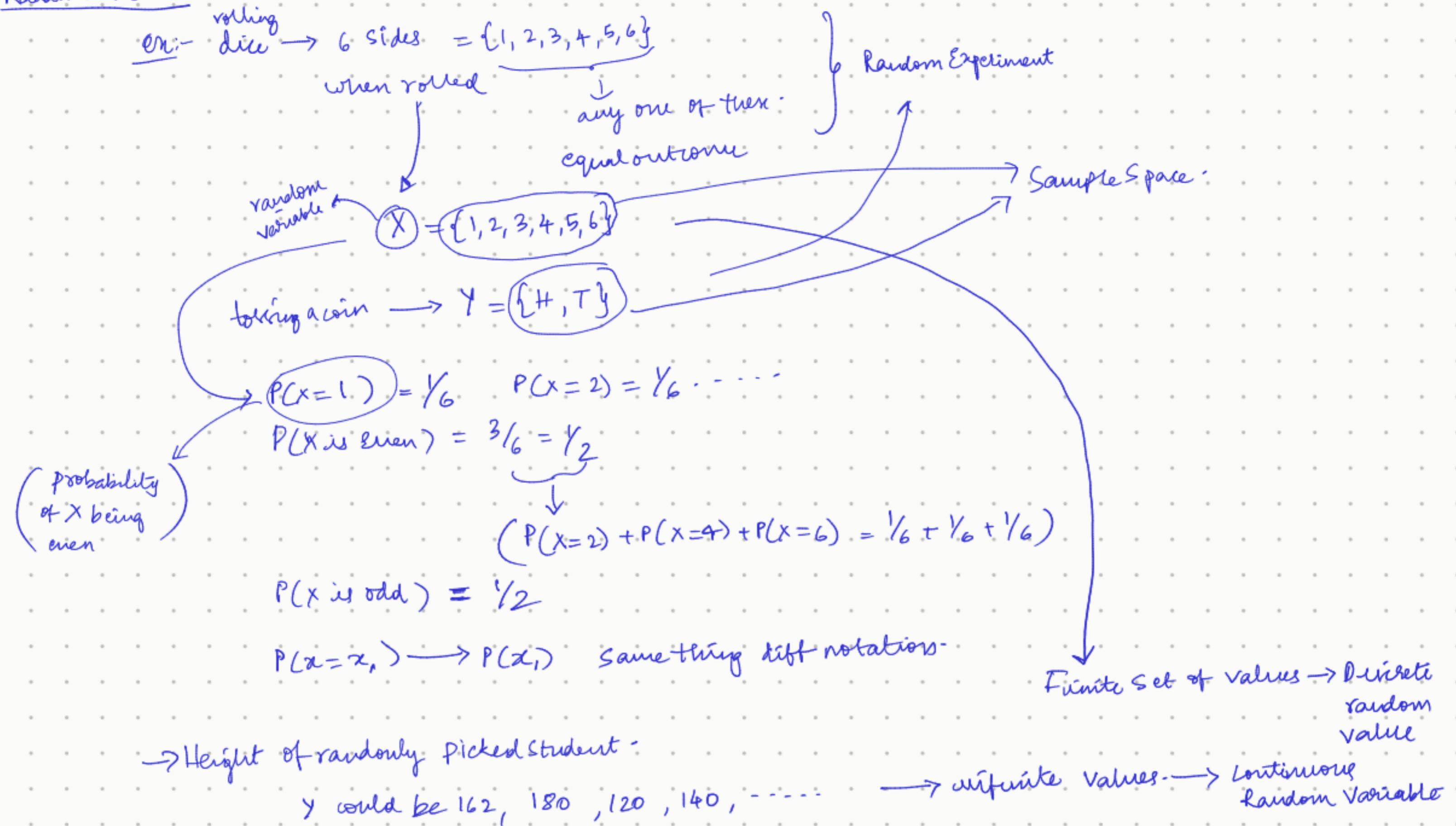


## Probability & Statistics :-

### Random Variable :-



### Outliers :-

$y$  : height of student-

$$\{122.2, 146.4, 132.5, \dots, 121.1, 156.3, 92.7, \dots\}$$

121.1

92.7

would be an outlier

outliers → could be human error

(or)

actual height

→ Outliers can corrupt data

→ A discrete value is obtained by counting

→ A continuous value is obtained by measuring.

Sample Space :- Set of all possible outcomes of an experiment.

→ A random variable value depends on the outcome of a random phenomenon.

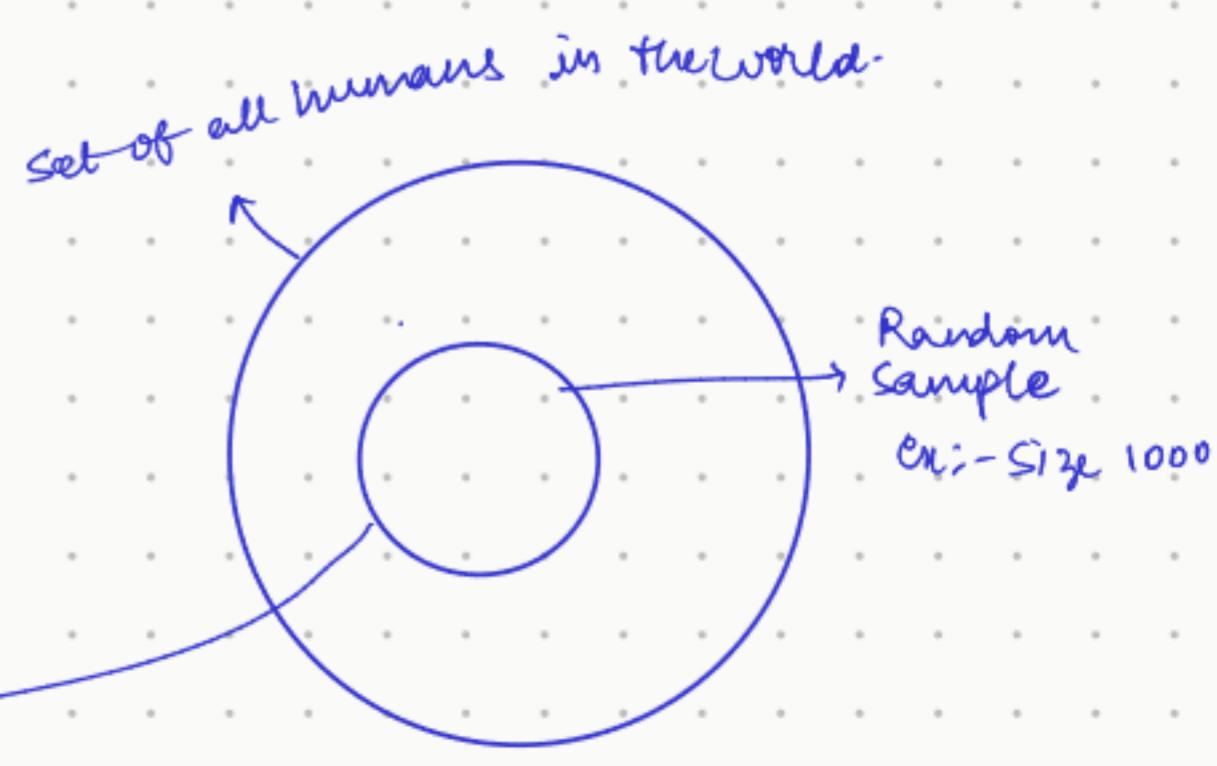
### Populations & Sample :-

→ Estimating the average height of human

$$\bar{x} = \frac{1}{\text{Pop}} \sum_{i=1}^{\text{Pop}} h_i \quad (\text{IMPOSSIBLE})$$

