Measure the Expectation Value of Pauli Matrices

Review

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A 2^n by 2^n Hermitian matrix can be decomposed into Pauli matrices:

$$\begin{bmatrix} 2 & 1+i \\ 1-i & 0 \end{bmatrix} = Z + X + Y + I$$

Review

ket:

colum vector

$$|\psi\rangle = \frac{4}{5}|0\rangle + \frac{3}{5}i|1\rangle = \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5}i \end{bmatrix}$$

bra:

Hermitian conjugate of ket

$$\langle\psi| = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5}i \end{bmatrix}$$

projection operator:

ket and bra $$|\psi\rangle\langle\psi|$$

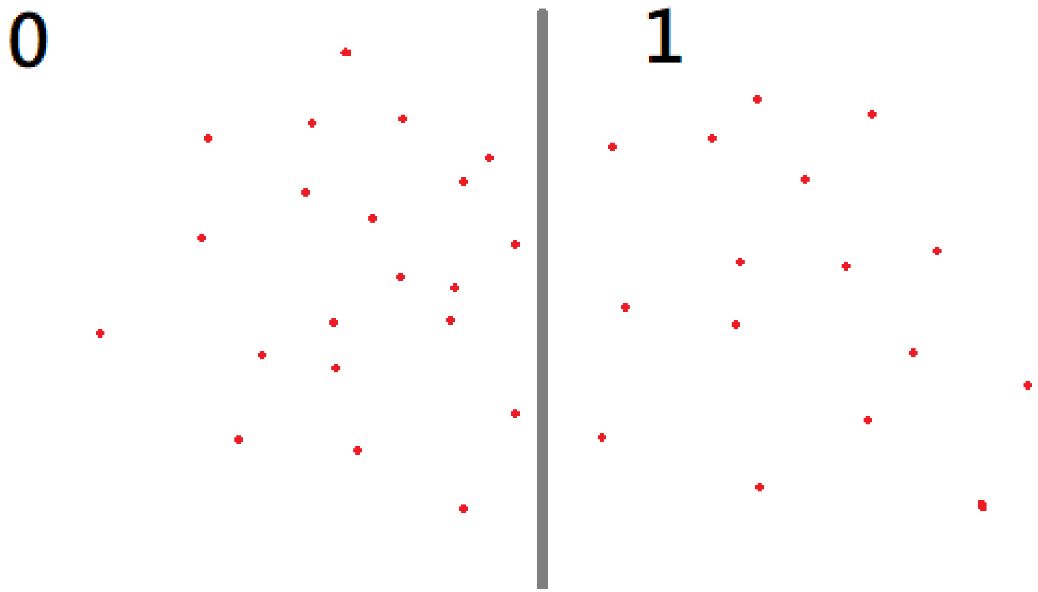
Expectation value:

$$\langle\psi|Z|\psi\rangle = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5}i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5}i \end{bmatrix} = \frac{7}{25}$$

Measurement

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

$$\mathbb{P}_0 = \langle\psi|0\rangle\langle 0|\psi\rangle = |a|^2$$
$$\mathbb{P}_1 = \langle\psi|1\rangle\langle 1|\psi\rangle = |b|^2$$



Pauli Z

$$|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|1\rangle\langle 1| = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{cc} |\mathbf{0}\rangle & |\mathbf{1}\rangle \end{array} \\ Z = & \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{array}{c} |\mathbf{0}\rangle \\ |\mathbf{1}\rangle \end{array} \end{array}$$

$$= |0\rangle\langle 0| - |1\rangle\langle 1|$$

Expectation Value of Z

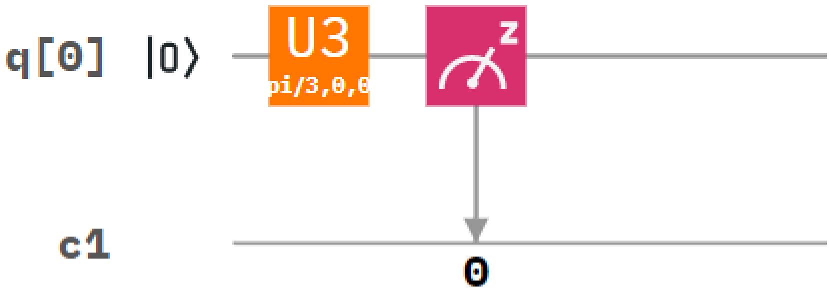
$$\langle\psi|Z|\psi\rangle = \langle\psi|0\rangle\langle 0|\psi\rangle - \langle\psi|1\rangle\langle 1|\psi\rangle$$
$$= \mathbb{P}_0 - \mathbb{P}_1$$

