1.a $\lim\limits_{n\to\infty} \frac{1+3+\dots+(2n-1)}{3n^2} = \frac{1}{3n^2} + \frac{3}{3n^2} + \dots + \frac{2n-1}{3n^2} =$

$= \cancel{\frac{1}{3n^2}} + \frac{3}{3n^2} + \dots + \frac{2n}{3n^2} - \cancel{\frac{1}{3n^2}} = \frac{1}{n^2} + \dots + \frac{2}{3n} = \frac{1}{\infty^2} + \dots +$

$+ \frac{2}{3\infty} = 0 + 0 = 0$

1.б $\lim\limits_{n\to\infty} \left(\frac{3n^2-n-4}{5n^2-2n+7}\right)^6 = \left(\frac{\frac{3n^2-n-4}{3n^2}}{\frac{5n^2-2n+7}{3n^2}}\right)^6 = \left(\frac{1-\frac{1}{3n}-\frac{4}{3n^2}}{\frac{5}{3}-\frac{2}{3n}+\frac{7}{3n^2}}\right)^6 =$

$= \left(\frac{1-\frac{1}{3\cdot\infty}-\frac{4}{3\cdot\infty^2}}{\frac{5}{3}-\frac{2}{3\infty}+\frac{7}{3\cdot\infty^2}}\right)^6 = \left(\frac{1}{\frac{5}{3}}\right)^6 = \left(\frac{3}{5}\right)^6 = \frac{729}{15625} \approx 0{,}0466$

1.в $\lim\limits_{n\to\infty} 3^{\frac{3n-4}{6n+2}} = 3^{\frac{\frac{3n-4}{3n}}{\frac{6n+2}{3n}}} = 3^{\frac{1-\frac{4}{3n}}{2+\frac{2}{3n}}} =$

$= 3^{\frac{1-\frac{4}{3\infty}}{2+\frac{2}{3\cdot\infty}}} = 3^{\frac{1-0}{2+0}} = 3^{\frac{1}{2}} = \sqrt{3}$

1.2 $\lim\limits_{n\to\infty} \left(\sqrt{n^2+2}-n\right) = n\sqrt{1+\frac{2}{n^2}} - n = \infty\sqrt{1+\frac{2}{\infty^2}} - \infty =$

$= \infty\sqrt{1+0} - \infty = \infty - \infty = 0$

Получили неопределенность $\infty - \infty$

Решение (продолжение) на стр. 2

1.g $\lim\limits_{n\to\infty} \dfrac{n^2}{1+5+9+\dots+(4n-3)} = \dfrac{\frac{n^2}{n^2}}{\frac{1+5+9+\dots+(4n-3)}{n^2}} =$

$= \dfrac{1}{\frac{1}{n^2}+\frac{5}{n^2}+\frac{9}{n^2}+\dots+\frac{4n}{n^2}-\frac{3}{n^2}} = \dfrac{1}{\frac{1}{\infty^2}+\frac{5}{\infty^2}+\frac{9}{\infty^2}+\frac{4}{\infty}-\frac{3}{\infty^2}} =$

$= \dfrac{1}{0} = \infty$

1.e $\lim\limits_{n\to\infty} \left(\dfrac{n+2}{2n+1}\right)^{n^2} = \left(\dfrac{\frac{n+2}{n}}{\frac{2n+1}{n}}\right)^{n^2} = \left(\dfrac{1+\frac{2}{n}}{2+\frac{1}{n}}\right)^{n^2} =$

$= \left(\dfrac{1+\frac{2}{\infty}}{2+\frac{1}{\infty}}\right)^{\infty^2} = \left(\dfrac{1}{2}\right)^{\infty^2} = 0$

1.2 (Продолжение)

$\lim\limits_{n\to\infty} \left(\sqrt{n^2+2}-n\right) = \dfrac{(\sqrt{n^2+2}-n)(n^2+2)^2}{(n^2+2)^2} =$

$= \dfrac{(\sqrt{n^2+2})(n^2+2)^2 - n(n^2+2)^2}{(n^2+2)^2} = \dfrac{(n^2+2)^{\frac{1}{2}}\cdot(n^2+2)^2 - n(n^2+2)^2}{(n^2+2)^2} =$

