COSC 6370 – Fundamentals of Medical Imaging
Computed Tomography Reconstruction

Members of the Team

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Introduction

Computed Tomography (CT) is a medical imaging technique that generates two-dimensional cross-sectional images from three-dimensional body structures. CT aims to differentiate tissues and structures present in the human body and obtain the image using a mathematical technique called reconstruction. A CT image, in other words, is a three-dimensional structure broken apart and mathematically put back together as a two-dimensional image. Digital image processing consists of multiple image manipulation techniques and has applications ranging from medicine to entertainment. Image reconstruction for CT produces tomographic images from X-ray projection data acquired from different angles around the patients with fundamental impact on image quality. Image processing for results obtained using CT remains a potential area of research. The tomography reconstruction problem lies in obtaining a tomographic slice from a set of projections. In this project, we aim to implement image reconstruction techniques and draw a comparison on the properties and performance of each algorithm which are further discussed in detail. The results are shown using a Graphical User Interface (GUI) that allows the user to provide a custom phantom image as input to undergo image reconstruction as per the user's requirement.

Objectives

1. **CT Image reconstruction:** To implement two significant image reconstruction methods: Simultaneous algebraic reconstruction method (SART) and the filtered back projection (FBP) using inverse radon transform.
2. **Compare algorithms:** The results obtained from the above objective are compared to discuss which algorithm performs better.
3. **GUI Implementation:** The reconstruction techniques developed in objective 1 are tested using the GUI to achieve objective 2.

Literature Review

The initial CT scanners used iterative reconstruction algorithms but, the computer technology in the early 1970s limited their usage. The CT images then, were constructed using an algebraic reconstruction technique (ART) which was replaced by simple analytic methods like the filtered back projection (FBP). This method was used until 2009 when the first iterative reconstruction algorithms were commercially available. The ART technique reconstructs the image from a series of angular projections (the sinogram) by successive estimates. The projections corresponding to the current estimate are compared with the measured projections and then, used to modify the current estimate, thereby creating a new estimate. This algorithm is based on an algebra equation that acts as a correction factor by changing pixel value for each iteration. FBP is based on back projection algorithm. However, FBP utilizes a filter process on the data projections (Rifqi Ramadhan, et al, 2018) by convolving it with certain filter frequency response before the interpolation to create the image result. In an experiment by Rifqi Ramadhan, et al (2018). the quantitative parameters of FBP produced better results than the ART due to noise calculation in ART algorithm. But the higher contrast in ART distinguishes between two different dielectric values better than the FBP.

The radon transform is an integral transform, and its inverse is used for medical CT image reconstruction to simplify the computation in the iterative approach. An approximated inverse radon transform will be used for the implementation. The ART-type algorithms are sequential and may contain salt and pepper appearance of the final reconstructed images. To overcome this, the Simultaneous algebraic reconstruction method (SART) has come into practice. This can be considered as an approach with the combination of formulation of the reconstruction problem as a set of linear equations and an iterative solver making the method computationally effective. These methods prove to be better approaches for CT Reconstruction, and we would draw a comparison between the performance of the FBP and the SART technique.

Methods

Filtered Back projection using Inverse Radon transform:

The Filtered Back Projection (FBP) Algorithm convolves using a one-dimensional integral equation to reconstruct a two-dimensional image. The FBP algorithm is also used as a sharpening filter as applying just a simple back projection on an image causes a blurred reconstructed result. The FBP reconstruction method follows the steps below:

- The function takes as input, the two-dimensional phantom object and generates the parallel sinogram (radon transform) for the object.
- The output matrix obtained from the radon transform is used to calculate the Fast Fourier Transform (FFT).
- The projections are then filtered in the frequency domain using the ramp filter. The filter that yields the reconstruction using this method is also referred as a ramp filter as it resembles a ramp when the filter is drawn in frequency space. To avoid the excessive noise amplification at these frequencies, a band limited filter called Ram-Lak filter is used in image reconstruction.
- Mathematically, the projection λ is the integral of the function *f (x)* where *f (x)* stands for the distribution of attenuation coefficients in the input object.

$$\lambda_{\phi}(p) = \int_{A} f(x)\delta\big(p - x.\hat{n}_{\phi}\big)d^2x$$

  Where the projection line is given by $p = x.\hat{n}_{\phi}$

- Back projection maps the data from the detector space to the image space in CT. Once the projections are weighted back projection is applied such that each radial sample taken at each angle and smeared along the path that we integrated.