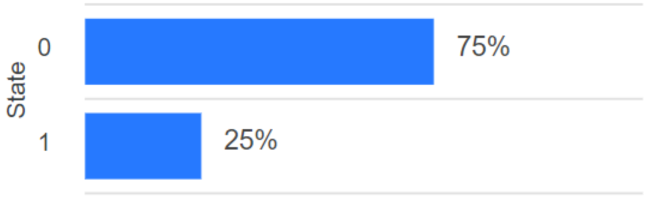
EX: $|\psi\rangle = \frac{1}{2}(\sqrt{3}|0\rangle + |1\rangle)$

Classical computer:

$$\langle\psi|Z|\psi\rangle = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{1}{2}$$

Quantum computer:





$$\langle\psi|Z|\psi\rangle = 0.75 - 0.25$$
$$= 0.5$$

Expectation Value of X&Y

Classical computer:

$$|\psi\rangle = \frac{1}{2}(\sqrt{3}|0\rangle + |1\rangle)$$

$$\langle\psi|X|\psi\rangle = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{\sqrt{3}}{2} = 0.8660\ldots$$

$$\langle\psi|Y|\psi\rangle = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = 0$$

Expectation Value of X

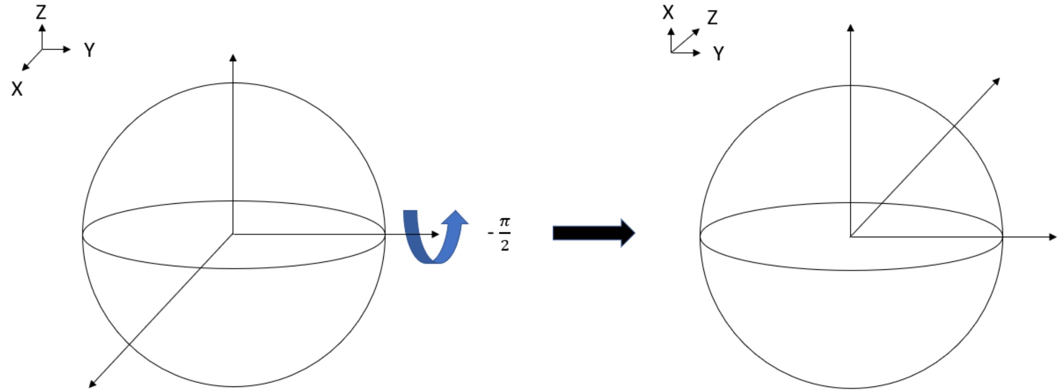
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\langle\psi|X|\psi\rangle = \langle\psi'|Z|\psi'\rangle$$

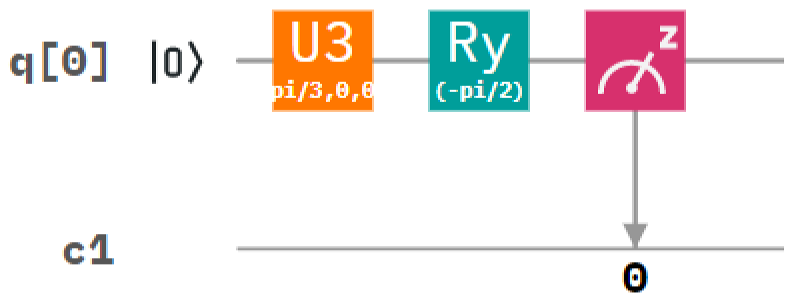
$$|\psi'\rangle = R_y(-\pi/2)|\psi\rangle$$

