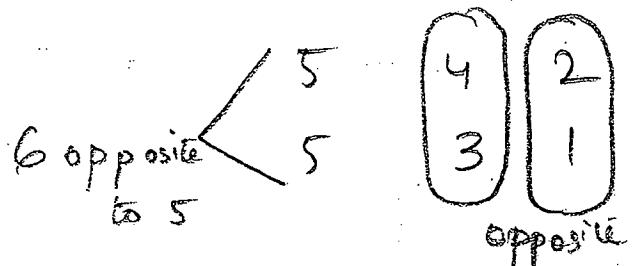
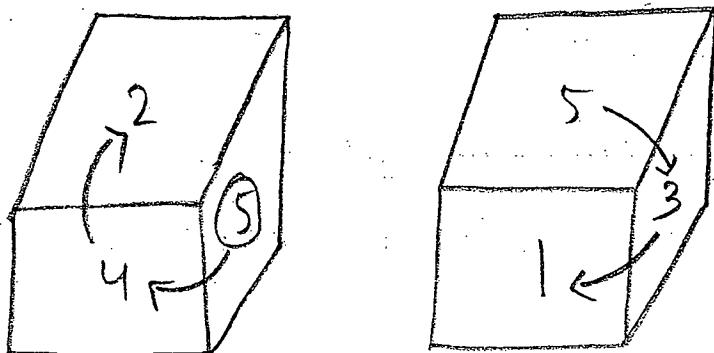
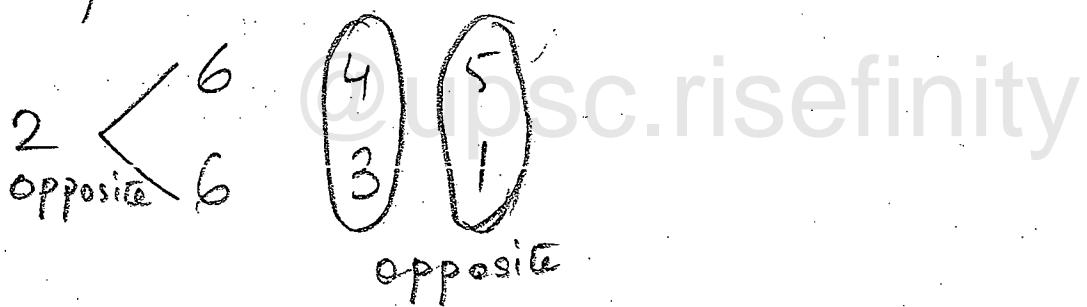
- If only one face is common in two different views of same dice then we can find all the three pairs of opposite faces.



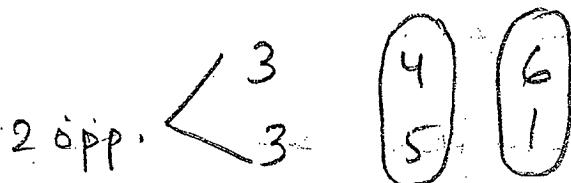
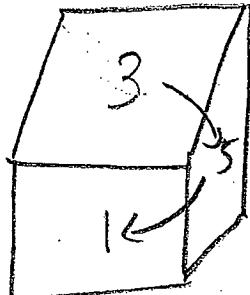
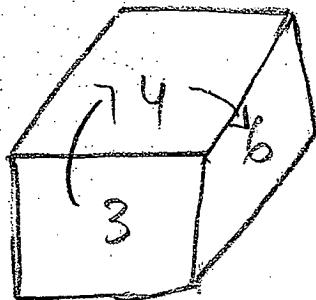
→ If 9 rotate once → rotating twice



reading from clockwise :-



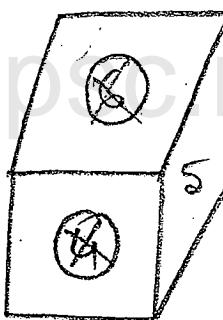
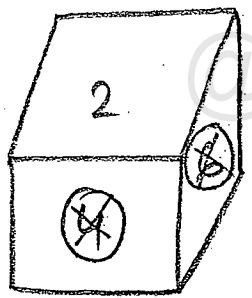
## # Master Property Method #7.



### Type : 2.

When two faces are common .

\* Never use property  
7, 8, 9 or master  
method #1



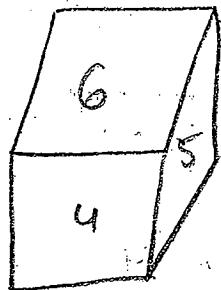
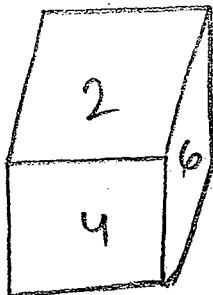
### Master Method #2 .

↳ cancel the common faces .

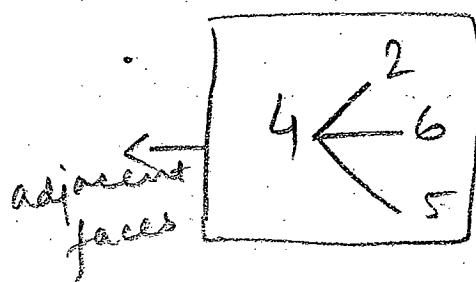
↳ in this case 2 is opposite to 5 .

### Property : 10

If two faces are common in two different views of same dice , then we cannot find all the three pairs of opposite faces but we can conclude only one pair of opposite faces .

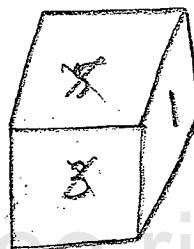
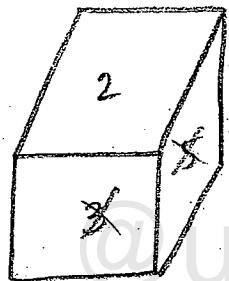


133  
⇒ opposite of 4?



\* either 1 or 3  
is opposite of  
4 in this case.

#

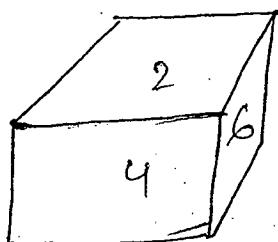


2  $\xrightarrow{\text{opposite}}$  1

Statement :

2, 4, 6 are adjacent to each other.,

→ in statement we  
cannot tell about  
the exact positions



4, 5, 6 are adjacent to each other.

② 1, 2, 5 are adjacent to each other.

2, 4, 5 are adjacent to each other

1 opposite to 4.

③ 2, 4, 6 are adjacent to each other.

3, 4, 5 are adjacent to each other.

Today's Handout:

Q20. ① Y, O, B

② 1, G<sub>1</sub>, Y } ?  $\xrightarrow{\text{opp.}}$  B - Pair

③ B, G<sub>1</sub>, Y }

④ O, V, B

Y, O, B }  
O, V, B }  $\xrightarrow{\text{Y opp.}} V \cdot P_2$

So, Orange  $\rightarrow$  Green.

→ Consider only 1<sup>st</sup> third statement

Y, O, B.

B, G<sub>1</sub>, Y

O  $\rightarrow$  G<sub>1</sub>.

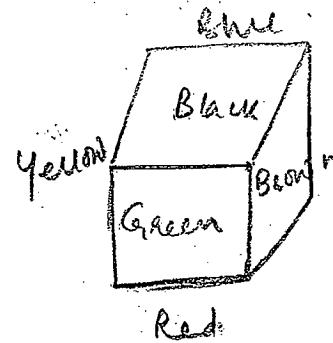
26

Blue adjacent Yellow

Brown adjacent blue.

