The Name of the Title is Hope

Abstract

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Keywords: Do, Not, Us, This, Code, Put, the, Correct, Terms, for, Your, Paper

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1 Introduction

We are in the midst of a hardware revolution, a new golden age for computer architecture [**?**]. The last decade has seen a shift in architectural paradigms, with the rise of GPUs and accelerators. This shift has been driven by the necessity to innovate in the post Moore's law and Dennard's scaling era. This transformation has also played a significant role in the success of modern deep learning models, as they enable scaling model training and inference to a massive number of threads. Such scalability would be essential for all performance critical applications, including other machine learning models that need to scale with increasing data sizes and model complexities.

Decision forest models remain the mainstay for machine learning over tabular data [**?**]. Their robustness, interpretability, and ability to handle missing data make them a popular choice for a wide range of applications. TODO a few more lines, classification, regression, etc. A recent survey [**?**].... The survey also finds that, the cost of inference is the most critical factor in the overall cost of deploying a machine learning model. This is because, in production settings, each model is trained once and often used for inference a few thousand times.

This paper is motivated by the need to accelerate decision tree inference to achieve portable performance on a variety of client hardware. In particular, we focus on portable performance across a range of commodity CPUs and low-to-mid-range GPUs, a class of hardware that has seen widespread adoption across client/edge devices used for inference.

Decision forest models are composed of a large collection of decision trees (100-1000), and inference involves traversing down each tree in the forest and aggregating the predictions. Inference is typically done in a batched setting, where multiple inputs are processed simultaneously.

Despite the simplicity of the model and the availability of multiple sources of coarse grain parallelism (parallelism across inputs in a batch and parallelism across trees), existing systems do not consistently scale well across different models. This is because existing systems exploit a limited set of optimizations and often specialize the implementation to a specific hardware platform, which limits their portability.

Evaluation on a diverse set of models highlights that the best implementation often requires a careful combination of many optimization strategies. For example, managing the working set of nodes across a set of trees is critical to get scalable performance. This requires a combination of techniques like data layout optimizations, loop transformations, and memory access optimizations. Existing systems each use specific data layouts for the underlying tree (XGBoost uses a sparse representation, RAPIDs FIL uses what is called the reorg representation and Tahoe uses a variation of the reorg representation) and propose simple loop transformations (XGBoost uses ...()) that together determine the accesses patterns and the working set. They do not perform consistently well across different models and hardware platforms. Managing the working set is just one of several optimizations that are critical for high-performance decision tree inference. TODO Improve the next line Others include tree ordering, incremental reduction of predicted values, interleaving across multiple trees, and so on.

This paper presents Treebeard, a novel schedule guided compilation infrastructure for decision tree inference on multiple target hardware. Treebeard is able to generate high-performance code for decision tree inference by exploring a large optimization space. The code generation is guided by a *schedule* object that allows us to effectively represent this solution space abstractly. The schedule object is written

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in Treebeard custom scheduling language that is expressive enough to represent a wide range of implementation strategies. We demonstrate that the language is sufficient to express the various optimizations proposed by prior work, and further generalizes them and incorporates several new optimizations. We also design and implement a heuristic that is able to quickly find high-performance schedules for the model being compiled. TODO Performance evaluation summary TODO Oganization

1.1 Contributions

- We present the design for a multi-target decision tree compiler infrastructure and implement several optimizations within this framework. We are also the first to implement an optimizing compiler for decision tree inference on GPUs.
- We identify that an extensive optimization space exists for the problem of decision tree inference. We design a scheduling language that allows us to effectively represent this solution space abstractly. This scheduling language is expressive enough to represent a wide range of implementation strategies proposed by prior work.
- To the best of our knowledge, we perform the first extensive characterization of the optimization space for decision tree inference on GPUs. Using some of the characteristics we identify, we design and implement a heuristic that is able to quickly find high-performance schedules for the model being compiled.
- We design and implement a general framework for expressing and optimizing reductions within MLIR. To the best of our knowledge, this is the first such framework.
- We evaluate our implementation by comparing it against RAPIDs and Tahoe, the state-of-the-art decision tree inference frameworks for GPU and report significant speedups. We also show that our compiler can effectively target different GPUs, including both NVIDIA and AMD GPUs.

2 Motivation

While a diverse set techniques have been proposed for optimization of decision tree inference on CPUs and GPUs [1–3, 6, 7, 9**?**], a very extensive design space of optimizations exists outside what has been proposed in the literature. Furthermore, decision tree inference is run on several platforms including CPUs and GPUs. The implementations used on each of these platforms are different and the techniques used to optimize them are different. To make matters even more complicated, several in-memory representations have been proposed for decision tree models. For example, XGBoost[3] uses a sparse representation, RAPIDs FIL[**?**] uses what is called the reorg representation and Tahoe uses a variation of the reorg representation. TODO Can we add some numbers here to show that different models/batch sizes need different optimizations?

To solve the problems of exploring the design space of optimizations for decision tree inference and enabling portable performance, we build several techniques in Treebeard, an open source compiler infrastructure for decision tree inference. To make Treebeard capable of unifying these different techniques and targets, we do the following.

- We design a scheduling language that encapsulates various optimization techniques and controls the structure of the generated code.
- We design an MLIR dialect to represent and optimize reductions and use this dialect within Treebeard to enable the generation of different variants of inference routines.
- We extend Treebeard's intermediate representations to include operations like caching. We were able to easily reuse and extend Treebeard's IR as it was built as an MLIR dialect.
- We design a plugin mechanism with which different in-memory representations can be composed with different optimizations.

3 Compiler Overview

Treebeard takes a serialized decision tree ensemble as input (For example XGBoost JSON, ONNX etc.) and generates an optimized inference function. Treebeard automatically generates an optimized inference function from the serialized model and can either target CPUs or GPUs. Figure 1 shows the structure of the Treebeard compiler. The inference computation is lowered through three intermediate representations – high-level IR (HIR), mid-level IR (MIR) and low-level IR (LIR). The LIR is finally lowered to LLVM and then JIT'ed to the specified target processor.

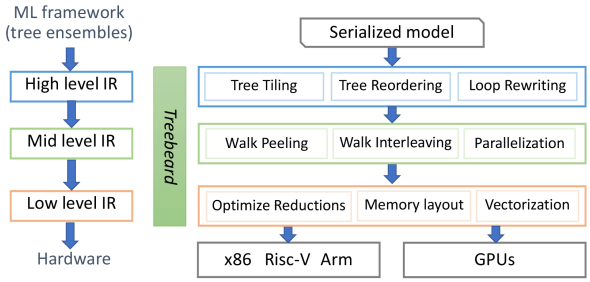


Figure 1. Treebeard compiler structure.