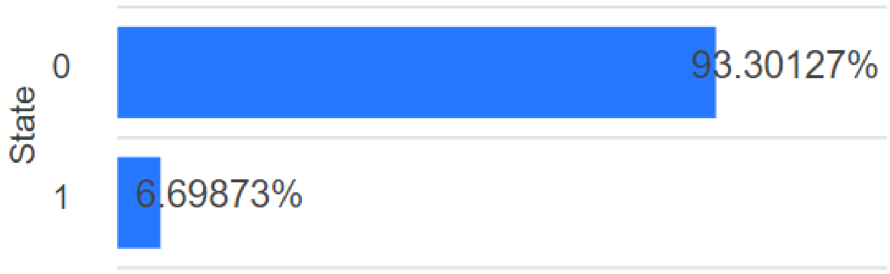
The rotation on the Bloch sphere:

$$R_y(-\pi/2) = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



Expectation Value of X





$$\langle\psi|X|\psi\rangle = 0.933\ldots - 0.067\ldots = 0.866\ldots$$

Expectation Value of Y

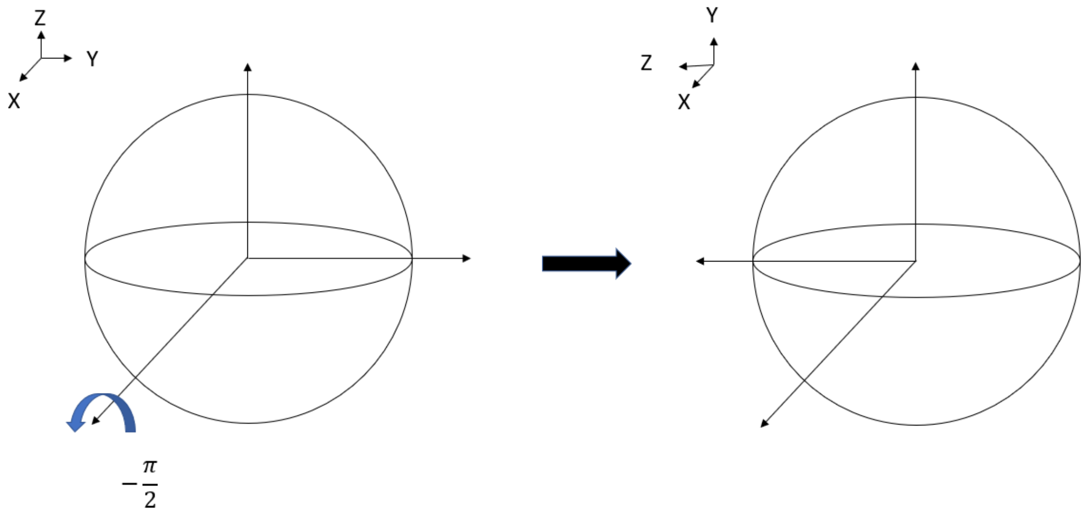
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\langle\psi|Y|\psi\rangle = \langle\psi'|Z|\psi'\rangle$$

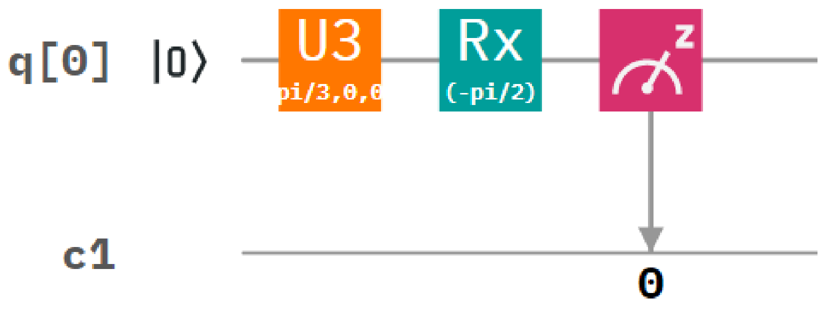
$$R_x(-\pi/2)|\psi\rangle = |\psi'\rangle$$

