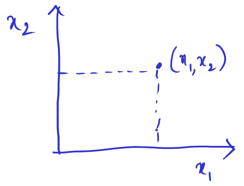
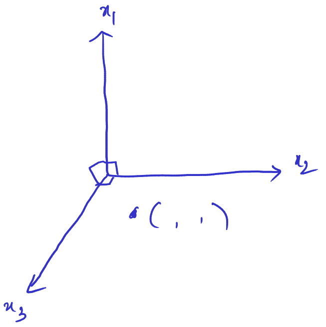
Linear Algebra :-

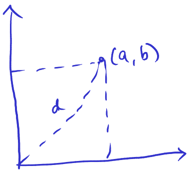
Point / Vector :-





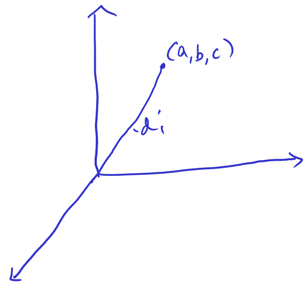
$\Rightarrow$ 4D, 5D, ... can't be visualized.

Distance of point from origin :-



$d = \sqrt{a^2 + b^2}$

(From pythagorean theorem)



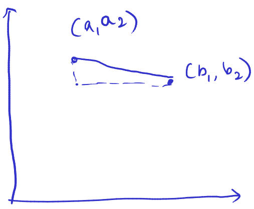
$d_1 = \sqrt{a^2 + b^2 + c^2}$

$\Downarrow$

nDimensional $\Rightarrow$ $d = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$ if $p = (a_1, a_2, a_3, \ldots\ldots a_n)$

whatever we are using in 1D, 2D we are able to generalize it to nD

Distance b/w two points :-



$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

$p = (a_1, a_2\ a_3)$

$q = (b_1, b_2, b_3)$

$\Downarrow$

$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

$\Downarrow$

For n dimensional points :-

$d = \text{sqrt}\left(\sum_{i=1}^{n} (a_i - b_i)^2 \right)$

row vector :- $A = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & & n \end{array} \\ [a_1, a_2, a_3, \ldots\ldots, a_n] \end{array}$

$1 \times n$ $\longrightarrow$ Number of columns. $\longrightarrow A_{1 \times n}$

$\downarrow$

Number of rows

Column vector :- $B = \begin{array}{c} 1 \\ 2 \\ \\ n \end{array} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \longrightarrow B_{n \times 1}$

$n \times 1 \longrightarrow$ column.

$\downarrow$

rows

Vector operations:-

(In general, unless specifically mentioned, assume it is a column vector)

$a = [a_1 \ a_2 \ a_3 \ \dots \ a_n]$

$b = [b_1 \ b_2 \ b_3 \ \ b_n]$

$c = a + b = [a_1+b_1 \quad a_2+b_2 \quad a_3+b_3 \quad \dots \quad a_n+b_n]$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} \qquad a^T = [a_1 \ a_2 \ a_3 \ \dots \ a_n]$$

Transpose.

Multiplication:- Two types.

(i) dot product

(ii) Cross product (Not often used in ML)

(i) Dot product:-

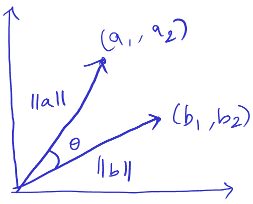
$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$$

$$= [a_1, a_2, a_3 \ , a_n]_{1\times n} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1}$$

Match

$$\Rightarrow a \cdot b = a \cdot b^T = \sum_{i=1}^{n} a_i b_i$$

Geometrical Intuition:-



$\|a\|$ = length of a from origin

$= \sqrt{a_1^2 + a_2^2}$

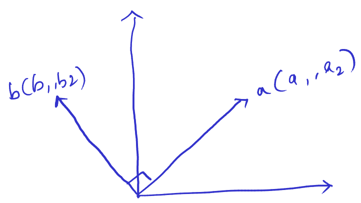
$$a \cdot b = \|a\| \, \|b\| \cos\theta$$

$$\boxed{\Rightarrow a_1 b_1 + a_2 b_2 = \|a\| \, \|b\| \cos\theta}$$

$\Rightarrow$ Angle between two vectors,

$$\boxed{\theta = \cos^{-1}\left\{ \frac{a_1 b_1 + a_2 b_2}{\|a\| \, \|b\|} \right\}}$$

→



$\Rightarrow a \cdot b = \|a\| \, \|b\| \cos\theta$

$= \|a\| \, \|b\| \cos 90^\circ$

$= \|a\| \, \|b\| \, (0) \ = 0$

→ This means that A is being scaled down by a factor equal to length of projection B on A.

