Deriving the Cartesian-Spherical coordinate conversion

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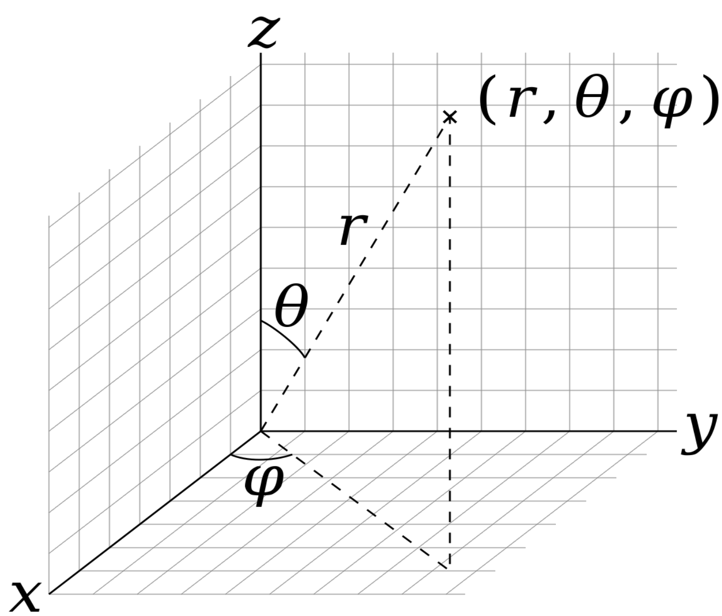
1 Introduction

Spherical coordinates are defined in the following way:

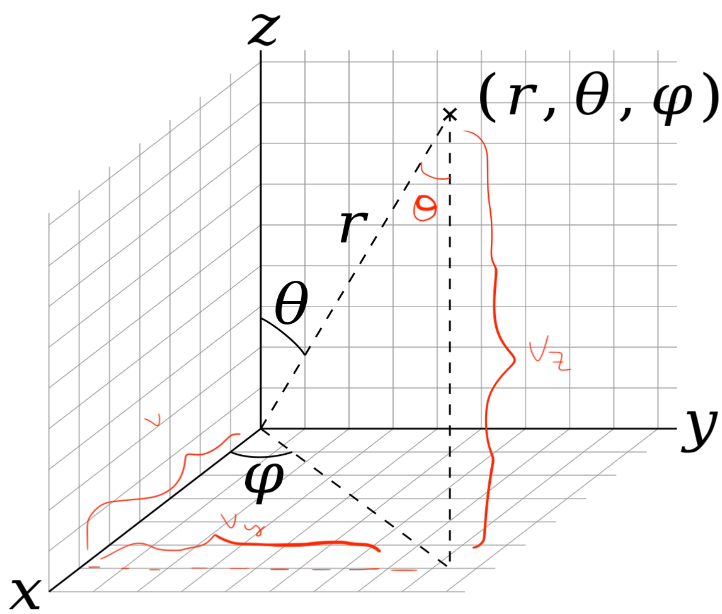


It consists of three parameters.

- r is the *radial distance*, defined as the distance from the coordinate to the origin
- θ is the *polar angle*, defined as the angle between the z-axis and the vector from the origin to the coordinate
- ϕ is the *azimuthal angle*, defined as the angle between the projection of the vector onto the x-y plane and the x-axis

Bear in mind this is the **physics** convention for spherical coordinates. Mathematicians typically swap the meaning of θ and ϕ.

2 Derivation



Consider the right triangle spanned by the the distance vector, its z component, and the projection of the distance vector onto the x-y plane. With some basic trig, we can deduce that the angle made by the vector and its z component is also θ, making the projection onto the x-axis $r \sin \theta$. Thus, the z-component of the vector is simply $r \cos \theta$.

Now, draw a triangle on the $x - y$ plane as pictured above. The legs of the right triangle are the x and y components of the distance vector and the angle is ϕ. The x component is thus $r \sin \theta \cos \phi$ and the y component is $r \sin \theta \sin \phi$. From which we get the familiar conversion:

- $x = r \sin \theta \cos \phi$
- $y = r \sin \theta \sin \phi$
- $z = r \cos \theta$