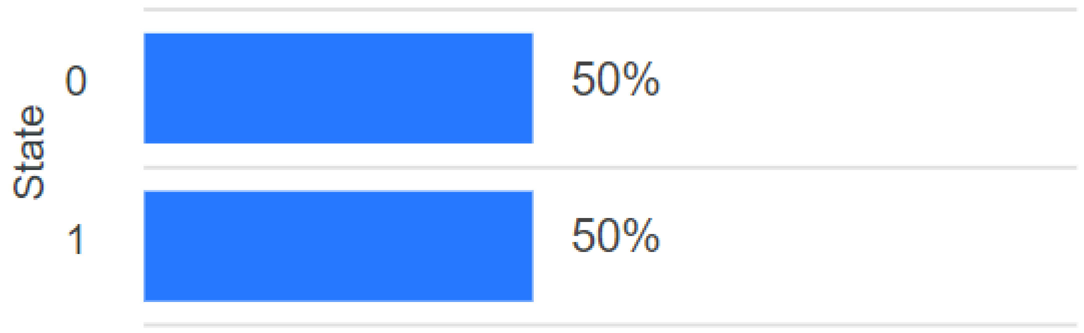
The rotation on the Bloch sphere:

$$R_x(-\pi/2) = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



Expectation Value of X



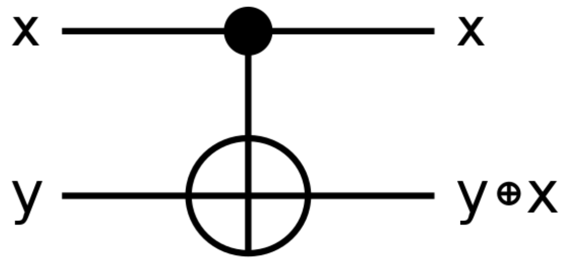


$$\langle\psi|Y|\psi\rangle = 0.5 - 0.5 = 0$$

CNOT gate

control qubit:

target qubit:



input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

This is a gate that can sum the information and store it in the target qubit.

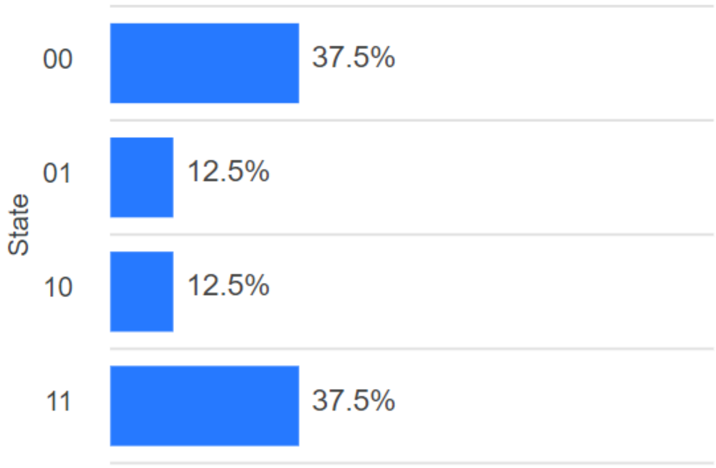
Scale UP

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

If the sum of the bit string is even(odd), the eigenvalue is 1(-1).