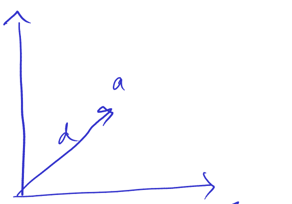
$\Rightarrow$ If dot product $= 0$, then angle b/w vectors $= 90^\circ$

↳ Applied to any dimensions.

→ $a \cdot a = a_1 a_1 + a_2 a_2 + \dots + a_n$

$= a_1^2 + a_2^2 + \dots \ a_n^2$

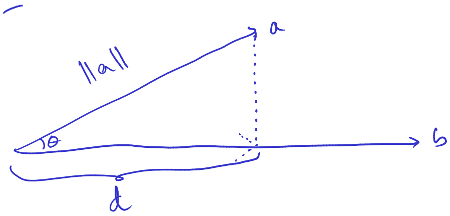
$= \|a\|^2$



$d = \sqrt{a_1^2 + a_2^2 + a_3^2 + \dots a_n^2}$

Dot product tells us how parallel two vectors are and cross product tells us how perpendicular they are. Since there is only one way of being parallel, dot product returns scalar.

Projection:-



projection.

$d = ||a|| \cos\theta \longrightarrow (1)$

$a \cdot b = \sum_{i=1}^{n} a_i b_i = ||a|| \, ||b|| \cos\theta$

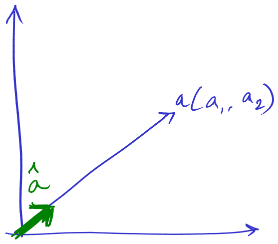
$\Rightarrow ||a|| \cos\theta = \dfrac{a \cdot b}{||b||}$

sub in (1)

$d = \dfrac{a \cdot b}{||b||}$

$$\boxed{\Rightarrow \text{Projection of 'a' on 'b'} = \frac{a \cdot b}{||b||}}$$

Unit vector:- ($\hat{a}$) → hat/cap



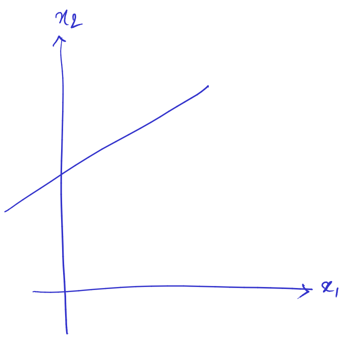
$\hat{a} = \dfrac{a}{||a||}$

→ $\hat{a}$ is in the same direction as 'a'

→ $||\hat{a}|| = 1$

Line :-

2D :-



$y = mx + c$
(or)
$ax + by + c = 0$

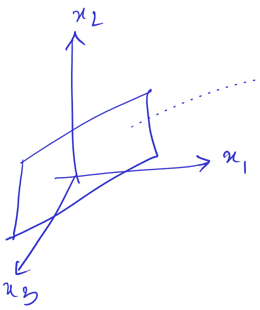
→ Both are the same.

→ This is called the general form.

$\Downarrow$

$ax_1 + bx_2 + c = 0$
$\Downarrow$
$w_1x_1 + w_2x_2 + w_0 = 0 \longrightarrow$ (2D)

line becomes plane



$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$ (Plane)

$\Downarrow$

For n dimensions, it is called Hyperplane.

equation of hyperplane $= w_0 + w_1x_1 + w_2x_2 + \dots + w_nx_n = 0$

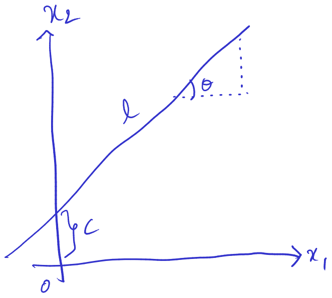
$\Rightarrow w_0 + \sum_{i=1}^{n} w_ix_i = 0$

$$\Rightarrow w_0 + \underbrace{\begin{bmatrix} w_1 & w_2 & w_3 & \dots & w_n \end{bmatrix}_{1\times n}}_{W} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}}_{X} = 0$$

$\Rightarrow W_0 + W^T X = 0$ → Generally, unless specified its a column vector.