$= \dfrac{(n^2+2)^1 - n(n^2+2)^2}{(n^2+2)^2} = \dfrac{\cancel{(n^2+2)}(1-n(n^2+2))}{(n^2+2)^{\cancel{2}}} =$

$= \dfrac{1-n(n^2+2)}{n^2+2} = \dfrac{1-n^3-2n}{n^2+2} = \dfrac{\frac{1-n^3-2n}{n^3}}{\frac{n^2+2}{n^3}} = \dfrac{\frac{1}{n^3}-1-\frac{2}{n^2}}{\frac{1}{n}+\frac{2}{n^3}} =$

$$= \frac{\frac{1}{\infty^3} - 1 - \frac{2}{\infty^2}}{\frac{1}{\infty} + \frac{2}{\infty^3}} = \frac{0-1-0}{0+0} = -\frac{1}{0} = -\infty$$

2.a $$\lim_{x \to 1} \frac{x^{100} - 2x+1}{x^{50} - 2x+1} = \frac{\frac{x^{100}-2x+1}{2x+1}}{\frac{x^{50}-2x+1}{2x+1}} = \frac{\frac{x^{100}}{2x+1} - 1}{\frac{x^{50}}{2x+1} - 1} =$$

$$= \frac{\frac{1}{2\cdot 1+1} - 1}{\frac{1}{2\cdot 1+1} - 1} = \frac{\frac{1}{3} - 1}{\frac{1}{3} - 1} = \frac{-\frac{2}{3}}{-\frac{2}{3}} = 1$$

2.б $$\lim_{x \to 0} \frac{\sqrt{x^2+1} - 1}{\sqrt{x^2+16} - 4} = \frac{(\sqrt{x^2+1} - 1)(\sqrt{x^2+1} + 1)}{(\sqrt{x^2+16} - 4)(\sqrt{x^2+1} + 1)} = \frac{x^2}{(\sqrt{x^2+16} - 4)(\sqrt{x^2+1} + 1)}$$

$$= \lim_{x \to 0} \frac{x^2}{\sqrt{x^2+16} - 4} \cdot \lim_{x \to 0} \frac{1}{\sqrt{x^2+1} + 1} = A$$

$$\lim_{x \to 0} \frac{x^2(\sqrt{x^2+16}+4)}{(\sqrt{(x^2+16)} - 4)(\sqrt{x^2+16}+4)} = \frac{\cancel{x^2}(\sqrt{x^2+16}+4)}{\cancel{x^2}+\cancel{16}-\cancel{16}} = \sqrt{x^2+16}+4$$

$$= \sqrt{16} + 4 = 8$$

$$\lim_{x \to 0} \frac{1}{\sqrt{x^2+1} + 1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$$A = 8 \cdot \frac{1}{2} = 4$$

2.2. $\lim\limits_{x \to \pi} \frac{\sin(x)}{\pi^2 - x^2} = \frac{\sin(\pi)}{\pi^2 - \pi^2} = \frac{0}{0}$

Используем правило Лопиталя:

$$\lim_{x \to \pi} \frac{\sin(x)}{\pi^2 - x^2} = \lim_{x \to \pi} \frac{\frac{d}{dx}(\sin(x))}{\frac{d}{dx}(\pi^2 - x^2)} = \frac{\cos(x)}{-2x} = \frac{\cos(\pi)}{-2\pi} =$$

$$= \frac{1}{2\pi}$$

2.g $\lim\limits_{x \to 0} \frac{\sin(\sin(x))}{\sin(5x)} = \frac{\sin(\sin(0))}{\sin(5 \cdot 0)} = \frac{0}{0}$

Воспользуемся правилом Лопиталя:

$$\lim_{x \to 0} \frac{\sin(\sin(x))}{\sin(5x)} = \lim_{x \to 0} \frac{\frac{d}{dx}(\sin(\sin(x)))}{\frac{d}{dx}(\sin(5x))} =$$

$$= \frac{\cos(x) \cdot \cos(\sin(x))}{5\cos(5x)} = \frac{\cos(0) \cdot \cos(\sin(0))}{5\cos(5 \cdot 0)} = \frac{1}{5}$$