The filtered back-projection formula for image reconstruction can be written as:

$$f(x, y) = \frac{1}{2}B(\phi * Rf)(x, y)$$

The parameters required for the reconstruction of the phantom object *f* are the data collected or read from the machine, convolution of *Rf* followed by its back projection with ϕ.

Simultaneous Algebraic Reconstruction Technique (SART):

Iterative image reconstruction methods are considered better methods especially in applications like CT but are computationally more expensive. They perform iterative algorithms to reconstruct 2D or 3D images. As mentioned earlier ART is the first iterative reconstruction method used in CT. The ART method follows an iterative approach for solving equations in linear system where the equations represent the relationship between pixels and projection data. The equations can be represented as:

$$
\begin{gathered}
w_{11}f_1 + w_{12}f_2 + w_{13}f_3 + \cdots + w_{1N}f_N = p_1 \\
w_{21}f_1 + w_{22}f_2 + w_{23}f_3 + \cdots + w_{2N}f_N = p_2 \\
w_{M1}f_1 + w_{M2}f_2 + w_{M3}f_3 + \cdots + w_{MN}f_N = p_M
\end{gathered}
$$

The above projections can be represented as:

$$\sum_{i=1}^{N} w_{i1}f_1 = p_i$$

With the ART method, the projection values are updated, and a correction term is added to obtain each pixel value. The below formula summarizes the ART reconstruction technique.

$$f_j^{(k+1)} = f_j^{(k)} + w_{ij}\frac{p_j - \sum_{i=1}^{N} f_1^{(k)} w_{i1}}{\sum_{i=1}^{N} w_{i1}^2},$$

Where,

p_j is the projection data, f_j is the pixel value at j^{th} position, w_{ij} is the weighted ratio of j^{th} pixel that i^{th} ray passes through, and k is the number of iterations.

The algorithm can thus be simplified as:

1. Initialize the image matrix with a custom input phantom.
2. Assign number of iterations *k.*
3. The projections p_j is calculated using the line integral equation.
4. The correction factor is calculated from the difference of projection data p_j.
5. The projection values are updated.

The last three steps (3, 4, 5) are iterated as per the value k to obtain the reconstructed image.

The SART algorithm is an improved iterative reconstruction algorithm. It is an improvement over the ART algorithm using the convergence concept of ART and additionally smoothing the noise. This helps return better results compared as compared to ART. The SART algorithm runs just like the ART algorithm but with an additional property discussed below. The SART's smoothing property is reflected in the below equation:

$$f_j^{(k+1)} = f_j^{(k)} + \lambda \frac{1}{\sum_{i \epsilon I_\theta} w_{ij}} \sum w_{ij} \frac{p_j - \sum_{i=1}^{N} f_1^{(k)} w_{i1}}{\sum_{i=1}^{N} w_{i1}^2}$$

Where, λ is the relaxation factors dependent on number of projects and noise.

The objective of SART algorithm is to reduce the salt and pepper noise that is usually associated with ART algorithm. It is computationally effective over ART as it generates a better reconstructed image with a single iteration.