So we estimate:

often represented by  $\bar{x} = \frac{1}{1000} \sum_{i=1}^{1000} h_i$



→ As sample size increases

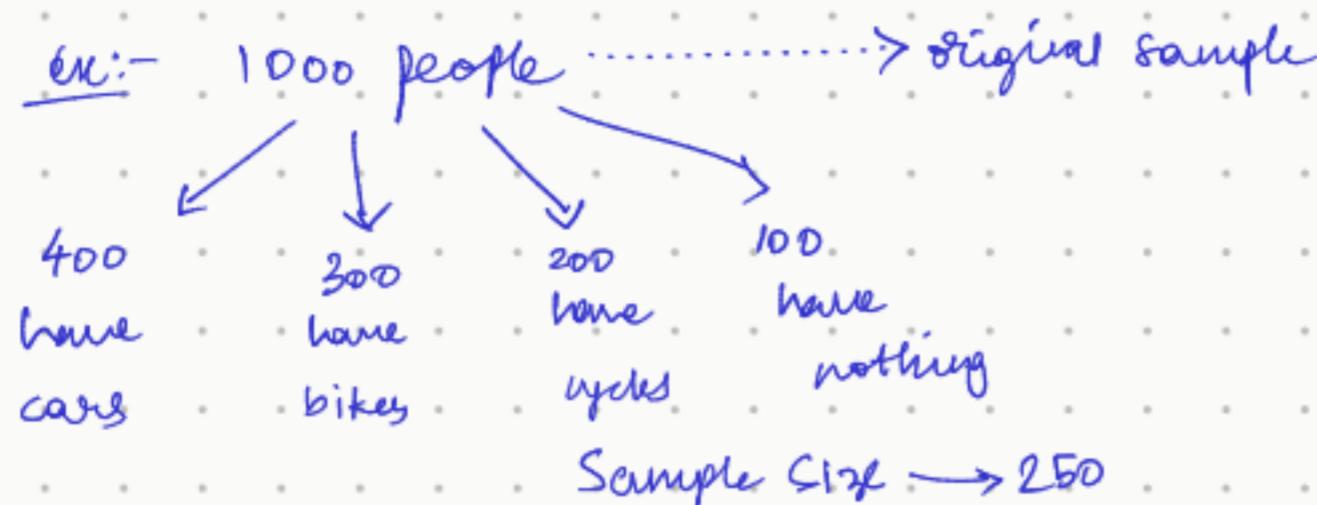
$$\bar{x} = \mu$$

↓  
Sample mean      ↓ original mean

Sampling is of two types:-

(i) Simple Sampling

(ii) Stratified Sampling → Unbiased Sampling & more accurate results.



### Simple Random Sampling

250 could have cars  
(or)

250 could have bikes  
(or)

100 bikes + 150 cars

### Stratified Random Sampling

Cars → 100  
bikes → 75  
cycles → 50  
nothing → 25

} These are random but equal in proportion to all classes

## Gaussian Distribution:- (AKA Normal Distribution)

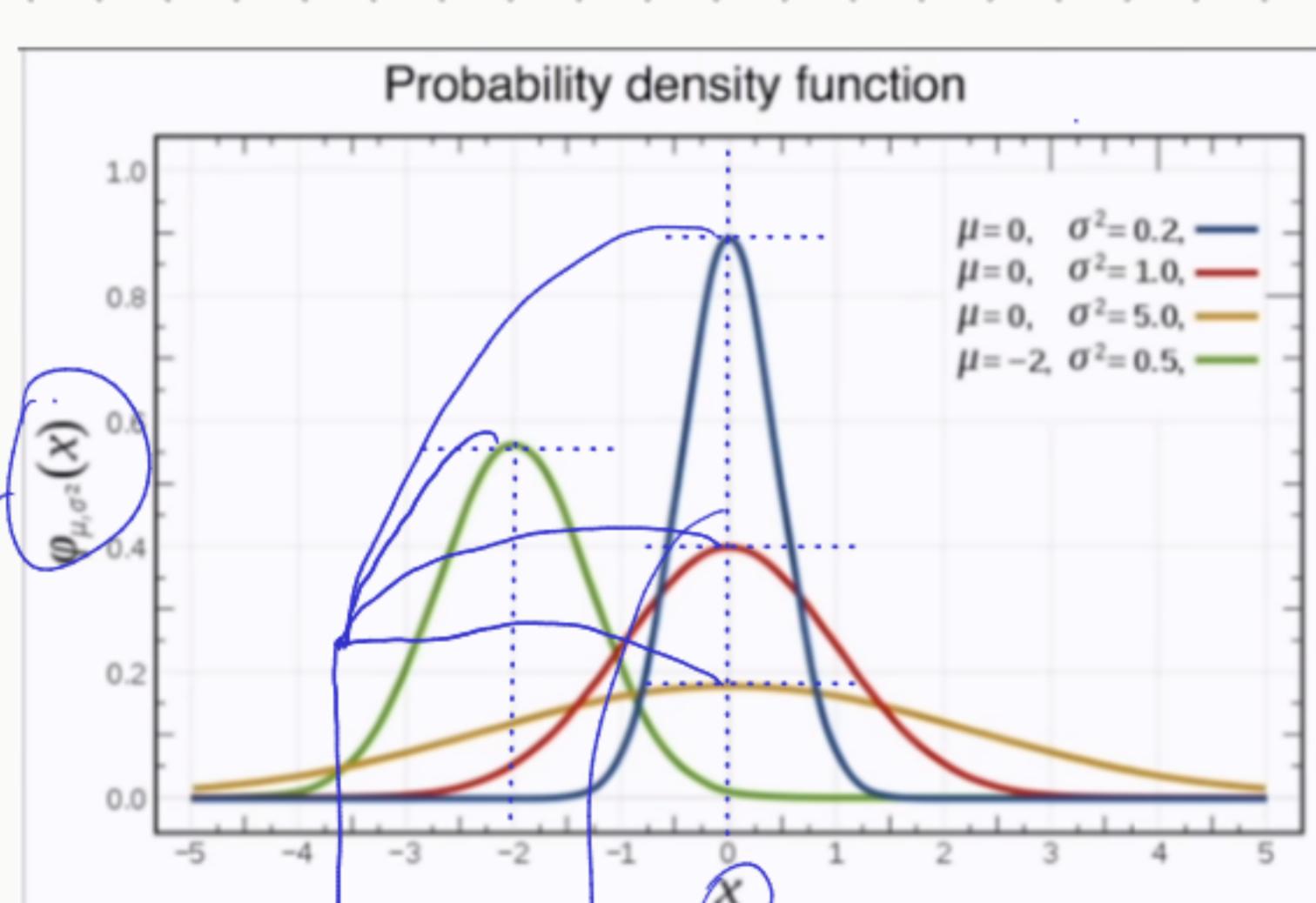
→ If  $X$  is a continuous random variable &  $X$  has a PDF curve ( $\mathcal{N}$ ), then we say  $X$  has a Gaussian distribution.

### Why learn?

- Stuff in nature tends to follow G.D.
- Height, weight of people follow it. (Natural phenomenon)
- They are simple models that summarize R.V.

PDF = Probability Density Functions

Mean, Median & Mode are all same for G.D. There are equal number of measurements above & below the mean



$\mu$  = mean  
 $\sigma^2$  = variance  $\rightarrow$  parameters

[If we are given  $\mu, \sigma^2$  & told that  $X$  follows G.D., we can plot PDF. We don't need the whole data.]

