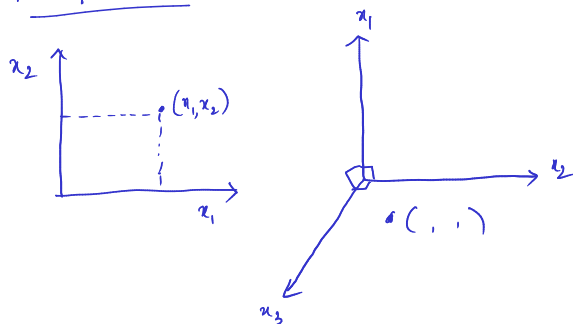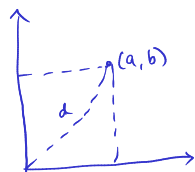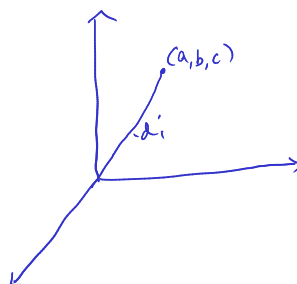# Linear Algebra :-

Point /vector :-

$\Rightarrow$ 4D, 5D,... can't be visualized.

Distance of point from origin :-

$d = \sqrt{a^2 + b^2}$
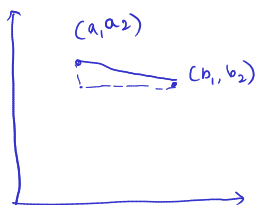
(From pythagorean theorem)

$d_1 = \sqrt{a^2 + b^2 + c^2}$

$\Downarrow$

nDimensional $\Rightarrow d = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$ if $p = (a_1, a_2, a_3, \ldots a_n)$

Whatever we are using in 1D, 2D we are able to generalize it to nD

Distance b/w two points :-

$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

$p = (a_1, a_2, a_3)$

$q = (b_1, b_2, b_3)$

$\Downarrow$

$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

$\Downarrow$

For n dimensional points :-

$d = sqrt\left(\sum_{i=1}^{n} (a_i - b_i)^2 \right)$

row vector :- $A = [\overset{1}{a_1}, \overset{2}{a_2}, \overset{3}{a_3}, \ldots, \overset{n}{a_n}]$

$\underset{\underset{\text{Number of rows}}{\downarrow}}{①} \times ⑩ \longrightarrow$ Number of columns.

$\searrow A_{1 \times n}$

Column vector :- $B = \begin{array}{c} 1 \\ 2 \\ \\ \\ n \end{array} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \longrightarrow B_{n \times 1}$

$\underset{\underset{\text{rows}}{\downarrow}}{⑩} \times ① \longrightarrow$ column.

Vector operations:-

$a = [a_1 \; a_2 \; a_3 \; \cdots \; a_n]$

$b = [b_1 \; b_2 \; b_3 \quad b_n]$

$c = a+b = [a_1+b_1 \quad a_2+b_2 \quad a_3+b_3 \quad \cdots \quad a_n+b_n]$

Multiplications:- Two types.

  (i) dot product
  (ii) cross product (Not often used in ML)

(In general, unless specifically mentioned, assume it is a column vector)

$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}$ $a^T = [a_1 \; a_2 \; a_3 \; \cdots \; a_n]$

Transpose.

(i) Dot product :-

$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n$

$= [a_1 \; a_2 \; a_3 \; , a_n] \underset{(n)}{} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}_{(n \times 1)}$

Match

$\Rightarrow a \cdot b = a \cdot b^T$

$= \sum_{i=1}^{n} a_i b_i$

Geometrical Intuition:-

$\|a\|$ = length of a from origin
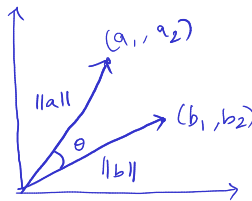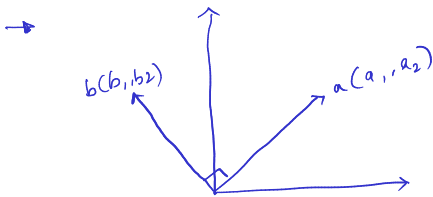
$= \sqrt{a_1^2 + a_2^2}$

$a \cdot b = \|a\| \; \|b\| \cos\theta$

$\Rightarrow a_1 b_1 + a_2 b_2 = \|a\| \|b\| \cos\theta$

$\Rightarrow$ Angle between two vectors, $\theta = \cos^{-1} \left\{ \dfrac{a_1 b_1 + a_2 b_2}{\|a\| \|b\|} \right\}$

$\Rightarrow a \cdot b = \|a\| \|b\| \cos\theta$

$= \|a\| \|b\| \cos 90°$

$= \|a\| \|b\| (0) = 0$

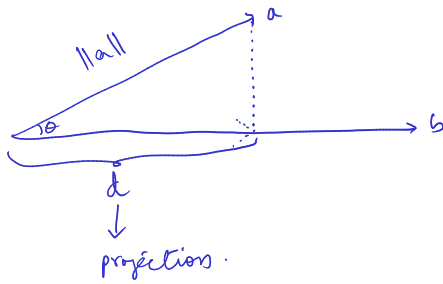$\Rightarrow$ If dot product $= 0$, then angle b/w vectors $= 90°$

$\quad\quad \hookrightarrow$ Applied to any dimensions.