In HIR, the model is represented as a collection of binary trees. This abstraction allows the implementation of optimizations that require the manipulation of the model or its constituent trees. In Figure **??**, A shows this representation for a model with three trees. In HIR, Treebeard tiles tree

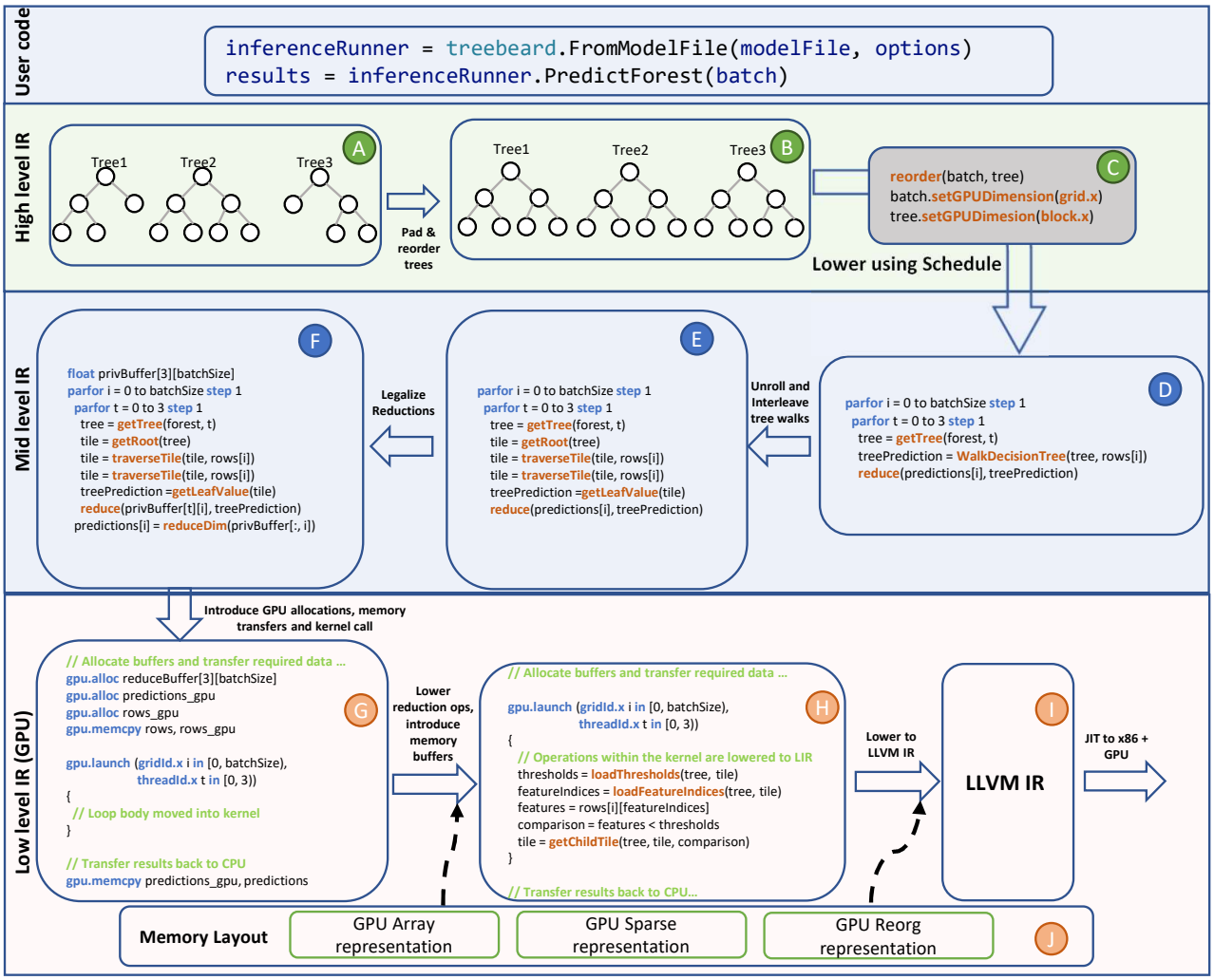


Figure 2. Treebeard IR lowering and optimization details: the three abstraction levels in Treebeard's IR are shown. The high level IR is a tree-based IR to perform model level optimization, the mid-level IR is for loop optimizations that are independent of memory layout and the low level IR allows us to perform vectorization and other memory layout dependent optimizations.

nodes together to convert the binary tree to an n-ary tree as shown in (B). Trees are also reordered to enable better code generation and padded to allow more efficient traversal as shown in (C). While these are the optimizations currently implemented in Treebeard, these are by no means the only ones that are enabled by HIR. It is not difficult to imagine optimizations such as tree pruning for specified accuracy levels for example. These optimizations and rewrites that are performed on the domain-specific HIR would have been much on a traditional loop based IR or even in other IRs within Treebeard.

After these model-level optimizations are performed on the HIR, the code is lowered to the mid-level IR (MIR) as dictated by a user specified schedule ((C) to (D)) in Figure **??**). The schedule specifies how the iteration space that goes over the trees and input rows is to be traversed. It specifies how the iteration space is to be tiled, which loops are to be parallelized, which loops are to be mapped to GPU grid and block dimensions etc. (Details in Section **??**). MIR is a loop-based IR that explicitly encodes details of the iteration space has to be traversed. However, it still abstracts details about the in-memory representation of the model. Optimizations such as tree-walk unrolling and interleaving are performed on the MIR ((E)). Subsequently, reduction operations are split and rewritten to correctly and explicitly implement reduction in the presence of parallel loops ((F)). More details of this

process that we call *legalization* are in Section **??**. Another important point to note is that MIR is independent of the target processor and therefore all optimizations on MIR can be reused across CPU and GPU compilation.

The MIR is then further lowered to a low-level IR (LIR). This is the level at which the compilation pipeline diverges for CPUs and GPUs. In the GPU compilation pipeline, the required memory transfers and kernel invocations are inserted into the LIR (J). Additionally, buffers to hold model values are inserted and tree operations are lowered to explicitly refer to these buffers. This lowering is controlled by a plugin mechanism where different in-memory representations can be added to the compiler by implementing an interface. These plugins provide information required for the lowering of MIR to LIR as well as the lowering to LLVM IR as shown in Figure **??**. Again, a significant amount of code is shared between the CPU and GPU pipelines for representations that are common between them (Array and Sparse representations). Vectorization of tree traversals is also explicitly represented in LIR.

To reiterate, the following are the salient points of Treebeard's design.

1. The compiler uses three intermediate representations (IRs) to represent the inference computation at different levels of abstraction. This allows different optimizations to be performed and also allows us to share infrastructure between compilation pipelines for different target processors.
2. The specification of how the inference computation is to be lowered to loops is not encoded directly in the compiler. Instead, this is specified as an input to the compiler using a scheduling language that is specialized for decision tree inference computations (Section **??**). This separation allows us to build optimizations and schedule exploration mechanisms independent of the core compiler (Section **??**). The scheduling language also exposes details like parallelization, optimization of reductions (vectorization, use atomics, shared memory on GPUs etc.).
3. Treebeard has been designed to keep the optimization passes and code generator independent of the in-memory representation finally used for the model. To achieve this, Treebeard specifies an interface to implement that provides the necessary capabilities to the code generator as a plugin. This interface abstracts several details on how model values are stored. In particular, it abstracts the actual layout of the memory buffers, how to load model values like thresholds and feature indices, how to move from a node to its children and determining whether a node is a leaf. This design allows us to write each memory representation as a standalone plugin and reuse the rest of the compiler infrastructure.

4 Treebeard IR Description

5 Scheduling Language

The goal of the scheduling language is to express the following

- The order in which the iteration space over the batch of inputs (batch) and trees in the forest (tree) is to be traversed. Note that there is a reduction over the tree dimension.
- Intra or inter tree optimizations that are to be performed on a tree or set of trees (tree walk unrolling, pipelining, SIMDize etc).

The reasons to use a scheduling language rather than a hard-coded lowering are as follows

- Making the scheduling specification external to the compiler allows us to more easily build auto-schedulers and auto-tuners.
- It is very hard to come up with a template loop nest that works for all models (for example, tree sizes may vary across the model making it necessary to iterate different number of trees at different times).
- A scheduling language will make writing newer locality optimizations faster since no changes to the compiler infrastructure will be needed.
- Adding support for additional hardware targets (GPUs, FPGAs), will be much easier with a scheduling language.

There are some simplifying assumptions and limitations in the current design

- Tree traversals are considered atomic. There is no way to express partial tree traversals or schedule individual node/level computations.
- Accumulation of tree predictions is done immediately (as opposed to, for example, collecting all predictions and performing a reduction later).

5.1 Language Definition

We broadly have three classes of directives in the language. The first is a set of loop nest modifiers that are used to specify the structure of the loop nest to walk the iteration space. The second is a set of clauses that specify intra and inter tree optimizations. Finally, we have a class of attributes that control how reductions are performed.

