

$$1.a \quad \left| \begin{array}{ccc} 4 & -3 & -3 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{array} \right| \sim \left| \begin{array}{ccc} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 4 & -3 & -3 \end{array} \right| \xrightarrow{(2)-(1), (3)-4(1)} \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & -11 & -7 \end{array} \right| \sim$$

$$\xrightarrow{(3)-1(2)} \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -18 \end{array} \right|; \quad \det A = 1 \cdot (-1) \cdot (-18) = 18$$

$$\det(A - \lambda E) = \left| \begin{array}{ccc} 4-\lambda & -3 & -3 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{array} \right| = (4-\lambda)((2-\lambda)^2 - 1) - (-3(2-\lambda) + 3) + (-3 - (-3)(2-\lambda)) = (4-\lambda)(4 - 4\lambda + \lambda^2 - 1) - (-6 + 3\lambda + 3) + (-3 - (-6 + 3\lambda)) = 4\lambda^2 - 16\lambda + 12 - \lambda^3 + 4\lambda^2 - 3\lambda - 6\lambda + 6 = -\lambda^3 + 8\lambda^2 - 25\lambda + 18 = -(\lambda-1)(\lambda^2 - 7\lambda + 18) = 0$$

$$\lambda_1 = 1; \lambda_2 = \pm i$$

$$15 \quad \left| \begin{array}{ccc} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{array} \right| \sim - \left| \begin{array}{ccc} 1 & -1 & 1 \\ -1 & 2 & 0 \\ 2 & -1 & 0 \end{array} \right| \xrightarrow{(2)+(1), (3)-2(1)} - \left| \begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & -2 \end{array} \right| \sim$$

$$\xrightarrow{(3)-(2)} - \left| \begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{array} \right|; \quad \det A = -(-3) = 3$$

$$\det(A - \lambda E) = \left| \begin{array}{ccc} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & 0 \\ 1 & -1 & 1-\lambda \end{array} \right| = (2-\lambda)(2-\lambda)(1-\lambda) + (-1)(1-\lambda) = (2-\lambda)^2(1-\lambda) - (1-\lambda) = (4 - 4\lambda + \lambda^2)(1-\lambda) - (1-\lambda) = 4 - 4\lambda - 4\lambda + 4\lambda^2 + \lambda^2 - \lambda^3 - 1 + \lambda = -\lambda^3 + 5\lambda^2 - 7\lambda + 3 =$$

$$= -(\lambda-1)(\lambda^2-4\lambda+3) = -(\lambda-1)(\lambda-1)(\lambda-3) = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = 3$$

(1.6)

$$\left| \begin{array}{ccc} 5 & -1 & -1 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{array} \right| \xrightarrow{(3)+\frac{1}{4}(2)} \left| \begin{array}{ccc} 5 & -1 & -1 \\ 0 & 4 & -1 \\ 0 & 0 & \frac{15}{4} \end{array} \right|$$

$$\det A = 5 \cdot 4 \cdot \frac{15}{4} = 75$$

$$\det(A - \lambda E) = \left| \begin{array}{ccc} 5-\lambda & -1 & -1 \\ 0 & 4-\lambda & -1 \\ 0 & -1 & 4-\lambda \end{array} \right| = (5-\lambda)((4-\lambda)^2 - 1) =$$

$$= (5-\lambda)(16 - 8\lambda + \lambda^2 - 1) = (5-\lambda)(\lambda^2 - 8\lambda + 15) = 5\lambda^2 - 40\lambda + 75 - \lambda^3 + 8\lambda^2 - 15\lambda =$$

$$= -\lambda^3 + 13\lambda^2 - 55\lambda + 75 = -(\lambda-3)(\lambda^2 - 10\lambda + 25) = -(\lambda-3)(\lambda-5)^2 = 0$$

$$\lambda_1 = 3$$

$$\lambda_2 = 5$$

2.a

$$A = \begin{pmatrix} 7 & -4 & -2 \\ -2 & 5 & -2 \\ 0 & 0 & 9 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} 7-\lambda & -4 & -2 \\ -2 & 5-\lambda & -2 \\ 0 & 0 & 9-\lambda \end{vmatrix} = (7-\lambda)(5-\lambda)(9-\lambda) +$$

$$+ 2(-4(9-\lambda)) = (7-\lambda)(45-5\lambda-9\lambda+\lambda^2) + 2(-36+4\lambda) =$$

$$= (7-\lambda)(\lambda^2-14\lambda+45) - 72 + 8\lambda = 7\lambda^2 - 98\lambda + 315 - \lambda^3 + 14\lambda^2 - 45\lambda +$$

$$+ 8\lambda - 72 = -\lambda^3 + 21\lambda^2 - 135\lambda + 243 = -(\lambda-3)(\lambda^2-18\lambda+81) =$$

$$= -(\lambda-3)(\lambda-9)^2 = 0$$

$$\lambda_1 = 3; \lambda_2 = 9$$

$$\lambda = 3$$

$$\left(\begin{array}{ccc|c} 4 & -4 & -2 & 0 \\ -2 & 2 & -2 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) \xrightarrow{(1):2} \left(\begin{array}{ccc|c} 2 & -2 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & -2 & -1 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) \sim$$

$$\xrightarrow{(2)-2(1)} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) \xrightarrow{(3)-6(2)} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} c$$

$$\lambda = 9$$

$$\begin{pmatrix} -2 & -4 & -2 & | & 0 \\ -2 & -4 & -2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{(1)-(2)} \begin{pmatrix} 0 & 0 & 0 & | & 0 \\ -2 & -4 & -2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 & | & 0 \\ 1 & 2 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$x_1 = -2x_2 - x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} c + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} c$$