Green in between

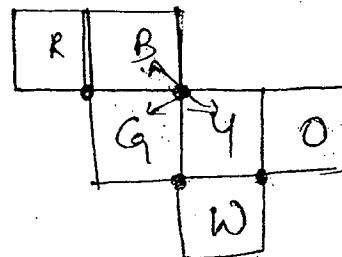
Red and Black.



Property: 11

⇒ opposite faces never have common vertex  
⇒ common vertex adjacent faces.

- If any vertex is common for two or three faces then this two or three faces will always be adjacent to each other.



@upsc.riseinfinity

white sharing common vertex

• W, G, Y are adjacent

Green, Y, B are adjacent

Since,

W G Y , since and Green & G Y, B . yellow common.

Blue is opposite to white.

If I change the position

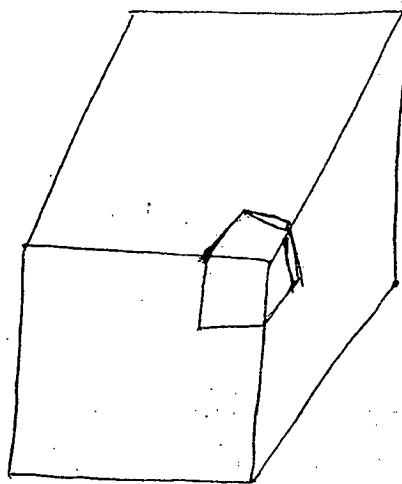
R, B, G adjacent

If I take 3 vertex

B, G, Y

Red opposite to yellow

40.



$$2 \times 6 \times 6 = 216$$

$$1 \times 1 \times 1 = 1 \text{ kg}$$

$$= 215 \text{ kg}$$

$$\frac{215}{216}$$

41.

$$512 - 1$$

$$\begin{array}{r} 64 \\ \times 8 \\ \hline 512 \end{array}$$

$$\frac{2511}{@upsc.riseinfinity}$$

$\Rightarrow$  A cube of length 3 cm is taken out  
from a cube of length 5 cm. 27

What is the weight of removed portion  $\frac{25}{27}$   
from the original | find the weight of  
the ratio of the removed is to remaining. 2

$$\underline{\underline{27/125}}$$

$$\begin{array}{r} 125 \\ - 27 \\ \hline 98 \end{array}$$

$$\text{Ratio} = 27 : 98$$

|   |   |   |
|---|---|---|
|   | 4 |   |
| 1 | 2 | 3 |
|   | 5 |   |
|   | 6 |   |

Adjacent to 2?

1, 4, 5, 3

### SERIES

CODING- DECODING

CHARACTER PUZZLE

Hit & Trial  
Brain Storming

1. 3, 5, 7, ? 9

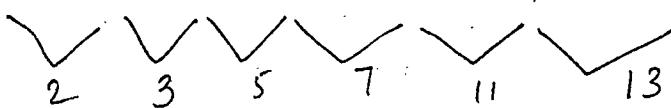
2. 2, 3, 5, 7, 11

3. 53, 59, 61, 67, 71, 73, 79, 83

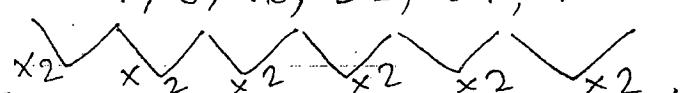
4. 4, 9, 25, 49, 121

$2^2, 3^2, 5^2, 7^2, 11^2$

5. 2, 4, 7, 12, 19, 30, 43



6. multiplication — 2, 4, 8, 16, 32, 64, 128



7.  $1, 2, 6, 24, 120, 720, \underline{5040}$

$\times 2$        $\times 3$        $\times 4$        $\times 5$        $\times 6$        $\times 7$

Factorial  $N! + 1$

$N! - 1$

$2, 3, 7, 25, 121, 721, 5041$

$1+1 \quad 2+1 \quad 6+1 \quad 24+1 \quad 120+1 \quad 720+1, 5040+1$

8.  $4, 12, 60, 420, 3780, \underline{41580}$

$\times 3$        $\times 5$        $\times 7$        $\times 9$        $\times 11$

9.  $720, 360, 120, 30, 6, \underline{1}$

$\div 2$        $\div 3$        $\div 4$        $\div 5$

10.  $4, 7, 10, 13, 16, 19, \underline{22}$

$+3$        $+3$        $+3$        $+3$        $+3$        $+3$

11.  $2, 5, 9, \underline{14}, 20, 27$

$+3$        $+4$        $+5$        $+6$        $+7$

12.  $5, 6, 9, 15, \underline{25} 40$

$+1$        $+2$        $+3$        $+10$        $+15$

$$50, 49, 46, 40, 30, \underline{15}$$

-1      -3      -6      -10      -15  
 -1    -1-2    -1-2-3    -1-2-3-4    -1-2-3-4-5

$$7, 15, 31, 63, 127, 255, \underline{511} \quad - \text{method-1}$$

x2+1    x2+1    x2+1    x2+1    x2+1    x2+1

$$7, 15, 31, 63, 127, 255, \underline{511}$$

8      16      32      64      128      256

- method 2.

$$8-1, 16-1, 32-1, 64-1, 128-1, 256-1, 512-1 \quad - \text{Method 3}$$

$$2^3-1, 2^4-1, 2^5-1, 2^6-1, 2^7-1, 2^8-1, 2^9-1$$

@upsc.risefinity

$$2, 8, 26, 80, 242, 728$$

$$3-1 \ 9-1 \ 27-1 \ 81-1 \ 243-1 \ 729-1$$

$$3^1-1 \ 3^2-1 \ 3^3-1 \ 3^4-1 \ 3^5-1 \ 3^6-1$$

$$2, 3, 8, 27, 112, \underline{565}$$

2x1+1      3x2+2      8x3+3      27x4+4      112x5+5

$$\begin{array}{l}
 2 \times 1 + 1 \\
 3 \times 2 + 2 \\
 8 \times 3 + 3 \\
 27 \times 4 + 4 \\
 112 \times 5 + 5
 \end{array}$$