2.b) $\lim_{x\to\infty} x^{3/2}\left(\sqrt{x+2}-2\sqrt{x+1}+\sqrt{x}\right) = -\sqrt{\frac{x^3\left(\sqrt{x+2}-2\sqrt{x+1}+\sqrt{x}\right)\left(\sqrt{x}+2\sqrt{x+1}-\sqrt{x+2}\right)}{\sqrt{x}+2\sqrt{x+1}-\sqrt{x+2}}} =$

$$= \frac{-\sqrt{\lim_{x\to\infty} 4x^3\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}}{\sqrt{x}+2\sqrt{x+1}-\sqrt{x+2}} = -\sqrt{\lim_{x\to\infty} \frac{4x^3\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{x}+2\sqrt{x+1}-\sqrt{x+2}\right)^2}} =$$

$$= -\sqrt{4\lim_{x\to\infty} \frac{x^3\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{x}+2\sqrt{x+1}-\sqrt{x+2}\right)^2}} = -\sqrt{4\lim_{x\to\infty} \frac{x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(1+\frac{2\sqrt{x+1}-\sqrt{x+2}}{\sqrt{x}}\right)^2}} =$$

$$= -\sqrt{4\lim_{x\to\infty} \frac{x^3\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{x}\left(x^{-1/2}\left(2\sqrt{x+1}-\sqrt{x+2}\right)+1\right)\right)^2}} = -\sqrt{\frac{4\lim_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\lim_{x\to\infty}\left(x^{-1/2}\left(2\sqrt{x+1}-\sqrt{x+2}\right)+1\right)^2}} =$$

$$= -\sqrt{\frac{4\lim_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\lim_{x\to\infty} x^{-\frac{1}{2}}\left(2\sqrt{x+1}-\sqrt{x+2}\right)+1\right)^2}} = -\sqrt{\frac{4\lim_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\lim_{x\to\infty}\left(\sqrt{\frac{\left(2\sqrt{x+1}-\sqrt{x+2}\right)^2}{x}}\right)+1\right)^2}} =$$

$$= -\sqrt{\frac{4\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\lim\limits_{x\to\infty}\sqrt{\frac{\left(2\sqrt{x+1}-\sqrt{x+2}\right)^2}{x}}+1\right)^2}} = -\sqrt{\frac{4\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\lim\limits_{x\to\infty}\frac{\left(2\sqrt{x+1}\left(1-\frac{1}{2}(x+1)^{-1/2}\sqrt{x+2}\right)\right)^2}{x}+1\right)^2}} =$$

$$= -\sqrt{\frac{4\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(4\left(\lim\limits_{x\to\infty}\frac{x+1}{x}\right)\left(\lim\limits_{x\to\infty}\left(1-\frac{1}{2}(x+1)^{-1/2}\sqrt{x+2}\right)^2+1\right)\right)^2}} = -\sqrt{\frac{\cancel{4}\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\lim\limits_{x\to\infty}\frac{1+\frac{1}{x}}{1}\cdot\lim\limits_{x\to\infty}\left(1-\frac{1}{2}(x+1)^{-1/2}\sqrt{x+2}\right)^2}+1\right)^2}} =$$

$$= -\sqrt{\frac{\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\lim\limits_{x\to\infty}\left(1-\frac{1}{2}(x+1)^{-1/2}\sqrt{x+2}\right)^2}+1\right)^2}} = -\sqrt{\frac{\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\left(\lim\limits_{x\to\infty}\left(\cancel{1}-\frac{1}{2}(x+1)^{-1/2}\sqrt{x+2}\right)^2\right)}+1\right)^2}} =$$

$$= -\sqrt{\frac{\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\left(\left(-\frac{1}{2}\right)\lim\limits_{x\to\infty}(x+1)^{-\frac{1}{2}}\sqrt{x+2}\right)^2+1}\right)^2}} = -\sqrt{\frac{\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\left(1-\frac{\lim\limits_{x\to\infty}\left(\sqrt{\frac{x+2}{x+1}}\right)}{2}\right)^2+1}\right)^2}} =$$

$$= -\sqrt{\frac{\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\left(1-\frac{\sqrt{\lim\limits_{x\to\infty}\frac{x+2}{x+1}}}{2}\right)^2+1}\right)^2}} = -\sqrt{\frac{\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{\left(1-\frac{\sqrt{\lim\limits_{x\to\infty}\frac{1+\frac{2}{x}}{1+\frac{1}{x}}}}{2}\right)^2+1}\right)^2}} =$$