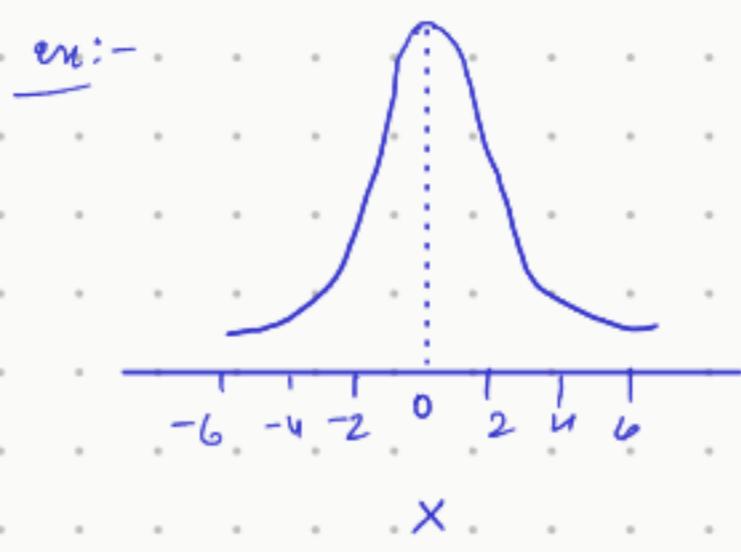
Variance is a measure of spread.

Continuous Random Variable  
Red, Yellow, Blue have  $\mu=0$  but varying variances.  
The peak of curve is usually at ' $\mu$ '.

[If this is not known, then we can]

→ The parameters of Gaussian Distribution are  $\mu$  &  $\sigma^2$

$X \sim N(\mu, \sigma^2)$  ( $\Rightarrow X$  follows Gaussian Distribution with  $\mu$  &  $\sigma^2$ )



$\rightarrow X \sim N(0, 1)$

$$\rightarrow P(X=x) = p(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left\{ \frac{-(x-\mu)^2}{2\sigma^2} \right\}$$

Probability Density at a point (x). PDF at any given point gives the probability density at that point. Probability of getting a single discrete value is 0.

ex:-  $\mu=0, \sigma^2=1, \sigma=1$

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp \left\{ \left( \frac{-1}{2} \right) x^2 \right\} = y$$

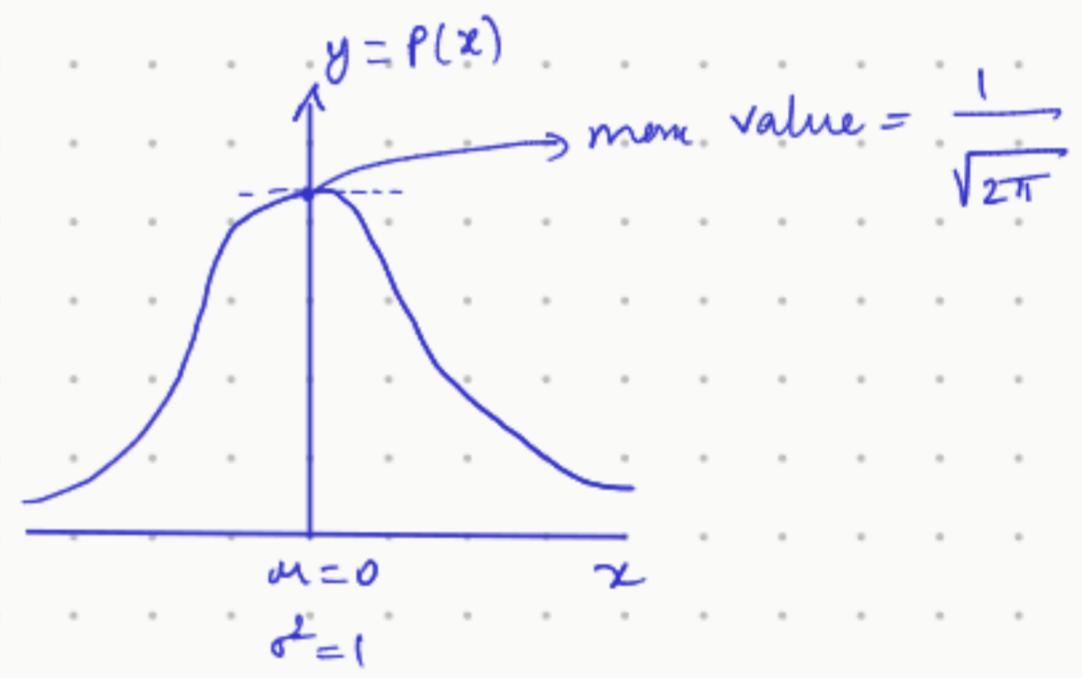
constants

further simplifying

$$y = \exp(-x^2)$$

↳ when plotted

as  $x$  increases  
y decreases.  
as  $x$  decreases  
y decreases.



conclusions :-

① x moves away from  $\mu$ , y decreases

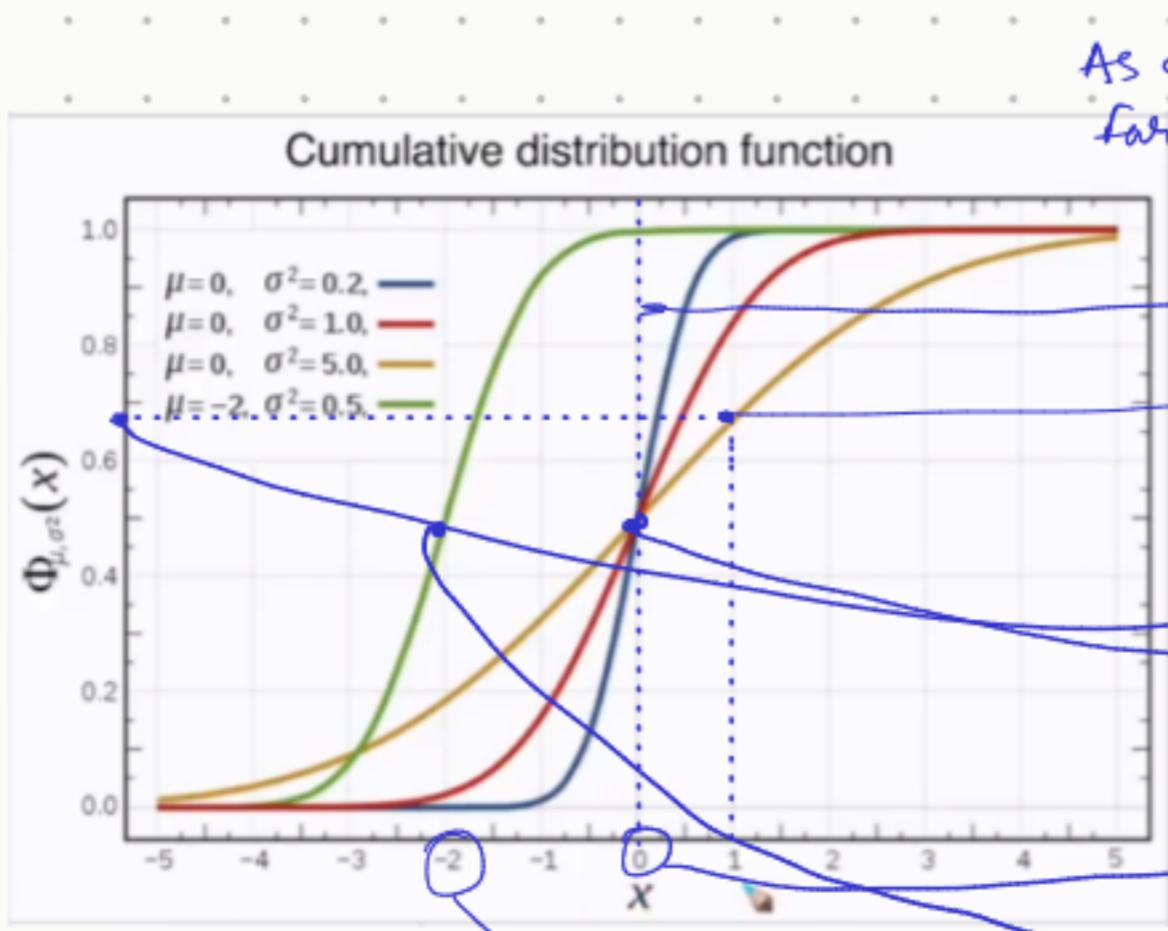
② Graph is symmetric.