$\longrightarrow$ This means that A is being scaled down by a factor equal to length of projection B on A.

$\longrightarrow a \cdot a = a_1 a_1 + a_2 a_2 + \cdots + a_n$

$= a_1^2 + a_2^2 + \cdots a_n^2$

$= \|a\|^2$

$d = \sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots a_n^2}$

Dot product tells us how parallel two vectors are and cross product tells us how perpendicular they are. Since there is only one way of being parallel, dot product returns scalar.

Projection:-

$$d = \|a\| \cos\theta \quad \longrightarrow \textcircled{1}$$

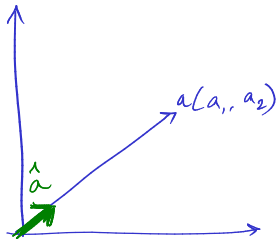$$a \cdot b = \sum_{i=1}^{n} a_i b_i = \|a\| \|b\| \cos\theta$$

$$\Rightarrow \|a\| \cos\theta = \frac{a \cdot b}{\|b\|}$$

sub in $\textcircled{1}$

$$d = \frac{a \cdot b}{\|b\|}$$

$$\Rightarrow \text{Projection of 'a' on 'b'} = \frac{a \cdot b}{\|b\|}$$

Unit vector:-

hat/cap

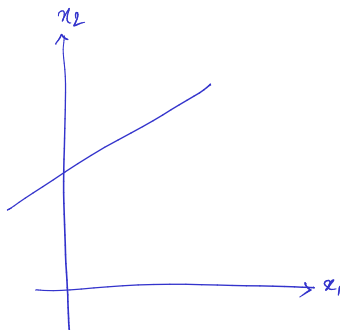$$\hat{a} = \frac{a}{\|a\|}$$

→ $\hat{a}$ is in the same direction as 'a'

→ $\|\hat{a}\| = 1$

Line:-

2D:-

line becomes plane
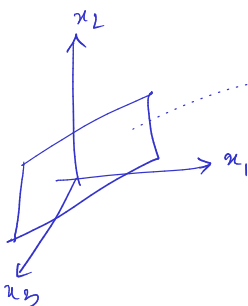
$y = mx + c$ (∇)

$ax + by + c = 0$

→ Both are the same.
→ This is called the general form.

$$ax_1 + bx_2 + c = 0$$

$$w_1 x_1 + w_2 x_2 + w_0 = 0 \longrightarrow \textcircled{2D}$$

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0 \quad (\text{Plane})$$

For n dimensions, it is called **Hyperplane.**

equation of hyperplane $= w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n = 0$

$$\Rightarrow W_0 + \sum_{i=1}^{n} W_i x_i = 0$$

$$\Rightarrow W_0 + \underbrace{[w_1 \; w_2 \; w_3 \cdots w_n]}_{W}{}_{1 \times n} \; \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{X}{}_{n \times 1} = 0$$

$$\Rightarrow W_0 + \boxed{W^T x} = 0 \qquad \longrightarrow \text{Generally, unless specified its a column vector}$$

$\longrightarrow$ Planes are typically denoted by Π

$$\Pi ; \quad W_0 + W^T x = 0 \qquad \longrightarrow \text{eqn of plane in n-dimensions.}$$
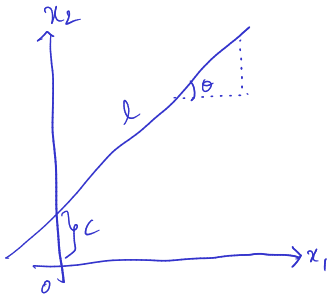
$y = \boxed{m}x + \boxed{C}$ — slope, $\rightarrow$ y-intercept.

$$W_1 x_1 + W_2 x_2 + W_0 = 0$$

$$a_2 = \frac{-W_0}{W_2} - \frac{W_1 x_1}{W_2}$$

$$m = \frac{-W_1}{W_2}, \quad c = \frac{-W_0}{W_2}$$

if line is passing through origin, $c = 0 \Rightarrow \frac{-W_0}{W_2} = 0 \Rightarrow W_0 = 0$

$$\Rightarrow \text{eqn becomes} \quad W_1 x_1 + W_2 x_2 = 0$$

$$\text{for 3D} \longrightarrow W_1 x_1 + W_2 x_2 + W_3 x_3 = 0 \longrightarrow \text{plane}$$

$$\vdots$$

$$nD \longrightarrow W_1 x_1 + W_2 x_2 + W_3 x_3 + \cdots + W_n x_n = 0$$

$$\longrightarrow W^T x = 0$$

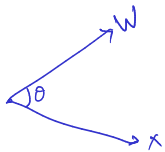equation of plane passing through origin.