5.1.1 Loop Modifiers. There are two special index variables – `batch` and `tree`. The clauses modify these index variables or index variables derived from these (through the application of clauses).

- **tile**: Tile the passed index variable using a fixed tile size.
- **split**: Split the range of the passed index variable into two parts. The range of the first part is specified by an argument.

Operation	Inputs	Outputs	Attributes	Description
`predictEnsemble`	**rows[]**	**result**	**ensemble** **predicate** **schedule**	Performs inference on the data in **rows[]** using the model specified by the **ensemble** attribute. The **schedule** attribute contains the schedule described in Section 5. **predicate** specifies the operator to use to evaluate nodes (Eg: $<$, $\leq$).
`walkDecisionTree`	**trees[]** **rows[]**	**results[]**	**predicate** **unrollDepth**	Represents an interleaved walk on all the element-wise pairs of **trees** and **rows**. **unrollDepth** specifies the number of hops to unroll. An array of tree walk results is returned.
`ensemble`		**ensemble**	**model**	Represents the forest of trees that constitute the model. The **model** attribute contains the actual trees model.
`getTree`	**ensemble** **treeIndex**	**tree**		Get the tree at the specified index (**treeIndex**) from the **ensemble**.
`getTreeClassId`	**ensemble** **treeIndex**	**classId**		Get the class ID for the tree at index **treeIndex** in the **ensemble**. This is used for multi-class models.
`getRoot`	**tree**	**rootNode**		Get the root node of the specified tree.
`isLeaf`	**tree** **node**	**bool**		Returns a boolean value indicating whether **node** is a leaf of **tree**.
`getLeafValue`	**tree** **node**	**value**		Returns the value of the leaf **node** in **tree**.
`traverseTreeTile`	**trees[]** **nodes[]** **rows[]**	**nodes[]**	**predicate**	Represents an interleaved traversal of the nodes in **nodes** based on the data in **rows**. **predicate** specifies the operator to use to evaluate nodes.
`cacheTrees`	**ensemble** **start** **end**	**ensemble**		Cache the trees in the **ensemble** between the specified **start** and **end** indices. The returned **ensemble** has the specified trees cached.
`cacheRows`	**rows[]** **start** **end**	**cachedRows[]**		Cache the rows in **rows[]** between the specified **start** and **end** indices. Returns an array of cached rows **cachedRows[]**.
`loadThreshold`	**buffer** **treeIndex** **nodeIndex**	**threshold**		Load the threshold value for the node specified by **nodeIndex** in the tree specified by **treeIndex** from **buffer**. Returns the loaded threshold.
`loadFeatureIndex`	**buffer** **treeIndex** **nodeIndex**	**threshold**		Load the feature index for the node specified by **nodeIndex** in the tree specified by **treeIndex** from **buffer**. Returns the loaded feature index.

Table 1. List of all the operations in the Treebeard MLIR dialect. These operations are used in conjunction with operations from other MLIR dialects like `scf`, `arith`, gpu etc. to represent and optimize decision tree inference.

- **unroll**: Unroll an index completely
- **reorder**: Reorder the specified indices. The specified indices must be successive indices in the current loop nest.
- **specialize**: Generate separate code for each iteration of the specified index variable. This is useful while parallelizing across trees and these trees have different depths.
- **gpuDimension**: Maps the specified index variable to represent a dimension in either the GPU kernel grid or thread block.

The default loop order (as currently generated by the compiler) is (batch, tree), i.e, for each row in the input batch, go over all trees.

The following are examples of how the loop modifiers can be used.

- The loop order used by XGBoost[3] is (tree, batch) – walk one tree for all inputs in the batch before moving to the next tree. The corresponding schedule would be

```
reorder(tree, batch)
```

- The below schedule computes 2 trees at a time over the whole batch.

```
tile(tree, t0, t1, 2)
reorder(t0, batch, t1)
```

- If we additionally only want to compute over 4 input rows (rather than the whole batch) for every 2 tree, and then move onto the next 2 trees for the same set of inputs, then the schedule is as follows.

```
tile(batch, b0, b1, 4)
tile(tree, t0, t1, 2)
reorder(b0, t0, b1, t1)
```

5.1.2 Optimizations.

The following clauses provide ways to optimize the inference routine being generated.

- **cache**: Cache the working set of one iteration of the specified loop corresponding to this index. This can be specified on either batch or tree loops. Specifying it on a batch loop leads to all rows accessed in a single iteration of the loop being cached. Similarly, specifying it on a tree loop leads to all trees accessed in one iteration of that loop being cached.
- **parallel**: Execute the loop corresponding to this index in parallel.
- **interleave**: Interleave the execution of the tree walks within the current index (must be applied on an inner most index).
- **unrollWalk**: Unroll tree walks at the current index.
- **peelWalk**: Peel the first n steps of the specified tree walk and don't check for leaves for that number of steps.

5.1.3 Reduction Optimization.

- **atomicReduce**: Use atomic memory operations to accumulate values across parallel iterations of the specified loop.
- **sharedReduce**: Only applies to GPU compilation. Specifies that intermediate results are to be stored in shared memory.
- **vectorReduce**: Use vector instructions with the specified vector width to reduce intermediate values across parallel iterations of the specified loop.

TODO Add examples for RAPIDS, Tahoe (Maybe show some strategies can be encoded?)

5.2 The XBoost Schedule

XGBoost[3] is a very popular gradient boosting library. It implements inference on the CPU by going over a fixed number of rows (64 in the previous version) for every tree and then moving to the next tree. When all trees have been walked for this set of rows, the next set of rows is taken up. Also, different sets of rows are processed in parallel.

The schedule used by XGBoost can be represented in Treebeard's scheduling language as follows.

```
tile(batch, b0, b1, CHUNK_SIZE)
reorder(b0, tree, b1)
parallel(b0)
```

5.3 Tahoe Schedules

Tahoe[9] has four strategies that it picks from for a given model. Each of these strategies can be encoded using Treebeard's scheduling language as we show below.

- **Direct Method**: In this strategy, a single GPU thread walks all trees for a given input row. The schedule for this strategy is as follows.

```
tile(batch, b0, b1, ROWS_PER_TB)
reorder(b0, b1, tree)
gpuDimension(b0, grid.x)
gpuDimension(b1, block.x)
```

  Here, `ROWS_PER_TB` is the number of rows that are processed by a single thread block.
- **Shared Data**: In this strategy, a thread block walks all the trees for a given row in parallel. If threads walk multiple trees, each thread accumulates partial results. Finally, a thread block wide reduction is performed to compute the prediction. The schedule for this strategy is as follows.

```
reorder(batch, tree)
gpuDimension(batch, grid.x)
gpuDimension(tree, block.x)
cache(batch)
```

- **Shared Forest**: In this strategy, the whole model is loaded into shared memory and subsequently, a single thread walks all trees for a particular row. The schedule for this strategy is as follows.

```
tile(batch, b0, b1, ROWS_PER_TB)
tile(tree, t0, t1, N_TREES)
reorder(b0, b1, t0, t1)
cache(t0)
gpuDimension(b0, grid.x)
gpuDimension(b1, block.x)
```

  Here, we create a placeholder single iteration loop `t0` so that we can specify that all trees are to be cached.
- **Shared Partial Forest**: In case the model is too large to fit into shared memory, the model is split into chunks and each chunk is loaded into shared memory. Again, as in the previous strategy, one thread walks all trees assigned to a thread block for a row. The schedule for this strategy is as follows.

```
tile(batch, b0, b1, ROWS_PER_TB)

tile(tree, t0, t0Inner, TREES_PER_TB)
tile(t0Inner, t1, t2, TREES_PER_TB)
cache(t1)
reorder(b0, t0, b1, t1, t2)

gpuDimension(b0, grid.x)
```

```
gpuDimension(t0, grid.y)
gpuDimension(b1, block.x)
```

6 HIR and MIR Optimizations

The Treebeard infrastructure was originally designed to target CPUs [8]. However, it implements several optimizations on the HIR and MIR that can be leveraged across target processors and we find that these are beneficial for GPUs as well. This reuse of optimizations is possible because the intermediate representations on which these optimizations are performed are abstract and are designed to be target-independent. We briefly review theese optimizations below.