③ In this particular graph, y is reducing exponential squared. ( $e^{-x^2}$ )

→ If  $p(x)$  is the probability density at a point 'x', the probability can be obtained by computing the integral of  $p(x)$  over a given interval.

i.e., probability of getting  $x \in [a, b]$  is  $\int_a^b p(x) dx$

Cumulative Distribution Function (CDF) of Gaussian Distribution/Normal Distribution:-



As  $\sigma^2$  increases, CDF goes far from unit line

$\rightarrow P(X \leq 1) = 0.65$

$\rightarrow \mu=0$ , center of CDF is at 0

$\rightarrow \mu=-2$ , center of CDF is at -2

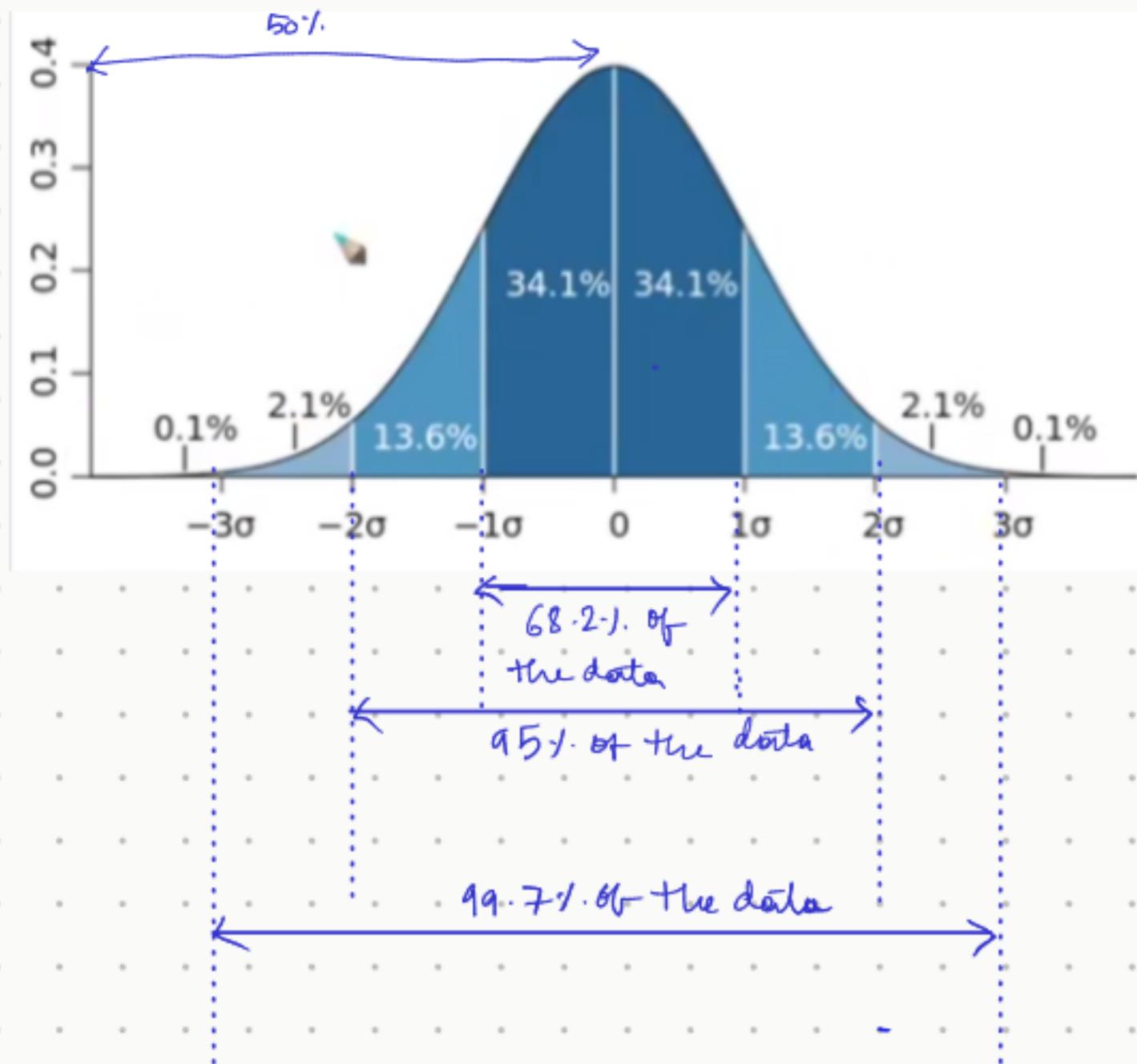
CDF of a random variable looks like

$$CDF = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x-\mu}{\sigma\sqrt{2}} \right) \right] \rightarrow \text{No need to memorize}$$

68-95-99.7 rule :-

$$\text{if } \mu=0, \sigma^2=4 \Rightarrow \sigma=2$$

$$X \sim N(0, 4)$$



How is this useful?

Ex:- if human population weight

$$X \sim N(150, 25)$$

$\downarrow \mu$        $\downarrow \sigma$

$\Rightarrow$  68.2% of human populations lies b/w  $(150-25, 150+25)$

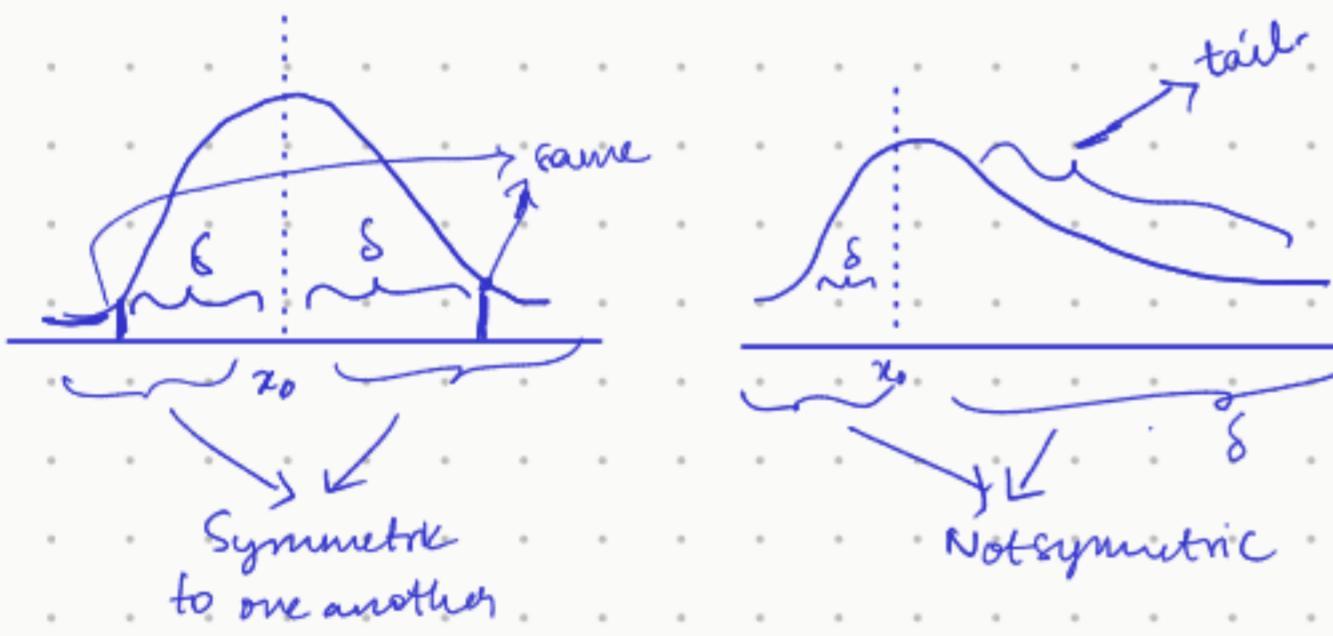
95% of people  $(150-50, 150+50)$

99.7% of people  $(150-75, 150+75)$

→ A standard gaussian distribution always has a mean of 0 & Variance 1.  
If it has other mean & variance, it's a non standard gaussian distribution.