$\nearrow$ plane passing through origin

$$\boxed{\Pi} : W^T x = 0$$

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$
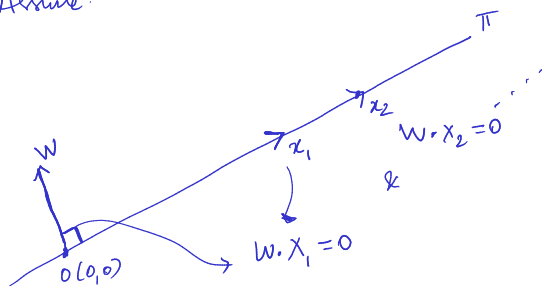
assume W & x as just two vectors

$$W \cdot x = W^T x = \|W\| \|x\| \cos\theta = \boxed{0}$$

$$W \perp x \Rightarrow \theta_{w,x} = 90°$$

Assume.

$W \cdot x_2 = 0$

$W \cdot x_1 = 0$

$$\Rightarrow \text{If } W \perp \Pi, \text{ then } W \cdot x_i = 0 \; \forall \; x_i \in \Pi$$
$$\left(W \cdot x_i \text{ is } 0 \text{ for all points } x_i \text{ on plane } \Pi \right)$$

$\longrightarrow$ w is just a vector that is perpendicular to the plane, at the origin.

$\longrightarrow$ w^T is called normal to the plane.

$$\rightarrow \hat{W} = \frac{W}{\|W\|} \qquad \hat{W} \cdot x_i = 0 \quad \forall \; x_i \in \Pi$$
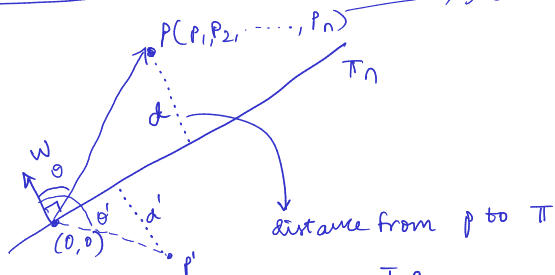
$\rightarrow$ For simplicity we always assume our plane passes through origin.

$\rightarrow$ The equation of plane with intercepts a, b, c on x, y, z-axis respectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Distance of point from plane :—

$\rightarrow$ Because point could be n-dimensional

$P(P_1, P_2, \ldots, P_n)$
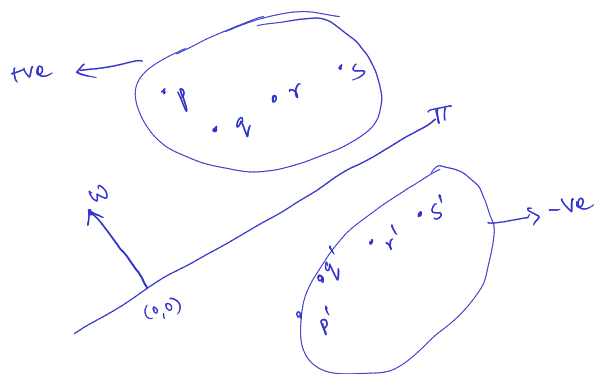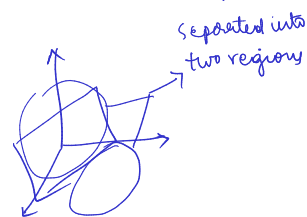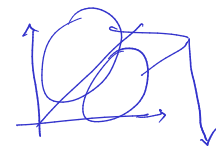
Π_n

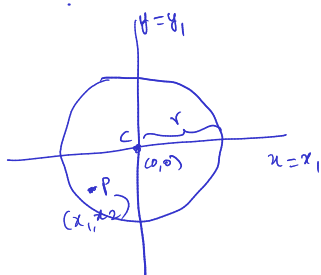distance from p to Π

$$d = \frac{W^T P}{\|W\|} = +ve$$

$$d' = \frac{W^T P'}{\|W\|} = -ve$$

since
$W \cdot P = \|W\| \; \|P\| \cos\theta$

This sign tells us on which side the point lies on the plane.