6.1 Optimizations on High-Level IR

We augment the existing Treebeard infrastructure with loop rewrites on the HIR that are implemented through the scheduling language (Section 5). We use these to implement the automatic scheduling described in Section 10. Additionally, the Treebeard infrastructure implements HIR transformations to reorder and pad trees. It also implements tree tiling transformations on the HIR [8]. We reuse the reordering and padding transformations on the HIR for GPUs. However, we found that tiling trees was not beneficial for GPUs. This is because the tiling transformations introduces redundant computation inorder to vectorize computation on CPUs where all lanes need to follow the same control flow. However, on SIMT GPUs, we find that the benefits of tiling (coalescing memory accesses) do not outweigh the cost of redundant computation. We leave an investigation of this for future work.

6.2 Optimizations on Mid-Level IR

The original Treebeard infrastructure implements optimizations like tree-walk unrolling, tree-walk interleaving, and parallelization on the MIR. These optimizations are beneficial for GPUs as well. and the design of Treebeard allows us to reuse the tree-walk unrolling and tree-walk interleaving optimizations on the MIR for GPUs. TODO Should we talk about how interleaving is implemented as a statemachine and therefore it can be used across representations and tile traversal techniques?

6.3 A Note on Low-level IR

Some changes to the original Treebeard design were required to get LIR to correctly lower to GPU code. The most important of these was the change to how the compiler implements support for in-memory representations of models (Section 8). With these design changes, we were able to reuse much of the CPU implementation while customizing some parts for GPUs (for example, buffers need to be allocated differently for CPU and GPU, caching is implemented differently etc.).

7 Reductions : Representation, Optimization and Lowering

Currently, existing reduction support in MLIR is insufficient to code generate and optimize the reductions Treebeard needs to perform while performing inference (sum up individual tree predictions to compute the prediction of the model). MLIR only supports reductions of value types and cannot directly represent and optimize inplace reductions of several elements of a memory buffer.

We design a mechanism to specify accumulating values into an element of a multi-dimensional array inplace. The accumulation is performed inside an arbitrary loop nest where several surrounding loops maybe parallel and the ultimate target machine maybe a CPU or a GPU. Because this problem is of general interest, we design this as an MLIR dialect.

The main abstraction we introduce is the `reduce` op. It models atomically accumulating values into an element of a multi-dimensional array (represented by a MLIR `memref`). The following example sums up the elements of the 1D memref `arr` into the first element of the memref `result`. It does this using in two concurrent iterations of a surrounding parallel loop.

```
builtin.func @ReduceVector(%arr: memref<num_elemsxf64>,
    %result: memref<1xf64>) -> void {
  par.for i0 in range(0 : num_elems/2 : num_elems) {
    for i1 in range(0 : num_elems/2)
      reduce(%result, 0, arr[i0 + i1]) <"+", 0.0>
  }
}
```

The semantics of the `reduce` op guarantee that all elements are correctly added and that there is no race between the parallel iterations of the loop.

The `reduce` op is defined for all associative and commutative reduction operations with a well-defined initial value. The reduction operator and the initial value are attributes applied on the `reduce` op.

The main differences between our `reduce` and the existing reductions in MLIR are the following: 1. Existing reductions only support scalar, by value reductions. It does not support accumulating inplace into memref elements. 2. Existing reduction support in MLIR does not provide a unified way to handle reductions in GPUs and CPUs. To the best of our knowledge, there is currently no way to model reductions on GPUs in MLIR. 2. Existing reductions have strict rules about where they can be written. For example, `scf.reduce` needs to be an immediate child `scf.parallel`. However, our `reduce` op can be generated anywhere in the loop nest.

Having modeled the reductions with an abstract op, the aim now is to lower this to a correct and optimized implementation on both CPU and GPU. In order to do this, we make the following observations. 1. The `reduce` op has a loop carried dependency on itself and loop carried dependences on other `reduce` ops that accumulate into the same

target array. A simple lowering to a sequence of load-add-store instructions is incorrect if any of these dependences are carried by a parallel loop. We call any such surrounding parallel loop a **conflicting loop** (TODO Change the name!) for the reduction. 2. There is a race between the parallel iterations of such a loop when naively accumulating values into target memref elements. To avoid this race, the result memref can be **privatized** wrt each surrounding conflicting loop. Subsequently, each privatized dimension can be reduced at the end of the conflicting loop it was inserted for. TODO We cannot do better than this in terms of memory usage TODO Need a proof.

Definition 7.1. A parallel loop surrounding one or more `reduce` ops is a **conflicting loop** for a target multi-dimensional array if this loop has a non-zero dependence distance for the dependence between any of the contained `reduce` ops.

In the context of Treebeard, this set of loops is exactly the set of surrounding parallel loops that are iterating over trees. The results can be privatized for each conflicting loop iteratively and reductions along each privatized dimension can be inserted immediately following the loop the dimension was inserted due to.

We illustrate this process through the example above. The `i0` loop is a conflicting loop for the reduction into the `result` array. We would therefore privatize the `result` memref for each iteration of the `i0` loop.

```
builtin.func @ReduceVector(%arr: memref<num_elemsxf64>,
    %result: memref<1xf64>) -> void {
  results_1 = memref<2x1xf64>
  par.for i0 = range(0 : num_elems/2 : num_elems) {
    for i1 = range(0 : num_elems/2)
      reduce(%result_1[i0/(num_elems/2), 0], arr[i0 +
   i1]) <"+", 0.0>
  }
  results = reduce_dimension(results_1, 0) <"+", 0.0>
}
```

The op `reduce_dimension` reduces values across the specified dimension of an n-dimensional memref. In the above example, the `reduce_dimension` op is reducing across all elements of the first dimension (index 0). Therefore, in this case, it produces a result memref with a single element (the first dimension with size 2 is collapsed).

Definition 7.2. **`reduce_dimension(targetMemref, memref, dim, [indices], [rangeStart], [rangeEnd])`**: Computes the reduction over the dimension specified by `dimension` and stores the result in `targetMemref`. `[indices]` must be a vector of `dim` elements (or empty if the dimension being reduced is the first dimension). `[rangeStart]` and `[rangeEnd]` represent the range of indices following the reduction dimension and must have the same number of elements. If both are `null` (not passed), all elements of these dimensions are reduced. The computation performed by the op is as follows.

$$targetMemref[\vec{indices}, \vec{k}] = \sum_{i=0}^{shape[dim]} memref[\vec{indices}, i, \vec{k}] \quad \forall \vec{k} \in [[rangeStart_0, rangeEnd_0), ..., [rangeStart_n, rangeEnd_n)]$$

Consider the following code with nested parallel loops. (A situation where trees are split across both threads and thread blocks could result in such generated code in Treebeard.)

```
builtin.func @ReduceVector(%arr: memref<num_elemsxf64>,
    %result: memref<1xf64>) -> void {
  par.for i0 = range(0 : num_elems/2 : num_elems) {
    par.for i1 = range(0 : num_elems/4 : num_elems/2) {
      for i2 = range(0 : num_elems/4)
        reduce(%result, 0, arr[i0 + i1 + i2]) <"+",
   0.0>
    }
  }
}
```

Here, the `i0` and `i1` loops are conflicting loops wrt the `result` memref. We **legalize** the reduction by privatizing the `result` array wrt the `i0` and `i1` loops. However, there are now two privatized dimensions and therefore, two dimensions need to be reduced to compute the final result. This multi-stage reduction is what enables us to model hierarchical reductions.