$$= -\sqrt{\frac{4\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2}{\left(\sqrt{4\left(\left(1-\frac{\sqrt{\frac{1}{1}}}{2}\right)^2\right)}+1\right)^2}} = -\sqrt{\frac{1}{4}\cdot 4\lim\limits_{x\to\infty} x^2\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2} =$$

$$= -\sqrt{\lim_{x\to\infty}\left(\left(x\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)\right)^2\right)} = -\sqrt{\lim_{x\to\infty}\frac{x\left(-3-2x+2\sqrt{x+1}\sqrt{x+2}\right)^2\left(-3-2x-2\sqrt{x+1}\sqrt{x+2}\right)}{-3-2x-2\sqrt{x+1}\sqrt{x+2}}} =$$

$$= -\sqrt{\lim_{x\to\infty}\left(\frac{x}{-3-2x-2\sqrt{x+1}\sqrt{x+2}}\right)^2} = -\sqrt{\lim_{x\to\infty}\left(\frac{x}{-2x-2\sqrt{x+1}\sqrt{x+2}}\right)^2} =$$

$$= -\sqrt{\lim_{x\to\infty}\frac{x}{\left(-2x\left(\frac{\sqrt{x+1}\sqrt{x+2}}{x}+1\right)\right)^2}} = -\sqrt{\lim_{x\to\infty}\left(-\frac{1}{2\left(\frac{\sqrt{x+1}\sqrt{x+2}}{x}+1\right)}\right)^2} = -\sqrt{-\lim_{x\to\infty}\left(\frac{1}{2\left(\frac{\sqrt{x+1}\sqrt{x+2}}{x}+1\right)}\right)^2} =$$

$$= -\sqrt{-\lim_{x\to\infty}\frac{\left(\frac{1}{\frac{\sqrt{x+1}\sqrt{x+2}}{x}+1}\right)^2}{2}} = -\sqrt{\frac{\left(-\frac{1}{\lim_{x\to\infty}\left(\frac{\sqrt{x+1}\sqrt{x+2}}{x}+1\right)}\right)^2}{2}} = -\sqrt{\left(\frac{-1}{2\lim_{x\to\infty}\left(\frac{\sqrt{x+1}\sqrt{x+2}}{x}+1\right)}\right)^2} =$$

$$= -\sqrt{\left(\frac{-1}{2\left(\sqrt{\lim\limits_{x\to\infty} \frac{(x+1)(x+2)}{x^2}}+1\right)}\right)^2} = -\sqrt{\left(\frac{-1}{2\left(\sqrt{\left(\lim\limits_{x\to\infty} \frac{2x+3}{2x}\right)}+1\right)}\right)^2} =$$

$$= -\sqrt{\left(\frac{-1}{2\left(\sqrt{\frac{1}{2}\lim\limits_{x\to\infty} \frac{2x+3}{x}}+1\right)}\right)^2} = -\sqrt{\left(\frac{-1}{2\left(\sqrt{\frac{1}{2}\lim\limits_{x\to\infty} \frac{2+\frac{3}{x}}{1}}+1\right)}\right)^2} = -\sqrt{\left(\frac{-1}{2\left(\sqrt{\frac{1}{2}\cdot 2}+1\right)}\right)^2} =$$

$$= -\sqrt{\left(-\frac{1}{4}\right)^2} = -\frac{1}{4}$$

3.a) $f(x) = \frac{2x^3 - 6x^2}{x-3} = \frac{2x^2\cancel{(x-3)}}{\cancel{x-3}} = 2x^2$

$\lim\limits_{x \to \infty} 2x^2 = 2 \cdot \infty^2 = \infty$

3.б) $f(x) = \begin{cases} x+7, & x \leq 2 \\ x^2+1, & -2 \leq x \leq 2 \\ 12-x, & x \geq 2 \end{cases}$

$\lim\limits_{x \to -2-0} (x+7) = -2+7 = 5$

$\lim\limits_{x \to -2+0} (x^2+1) = (-2)^2 + 1 = 5$

$x = -2$, Точка устранимого разрыва

$\lim\limits_{x \to 2-0} (x^2+1) = 2^2+1 = 5$

$\lim\limits_{x \to 2+0} (12-x) = 12-2 = 10$

$x = 2$, Точка разрыва первого рода