17. 4, 13, 41, 126, 382, 1151

$$\begin{array}{r} 3 \times 1 + 1 \\ 3 \times 4 + 1 \end{array}$$

$$4 \times 3 + 1 = 13$$

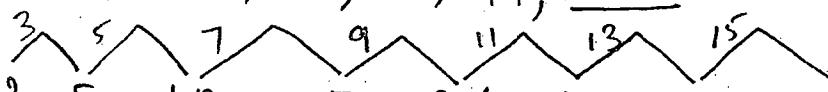
$$13 \times 3 + 2 = 43$$

$$41 \times 3 + 3 = 126$$

$$126 \times 3 + 4 = 382$$

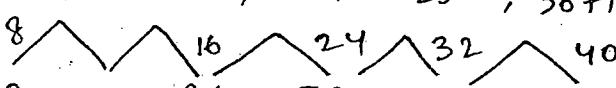
$$382 \times 3 + 5 = 1151$$

18. 1, 4, 9, 16, 25, 36, 49, 81



19. 2, 5, 10, 17, 26, 37, 50, 65

1H 4+1 9+1, 16+1 25+1, 36+1 49+1, 64+1



19. 2, 10, 26, 50, 82, 122

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

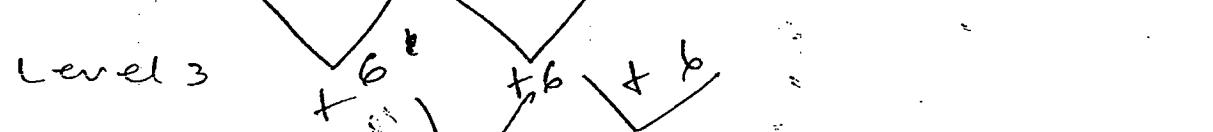
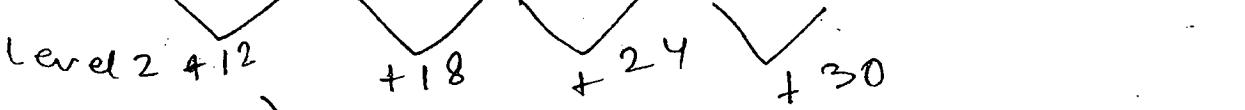
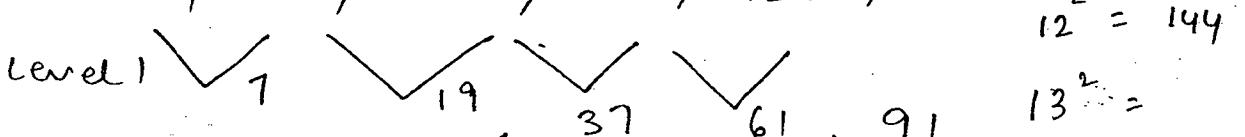
20. 3, 10, 29, 66, 127, 218

1+2, 8+2, 27+2, 64+2, 125+2 + 216+2 ~~3+2~~

1<sup>3</sup>+2, 2<sup>3</sup>+2, 3<sup>3</sup>+2, 4<sup>3</sup>+2, 5<sup>3</sup>+2 + 6<sup>3</sup>+2 ~~3+2~~

### Triangular Pattern

1. 3, 10, 29, 66, 127,



2. 6, 25, 62, 123, 214, 341

8-2, 27-2, 64-2, 125-2, 216-2, 343-2

$2^3-2, 3^3-2, 4^3-2, 5^3-2, 6^3-2, 7^3-2$

$$\begin{array}{r}
 12 \\
 21 \\
 16 \\
 \hline 49 \\
 13 \\
 17 \\
 14 \\
 \\ 12 \\
 25 \\
 16 \\
 \hline 49 \\
 15 \\
 15 \\
 19 \\
 \hline 49
 \end{array}$$

### Alphabetical

1. T, R, P, N, L, J, H, E

|    |    |    |    |    |    |    |    |    |    |  |
|----|----|----|----|----|----|----|----|----|----|--|
| A  | B  | C  | D  | E  | F  | G  | H  | I  | J  |  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |
| K  | L  | M  | N  | O  | P  | Q  | R  | S  | T  |  |
| 24 | 28 | 23 | 24 | 25 | 26 |    |    |    |    |  |
| U  | V  | W  | X  | Y  | Z  |    |    |    |    |  |

|   |    |    |    |    |
|---|----|----|----|----|
| E | J  | O  | T  | Y  |
| 5 | 10 | 15 | 20 | 25 |

|   |    |    |
|---|----|----|
| C | M  | W  |
| 3 | 13 | 23 |

2. R, U, X, A, D, G, J, M

18, 21, 24, 27, 4

+3 +3 +3 +3 +3 +3 +3

3. H, I, K, N, R, W, C

8, ~~9~~, 11, 14, 18, 23, 29

+1 +2 +3 +4 +5 +6

4. AZ, GT, MN, SH, YB

A G M S Y Z, T, N, ~~H~~, B  
 1 7 13 19 25 26, 20, 14 8, 2  
 +6 +6 +6 +6 -6 -6 -6 -6

\* After Z  
which is 26

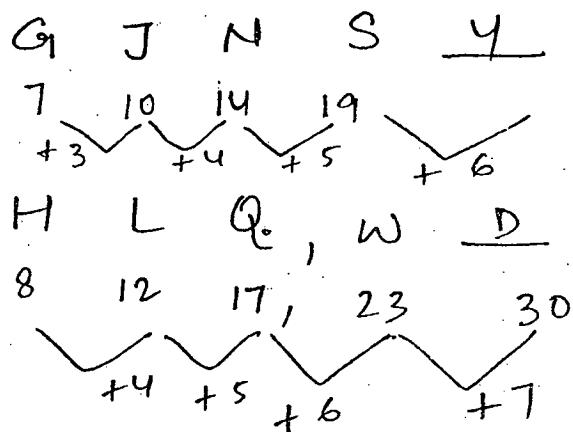
A is 27

B " 28

C " 29

G H, J L, N Q, S W, Y D

$\frac{19}{6}$   
25.



### CODING - DECODING

1. E = 5

$16+5+14$

PEN = 35

P = 16

N

E = 5

$16+1+5+7$

PAGE - 29.

$\frac{17}{12}$   
29 :

$$\begin{array}{r} 18 \\ \times 5^{\text{4}} \\ \hline 90 \\ \times 4 \\ \hline 36^{\text{0}} \end{array}$$

2. RED - 6720

$18 \ 5 \ 4$   
 $+2 +2 +2 = 2076 = 6720$

GREEN - REVERSE

G - 7

R - 18

E - 5

E - 5

N - 14

7 - 18 5 5 14  
 $+2 +2 +2 +2 +2$

= 9 20 7 7 16

16 7 7 20 9.

3. D E E R 34417 H I G H 7867 HELL  
 4 5 5 18 8 9 7 8 8 5 12 12  
 $-1 -1 -1 -1 = 34417 -1 -1 -1 -1 = 7867 -1 -1 -1 -1$   
 $\underline{2741111}$

4. GO = 32 SHE = 49  $\frac{15}{\underline{22}}$   
 $\begin{array}{r} 7 \ 15 \\ \swarrow \\ 20 + 12 \end{array}$   $\begin{array}{r} 19 \ 8 \ 5 \\ = 8 + 19 + 22 \end{array}$

SOME = 56 S = 27 - 19 = 8 Ranking from starting  
 19 15 13 5 O = 27 - 15 = 12 Ranking from last =

$$M = 27 - 13 = 14$$

$$E = 27 - 5 = \frac{22}{56}$$

TN + 1

$$R_s + R_L = 26 + 1$$

$$G = \text{starting} = 7 \quad S = \frac{56}{27-19} = 8$$

$$\text{from last} = 27 - 7 \quad H = 27 - 8 = 19$$

$$= 20 \quad E = 27 - 5 = 22$$

$$O = 27 - 15 = 12$$

5. AT = 20 BAT = 40  $\frac{27-1}{26}$ .  
 $1 \times 20 = 20$   $2 \times 1 \times 20 = 40$   $\frac{27-20}{2}$

ZAT  $\frac{19}{26 \times 1 \times 20} = 520$  CSA T = 1140  $\frac{19}{2} \frac{2}{38} \frac{2}{3} \frac{1}{1140}$   
 $\frac{19}{2} \frac{2}{38} \frac{2}{3} \frac{1}{1140}$

$$\begin{array}{r}
 6. \quad \begin{array}{r} 2 & 1 & 1 \\ | & | & | \\ 26 & 9 & 86 \\ & & \end{array} = 153 \\
 \begin{array}{r} 51 \\ \times 3 \\ \hline 153 \end{array}
 \end{array}$$

$$ZAP - 129$$

$$26 \times 1 \times 16 = 43 \times 3$$

$$= 129.$$

$$\begin{array}{r}
 \begin{array}{r} 26 \\ \times 16.3 \\ \hline 156 \\ 26 \times \\ \hline 416 \end{array}
 \quad
 \begin{array}{r} 26 \\ \times 9 \\ \hline 16. \\ \hline 51 \end{array}
 \end{array}$$

$$ZAT = \overline{141}$$

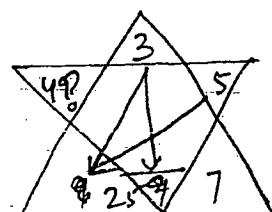
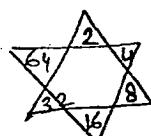
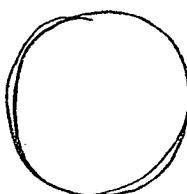
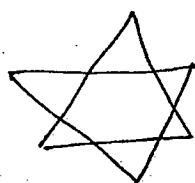
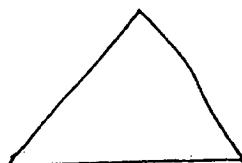
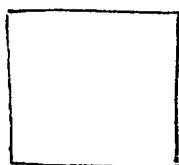
7. chu cha chi — 123  
2.

chu    chu    chul — 422.  
      2            2            ↓  
      4

$\text{Cha} \rightarrow$  can either 1 or 3

Chi → can either 1 or 3.