→ Planes are typically denoted by π

$\pi : W_0 + W^T X = 0$ → eqn of plane in n dimensions.



$y = mx + c$ → slope (m), → y intercept (c).

$w_1 x_1 + w_2 x_2 + w_0 = 0$

$x_2 = \frac{-w_0}{w_2} - \frac{w_1}{w_2} x_1$

$m = \frac{-w_1}{w_2}$, $c = \frac{-w_0}{w_2}$

if line is passing through origin, $c = 0 \Rightarrow \frac{-w_0}{w_2} = 0 \Rightarrow w_0 = 0$

$\Rightarrow$ eqn becomes $w_1 x_1 + w_2 x_2 = 0$

for 3D $\rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 = 0$ → plane

⋮

nD $\rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n = 0$

$\rightarrow W^T X = 0$

equation of plane passing through origin.

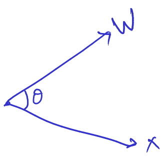
plane passing through origin

π : $W^T X = 0$

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

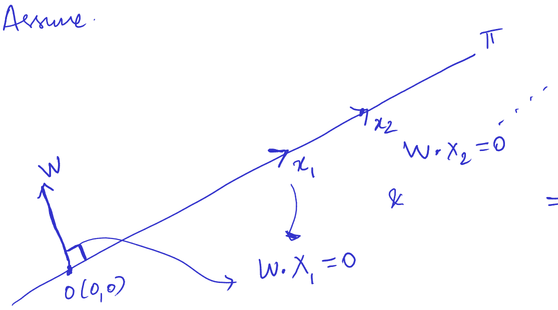
assume W & X as just two vectors

$W \cdot X = W^T X = \|W\| \, \|X\| \cos\theta = 0$



$W \perp X \Rightarrow \theta_{w,x} = 90°$

Assume.



$\Rightarrow$ If $w \perp \pi$, then $w \cdot x_i = 0 \; \forall \; x_i \in \pi$

($w \cdot x_i$ is 0 for all points x_i on plane π)

→ w is just a vector that is perpendicular to the plane, at the origin.

→ w^T is called normal to the plane.

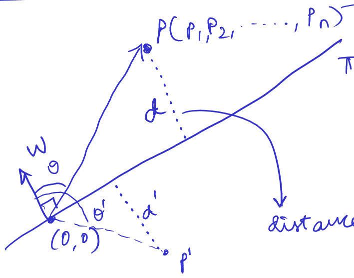
$\rightarrow \hat{W} = \frac{W}{\|W\|}$ $\qquad \hat{W} \cdot x_i = 0 \quad \forall \; x_i \in \pi$

→ For simplicity we always assume our plane passes through origin.

→ The equation of plane with intercepts a, b, c on x, y, z-axis respectively is

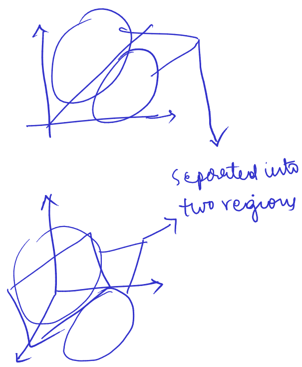
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Distance of point from plane :-



→ Because point could be n dimensional

distance from p to π



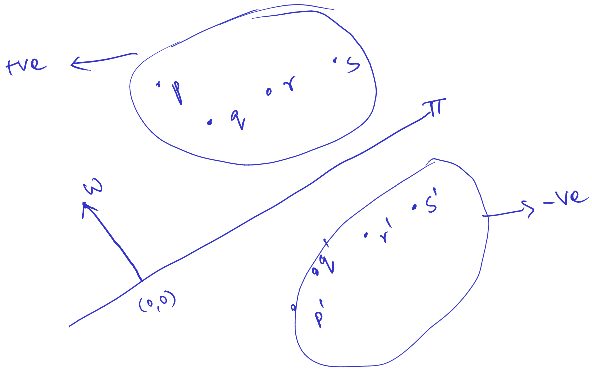
separated into two regions

$$d = \frac{W^T p}{\|W\|} = +ve$$

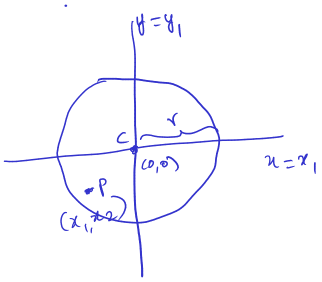
$$d' = \frac{W^T p'}{\|W\|} = -ve$$

since
$W \cdot P = \|W\| \, \|P\| \cos\theta$

This sign tells us on which side the point lies on the plane.