$$\langle\psi|ZZ|\psi\rangle = |a|^2 - |b|^2 - |c|^2 + |d^2|$$

Two qubit case

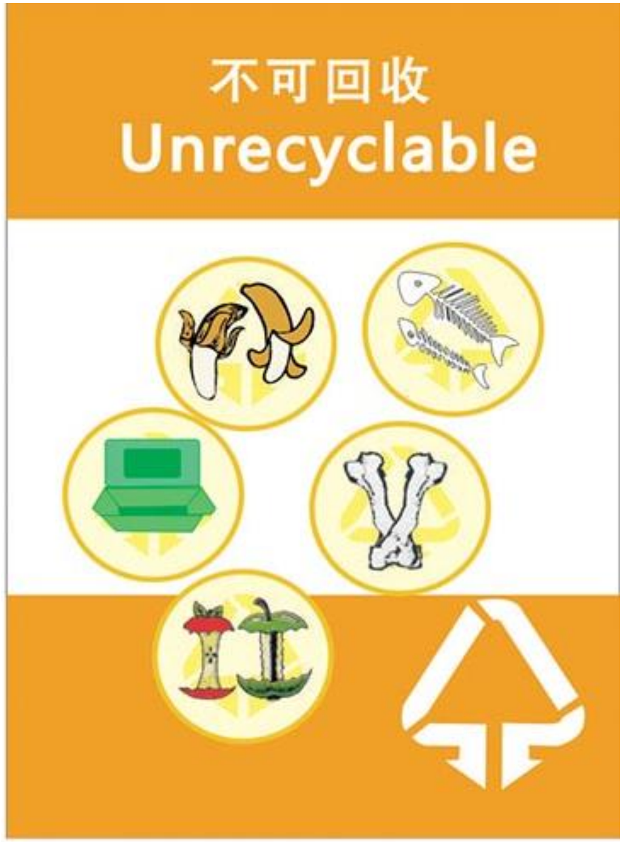


Five qubit case

N qubit case

efficient?

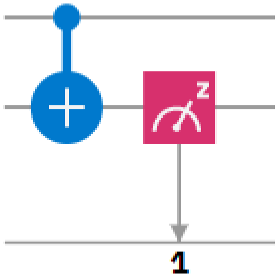

可回收
Recyclable
塑料
金属
纸类
纸类
织物


不可回收
Unrecyclable

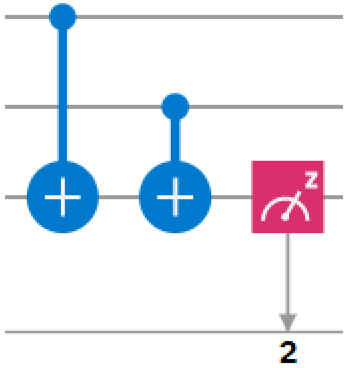
Scale UP!

We can classify the sum of the bit string even or odd via the CNOT gates.

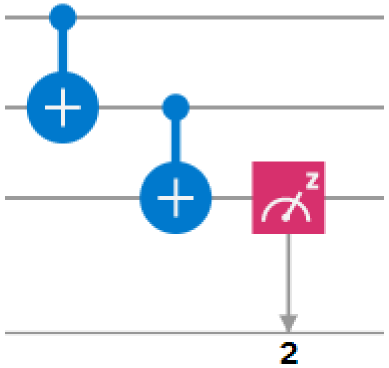
$$\langle\psi|ZZ|\psi\rangle$$

$|q_1, q_2\rangle$  $|q_1, q_1 \oplus q_2\rangle$

$$\langle\psi|ZZZ|\psi\rangle$$



or



$$\langle\psi|ZZ....|\psi\rangle = \mathbb{P}_0 - \mathbb{P}_1$$

If you want to measure other Pauli matrices, operate the corresponding rotation gate before the CNOT gates.