## 8. Character Puzzle



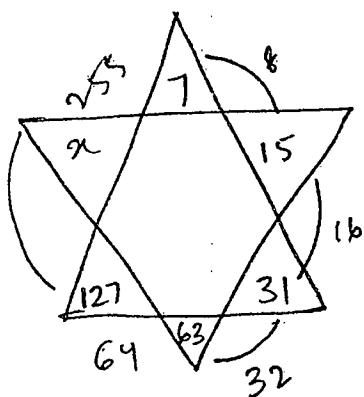
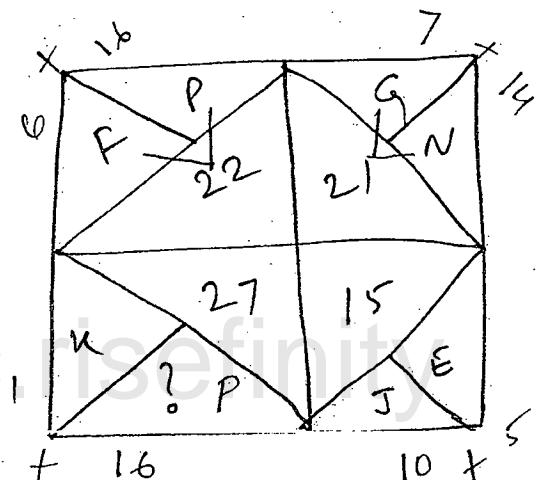
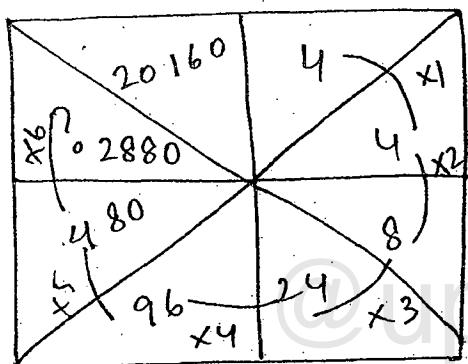
|   |    |    |
|---|----|----|
| 3 | 10 | 13 |
| 1 | 14 | 15 |
| 7 | 12 | 19 |

|   |    |    |
|---|----|----|
| 2 | 3  | 2  |
| 1 | 4  | 7  |
| 5 | 25 | 53 |

$$\begin{array}{r}
 \downarrow \quad \downarrow \\
 \begin{array}{r}
 2 \\
 +1 \\
 - \\
 \hline
 5
 \end{array} \quad \begin{array}{r}
 2 \\
 2 \\
 +4 \\
 \hline
 9
 \end{array} \quad \begin{array}{r}
 3 \\
 2 \\
 2 \\
 \hline
 16
 \end{array}
 \end{array}$$


---

2.



→, 87, 15, 31, 63, 127

$$x \times 2 + 1 = 7$$

$$2x + 1 = 7$$

$$2x = 6$$

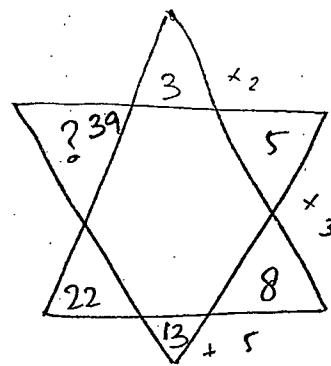
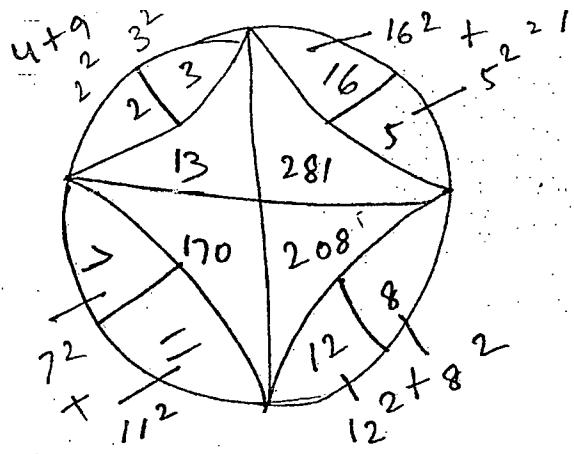
$$x = 3$$

$$\begin{array}{r}
 7 \times 2 + 1 \\
 7 \times 4 - 2 \\
 \hline
 26
 \end{array}$$

$$\begin{array}{r}
 15 \times 2 + 1 \\
 231
 \end{array}$$

$$31 \times 2 + 1 = 63$$

255 & 3 both can be  
correct.



$$\frac{22}{11} \\ \underline{33}$$

$$121 \\ \cdot \frac{99}{170}$$

a) 35

b) 39.

c) 43

d) 47

3, 5, 8, 13, 21

2, 3, 6, 18, 108, —

$$3 \times 2 - 1 = 5$$

$$5 \times 2 - 2 = 8.$$

$$8 \times 2 - 3 = 13$$

$$13 \times 2 - 4 = 22$$

$$22 \times 2 - 5 = 39.$$

if diagonally:

$$3 \times 4 + 1 = 13$$

$$5 \times 4 + 2 = 22$$

$$8 \times 4 + 3 = 35$$

$$12+2 = 12$$

$$10-2 = \underline{8}$$

$$12-2$$

$$10-2$$

$$\frac{16}{16}$$

$$\frac{25}{21}$$

$$\frac{25}{25}$$

$$\frac{50 \times}{525}$$

## CALENDAR

Leap Year - for century year we divide by 400.

Non-century year we divide by 4, check divisibility of 4.

Odd days.

$$365 \div 7$$

$\therefore$  52 weeks + 1 odd day.

$$366 \div 7$$

$\therefore$  52 weeks + 2 odd days

Jan = 31.

$$31 \div 7 =$$

$\therefore$  4 weeks + 3 days.

Odd days with respect to month,

A J S N, - 30 days.

J M M J A O D, - 31 days months.

Feb - either 28 or 29

July Aug }  
Dec Jan } 2 consecutive  
months with 31 days.

Jan - 31 days - 3 odd  $\frac{3}{3}$  leap year.

Feb - 28 u 2 0 " 1

Mar - 31 u = 3 odd 3

Apr - 30 u = 2 odd 2

### Leap year

|        |         |         |   |
|--------|---------|---------|---|
| May -  | 31 days | - 3 odd | 3 |
| June - | 30 "    | - 2 "   | 2 |
| July - | 31 "    | - 3 4   | 3 |
| Aug -  | 31 "    | - 3 4   | 3 |
| Sep -  | 30 "    | - 2 4   | 2 |
| Oct -  | 31 "    | - 3 4   | 3 |
| Nov -  | 30 "    | - 2 4   | 2 |
| Dec -  | 31 "    | - 3 4   | 3 |

Monday - 1

Tuesday - 2

Wednesday - 3

Thurs - 4

Friday - 5

Sat - 6

Sun - 7 or 0.

