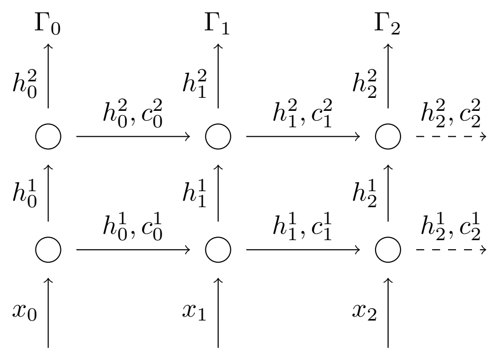
LSTM

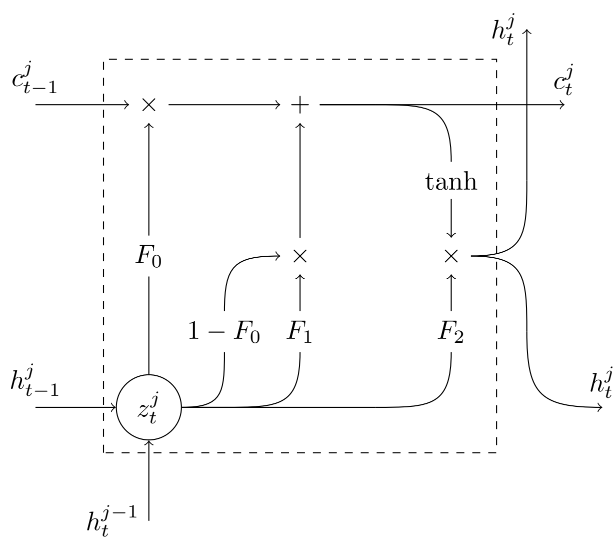
A long short-term network, LSTM, is a series of (the same) recurrent neural network, RNN, with an additional memory variable. The network may be depicted as



The x's are the inputs and we will also denote them by $h_t^0 = x_t$.

Cell

Each layer in the each of the T'th network is also callled a cell. Since each cell not only transform and propagates input, but also the memory variable, c_t^j, the inner workings of each cell is slightly more complicated then that off a RNN. It may be depicted as



$$z_t^j = W^j(h_{t-1}^j, h_t^{j-1}) + b^j$$

$$c_t^j = c_{t-1}^j F_1(z_t^j) + (1 - F_1(z_t^j))F_2(z_t^j)$$

$$h_t^j = F_4(c_j^t)F_3(z_t^j)$$

Γ_r er cost-funktionen i det r'te netværk.

$$\delta_{t,r}^j = \frac{d\Gamma_r}{dz_t^j} = \frac{d\Gamma_r}{dz_t^{j+1}}\frac{dz_t^{j+1}}{dz_t^j} + \frac{d\Gamma_r}{dz_{t+1}^j}\frac{dz_{t+1}^j}{dz_t^j} + \frac{d\Gamma_r}{dc_t^j}\frac{dc_t^j}{dz_t^j}$$

$$\tau_{t,r}^j = \frac{d\Gamma_r}{dc_t^j} = \frac{d\Gamma_r}{dc_{t+1}^j}\frac{dc_{t+1}^j}{dc_t^j} + \frac{d\Gamma_r}{dz_t^{j+1}}\frac{dz_t^{j+1}}{dc_t^j} + \frac{d\Gamma_r}{dz_{t+1}^j}\frac{dz_{t+1}^j}{dc_t^j}$$

$$\frac{dc_{t+1}^j}{dc_t^j} = F_1(z_{t+1}^j)$$

$$\frac{dz_t^{j+1}}{dc_t^j} = W^{j+1}(0, F_4'(c_j^t)F_3(z_t^j))$$

$$\frac{dz_{t+1}^j}{dc_t^j} = W^{j+1}(F_4'(c_t^j)F_3(z_t^j))$$

$$\frac{dc_t^j}{dz_t^j} = F_2'(z_t^j)(1 - F_1(z_t^j)) + F_1'(z_t^j)(c_{t-1}^j - F_2(z_t^j))$$

$$dz_t^{j+1} = W^{j+1}(h_{t-1}^{j+1}, h_t^j) = W^{j+1}(F_4(c_{t-1}^{j+1})F_3(z_{t-1}^{j+1}), F_4(c_t^j)F_3(z_t^j))$$

So since

$$c_t^j = c_t^j(z_t^j) = c_{t-1}^j F_1(z_t^j) + (1 - F_1(z_t^j))F_2(z_t^j)$$

thus

$$\frac{dz_t^{j+1}}{dz_t^j} = W^{j+1}(0, (c_t^j)'(z_t^j)F_4'(c_t^j)F_3(z_t^j) + F_4(c_t^j)F_3'(z_t^j))$$

where

$$(c_t^j)'(z_t^j) = c_{t-1}^j F_1'(z_t^j) + (1 - F_1(z_t^j))F_2'(z_t^j) - F_1'(z_t^j)F_2(z_t^j)$$

Likewise

$$\frac{dz_{t+1}^j}{dz_t^j} = W^j((c_t^j)'(z_t^j)F_4'(c_t^j)F_3(z_t^j) + F_3'(z_t^j)F_4(c_t^j), 0)$$