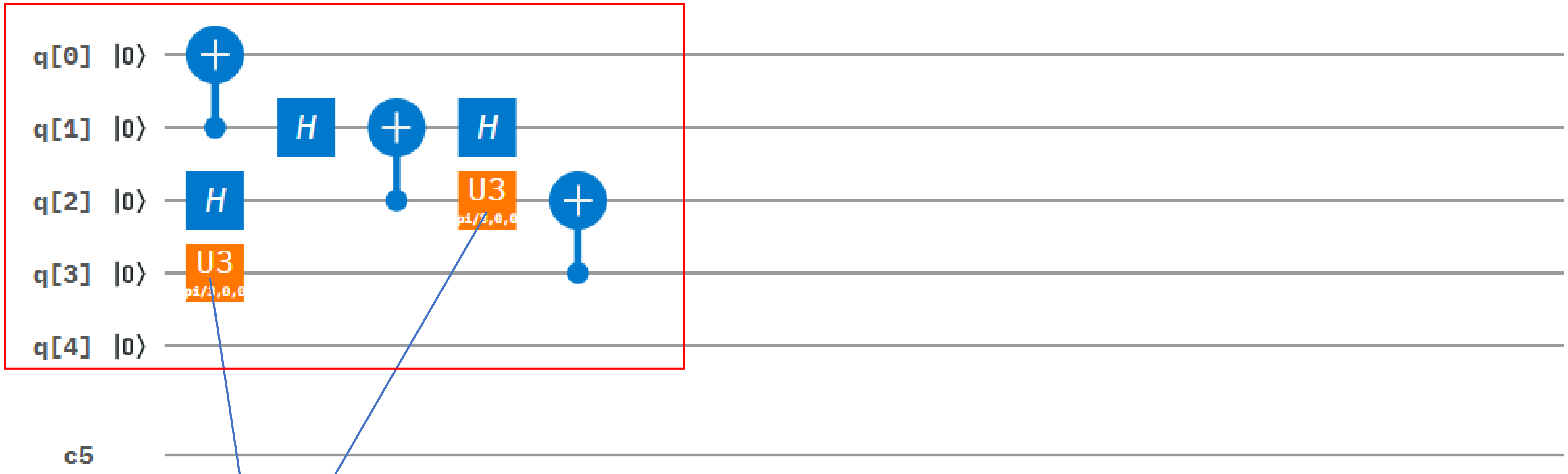
Exercise

$$\langle\psi|X_0Y_1Z_2Z_3|\psi\rangle =$$

1.ibmq_qasm_simulator
2.ibmqx4

Jupyter notebook