### Navigation

1. Yearly Navigation
2. Monthly Navigation
3. Date Navigation

If 1st Jan 2017 - Mon

1st Jan 2086

4<sup>th</sup> April - 2017 - Mon.

14<sup>th</sup> Nov, 2047 - ?

If 1<sup>st</sup> Jan 2017 - Mon

1<sup>st</sup> Jan 2027 - ?

1<sup>st</sup> Jan 2017 ~~Wednesday~~ Monday

1<sup>st</sup> Jan 2018 - Tuesday.

1<sup>st</sup> Jan 2019 - Wednesday

1<sup>st</sup> Jan 2020 - Thursday

1<sup>st</sup> Jan 2021 - Saturday.

$$\text{Step 1} : 2027 - 2017 = 10 \text{ yrs}$$

$$\text{Step 2} : 2 \text{ leap year } 2 \times 2 = 4$$

$$8 \text{ normal year } 8 \times 1 = \underline{8}$$
  
$$12$$

1<sup>st</sup> Jan 2011 - Mon

1<sup>st</sup> Jan 2031 - ?

$$\text{Step 1} : 2031 - 11$$
  
$$= 20 \text{ yrs}$$

$$\text{Step 3} : \frac{12}{7} = 1 \text{ week } + 5 \text{ days}$$

Step 2 : 12, 16, 20,  
24, 28

Step 4 = Mon

+ 5

= 6 - Saturday.

$$5 \text{ LP } = 5 \times 2 = 10$$

$$15 \text{ NY } = 15 \times 1 = \underline{\frac{15}{25}}$$

$$\text{Step 3} : 3 \div \frac{25}{7}$$

=  $\frac{3}{4}$  weekly 4 days.

$$\text{Step 4 : Mon} = 1$$
  
$$+ 4$$
  
$$\underline{5}$$

= Friday

1st Jan 2011 - Mon

1st Jan 2041 -

Step 1 :  $41 - 11 = 30$

Step 2 :  $8 \text{ LY} = 8 \times 2 = 16$

$$\begin{array}{r} 22 \times 1 \\ \hline 22 \\ + 38 \\ \hline 38 \end{array}$$

Step 3 :  $\frac{38}{7} = 3 \text{ odd days.}$

$$\begin{array}{r} 26 \\ 11 \\ \hline 15 \\ - 4 \\ \hline 01 \end{array}$$

1st Jan 2011 - Mon

2012

1st Jan 2026 -

2016

2020

2024

4 LP  $4 \times 2 = 8$

$$\begin{array}{r} 11 \times 1 \\ \hline 11 \\ + 19 \\ \hline 19 \end{array}$$

$$\begin{array}{r} 19 \\ 7 \\ \hline 2 \text{ weeks} \\ + 5 \end{array}$$

Mon = 1

+ 5

2. Saturday

$$\begin{array}{l}
 \text{1st Jan 2011 - Mon} \\
 + 0 \\
 \hline
 \text{1st Jan 2061 - ? Mon}
 \end{array}
 \quad
 \begin{array}{r}
 4, 8, 12 \dots 18 \text{ Jan. 2061} \\
 15 \quad 3 \times 2 \\
 \hline
 51 \quad 12 \\
 \hline
 18
 \end{array}$$

Step 1 :  $61 - 11$

$\approx 50 \text{ yrs}$

Step 2 : Leap year - 13 yrs

Normal =  $50 - 13$

$\approx 37$

2012

2016

2020

2024

2028

2032

2036

2040

Step 3 : ~~80/~~  $13 \times 2 = 26$

~~80/~~  $37 \times 1 = 37$

$^2 \text{ leap year} - \text{first leap year}$

$^2 \frac{63}{7} = \underline{63}$

$^2 \text{ last year - first leap year} + 1$

Common difference

$^2 0 \quad ^2 2060 - 12 + 1$

Step 4 : No change.

$$\begin{array}{r}
 4 \\
 = \frac{48}{4} + 1 \\
 12 + 1 = 13
 \end{array}$$

1st Jan 2011 - Mon + 1

+ 4

1st Jan 2081  $\approx$  ~~100 yrs~~

$$\frac{2080 - 12 + 1}{4}$$

Friday

$18 \times 2 = 36$

$52 \times 1 = 52$

$\underline{88}$

$$\frac{80 - 12}{4} + 1$$

$$\frac{68}{4} + 1$$

$88/7^2$  4 odd days

$17 + 1 = 18$

1<sup>st</sup> Jan, 2015 Mon + 1  
+ 2

$$\begin{array}{r} 60 \\ 12 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 61 \\ 15 \\ \hline 46 \end{array}$$

1<sup>st</sup> Jan, 2061 Wed

$$\begin{array}{r} 46 \\ 26 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 13 \times 2 = 26 \\ 20 \times 1 = 20 \\ \hline 46 \end{array}$$

$$46/4 =$$

$$\begin{array}{r} 60 - 18 + 1 \\ 4 \\ 48 + 1 \\ \hline 49 \\ 12 + 1 \\ = 13. \end{array}$$

$$12 \times 2 = 24$$

$$\begin{array}{r} 34 \times 1 = 34 \\ \hline 58 \end{array}$$

58/7 = 2 days Remainder

1<sup>st</sup> Jan, 2015 - Mon + 1

1<sup>st</sup> Jan, 2055 - Tuesday

Step 1: ~~20~~ 55 - 15 = 40 yrs.

When the gap is exactly divisible

$$\text{by } 4 = \frac{40}{4} = 10$$

$$10 \times 2 = 20$$

$$\begin{array}{r} 30 \times 1 = 30 \\ \hline 50 \end{array}$$

50/7 = 1 day

1st Jan, 2015 - Mon + 1

+ 8.5

$$\begin{array}{r} 60 \\ - 15 \\ \hline 45 \\ 15 \\ \hline 60 \\ 4 \end{array}$$

1st Jan, 2015 - Saturday

$$15 \times 2 = 30$$

$$\begin{array}{r} 45 \times 1 = 45 \\ \hline 75 \end{array}$$

$$75/7 = 5 \text{ day}$$

~~Total~~ Total odd ~~glosses~~ days = Total year + leap year

1st Jan, 2014 - Mon

1st Jan, 2051 = 40 yrs + 1

Tuesday

1st Jan, 2015 = Mon 1  
+ 2.

1st Jan, 2095 = Wed

$$95 - 15 = 80$$

$$\frac{80}{4} = 20$$

$$80 + 20$$

$$= 100$$

$$100/7 = 2 \text{ days}$$

1st Jan 2017, Mon + 5

$$\begin{array}{r} 77 \\ 17 \\ \hline 60 \\ -15 \\ \hline 45 \end{array}$$

1st Jan 2017, Sunday

$$\frac{60}{4} = 15$$

$$15 \times 2 = 30 \quad 45 \times 1 = 45$$

$$60 + 15$$

$$= 75 / 7$$

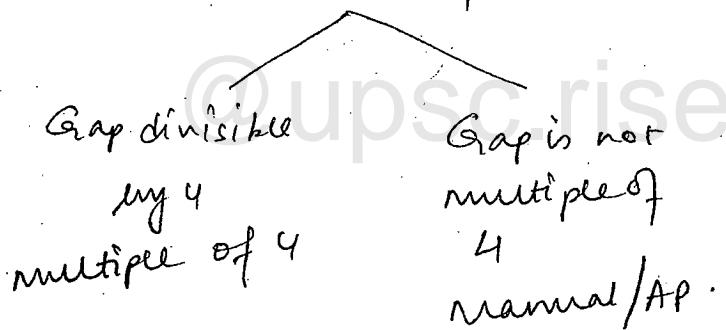
$$= + 5$$

Step 1 : Total year

Step 3 : Total year + leap

Step 2 : Leap year.