The following code shows how the reduction above is legalized. We introduce a new op, `reduce_dimension_inplace` which reduces a dimension of the input memref and stores results in the same array. This helps saves memory by removing the need to create multiple intermediate arrays to store results. Only the final dimension reduction uses the `reduce_dimension` op.

```
builtin.func @ReduceVector(%arr: memref<num_elemsxf64>,
    %result: memref<1xf64>) -> void {
  results_1 = memref<2x2x1xf64>
  par.for i0 = range(0 : num_elems/2 : num_elems) {
    par.for i1 = range(0 : num_elems/4 : num_elems/2) {
      for i2 = range(0 : num_elems/4)
        index0 = i0/(num_elems/2)
        index1 = i1/(num_elems/4)
        reduce(%result_1[index0, index1, 0], arr[i0 +
   i1 + i2]) <"+", 0.0>
    }
    // result_1[i0/(num_elems/2), 0] = sum(result_1[i0
   /(num_elems/2), :])
    reduce_dimension_inplace(%result_1, 1, i0/(
   num_elems/2))
  }
  // result = sum(result[:, 0])
  %result = reduce_dimension(%result_1, 0)
}
```

The behavior of the `reduce_dimension_inplace` op is similar to the `reduce_dimension` op except that it updates the input array inplace rather than writing results to a target array. The definition of the op is as follows.

Definition 7.3. **`reduce_dimension_inplace(memref, dim, [indices], [rangeStart], [rangeEnd])`**: Computes the reduction over the dimension specified by `dimension` and stores the result at index 0 of that dimension. `[indices]` must be a vector of `dim` elements (or empty if the dimension being reduced is the first dimension). `[rangeStart]` and `[rangeEnd]` must have the same number of elements. If

both are `null` (not passed), all elements of the corresponding dimension are reduced.

The computation performed by the op is defined by the following equation.

$$memref[\vec{indices}, 0, \vec{k}] = \sum_{i=0}^{shape[dim]} memref[\vec{indices}, i, \vec{k}]$$
$$[[rangeStart_0, rangeEnd_0), ..., [rangeStart_n, rangeEnd_n)]$$

7.1 Lowering Reduction Operations

We implement lowering of the operations defined above to both the CPU and GPU. Since the compilation pipeline diverges after the reductions are legalized, we can implement lowering and optimization of our reduction dialect to CPUs and GPUs simply using different MLIR rewrite patterns. We now briefly describe how these operations are lowered to the CPU and GPU.

7.1.1 Lowering to CPU. The lowering of the reduction operations to CPU is fairly straightforward. We lower the two operations listed above, `reduce_dimension_inplace` and `reduce_dimension` to a simple loop nest that goes over the specified subset of the input array, performs the reduction and writes the result into the appropriate location of the target array. If the schedule specifies that the reduction is to be vectorized, then as many elements as specified by the vector width are read from the input array as a vector, accumulated as a vector, and finally written back to the target array. In general, this works well because reductions are typically being performed on dimensions other than the inner-most dimension and therefore, this strategy loads successive elements from memory maximizing memory bandwidth utilization.

TODO explain atomic reduction

7.1.2 Lowering to GPU. The lowering on GPU is slightly more involved than the lowering on CPUs. However, we can lower the same abstractions to efficient implementations and therefore simplify high-level code generation. The lowering for the inplace and non-inplace operations are essentially the same, except for the target array and we do not distinguish between them except for finally storing the result.

The lowering of the `reduce_dimension_*` ops is distinct from existing work on implementing reductions efficiently on GPUs [**?**] because our abstractions potentially represent several independent reductions (independent for different output elements). Therefore, we can either exploit parallelism across the independent reductions or the inherent parallelism in the reduction by performing a divide and conquer reduction.

The reduction pass for GPU can follow one of two paths. If the lowering pass determines that there are enough independent reductions to keep all threads in a thread block busy, then it simply generates code that performs one (or multiple) reductions completely in a thread. If however there are not enough independent reductions, then the lowering pass generates a tree style reduction where multiple threads cooperate to perform a single reduction using inter-thread shuffles.

Another feature specific to GPU reductions is the use of shared memory. If the schedule specifies that the reduction needs to be performed using shared memory, the privatized buffer is allocated in shared memory. Also, the compiler ensures that only as much shared memory is allocated as needed to hold values processed by a single thread-block and index offsets are appropriately rewritten to handle the differences between the indexing of the target memref and the shared memory array. Our abstractions allow our lowering passes to be written completely independent of whether we use shared memory and therefore allow us to enable or disable shared memory use independently from the other parts of the compiler.

7.2 Use in Treebeard

We now present an example specific to Treebeard. The schedule with which code is generated is as below. `N_t` is the number of trees and `batch_size` is the batch size. The schedule tiles both the batch loop and the tree loop and parallelizes the outer batch and tree loops.

```
IndexVar i0, i1, t0, t1;
auto& batch = schedule.GetBatchIndex();
auto& tree = schedule.GetTreeIndex();
schedule.Tile(batch, i0, i1, batch_size/2);
schedule.Tile(tree, t0, t1, N_t/2);
schedule.Reorder({i0, t0, t1, i1});
schedule.Parallel(t0);
schedule.Parallel(i0);
```

The loop-nest generated by Treebeard for the above schedule is as follows.

```
builtin.func @Prediction_Function(%arg0: memref<
    batch_sizexnum_featuresxf64>) -> memref<
    batch_sizexf64> {
  %result = memref.alloc <batch_sizexf64>
  %0 = #decisionforest<ReductionType = 0, #Trees = N_t,
    resultType = memref<batch_sizexf64>>
  par.for i0 = range(0 : batch_size/2 : batch_size) {
    par.for t0 = range(0 : N_t/2 : N_t) {
      for t1 = range(0 : N_t/2) {
        for i1 = range(0 : batch_size/2) {
          %2 = GetTree(%0, t0 + t1)
          %3 = WalkDecisionTree(%2, %arg0[i0+i1])
          reduce(%result, i0+i1, %3)
        }
      }
    }
  }
}
```

Treebeard determines that the `t0` loop is a conflicting loop for the `result` array and therefore legalizes the reduction by inserting a privatized array `result_1`. The privatized dimension of this array is reduced at the end of the `t0` loop.

```
builtin.func @Prediction_Function(%arg0: memref<
    batch_sizexnum_featuresxf64>) -> memref<
    batch_sizexf64> {
  %result = memref.alloc <batch_sizexf64>
  %result_1 = memref.alloc <2xbatch_sizexf64>
```

```
    %0 = #decisionforest<ReductionType = 0, #Trees = N_t,
      resultType = memref<batch_sizexf64>>
    par.for i0 = range(0 : batch_size/2 : batch_size) {
      par.for t0 = range(0 : N_t/2 : N_t) {
        for t1 = range(0 : N_t/2) {
          for i1 = range(0 : batch_size/2) {
            %2 = GetTree(%0, t0 + t1)
            %3 = WalkDecisionTree(%2, %arg0[i0+i1])
            reduce(%result, i0+i1, %3)
          }
        }
      }
      %result[i0 : i0+step] = reduce_dimension(%result_1,
      0, i0 : i0+step)
    }
  }
```

While legalizing the reduction, the compiler determines that the `reduce_dimension` operation must only process a subset of the final result that is computed within the current parallel iteration of the `i0` loop. Once this process is complete, the `reduce` ops in the result IR can be lowered to a simple "read-accumulate-write" sequence of instructions

Finally, we note that in our experiments, we found that our current implementation of lowering the reduction operations was sufficient and reduction is not the bottleneck in our generated code. However, we believe this approach to enabling higher level code generators to easily generate reductions through simple abstractions and then having the compiler automatically lower them to efficient implementation is an important area for future work with applicability in several domains.