GUI Implemented for the project

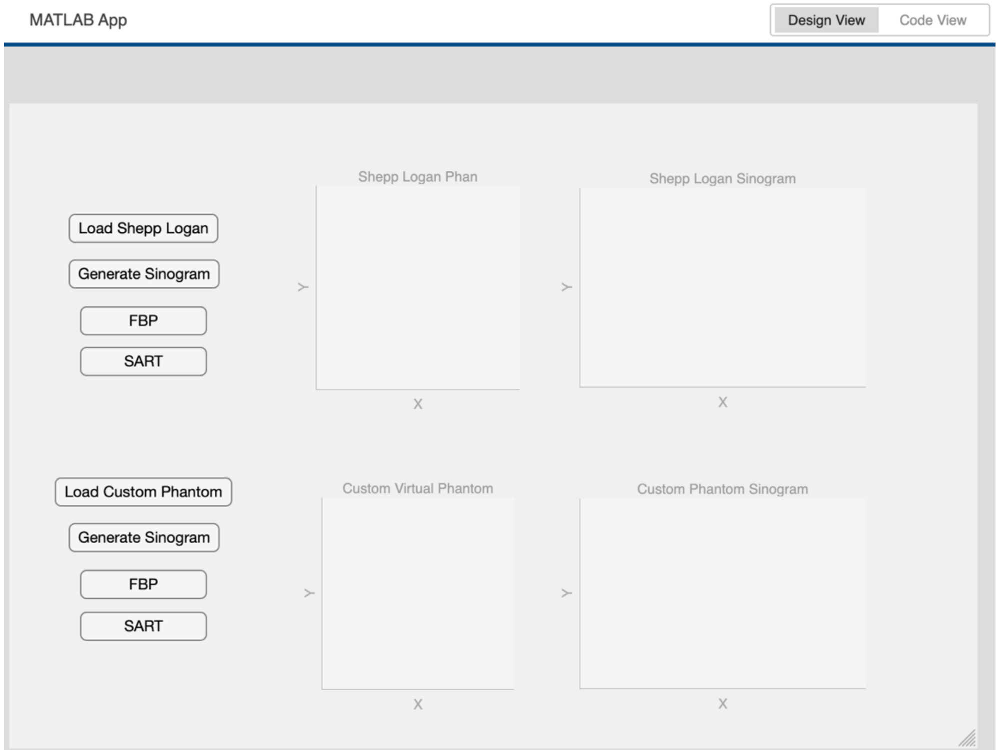


Fig. 1. GUI implemented for the project.

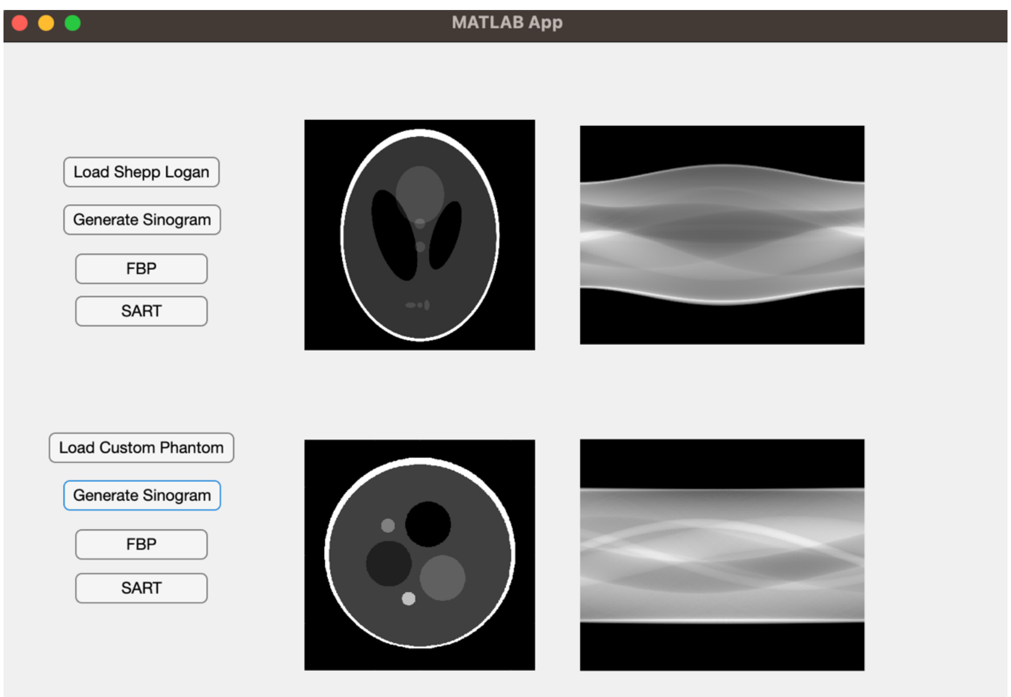


Fig. 2. GUI displaying sinograms for Shepp Logan and custom Phantom.

The entire code for the project is written and executed using MATLAB. This includes the reconstruction methods and the GUI implementation. As shown in figure 2, the GUI designed tests the image reconstruction techniques FBP and SART using Shepp Logan and a custom Phantom. The above figure displays the generated sinogram for both the input objects.