3. error mitigation

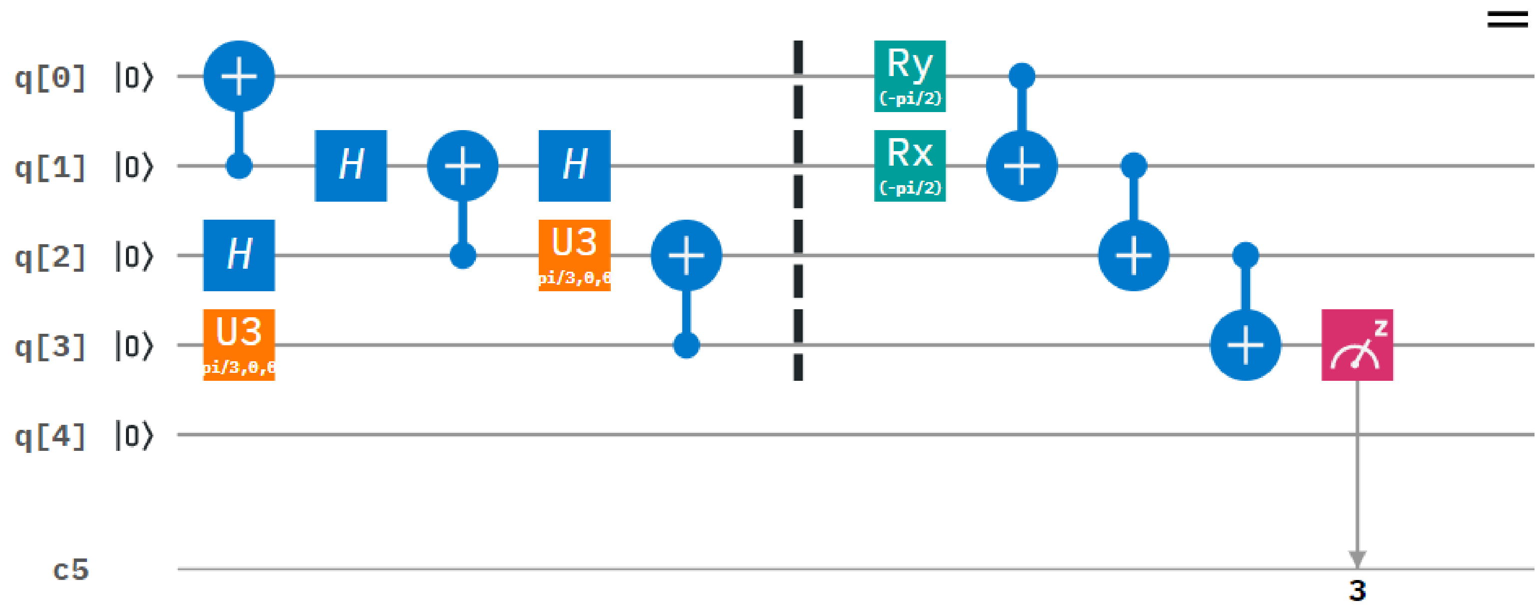
$|\psi\rangle =$

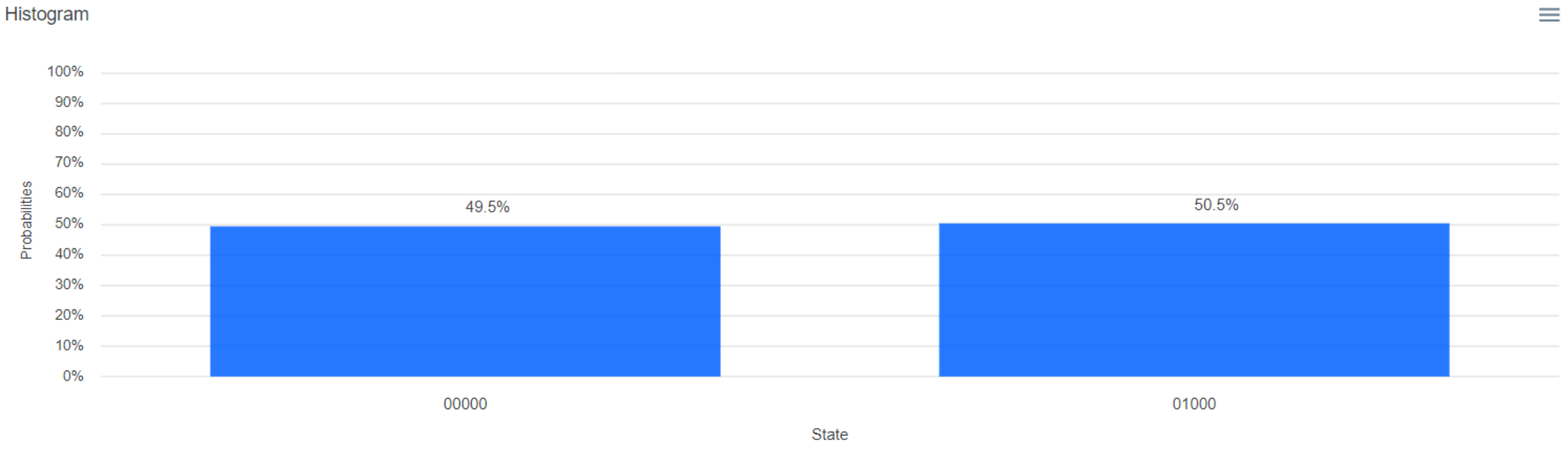


(Pi/3,0,0)