TODO Should we mention how we handle multi-class models?

8 Representations

The design of the Treebeard compiler allows the implementation of different strategies for the in-memory representation of the model. The compiler currently has implementations for the three representations shown in Figure 3. The array and sparse representations are the ones described in the Treebeard paper[8]. The reorg representation is the representation used by the RAPIDs library[?]. The **array representation** is the simplest representation where the trees are stored in an array in level order. The **sparse representation** stores the trees in a sparse format where only the non-zero nodes are stored and nodes contain pointers to their children. The **reorg representation** interleaves the array representation of each tree in the model: all root nodes are stored first, then the left children of all the roots and so on.

One of the major changes we make to the original design of Treebeard [8] is to separate the implementation of representations from the rest of the compiler. This allows us to implement representations as plugins to the compiler. We do this by defining an interface that each representation needs to implement. The code generator is implemented using methods on the representation interface thus hiding

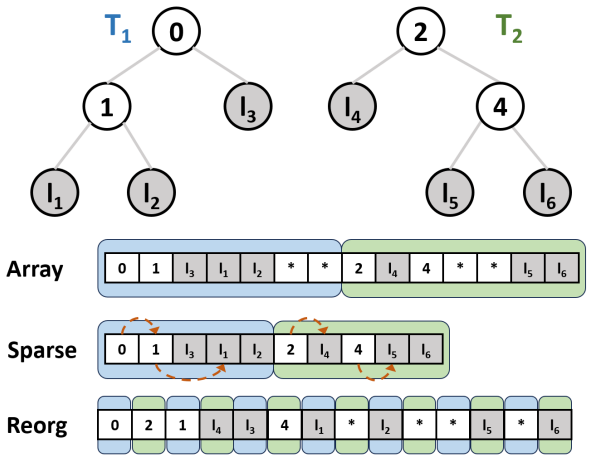


Figure 3. The three representations supported by Treebeard.

details of the actual representation from the core compiler. The interface abstracts the following details.

- **Buffer allocation:** The representation object exposes methods that generate buffer allocations in the IR.
- **Moving to child nodes:** Given the value of the predicate at a node (or tile), generate code to move to the appropriate child node of the current node.
- **Leaf representation:** The interface abstracts determining whether the current node is a leaf (which is needed to terminate walks) and how to get the value of the leaf (which is needed to get a tree's prediction).
- **Caching trees:** Since reading the trees into shared memory on GPU (or prefetching) them on CPU require knowledge of the buffer layout, the task of generating caching code for trees is delegated to the representation object.
- **Loading thresholds and feature indices:** The representation object provides MLIR rewrite patterns to lower operations that load thresholds and feature indices to LLVM IR. This is necessary because the details of how to load the thresholds and feature indices from the model buffers is representation-specific.

In summary, the representation interface abstracts the details of how the model is stored in memory and allows the compiler to generate code without having to explicitly know the details of the representation. This design allows us to implement new representations without changing the core compiler infrastructure. Implementing the representations as plugins also allows us to reuse the implementations across different lowering pipelines. For example, the code for the array and sparse representations is almost fully shared between the CPU and GPU lowering pipelines.

9 Caching Implementation

Treebeard provides mechanisms to cache both trees and input rows on both the CPU and GPU. As described in Section 5, the user can specify that the working set of an iteration of an index variable needs to be cached using the `Cache()` directive. This provides a unified way to specify caching of both trees and input rows.

Treebeard implements caching at the granularity of a tree or a row. Also, the semantics of caching depends on the target processor. For the CPU, caching is implemented as prefetching, while for the GPU, caching is implemented using shared memory.

9.1 IR Representation of Caching

Caching is encoded in the mid-level IR using the `cacheTrees` and `cacheRows` operations. These operations are generated when the HIR lowered to MIR and `Cache()` is specified on an index variable in the schedule. While the HIR is being lowered and a cached index variable is encountered, the compiler generates a `cacheTrees` or `cacheRows` operation depending on whether the index variable is a tree or a batch index variable. Additionally, as a part of the lowering process, Treebeard determines the working set of the loop with caching enabled and generates a caching operation with the appropriate limits.

Each of the caching operations take parameters that specify the set of trees or rows that need to be cached. The caching operations are defined as follows.

- **`cacheTrees(forest, start, end)`:** This operation caches the trees in ensemble `forest` from `start` to `end`. The trees are cached in the order in which they are specified in the ensemble.
- **`cacheRows(data, start, end)`:** This operation caches the rows in the input array `data` from `start` to `end`. The rows are cached in the same order as in the input array.

9.2 Lowering of Caching Operations

When the MIR is lowered to LIR, the cache ops are lowered to target-specific code. Each of the two caching operations is lowered differently for the CPU and the GPU.

For the `cacheRows` operation, Treebeard uses pre-implemented lowerings for both CPU and GPU. This is possible because the input is currently assumed to be a dense array format. Therefore, regardless of any other configuration choices (like what representation is used for the model itself), the lowering of the `cacheRows` operation is the same. For the CPU, the `cacheRows` operation is lowered to prefetches. For the GPU, a series of coalesced loads read the rows into shared memory.

For the `cacheTrees` operation, the lowering is representation-specific. Each representation provides a lowering to the target-specific code generator to lower the `cacheTrees` op when that representation is used. However, the Treebeard infrastructure does provide helpers to generate caching code to cache contiguous regions of memory. These helpers are reused as required across different representations.

10 Exploring the Schedule Space

The set of schedules that can be constructed using the scheduling language described in Section 5 is vast. Searching this schedule space to find a schedule that provides good performance is a non-trivial task. To simplify this process, we identify a template schedule for GPUs that encompasses several strategies published in prior work. Our template schedule assigns a fixed number of rows to each thread block and to each thread. It distributes the trees across a fixed number of threads and can cache trees and input rows if required. Unrolling and interleaving of tree walks is also supported.

The template schedule exposes a number of parameters that can be tuned to find a high-performance schedule.

- **Number of rows per thread block (Integer):** The number of rows that are processed by each thread block.
- **Number of rows per thread (Integer):** The number of rows processed by each thread.
- **Number of tree threads (Integer):** The number of threads across which the trees are distributed.
- **Cache rows (Boolean):** Whether the input rows are cached in shared memory.
- **Cache trees (Boolean):** Whether the trees are cached in shared memory.
- **Unroll walks (Boolean):** Whether the tree walks are unrolled.
- **Tree walk interleave factor:** The number of tree walks that are interleaved.
- **Shared memory reduction:** Whether the reduction across tree threads is done in shared memory.

While the template schedule simplifies optimization of generated inference code, it is important to note that the Treebeard compiler itself does not place any restrictions on the schedule. The user is free to specify any schedule they wish. The auto-scheduler that implements the template schedule is implemented as a module outside the core Treebeard compiler. Users are also free to implement other auto-schedulers that generate schedules different from the template schedule.

Figure 4 shows the distribution of normalized execution times for all benchmark models with different parameter values for the template schedule. The normalized execution time is the inference time of the model with a given set of parameter values divided by the best inference time of the model. The histogram shows that even within the variants of the template schedule, there is a significant amount of variation in performance. Very few schedules perform close to the best while a vast majority of schedules perform poorly.