Circle, sphere & hypersphere:-



eqn of circle:-

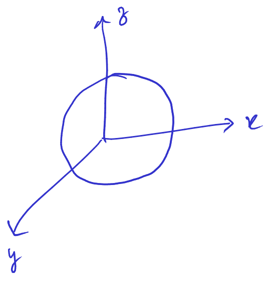
$x^2 + y^2 = r^2$ (if it's center is origin) else

$(x-h)^2 + (y-k)^2 = r^2$ where (h, k) = center

if $x_1^2 + x_2^2 < r^2$, then p lies inside the circle
$x_1^2 + x_2^2 > r^2$, then p lies outside the circle
$x_1^2 + x_2^2 = r^2$, then p is on the circle.

→ can be extended to all dimensions.

3D :-



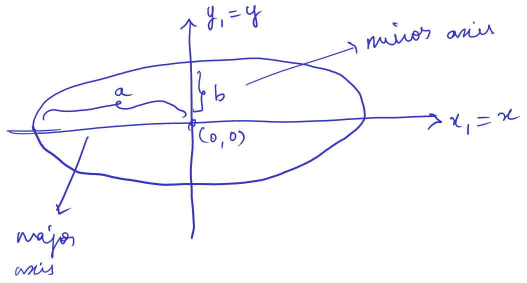
$x_1^2 + x_2^2 + x_3^2 = r^2$ → sphere

$nD \Rightarrow x_1^2 + x_2^2 + \cdots\cdots + x_n^2 = r^2$ ⟶ hypersphere

$$\sum_{i=1}^{n} x_i^2 = r^2$$

Ellipse

2D :-



Not that often used in ML

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$ ⟶ (x, y) lies inside ellipse

> 1 ⟶ " " outside ellipse

$= 1$ ⟶ on the ellipse

} ⟶ applied to all dimensions.

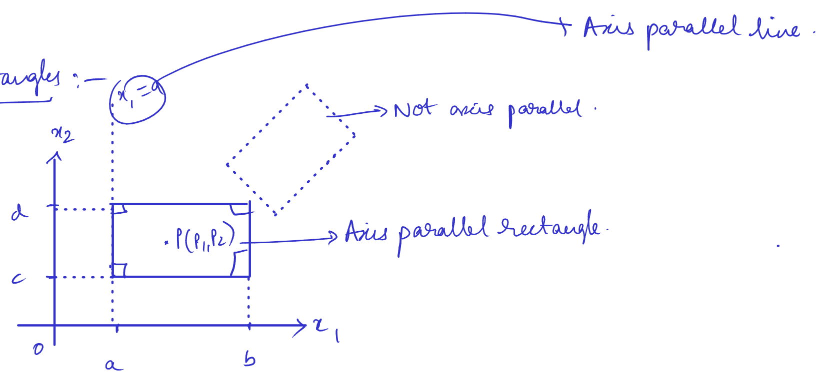
3D :- (ellipsoid) ⟶ American Football Ball.

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$$

nD :- (Hyperellipsoid) ⟶ $\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \cdots + \frac{x_n^2}{a_n^2} = 1$

Square, Rectangles :-

2D :-



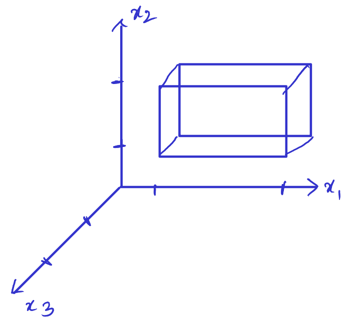
if $p_1 < b$ && $p_1 > a$

if $p_2 > c$ && $p_2 < d$

then p lies inside rectangle.

if its not axis parallel, then we take the equations of the sides & check which side of the line the point is

3D :- (cuboid)



similar rule can be made for 3D

nD :- hypercuboid.