### Symmetric Distribution, Skewness & Kurtosis :-

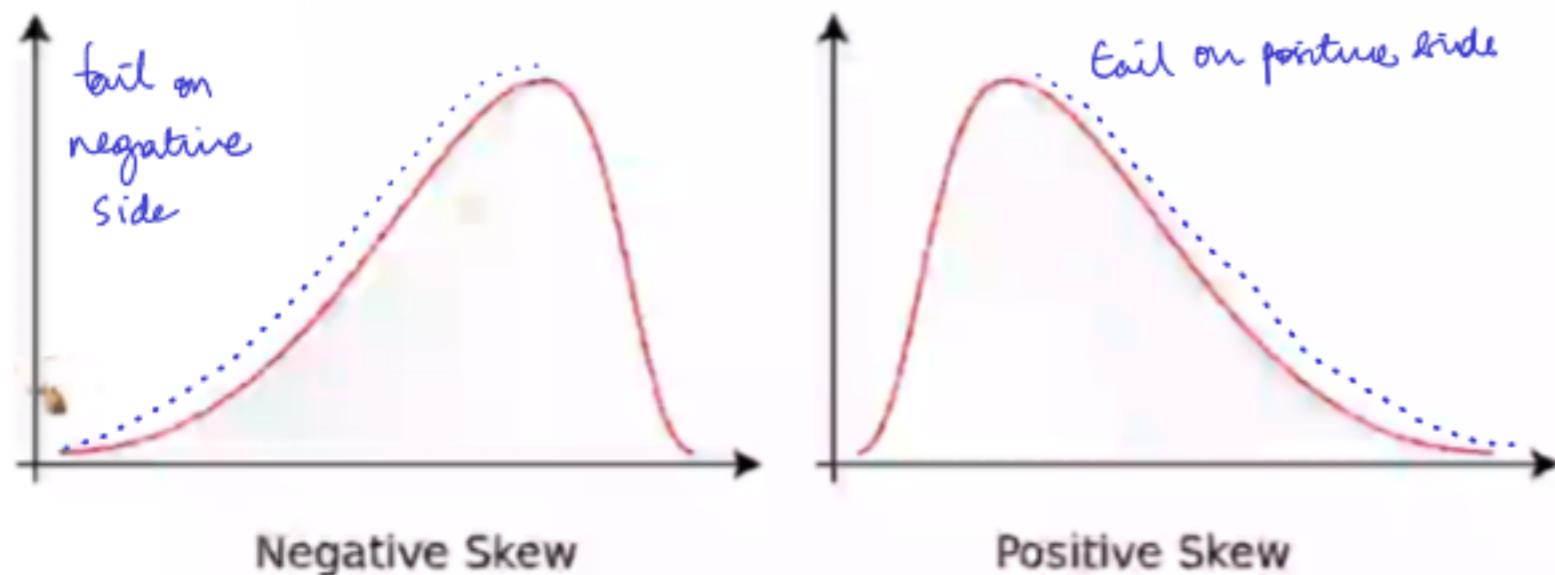
→ They help understand shape of PDF.



→ A probability distribution is said to be symmetric if and only if there exists a value  $x_0$  such that  $f(x_0 - \delta) = f(x_0 + \delta)$  for all real numbers  $\delta$ .  
 $f(x)$  is the height of PDF at any point  $x$ .

### Skewness :-

Skewness is a measure of asymmetry.



$$x = [x_1 \\ x_2 \\ \vdots \\ x_n] \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\text{Skewness} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3 \quad \text{if } 2 = \text{Variance}$$

$$\left[ \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \right]^{3/2}$$

sample std-deviation

## Kurtosis :-

→ Measure of tailedness

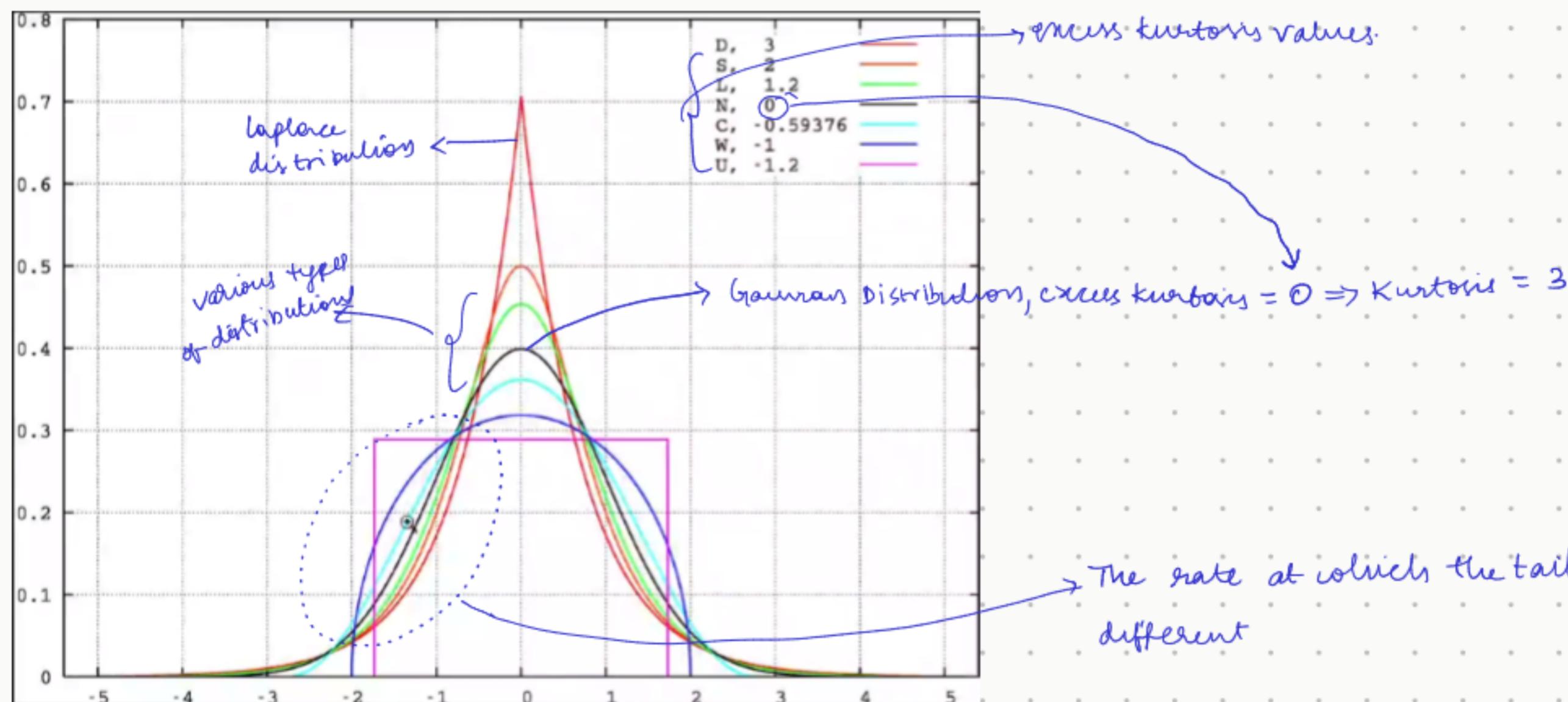
$$\text{excess kurtosis} = \text{kurtosis} - 3$$

$$\text{excess kurtosis} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left( \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2} - 3$$

Variance

Kurtosis of Gaussian Distribution = 3

→ Kurtosis is not a measure of peakedness. Might look like it -



→ Kurtosis tells us if there are outliers are there in the data.