Набор базис $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\varphi(F_1) = 3F_1, \quad \varphi(F_2) = 9F_2, \quad \varphi(F_3) = 9F_3$$

$$\tilde{A} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

2.5

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} 3-\lambda & 0 & 0 \\ 1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = (3-\lambda)((2-\lambda)^2 - 1) =$$

$$= (3-\lambda)(4-4\lambda+\lambda^2-1) = (3-\lambda)(\lambda^2-4\lambda+3) = 3\lambda^2 - 12\lambda + 9 - \lambda^3 + 4\lambda^2 -$$

$$-3\lambda = -\lambda^3 + 7\lambda^2 - 15\lambda + 9 = -(\lambda-1)(\lambda^2 - 6\lambda + 9) = -(\lambda-1)(\lambda-3)^2 = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = 3$$

$$\lambda = 1$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right) \xrightarrow{(1):2} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right) \xrightarrow{(3)-(1)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array} \right) \sim$$

$$\xrightarrow{(2)+(3)} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array} \right)$$

$$x_1 = 0$$

$$-x_2 + x_3 = 0$$

$$x_2 = x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda = 3$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & -1 & -1 & 0 \end{array} \right) \xrightarrow{(2)-(3)} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 \end{array} \right)$$

$$x_1 - x_2 - x_3 = 0, \quad x_1 = x_2 + x_3$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} c + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} c$$

Наборът базис $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\varphi(F_1) = F_1 \quad \varphi(F_2) = 3F_2 \quad \varphi(F_3) = 3F_3$$

$$\tilde{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$2.6 \quad A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -2 \\ 1 & -1 & 2 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} 4-\lambda & 1 & -1 \\ 2 & 3-\lambda & -2 \\ 1 & -1 & 2-\lambda \end{vmatrix} = (4-\lambda)((3-\lambda)(2-\lambda) - 2) -$$

$$-(2(2-\lambda)+2) - (-2-3+\lambda) = (4-\lambda)(6-5\lambda+\lambda^2-2) - (4-2\lambda+2) -$$

$$-\lambda+5 = 24 - 20\lambda + 4\lambda^2 - 8 - 6\lambda + 5\lambda^2 - \lambda^3 + 2\lambda - 6 + 2\lambda - \lambda + 5 =$$

$$= -\lambda^3 + 9\lambda^2 - 23\lambda + 15 = -(\lambda-1)(\lambda^2 - 8\lambda + 15) =$$

$$= -(\lambda-1)(\lambda-3)(\lambda-5) = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = 3$$

$$\lambda_3 = 5$$

$$\lambda = 1$$

$$\sim \left(\begin{array}{ccc|c} 3 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right) \xrightarrow{(2)\cdot 2} \left(\begin{array}{ccc|c} 3 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right) \xrightarrow{\text{Row operations}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 3 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right) \xrightarrow[(3)-1]{(2)-3(1)} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & -2 & 2 & 0 \end{array} \right) \xrightarrow{(3)-(2)} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(2)\cdot -2} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) -$$

$$\xrightarrow{(1)-(2)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda = 3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ 1 & -1 & -1 & 0 \end{array} \right) \xrightarrow[\text{(3)-(1)}]{(2)-2(1)} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array} \right) \xrightarrow{(3)-(2)} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{(2)\cdot -2} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = 5$$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 2 & -2 & -2 & 0 \\ 1 & -1 & -3 & 0 \end{array} \right) \xrightarrow{(2)+2(1)} \sim \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -4 & 0 \end{array} \right) \xrightarrow{(3)+(1)} \sim \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{(2) \cdot -4} \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(1)+(2)} \sim \left(\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(1) \cdot -1} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_1 - x_2 = 0$$

$$x_3 = 0$$

$$x_1 = x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Ноборъ даде съ

$$\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$\tilde{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \quad \varphi(F_1) = F_1 \quad \varphi(F_2) = 3F_2 \quad \varphi(F_3) = 5F_3$$

$$\det(A - \lambda E) = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} (1-\lambda) & 1 & 1 & 0 \\ 1 & (1-\lambda) & 0 & 1 \\ 1 & 0 & (1-\lambda) & 1 \\ 0 & 1 & 1 & (1-\lambda) \end{vmatrix} = (1-\lambda) \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}$$