Separated into two regions

$+ve \leftarrow$

$\cdot p \quad \cdot r \quad \cdot s$
$\cdot q$

Π

W

$(0,0)$

$\cdot r' \quad \cdot s'$
$\cdot q' \quad \rightarrow -ve$
$\cdot p'$

Circle, Sphere & hypersphere :—

$y = y_1$

r
c
$(0,0)$ $\quad x = x_1$

$\cdot P$
(x_1, x_2)

eqn of circle :—

$x^2 + y^2 = r^2$ (if its center is origin) else

$(x-h)^2 + (y-k)^2 = r^2$ where $(h, K) = $ center

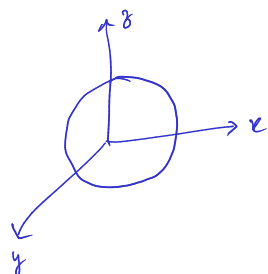if $x_1^2 + x_2^2 < r^2$, then p lies inside the circle

$x_1^2 + x_2^2 > r^2$, then p lies outside the circle

$x_1^2 + x_2^2 = r^2$, then p is on the circle.

$\rightarrow$ can be extended to all dimensions.

3D :—

z

x

y

$x_1^2 + x_2^2 + x_3^2 = r^2 \longrightarrow$ sphere

$nD \Rightarrow x_1^2 + x_2^2 + \cdots + x_n^2 = r^2$ ——→ hypersphere

$$\sum_{i=1}^{n} x_i^2 = r^2$$

Ellipse

Not that often used in ML

2D :-

minor axis
$y_1 = y$
a
b
$(0,0)$
$x_1 = x$
major axis
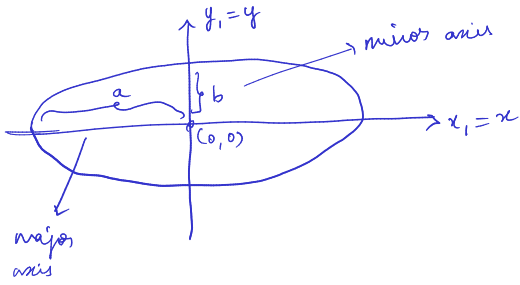
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$ ——→ (x, y) lies inside ellipse

$\phantom{\frac{x^2}{a^2}} > 1$ ——→ " " outside ellipse

$\phantom{\frac{x^2}{a^2}} = 1$ ——→ on the ellipse

$\}$ ——→ applied to all dimensions.
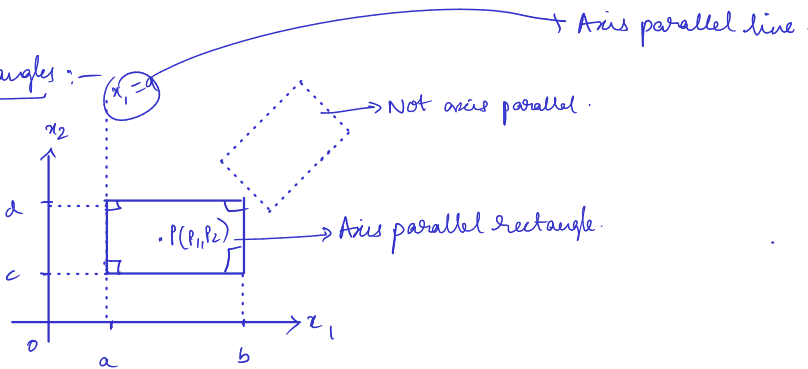
3D :— (ellipsoid) ——→ American Football Ball.

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$$

nD :— (Hyper ellipsoid) ——→ $\dfrac{x_1^2}{a_1^2} + \dfrac{x_2^2}{a_2^2} + \cdots + \dfrac{x_n^2}{a_n^2} = 1$

Squares, Rectangles :—

→ Axis parallel line.

2D :-

$x_1 = \alpha$

→ Not axis parallel.

x_2
d
c
$\cdot P(P_1, P_2)$ → Axis parallel rectangle.
0
a
b
x_1

if $P_1 < b$ && $P_1 > a$

if $P_2 > c$ && $P_2 < d$

then p lies inside rectangle.

3D :— (cuboid)

x_2
x_1
x_3
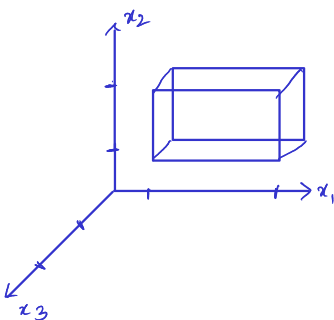
nD :— hypercuboid.

Similar rule
can be made
for 3D