Custom Phantom creation:

As the GUI tests the implemented reconstruction methods using a custom phantom input, we generated the input image using the below code snipped.

```
disk = [ 0  -5  210  8;
         0   0  200  2;
       -68  18   50  1;
        18 -68   50  0;
        50  50   50  3;
       -25  95   15  6;
       -70 -65   15  4];

row = 512; col = 512; dx = 1;

x = dx * ((1:row)'-(row+1)/2); y = -dx * ((1:col)'-(col+1)/2);
xx = x(:,ones(1,row)); yy = y(:,ones(1,col))';

phantom = zeros(row,col);
for index=1:size(disk,1)
    x_cord = disk(index,1); y_cord = disk(index,2);
    radius = disk(index,3); signal_intensity = disk(index,4);
    t = find( ((xx-x_cord)/radius).^2 + ((yy-y_cord)/radius).^2 <= 1 );
    phantom(t) = signal_intensity * ones(size(t));
end

figure(1)
imagesc(x, y, phantom'); colormap('gray'); axis('square');
title('Disk Phantom');
```

Fig.2. Code used for creating custom phantom

The input phantom image for the test case is shown below:

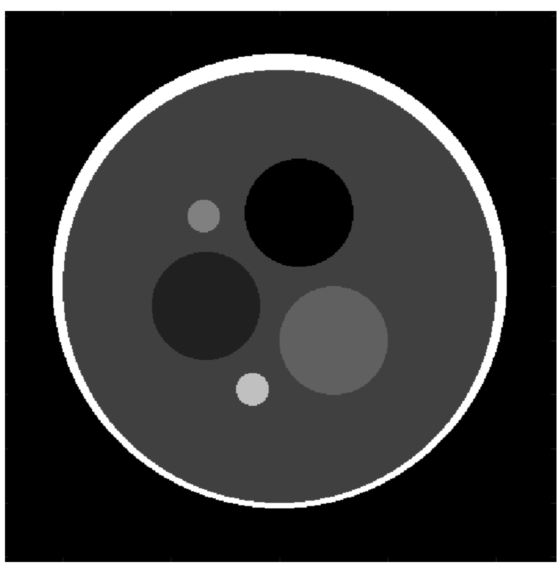


Fig.3. Custom Phantom Input – Disk Phantom

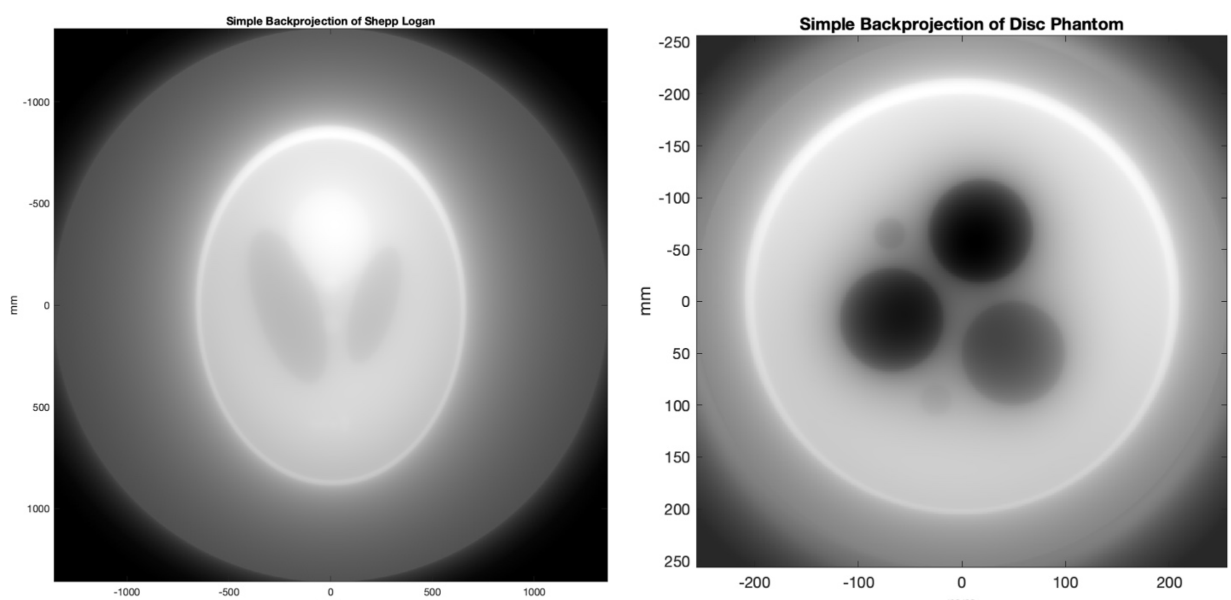
Results

The GUI designed for this project requires input images to perform the image reconstruction methods FBP and SART. The input images are shown below:

(a) (b)

Fig. 4. (a) Shepp-Logan Phantom, (b) Custom Disk Phantom

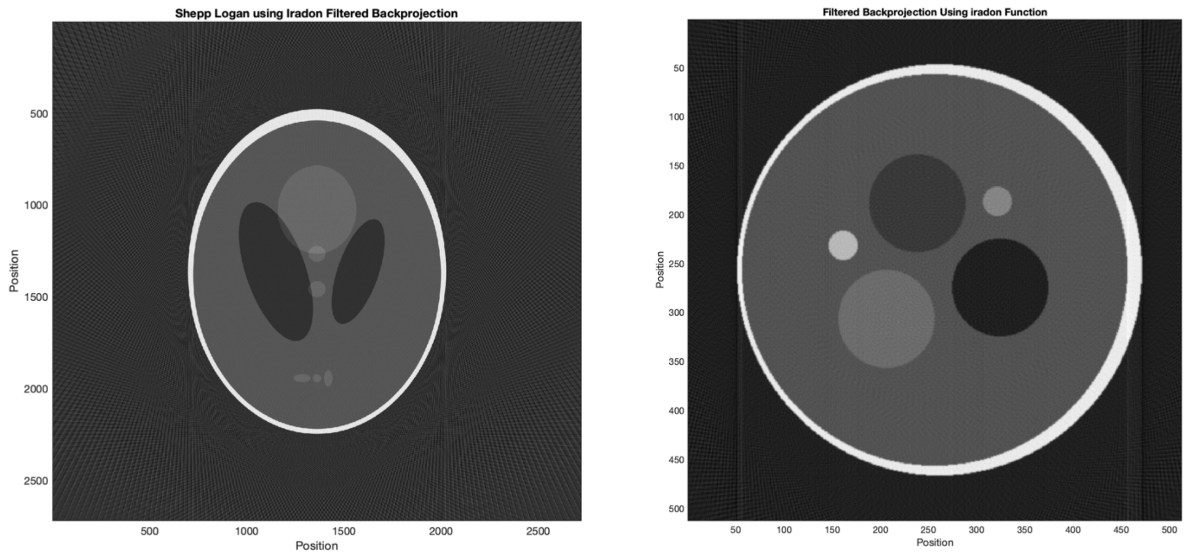
Simple back projection is applied to the images to test the algorithms and compare between a simple back projection and a filtered back projection on the input phantom images. It is observed that just a simple back projection results in a blurry reconstructed image.



(a) (b)

Fig. 5. (a) Simple back projection on Shepp-Logan Phantom, (b) Simple back projection on Disk Phantom

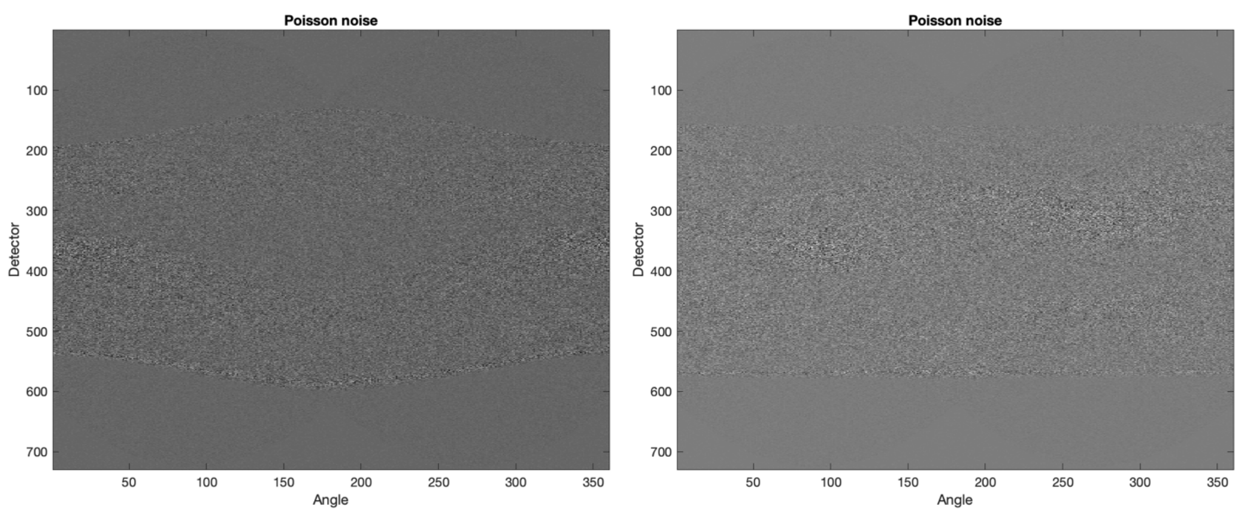
The Filtered Back projection algorithm using iradon transform reconstructs the image with better quality eliminating the blurriness as observed earlier. Hence it is also referred as a sharpening filter.