Year = Total odd day



Step 4 :  $\frac{\text{Total days}}{7}$

## Monthly Navigation

Jan - 21. 10 Jan 2017 → Mon

Feb - 28

Mar - 31. 10 Jun 2017 →

Apr - 30

June - 10.

$$\frac{120}{7}$$

For a monthly navigation question, for whatever month we cross the last date we take odd days for that month only.

1st Jan 2018 Mon +3  
1st Feb 2018 Thursday +0  
1st Mar 2018 Thursday - 4<sup>th</sup> day +3  
1st April 2018 Sunday

-3  
-0  
1-3  
+2  
1-3  
F-2  
13/7  
= 6. 10<sup>th</sup> Jan 2017 Mon - 1  
10<sup>th</sup> Jun 2017 Friday +4.  $\frac{11}{7}$   
= 4 days.

26<sup>th</sup> Jan 2018 - Mon - 1  
26<sup>th</sup> June 2018 - Friday +4  $\frac{11}{4}$

+ Jan 1  
increasing ← 31<sup>st</sup> Jan 2018 - Mon - 1  $\frac{13}{7}$ .  
+ not 31<sup>st</sup>  
July. 31<sup>st</sup> July 2018 - Sunday = 6.

7<sup>th</sup> April 2018 - Mon - 1

7<sup>th</sup> Nov 2018 - Friday + 4

2 A  
3 M  
2 J  
3 J  
3 A

7<sup>th</sup> Jan, 2016 - Mon - 1

18/7

2 S.  
3 O

7<sup>th</sup> Jan 2016 - + 5 = 4 days

18.

12/7.

~~10/7.~~

Feb - 1 odd day.

2 + 5

2016 - leap year

### Monthly & Yearly Navigation

4<sup>th</sup> Jan 2017 - Mon

57  
- 17.  
40

4<sup>th</sup> April 2057 - Mon

40  
4  
210.

4<sup>th</sup> Jan 2017 - Mon + 1

40 + 10  
250

4<sup>th</sup> April 2057

50/7 = 1 odd day.

4<sup>th</sup> Jan 2017 = + 6.

7

1 week after

Mon

=

7th Nov 2018 - Mon - 1

+ 6.

27th Nov 2018 - Sunday

27 - 7

= 20/7 = 6.

4th April 2018 Mon - 1

+ 4

22nd April 2018 Friday.

22 - 4 = 18/7

4th Jan 2016 - Mon - 1

$\frac{51}{11} \\ \frac{40}{}$

14th April 2016 - Thursday

$\frac{40}{4}$

210

1 + 6 + 3

14 - 4

= 10/7

2

Jan

$\frac{40}{+10}$

Jan 3

$\frac{50}{}$

Feb. 0

Mar - 3.50

/ 7 21 odd

April - 2

May - 3

day.

2 3 odd

days - 1/7 = 4 odd day.

1st Jan 2001 — Mon

18.

3rd Jun 2019 — Monday.

Monthly Navigation

$\begin{array}{r} 3 \\ 3 \\ 2 \\ 3 \\ \hline 11 \end{array}$

70

25/7

1 + 4 + 2

2 7

$\begin{array}{r} 1/2 \\ + 1/1 \\ \hline 2/3 \\ + 2/ \\ \hline 2/5 \end{array}$

~~4x2=8~~

~~18~~

$4 \times 2 = 8$

$$18 - 8 = 14.$$

22/7 1 day  
11/7 4 day

$3 - 1 = 2$  day = 2.

### Backward Movement -

-1 [ 29<sup>th</sup> Aug - Wednesday ] 1

+1 [ 30<sup>th</sup> Aug - Thursday ] 4

+1 [ 31<sup>st</sup> Aug - Friday ] 1

1st Jan, 2018 - Sunday ] 7

1st Jan 1 odd day.

1st Jan 2017 - Saturday ] -1

6.

$18^{\text{th}} \text{ Jan, 2017} - \text{Sun}] = 7$   
 $18^{\text{th}} \text{ Jan, 2007} = \underline{\text{Monday}}$   
 $\underline{\underline{-}} = 1$

$17 - 7$

$$210 + 3 = 13.$$

$16, 12, 8, 4$      $13/7^2$  6 odd days

$18^{\text{th}} \text{ Jan, 2021} - \text{Friday} = 5$

$18^{\text{th}} \text{ Jan, 2001} - \underline{\text{Monday}} = 4$   
 $\underline{\underline{-}} = 1$

$2021 - 2001$

$2$     $20$     $\text{@upsc.risefinity}$

$20, 16, 12, 8, 4$        $18^{\text{th}} \text{ Jan } 1905 - \text{Mon}$

$20+5$

$31^{\text{st}} \text{ Dec. } 1909$

$25/7$

$\begin{array}{r} 4 \\ +1 \\ \hline 57 \end{array}$

$2$     $4$

$26 \text{ Jan } 2018 - \text{Sun} = 7$

$15 \text{ Aug } 2018 - \text{Tuesday}$

$3: 7$

$0: F$

$3: M$

$2: A$

$3: M$

$2: J$

$3: J$

$16/7.$

$2 \text{ odd}$

Mon Jan 1 2007

Jan 1 2010

$3:$

$+ 2: 1$

$5:$

## Repeation of Calendar.

1<sup>st</sup> Jan 2017 - Mon

4<sup>th</sup> April 2017 - Thurs.

7<sup>th</sup> Nov 2017 - Sun.

① Normal Year calendar repeat in Normal year

Leap Year calendar repeat in Leap Year only.

② 1<sup>st</sup> Jan of both year should have same day.

③ 1<sup>st</sup> Jan, 2021 - Mon  
 2022 + 1 odd day  
 2023 + 1  
 UP | 2024 + 1  
 2025 + 2  
 2026 + 1  
 2027 + 1  


---

 7.

2027 - Mon.

④ 1<sup>st</sup> Jan, 2019 - Mon

1<sup>st</sup> Jan, 2020, 2020 + 1.

will be Mon. 2021 + 2.

2022 + 1.

2023 + 1.

2024 + 1.

2025 + 2.

2026 + 1.

2027 + 1.

2028 + 1.

2029 + 2.

- 1<sup>st</sup> Jan 2023 - Mon

9 of 1<sup>st</sup> Jan, 2017 - Mon -

① 1<sup>st</sup> Jan, 2018 > +1

1<sup>st</sup> Jan, 2019 > +1 Wed

1<sup>st</sup> Jan, 2020 - Thurs.  
 LP > +2

1<sup>st</sup> Jan, 2021 - Sat

1<sup>st</sup> Jan, 2022 - Sun

1<sup>st</sup> Jan, 2023 - Mon.

2017 .

18 +1

19 +1.

LP | 20 +1.

21 +2

22 +1

23 +1

7

③ 1<sup>st</sup> Jan, 2018 - Mon.

2024 | +2 2019 +1.

2025 | +1 2020 +1

2026 +1

2027 +1 2021 +2

2028 +1 2022 +1

2029 +1 2023 +1

Mon? - 2024 +1

8

7.

Second, condition satisfied  
 but not the first,  
 So, it cannot happen.