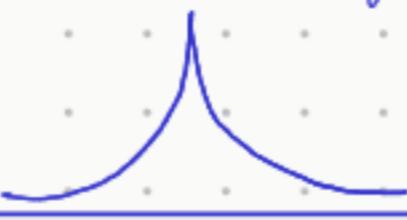
→ Kurtosis is used in analysis of trading, finance etc.)

→ Kurtosis measures how much density/weight is in tails compared to middle parts

- If there are some stocks. Some stocks fall ↓
- some go up ↑
- some go up enormously. ↑

To estimate risk after buying some stocks, we model data - It's generally assumed that they follow gaussian distributions for ease of calculations (Pre 2008 financial crisis). As to gaussian distributions chances of losses are less & has good chance of high profits. But now, instead of assuming the data is a gaussian distribution, we look at the actual data & calculate the kurtosis

If kurtosis is  $> 3 \rightsquigarrow$



⇒ chances of extreme profits are high & chances of losses are also high.

If kurtosis = 3  $\Rightarrow$  Follow gaussian distribution

If kurtosis  $< 3 \Rightarrow$



⇒ chances of extreme profits are low & chances of losses are also fairly low.

→ Kurtosis has a degree of 4. So if there is large deviations, then it will lead to higher kurtosis value.

→ High kurtosis is caused by infrequent extreme deviations rather than frequent modestly sized deviations.

→ In order to compare kurtosis b/w two curves, they need to have the same variance.

## Standard Normal Variate:- (z)

$$Z \sim N(0, 1)$$

$$\mu = 0$$

$$\sigma^2 = 1$$

Let  $X \sim N(\mu, \sigma^2)$

$$X = [x_1, x_2, x_3, \dots, x_{50}]$$

$$\text{Standardization: } z_i' = \frac{x_i - \mu}{\sigma}$$

$$\Rightarrow z_i' \sim N(0, 1)$$

↑ Standard Normal Variate.

Given any random variable  $X$ , where  $X \sim N(\mu, \sigma^2)$

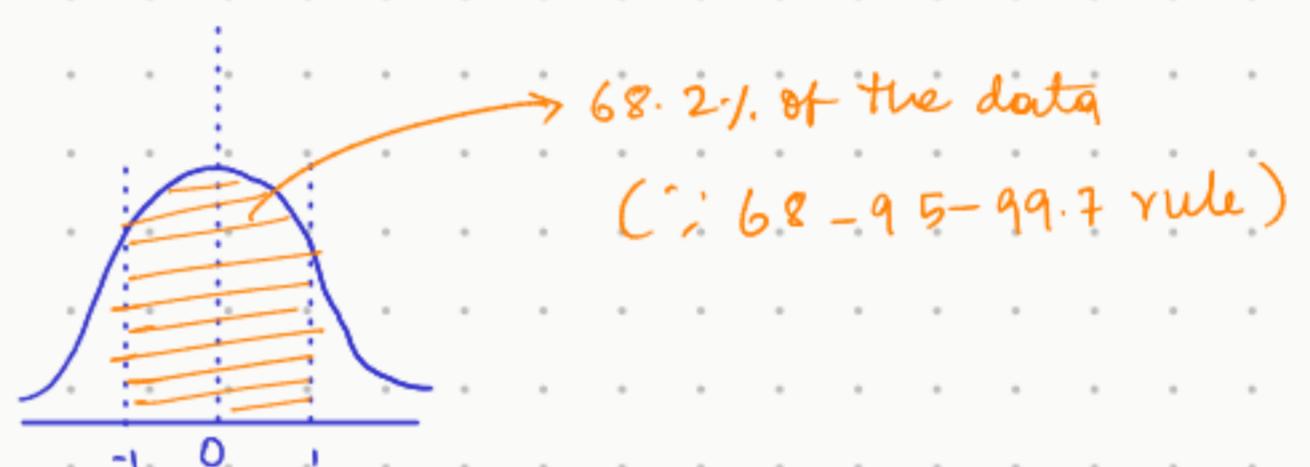
$$z = \frac{x - \mu}{\sigma}$$

$$\Rightarrow Z \sim N(0, 1)$$

why? :-

① After standardization, PDF becomes

This is also called as basic z-score formula. This basically tells us how many  $\sigma$  is  $x$  away from  $\mu$ .



② If we are comparing multiple GDS with different  $\mu$  &  $\sigma^2$ , doing this helps understand better & interpret better

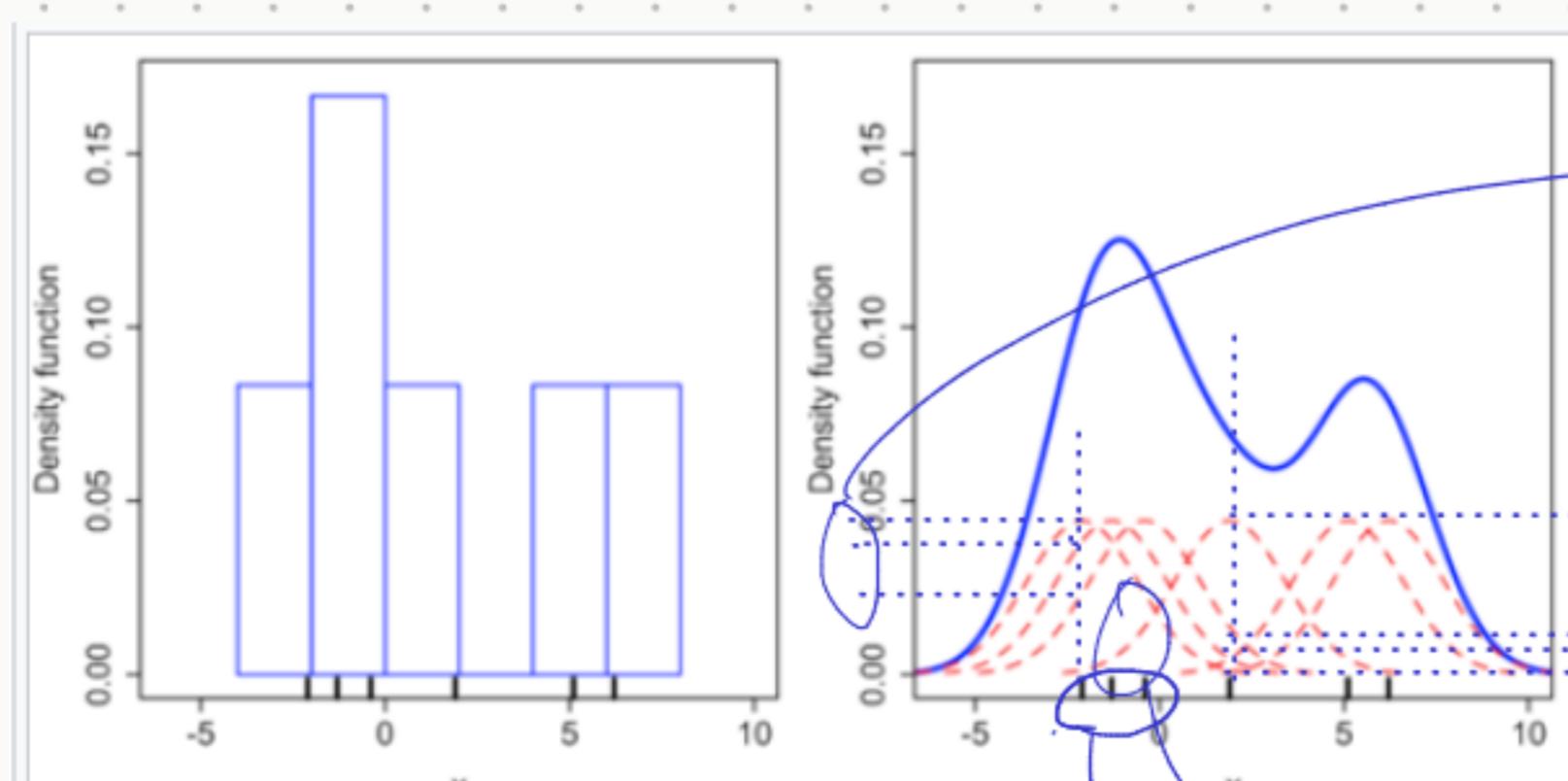
→ We use StandardScalar for standardization & MinMaxScaler for Normalization