$$- \begin{vmatrix} 1 & 0 & 1 \\ 1-\lambda & 1 & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} + \begin{vmatrix} 1 & 1-\lambda & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = (1-\lambda) \left((1-\lambda)((1-\lambda)^2 - 1) + \right.$$

$$+ (0 - (1-\lambda)) - \left((1-\lambda)^2 - 1 + 1 \right) + \left(-1 - 1((1-\lambda)^2 - 1) \right) =$$

$$= (1-\lambda) \left((1-\lambda)(\lambda - 2\lambda + \lambda^2 - \lambda) - 1 + \lambda \right) - \left(1 - 2\lambda + \lambda^2 \right) - 1 - (1 - 2\lambda + \lambda^2) =$$

$$= (1-\lambda) \left(-2\lambda + \lambda^2 + 2\lambda^2 - \lambda^3 - 1 + \lambda \right) - 1 + 2\lambda - \lambda^2 - 1 - \lambda + 2\lambda - \lambda^2 + \lambda =$$

$$= (1-\lambda) \left(-\lambda^3 + 3\lambda^2 - \lambda - 1 \right) - 2\lambda^2 + 4\lambda - 2 = -\lambda^3 + 3\lambda^2 - \lambda - 1 + \lambda^4 - 3\lambda^3 +$$

$$+ \lambda^2 + \lambda - 2\lambda^2 + 4\lambda - 2 = \lambda^4 - 4\lambda^3 + 2\lambda^2 + 4\lambda - 3 = (\lambda-1)(\lambda^3 - 3\lambda^2 - \lambda + 3) =$$

$$= (\lambda-1)^2(\lambda^2 - 2\lambda - 3) = (\lambda-1)^2(\lambda+1)(\lambda-3) = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = -1$$

$$\lambda_3 = 3$$

$$\lambda_1 = 1$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array} \right) \xrightarrow{(1)-(4)} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array} \right) \xrightarrow{(2)-(3)} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array} \right)$$

$$x_1 + x_4 = 0$$

$$x_2 + x_3 = 0$$

$$x_1 = -x_4$$

$$x_2 = -x_3$$

$$X = \begin{pmatrix} -x_4 \\ -x_3 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} c + \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} c$$

$$\lambda_2 = -1$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{array} \right) \xrightarrow{(1)-2(2)} \left(\begin{array}{cccc|c} 0 & -3 & 1 & -2 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{array} \right) \xrightarrow{(3)-(2)} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & 1 & -2 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{array} \right) \xrightarrow{(3)+2(4)} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & 1 & -2 & 0 \\ 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 3 & 0 \end{array} \right) \xrightarrow{(3):4} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right) \xrightarrow{(4):\frac{1}{3}} \sim$$

$$(4)-(3) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(2)-(3)} \left(\begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -3 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\text{A1} + \frac{2}{3}\text{(2)}} \left(\begin{array}{cccc|c} 1 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -3 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(1)-\frac{2}{3}(3)} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & -3 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(2):-3}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_1 - x_4 = 0$$

$$x_2 + x_4 = 0$$

$$x_3 + x_4 = 0$$

$$x_1 = x_4$$

$$x_2 = -x_4$$

$$x_3 = -x_4$$

$$x_4 = x_4$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\lambda = 3$$

$$\left(\begin{array}{cccc|c} -2 & 1 & 1 & 0 & 0 \\ 1 & -2 & 0 & 1 & 0 \\ 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 0 \\ -2 & 1 & 1 & 0 & 0 \\ 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array} \right) \xrightarrow{(2)+2(1)} \left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 0 \\ 0 & -3 & 1 & 2 & 0 \\ 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array} \right) \xrightarrow{(3)-(1)} \left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 0 \\ 0 & 2 & -2 & 0 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array} \right)$$

$$(2)-2(4) \sim \left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 0 \\ 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & -4 & 4 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array} \right) \xrightarrow{(4)+\frac{1}{3}(2)} \left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 0 \\ 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & \frac{4}{3} & -\frac{4}{3} & 0 \end{array} \right) \xrightarrow{(4)-\frac{4}{3}(3)} \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$(2)\xrightarrow{3(3)} \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & -3 & 3 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(2):-3} \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{(1)+(2)} \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\sim \begin{pmatrix} 1 & -1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & -1 & | & 0 \\ 0 & 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{(1)+(2)} \begin{pmatrix} 1 & 0 & 0 & -1 & | & 0 \\ 0 & 1 & 0 & -1 & | & 0 \\ 0 & 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 - x_4 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 - x_4 = 0$$

$$x_4 = x_4$$

$$x_1 = x_4$$

$$x_2 = x_4$$

$$x_3 = x_4$$

$$x_4 = x_4$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Набор базисов

$$\left\{ \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\}$$

$$\varphi(F_1) = F_1 \quad \varphi(F_2) = F_2 \quad \varphi(F_3) = -F_3$$

$$\varphi(F_4) = 3F_4$$

$$\tilde{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$