So in this case 2029 will be

1st Jan, 2013 Mon  
 2014 +1  
 2015 +1  
 2016 +1  
 2017 +2  
 2018 +1  
 2019 +1  
 $\frac{7}{}$

1st Jan 2019, Mon.

Within a century, calendar  
 a leap year repeats  
 in every 28 years.

Calendar of 2016  
 $+28$   
 $\frac{044}{-28} \quad \frac{2020}{072}$   
 $\frac{+28}{2048} \quad \frac{2012}{+28}$   
 $\frac{2040}{+28}$   
 $\frac{2068}{}$

1st Jan 2092 Monday.  
 8 +1      2093 +2  
 9 +1      2094 +1  
 0 +1      2095 +1  
 1 +1      2096 +1  
 2 +1      2097 +2  
 $\frac{7}{}$   
 14.  
 1st Jan, 2104 - Mon

(6) 1st Jan, 2016 - Sun  
 2017 +2  
 2018 +1  
 2019 +1  
 2020 +1  
 2021 +2  
 $\frac{7}{}$   
 1st Jan, 2044 Sun

2021      2028 +1  
 2022 +1      2029 +2  
 2023 +1      2030 +1  
 2024 +1      2031 +1  
 2025 +2      2032 +1  
 2026 +1      2033 +2  
 2027 +1      2034 +1  
 $\frac{7}{}$

alternate method  
 $2016 \quad \left. \begin{matrix} 2020 \\ 2024 \end{matrix} \right\} +5$   
 $2028 \quad \left. \begin{matrix} 2032 \\ 2036 \end{matrix} \right\} +5$   
 $2040 \quad \left. \begin{matrix} 2044 \end{matrix} \right\} +5$   
 $35$

472123  
 $\frac{8}{7}$

Divisible by 7.

5 odd days = 4 yrs

1 odd day =  $\frac{4}{5} \frac{1}{7}$

35 odd day =  $\frac{4}{8} \times 35$

= 28 yrs

Alternate,

2092 > +5  
2096 > +5  
2100 > +5  
2104 > +4

14. 1st Jan, 209 will be Mon,  
not necessary to be added 28.

6.9.18.

Father/Mother  
Uncle/Aunt

## BLOOD RELATIONS

Cousin. I → Brother/Sister — Husband/Wife.

brother in law/  
sister in law

Father in law/  
Mother in law

Grand father/  
Grand mother  
Father/mother  
Uncle/Aunt  
Nephew/Niece

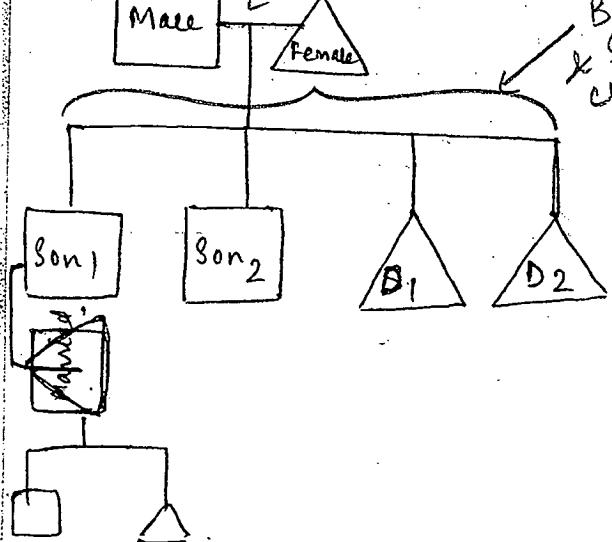
connected  
horizontally  
Husband &  
wife

Brother Son/Daughter  
& Sister chain

① Grand Son / Grand Daughter

How is X related to  
Y?

- i) Sister
- ii) Niece
- iii) Uncle
- iv) Aunt.

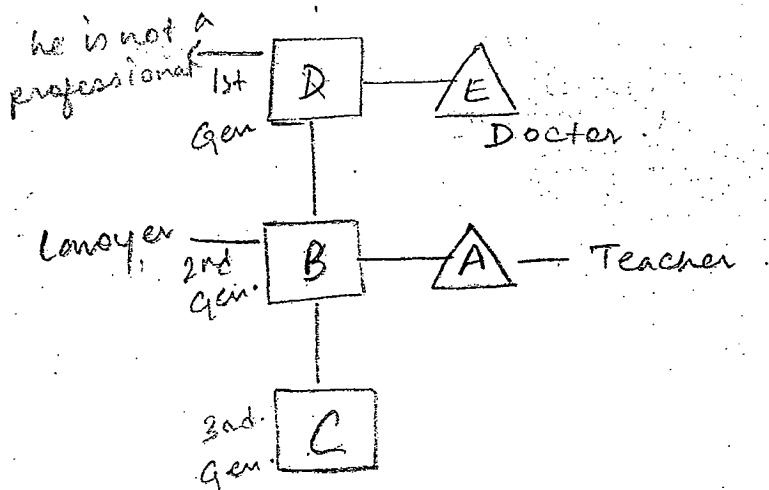


4

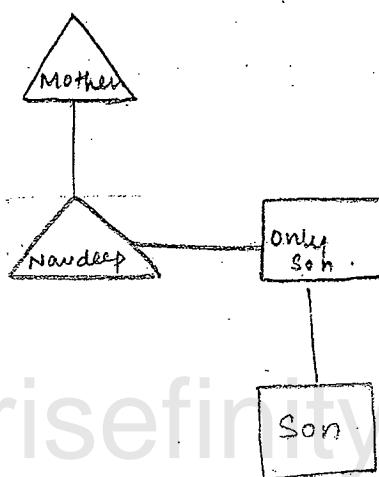
Based on level we can  
eliminate options:

## Handout

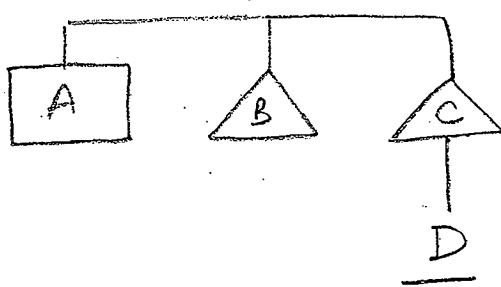
### Type : 1



### Type : 2

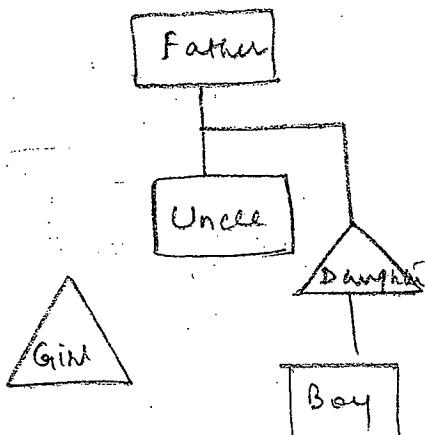


3.



Gender  
cannot be  
determined  
whether male  
or female

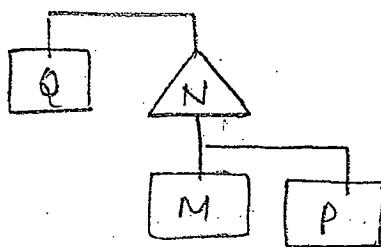
5.