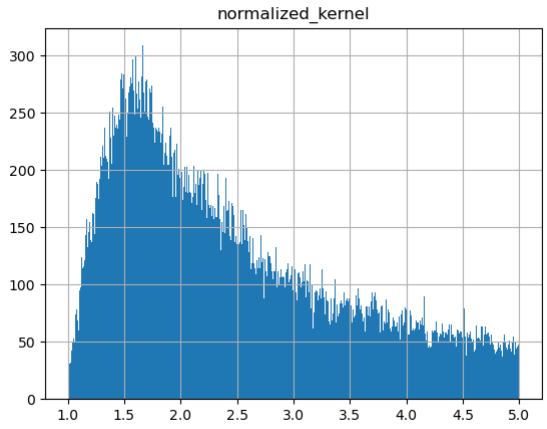


Figure 4. Distribution of normalized execution times for all benchmark models with different parameter values for the template schedule. TODO We need to describe exactly what the considered set of parameter values are.

Exploring the schedule space extensively even for a reasonable set of parameter values is very expensive. We explored the set of parameter values listed in Table ?? for our benchmarks and found that it took anywhere between thirty minutes up to a few hours to explore the entire space. Performing this extensive search for every model being compiled is infeasible in practise. We therefore need a better mechanism to guide the search for a good schedule.

We design a heuristic to narrow down the set of schedules to explore based on the following observations on high-performance schedules.

- For small batch sizes, the best schedules tend to have a small number of rows per thread block and partition the trees across a larger number of threads. This is intuitive since the amount of data parallelism across the rows is limited for small batch sizes.
- Always cache rows in shared memory and never cache trees. We find that caching rows when possible (i.e., when the number of features is small enough to fit in shared memory) almost always improves performance. Caching trees on the other hand almost always degrades performance. This is because the one time cost of loading trees into shared memory is not sufficiently amortized when the whole of the tree is not accessed during inference.
- Models with a large number of features tend to benefit from partitioning the trees across more threads even at larger batch sizes. This is because processing fewer rows at a time allows us to keep them in shared memory. We empirically find that the threshold for when we should start partitioning the trees across more threads is when the number of features is greater than 100.
- We find that when a model prefers schedules with shared reduction, the same schedules without shared reduction are among the best performing schedules without shared reduction. We therefore are able to separate the evaluation of shared reduction by collecting the best schedules without shared reduction and only evaluating shared reduction on them. Evaluating the top 3 schedules for shared reduction is sufficient in practice.

Treebeard uses these observations to narrow down the set of schedules to explore. The pseudo-code for the current heuristic is shown in Algorithm 1. The algorithm first computes a subset of thread block configurations in the function `TBConfigs`. A set of schedules based on these thread block configurations is then computed (`schedules`). The model is compiled with each of these schedules and then the resulting inference code is profiled. The three best performing schedules are collected and shared reduction is enabled on them and the resulting schedules evaluated. The best schedule among all the evaluated schedules is selected as the schedule to use. We find that this heuristic is able to find schedules that are close to the best schedules but improves the search time by two orders of magnitude as we show in Section ??.

11 Experimental Evaluation

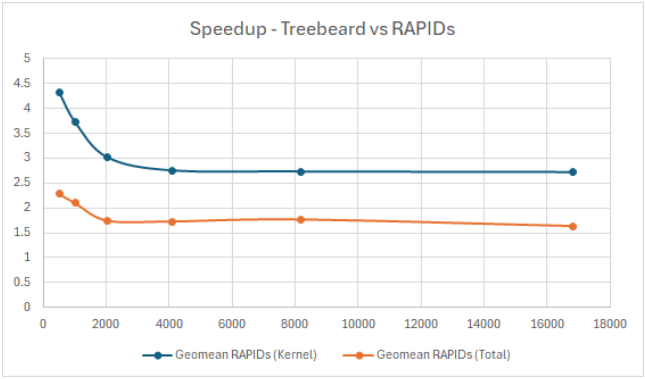


Figure 5. Treebeard vs RAPIDs Speedup on NVIDIA RTX 4060.

12 Citations and Bibliographies

Artifacts: [5] and [4].

References

[1] [n. d.]. Treelite : model compiler for decision tree ensembles. https://treelite.readthedocs.io/en/latest/. Accessed: 2022-04-16.

[2] Nima Asadi, Jimmy Lin, and Arjen P. de Vries. 2014. Runtime Optimizations for Tree-Based Machine Learning Models. *IEEE Transactions on Knowledge and Data Engineering* 26, 9 (2014), 2281–2292. https://doi.org/10.1109/TKDE.2013.73

Algorithm 1 Heuristic to guide the search for a good schedule

procedure TBConfigs(N_{batch}, N_f)
  $T_{batch} \leftarrow 2048, T_f \leftarrow 128$
  if $N_{batch} \leq T_{batch}$ **then**
    $rowsPerBlock \leftarrow \{8, 32\}$
    $treeThreads \leftarrow \{20, 50\}$
  else
    if $N_f \leq T_f$ **then**
      $rowsPerBlock \leftarrow \{32, 64\}$
      $treeThreads \leftarrow \{2, 10\}$
    else
      $rowsPerBlock \leftarrow \{8, 32\}$
      $treeThreads \leftarrow \{20, 50\}$
    end if
  end if
  return $rowsPerBlock, treeThreads$
end procedure

$bestSchedules \leftarrow PriorityQueue(3)$
$rowsPerTB, treeThreads \leftarrow TBConfigs(N_{batch}, N_f)$
$cacheRows \leftarrow$ True
$cacheTrees \leftarrow$ False
$interleaveFactors \leftarrow \{1, 2, 4\}$
$reps \leftarrow \{array, sparse, reorg\}$
$schedules \leftarrow (rowsPerTB, treeThreads,$
  $cacheRows, cacheTrees,$
  $interleaveFactors)$
for $(sched, rep) \in schedules \times reps$ **do**
  $time \leftarrow EvaluateSchedule(sched, rep)$
  $bestSchedules.insert(time, sched, rep)$
end for
$shMemSchedules \leftarrow \emptyset$
for $sched, rep \in bestSchedules$ **do**
  $EnableSharedReduction(sched)$
  $time \leftarrow EvaluateSchedule(sched, rep)$
  $shMemSchedules.insert(time, sched, rep)$
end for
return $min(shMemSchedules \cup bestSchedules)$

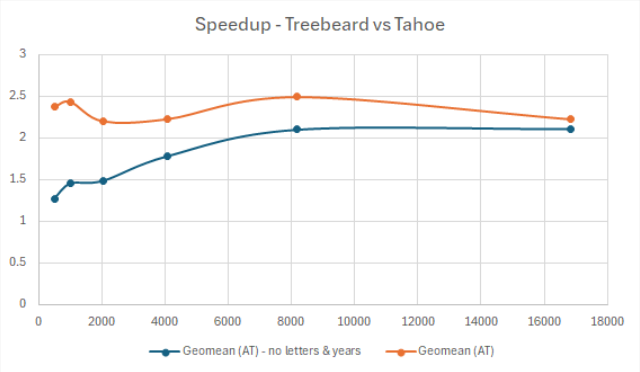


Figure 6. Treebeard vs Tahoe Kernel Time Speedup.

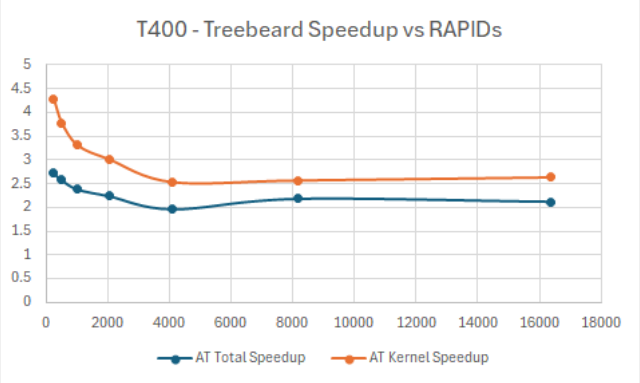


Figure 7. Treebeard vs RAPIDs Speedup on T400.