(a) (b)

Fig. 6. (a) FBP on Shepp-Logan Phantom, (b) FBP on Disk Phantom

The noise added to the images is shown below:



(a) (b)

Fig. 7. (a) Poisson noise added to Shepp-Logan (b) Poisson noise added to Custom Disk Phantom

The SART algorithm operates for k iterations on the input images. The iterations k is set to 200. The output images are shown below for few iterations to help visualize the difference observed as the iterations are increased on the input phantom images.

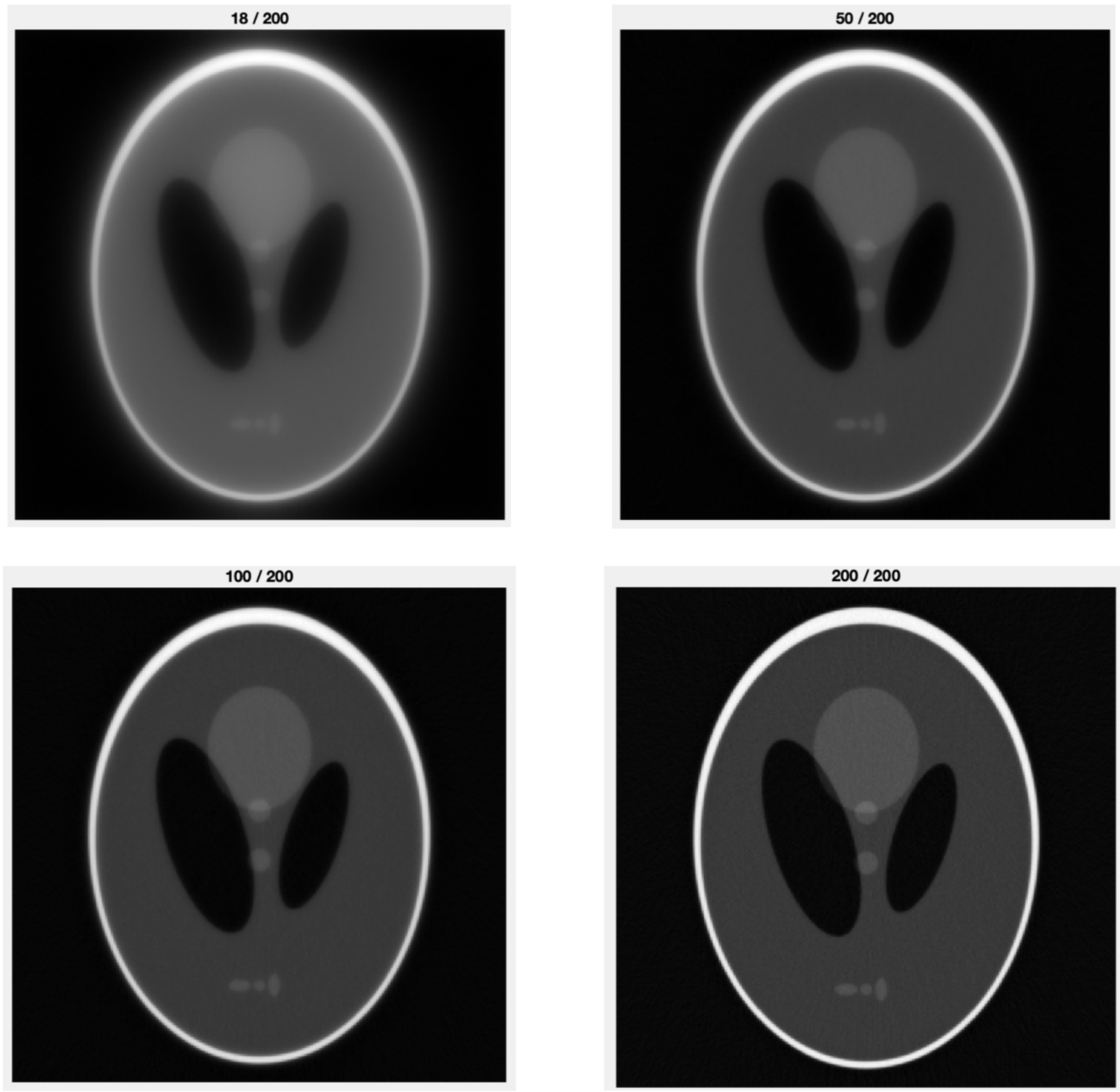


Fig. 8. SART on Shepp-Logan Phantom for k=18, 50, 100, and 200 iterations

It can be clearly observed from the output images above how the images obtain better quality after each iteration. After 200 iterations, the image is reconstructed with a much better quality.

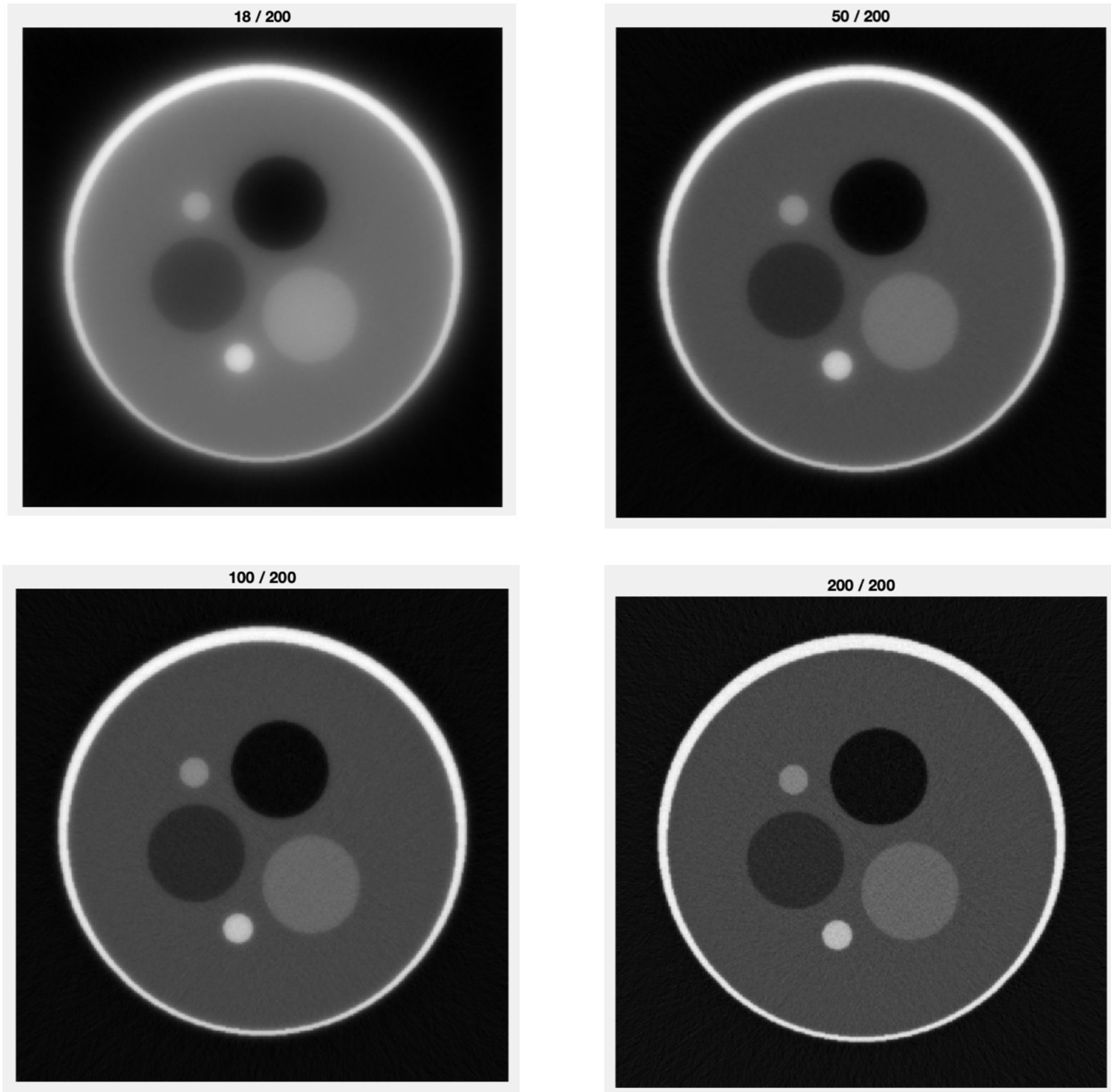
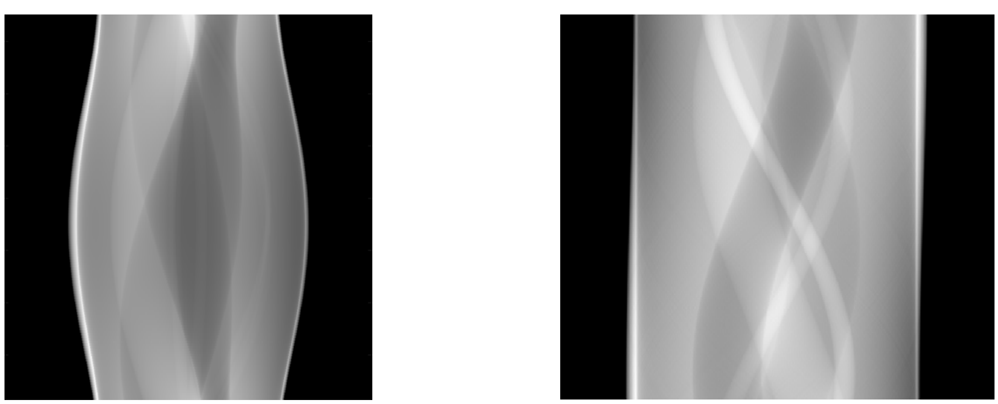