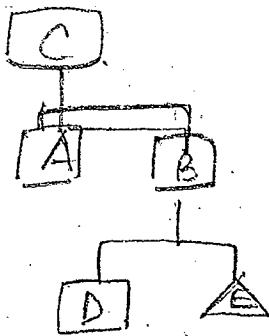
Uncle is not  
confirmed whether  
paternal or maternal.  
So it can either  
be brother/son/

2.

$$A. Q = N + M \times P$$



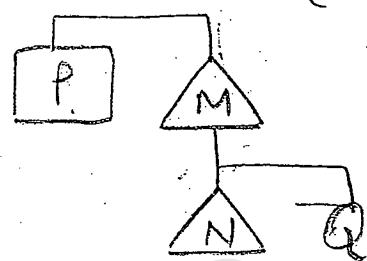
Q4



Female

$$B. P + S \times N = Q$$

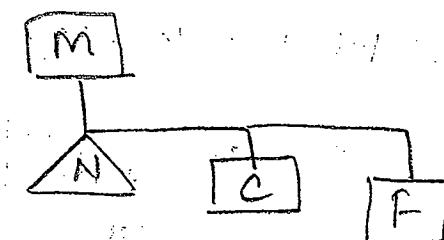
$$C. P - M + N \times Q$$



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4.

$$A. M$$



9.18.

## DATA INTERPRETATION

Based on:

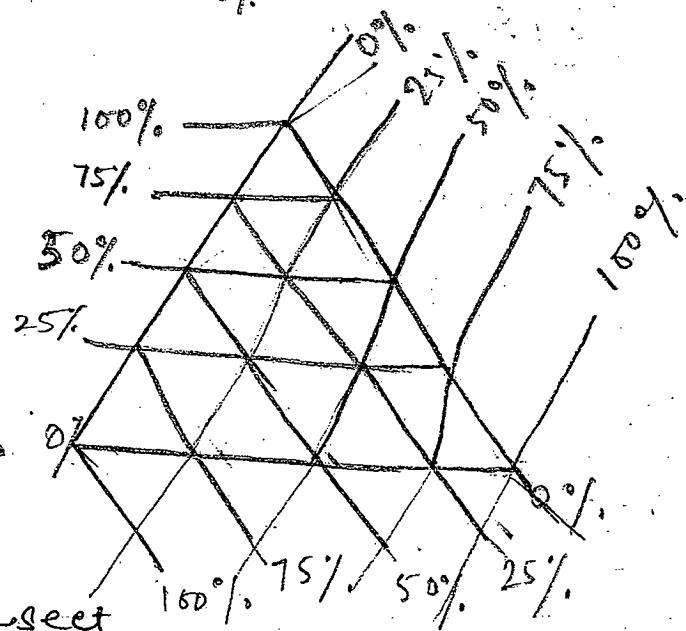
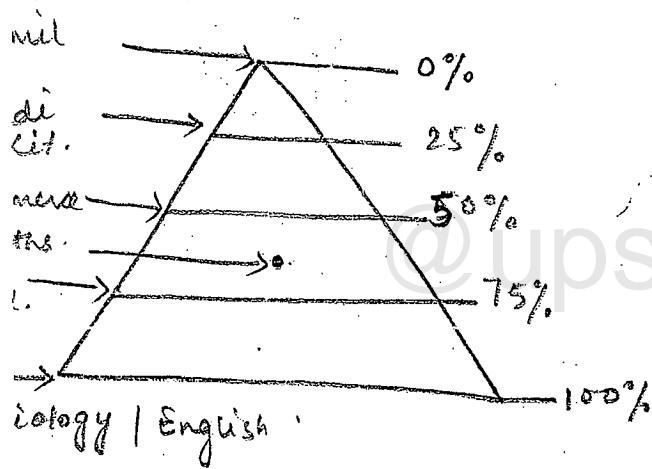
1. Line Graph
2. Bar Graph
3. Mixed Graph
4. Pie Chart
5. Table
6. Misc.

1. Number System
2. Ratio & Proportion
3. Average
4. Percentage

Triangle based Data Interpretation

Venn Diagram based D.I.

Misc.



1. You cannot intersect the scale

2. you have to go parallel to the scale.

| <u>Assignment</u> |             |                              | $\frac{75}{2}$                |
|-------------------|-------------|------------------------------|-------------------------------|
| Emerald School    | 500         | $50\% \text{ of } 500 = 250$ | $\frac{250}{150} \times 75\%$ |
| DPS               | 750         | $20\% \text{ of } 750 = 150$ | $\frac{150}{75} \times 25\%$  |
| Sawan Public      | 470         | $40\% \text{ of } 470 = 188$ | $\frac{188}{94} \times 25\%$  |
|                   | <u>1720</u> | <u>588</u>                   | <u>94</u>                     |
|                   |             |                              | <u>619</u>                    |

3. DPS : Emerald Public  
~~75155~~ : ~~30152~~  
~~375~~ : ~~150~~

$$\begin{array}{r} 75 \\ \times 375 \\ \hline 35 \\ 25 \\ \hline 25 \\ \times \end{array}$$

4.  $\frac{619}{172} \times 100$

$$\begin{array}{r} 6190 \\ \hline 172 \\ 172 \\ \hline 0 \end{array}$$

(Total) read with total you will be comparable.

Q14. what % of L size shirts belong to Brand A?

Total of Large size = 185

$$\frac{185}{2}$$

Brand A large size = 91

$$\% = \frac{91 \times 100}{185}$$

Short cut

$$\text{check} = \frac{185}{2}$$

$$= 92.5$$

$$= 91.$$

50% of 91

2 is half

$$\text{Total} = \cancel{100} 252$$

$$\text{Brand B} = 94$$

$$\% = \frac{94}{185} \times 100$$

$$\frac{94}{252} \times 100$$

$$L \text{ and } XL = 6\%$$

$$M = 40\%$$

$$40\% \text{ of } 560 = \frac{40}{100} \times 560$$

$$= 224$$

|                          | <u>Harish</u>               | <u>Sanjay</u>                               | <u>Total</u>   |
|--------------------------|-----------------------------|---|--|
| 1991                     | 1800                        | 2200  | 1000 kg/ha<br>Total = 4000   |
| 1992                     | 2500                        | 2700  | 5000   |
| 1993                     |                             |   | 6000   |
| 1994.                    | 3200                        | 4800  | 8000   |
| 1995                     | 9000<br>$\downarrow$<br>75% | 25% of 9000.<br>3000<br>$\downarrow$<br>25% | 12000<br>$\begin{array}{r} 4000 \\ -1800 \\ \hline 2200 \end{array}$ |
| 18. Ratio of production. |                             |   |  |

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If 9,000 is 75%.

$$\frac{60}{100} \times 8000.$$

$$\frac{75}{100} \times 9000 = 1$$

$$4800$$

$$\frac{75}{100} =$$

$$25\%$$

$$2 \frac{25}{100} \times 9000$$

$$\frac{25}{100} \times 92000 = 23000$$

$$\frac{21}{1800} : \frac{45}{9000}$$

|          |       |     |     |     |     |
|----------|-------|-----|-----|-----|-----|
| 65       | 35    | 50  | 77  | 80  | 80  |
| 150      | 130   | 120 | 100 | 100 | 100 |
| ↓        | ↓     | ↓   | ↓   | ↓   | ↓   |
| 90 + 7.5 | 35% = | 60  | 77  | 80  | 80  |
| 45.5     |       |     |     |     |     |
| 2. 97.5  |       |     |     |     |     |

$$\frac{97.5 + 45.5 + 60 + 77 + 80 + 80}{700} \times 100$$

7) 460

$$\frac{460}{700} \times 100^2$$

63%

2006 → 51%

Total - 384 in lakh.  
= 72%

51% of 72% of 384.

51 = 49% of 72% of 384

$$\frac{49}{100} \times \frac{72}{100} \times 3840000$$

2.0

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