Two techniques of feature scaling -

$$\text{Normalization} \rightarrow x_i' = \frac{x_i - x_{\min}}{(x_{\max} - x_{\min})}$$

## Kernel Density Estimation:-

→ Used for smoothing histograms to obtain PDFs



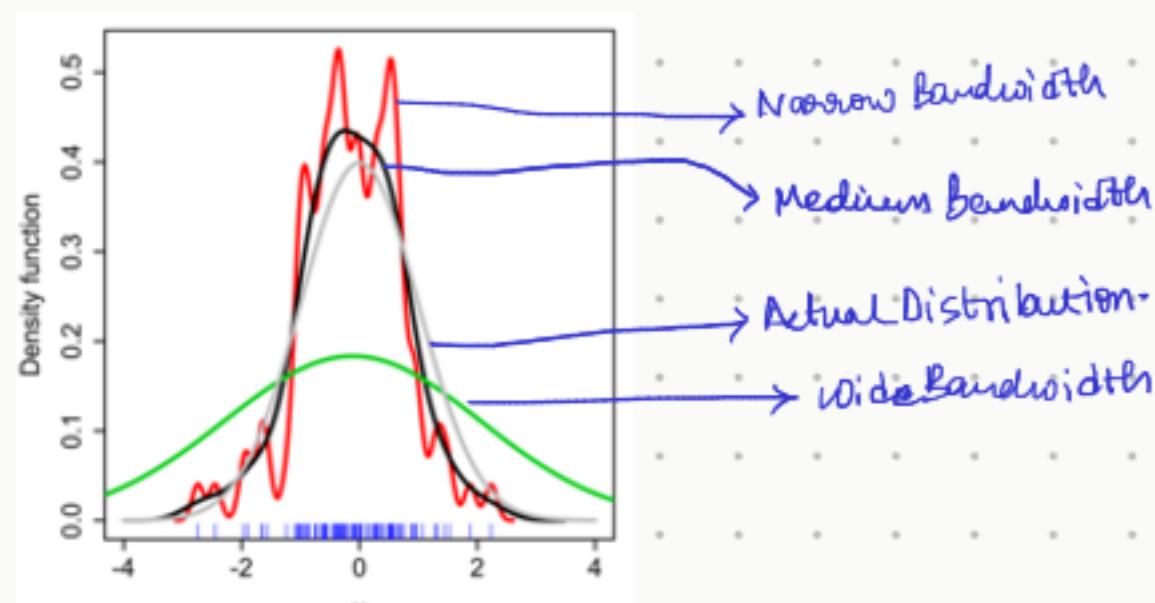
Comparison of the histogram (left) and kernel density estimate (right) constructed using the same data. The six individual kernels are the red dashed curves, the kernel density estimate the blue curves. The data points are the rug plot on the horizontal axis.

→ gives height at that point

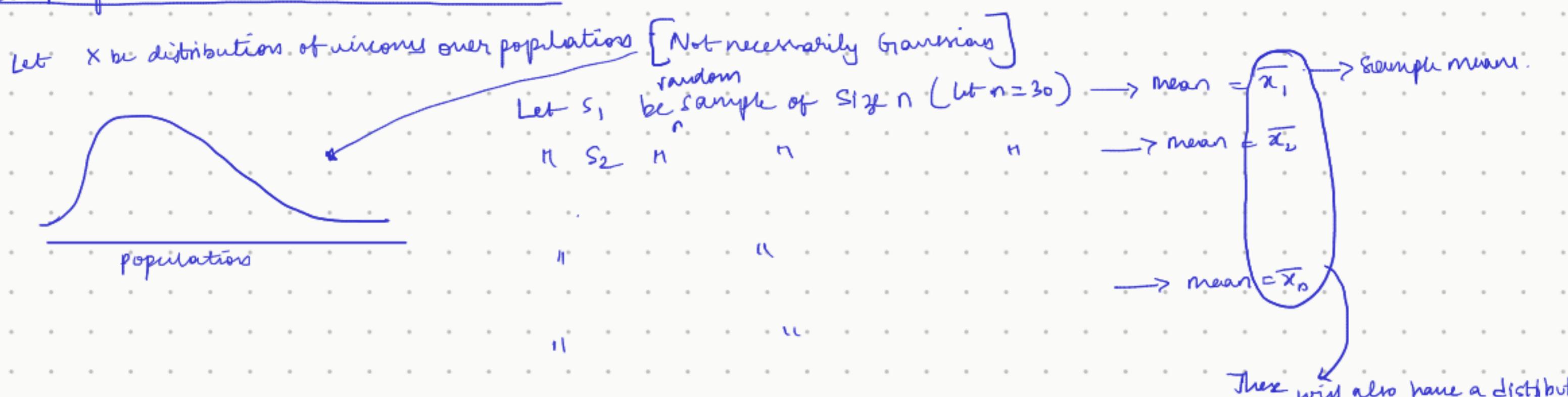
→ Variance of the gaussian kernel is known as bandwidths.

→ Gaussian Kernel -

→ Since there are many points here, the number of means will be high. So the PDF will be high. In natural histogram,



## Sampling Distributions & Central Limit Theorem



These will also have a distribution

The distribution  $\bar{x}_n$  = Sampling distribution of sample means.

Central Limit Theorem :- If original distributions 'X' has finite mean (there can be infinite mean ex:- parabola) & variance  $\sigma^2$  Samples are created of size 'n' where sample means are  $\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots$  whose distribution is called sampling distribution of sampling mean, central limit theorem states that

$$\bar{x}_n \rightarrow N(\mu, \frac{\sigma^2}{n}) \text{ as } n \rightarrow \infty$$

(But IRL if  $n \geq 30$ , then it becomes gaussian. Rule of thumb)

mean is same as original or that of distribution

Why? → By using just  $m \times n$  datapoints, we are able to estimate  $\mu$  &  $\sigma^2$  of any distribution, if we just know that they are finite.

## Quantile Quantile Plot (Q-Q-Plot) :-

Let  $X$  be a random variable

$$X: x_1, x_2, x_3, \dots, x_{500}$$

Question :- Is  $X$  Gaussian Distributed. Q-Q-Plot helps us identify other techniques such as statistical testing (KS testing etc) also exist.

graphical method

more powerful

How? :- ① Sort  $X$  & calculate percentiles.