Fig. 9. SART on the custom disk phantom for k=18, 50, 100, and 200 iterations

The sinograms are also generated using the GUI designed with the algorithm implemented in the project.



(a) (b)

Fig. 10. (a) Sinogram generated for Shepp-Logan, (b) Sinogram generated for Disk Phantom

Conclusion

To compare the reconstructed images obtained from FBT and SART we can use metrics like similarity of images using structural information (SSIM) and Signal to Noise Ratio(SNR). Since we introduced noise into the input images before performing the SART, the output images are grainy. This makes our FBP outputs look well reconstructed. But with the increase in the number of iterations SART can produce better reconstruction than FBP. As observed from the results, the SART algorithm when applied on the input images accurately reconstructs the images as compared to the FBP algorithm. The FBP method as discussed earlier implements the image reconstruction using Fourier transforms which might result in some artifacts in the Fourier domain. The FBP algorithm is a most used technique for image reconstruction as it is computationally faster and provides quality outputs faster. The SART algorithm with its iterative method of solving the algebraic equations is a computationally complex method as compared to FBP but proves to provide better results with increased iterations.

Appendix A

The MATLAB code is added to a public Git repository. The repository is found here: https://github.com/asriraj/COSC6370_Project

The list of files is given below:

- **ProjectGUI.mlapp:** code for MATLAB GUI interface. Run this file to start the program.
- **Generatesinogram.m:** generates the sinogram for the input images.
- **fbp_discphantom.m:** this function creates a disk phantom and reconstructs the CT using simple back projection, filtered back projection, and filtered back projection using iradon transform.
- **fbp_shepplogan.m:** this script uses the Shepp-Logan phantom to reconstruct CT using back projection and filtered back projection using iradon transform.
- **fbp_shepplogan.m:** this script uses the Shepp-Logan phantom to reconstruct CT using back projection and filtered back projection using iradon transform.
- **fbp_shepplogan.m:** this script uses the Shepp-Logan phantom to reconstruct CT using back projection and filtered back projection using iradon transform.
- **sart_shepplogan.m:** this script reconstructs the Shepp-Logan phantom using Simultaneous Algebraic Reconstruction Technique SART.
- **Shepplogannorm.mat:** The sart_shepplogan.m uses the normalized (0 to 0.0309) mat file of the disk to perform the SART.

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