

Qiskit Finance Portfolio Optimization

Background

- Portfolio:
 - “A **portfolio** is a **collection of investments** held by an investment company, hedge fund, financial institution or individual.” — wikipedia
- Optimization

Don't put all your eggs in one basket.



Background

- For N assets, an N -dim vector $x=(x_1, x_2, \dots, x_{N-1}) \in \{0,1\}^N$ represents an investment portfolio.
($x_i=1$ indicates picking the i -th asset, and vice versa.)
- $\mu \in \mathbf{R}^N$ denotes the expected returns for the assets.
- $\Sigma \in \mathbf{R}^{N \times N}$ denotes covariance between and variance of assets.
(Variance/Covariance are indicators of risk.)

Mathematical Formulation

- Expected profit function (P):

$$P = \mu^T x - q x^T \Sigma x$$

where $q > 0$ is the “risk appetite” of the decision maker.

- Constraint (limited budget B)

Assume that

- all assets have same price (= 1),
- full budget has to be used.

$$(1, 1, \dots, 1)x = B$$

Cost Function

- The cost function to be minimized consists of $-P$ and penalty term for constraint with scaling factor p .

$$C = qx^T \Sigma x - \mu^T x + p|(1, 1, \dots, 1)x - B|^2$$

- Goal: Find x^* that minimizes C :

$$x^* = \min_{x \in \{0,1\}^N} C(x)$$

Encoding to Pauli Matrices

- x_i is either 0 or 1; while Pauli-Z has eigenvalues +1 and -1.

Encoding: $x_i \rightarrow (1-Z_i)/2$

$$0 \rightarrow Z_i = 1$$

$$1 \rightarrow Z_i = -1$$

- Substitute it into $C(x)$:

$$C(Z) = \sum_{i \neq j} c_{ij} Z_i Z_j + \sum_i c_i Z_i + \text{const.}$$

VQE (Variational Quantum Eigensolver)

- Use VQE method to minimize cost function $C(Z)$.