Result

Exact expectation value=0

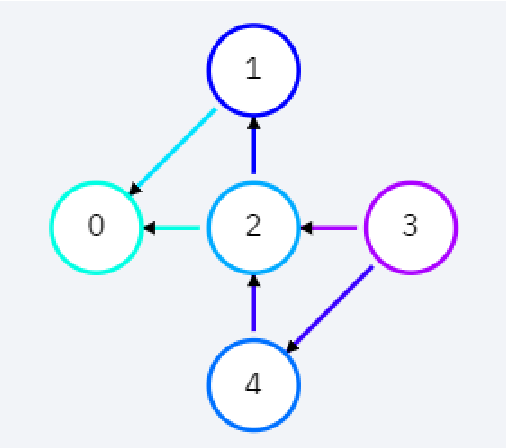


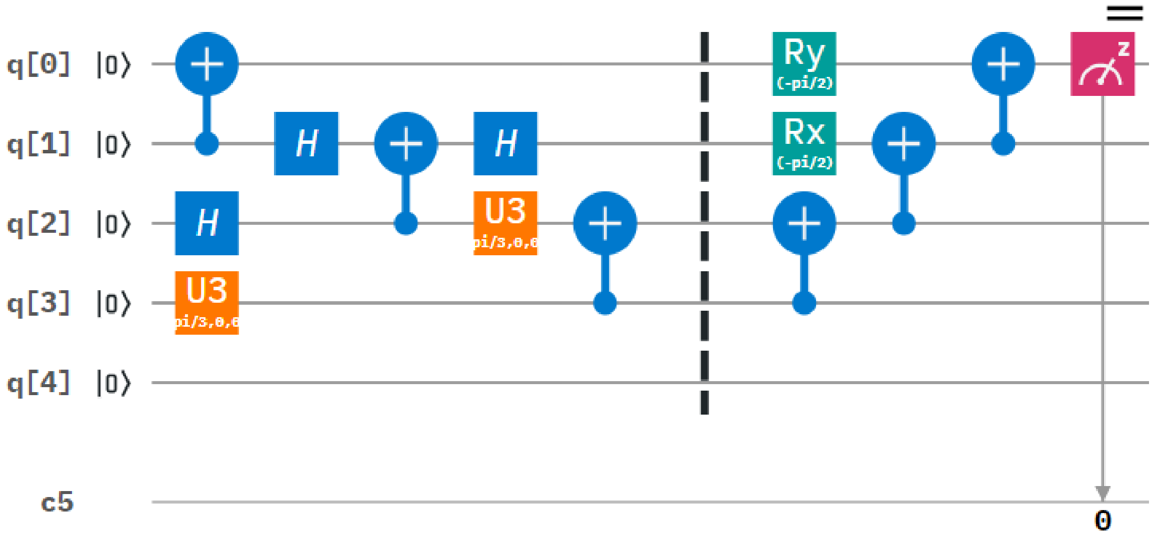


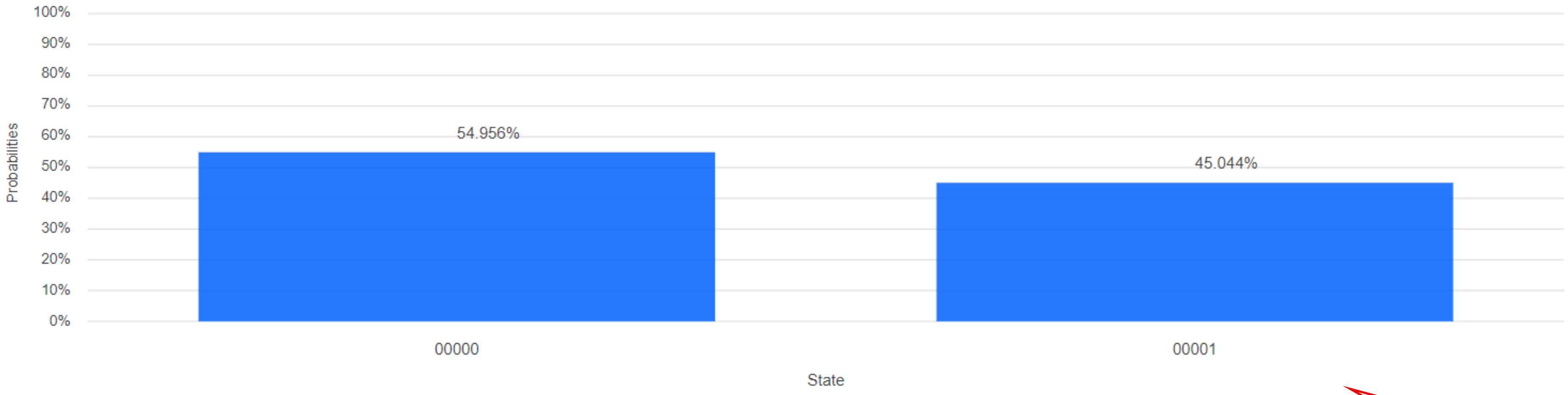
ibmq_qasm_simulator:

$$\langle\psi|X_0Y_1Z_2Z_3|\psi\rangle = 0.01$$

Real device







ibmqx4:

$$\langle\psi|X_0Y_1Z_2Z_3|\psi\rangle = 0.09912$$

error mitigation

0.030