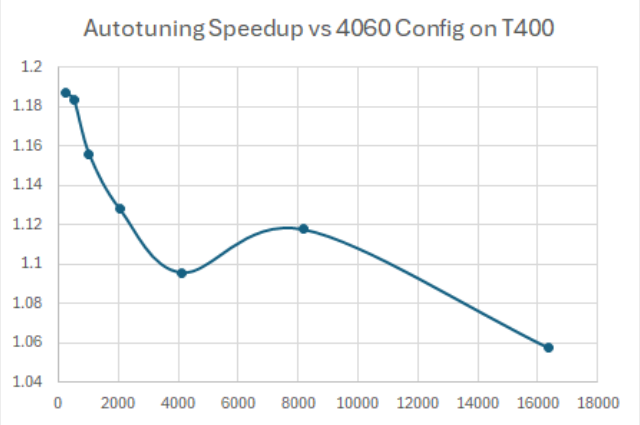


Figure 8. Autotuning heuristics speedup vs best 4060 schedule on T400.

[3] Tianqi Chen and Carlos Guestrin. 2016. XGBoost: A Scalable Tree Boosting System. In *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining* (San Francisco, California, USA) *(KDD '16)*. Association for Computing Machinery, New York, NY, USA, 785–794. https://doi.org/10.1145/2939672.2939785

[4] C. Lattner and V. Adve. 2004. LLVM: a compilation framework for lifelong program analysis amp; transformation. In *International Symposium on Code Generation and Optimization, 2004. CGO 2004.* 75–86. https://doi.org/10.1109/CGO.2004.1281665

[5] Chris Lattner, Mehdi Amini, Uday Bondhugula, Albert Cohen, Andy Davis, Jacques Pienaar, River Riddle, Tatiana Shpeisman, Nicolas Vasilache, and Oleksandr Zinenko. 2021. MLIR: Scaling Compiler Infrastructure for Domain Specific Computation. In *2021 IEEE/ACM International Symposium on Code Generation and Optimization (CGO)*. 2–14. https://doi.org/10.1109/CGO51591.2021.9370308

[6] Claudio Lucchese, Franco Maria Nardini, Salvatore Orlando, Raffaele Perego, Nicola Tonellotto, and Rossano Venturini. 2015. QuickScorer: A Fast Algorithm to Rank Documents with Additive Ensembles of Regression Trees. In *Proceedings of the 38th International ACM SIGIR Conference on Research and Development in Information Retrieval* (Santiago, Chile) *(SIGIR '15)*. Association for Computing Machinery, New York, NY, USA, 73–82. https://doi.org/10.1145/2766462.2767733

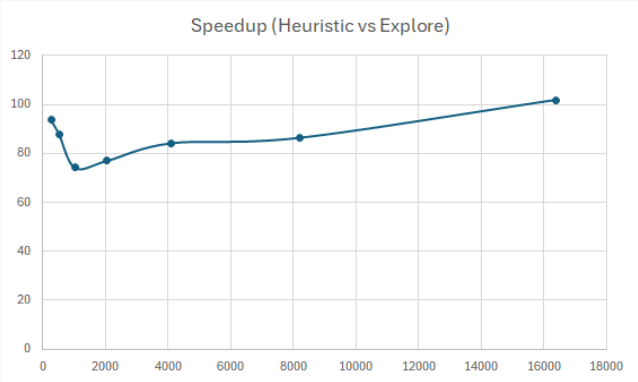


Figure 9. Autotuning heuristic compile time speedup vs full schedule exploration.

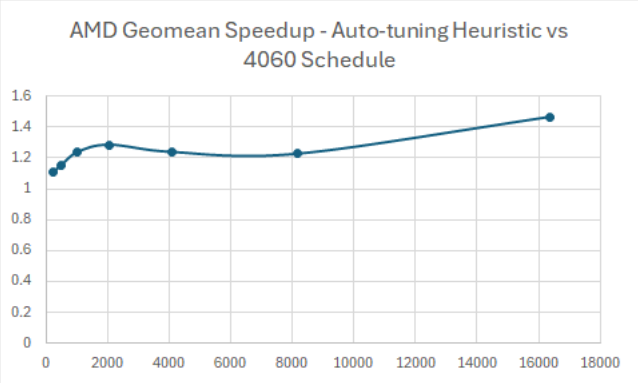


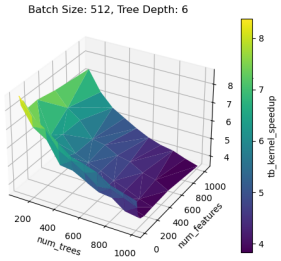
Figure 10. Autotuning heuristics speedup vs best 4060 schedule on MI210.

[7] Supun Nakandala, Karla Saur, Gyeong-In Yu, Konstantinos Karanasos, Carlo Curino, Markus Weimer, and Matteo Interlandi. 2020. A Tensor Compiler for Unified Machine Learning Prediction Serving. In *14th USENIX Symposium on Operating Systems Design and Implementation (OSDI 20)*. USENIX Association, 899–917. https://www.usenix.org/conference/osdi20/presentation/nakandala

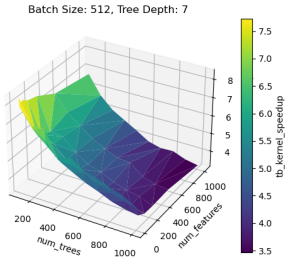
[8] Ashwin Prasad, Sampath Rajendra, Kaushik Rajan, R Govindarajan, and Uday Bondhugula. 2022. Treebeard: An Optimizing Compiler for Decision Tree Based ML Inference. In *2022 55th IEEE/ACM International Symposium on Microarchitecture (MICRO)*. 494–511. https://doi.org/10.1109/MICRO56248.2022.00043

[9] Zhen Xie, Wenqian Dong, Jiawen Liu, Hang Liu, and Dong Li. 2021. Tahoe: Tree Structure-Aware High Performance Inference Engine for Decision Tree Ensemble on GPU. In *Proceedings of the Sixteenth European Conference on Computer Systems* (Online Event, United Kingdom) *(EuroSys '21)*. Association for Computing Machinery, New York, NY, USA, 426–440. https://doi.org/10.1145/3447786.3456251

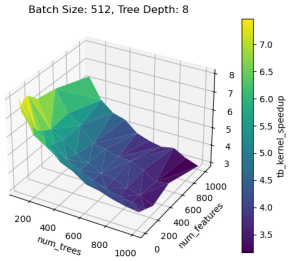
Received 20 February 2007; revised 12 March 2009; accepted 5 June 2009



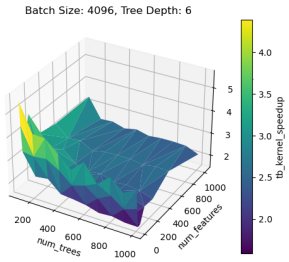
(a) Kernel Speedup for batch size 512, depth 6.



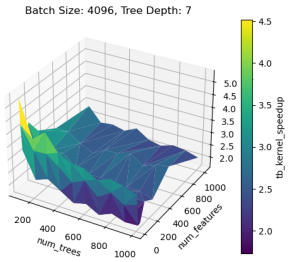
(b) Kernel Speedup for batch size 512, depth 7.



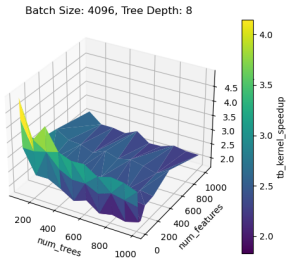
(c) Kernel Speedup for batch size 512, depth 8.



(d) Kernel Speedup for batch size 4096, depth 6.



(e) Kernel Speedup for batch size 4096, depth 7.



(f) Kernel Speedup for batch size 4096, depth 8.