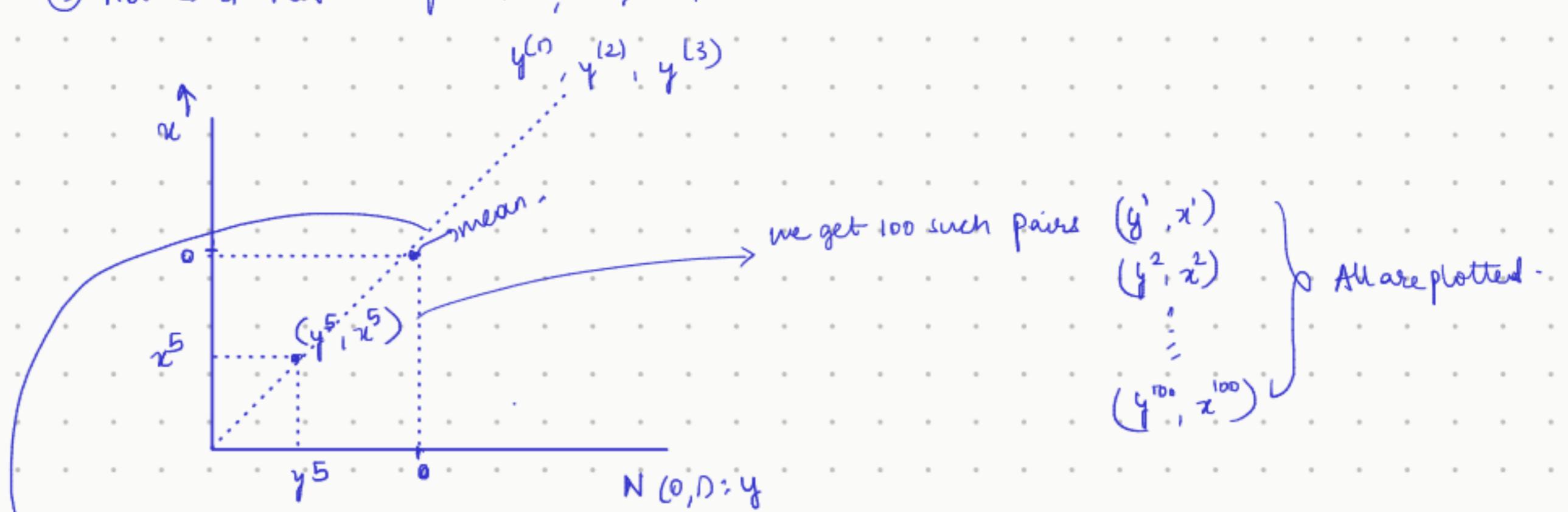
$$\begin{aligned} x'_1, x'_2, x'_3, \dots, x'_{500} \\ x'_1 \rightarrow 1^{st} \text{ percentile} \rightarrow x^{(1)} \\ x'_{10} \rightarrow 10^{th} \text{ percentile} \rightarrow x^{(2)} \\ \vdots \\ x'_{500} \rightarrow 100^{th} \text{ percentile} \rightarrow x^{(100)} \end{aligned}$$

② take  $Y \sim N(0, 1)$

$$\begin{aligned} Y: y_1, y_2, y_3, \dots, y_{1000} \\ \downarrow \\ y'_1, y'_2, y'_3, \dots, y'_{1000} \\ \downarrow \\ y^{(1)}, y^{(2)}, y^{(3)}, \dots, y^{(100)} \end{aligned}$$

These are called as Theoretical Quantiles

③ Plot Q-Q-Plot using  $x^{(1)}, x^{(2)}, x^{(3)}, \dots$



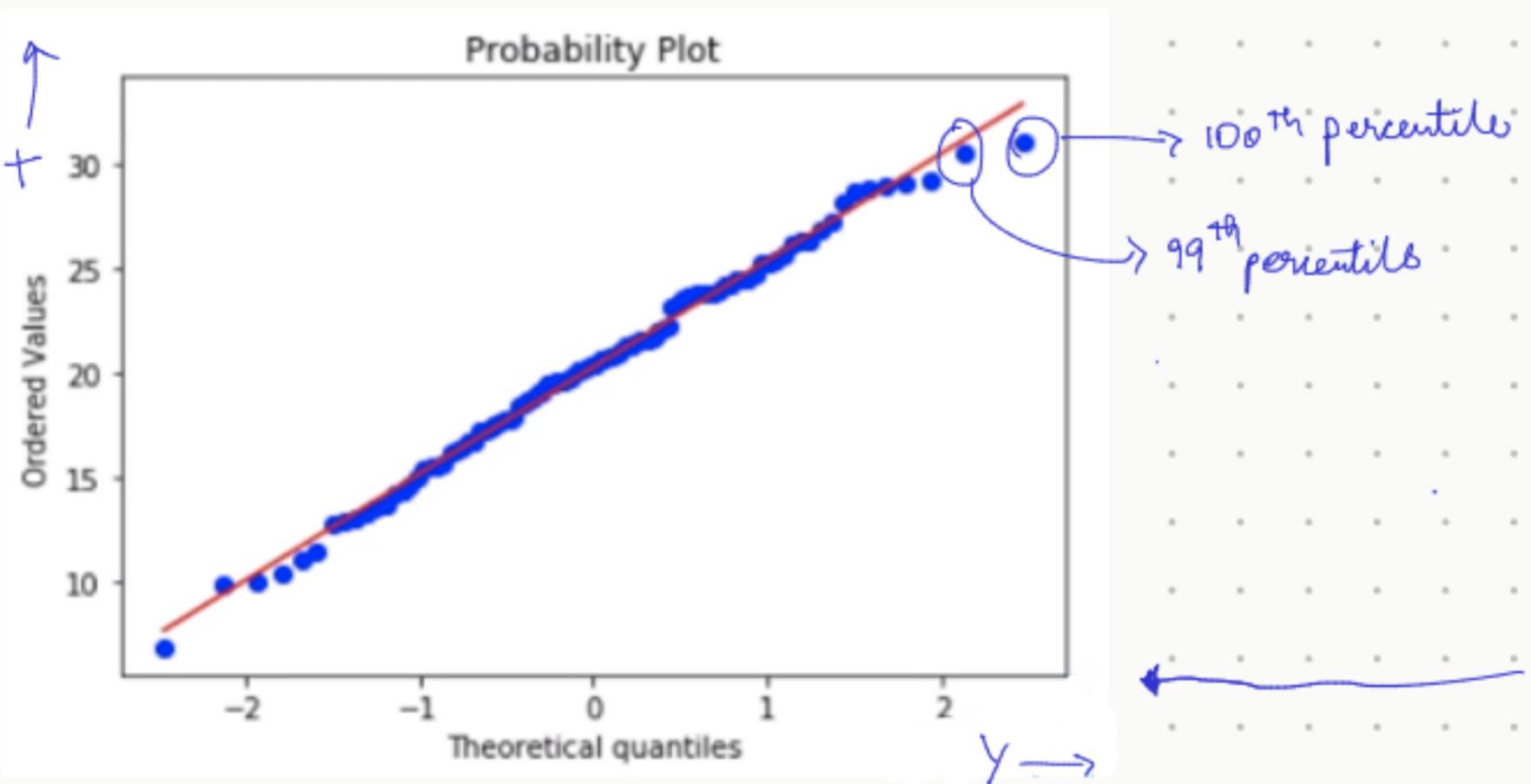
If  $(y^i, x^i)$  for  $i \geq 100$ , lie roughly on a straight line, then  $y \sim x$  have similar distributions.

$\Rightarrow x$  also has gaussian distribution.

→ stats.probplot()

→ If size of  $x$  is small, it is hard to interpret Q-Q plot. Won't be a straight line.

→ Another use of Q-Q plot is: given  $X, Y$ , does  $X \sim Y$  have the same distribution?



Generating  $Y$  :- (theoretical Quantiles)

```
#Q-Q plot
import numpy as np
import pylab
import scipy.stats as stats
# N(0,1)
std_normal = np.random.normal(loc=0, scale=1, size=1000)
```

mean std-dev

Generating  $X$  :-

```
# generate 100 samples from N(20,5)
measurements = np.random.normal(loc=20, scale=5, size=100)
#try size=1000
```

$\mu$

```
stats.probplot(measurements, dist="norm", plot=pylab)
pylab.show()
```

How & where to use distributions:-

→ All probability concepts are used for EDA most of the time.

→ If data is gaussian Distributed, many assumptions can be made by plotting PDF & CDF

Margin of error - How much deviation from original numbers is allowed in samples.

Chebyshew's Inequality :-

→ If we know that data is gaussian Distributed, we can apply 68-95-99.7 rule.

→ But if we don't know the distribution, but we know  $\mu, \sigma$  can we make assumptions like this?

$\uparrow$  (finite)  
 $\uparrow$  (nonzero & finite)

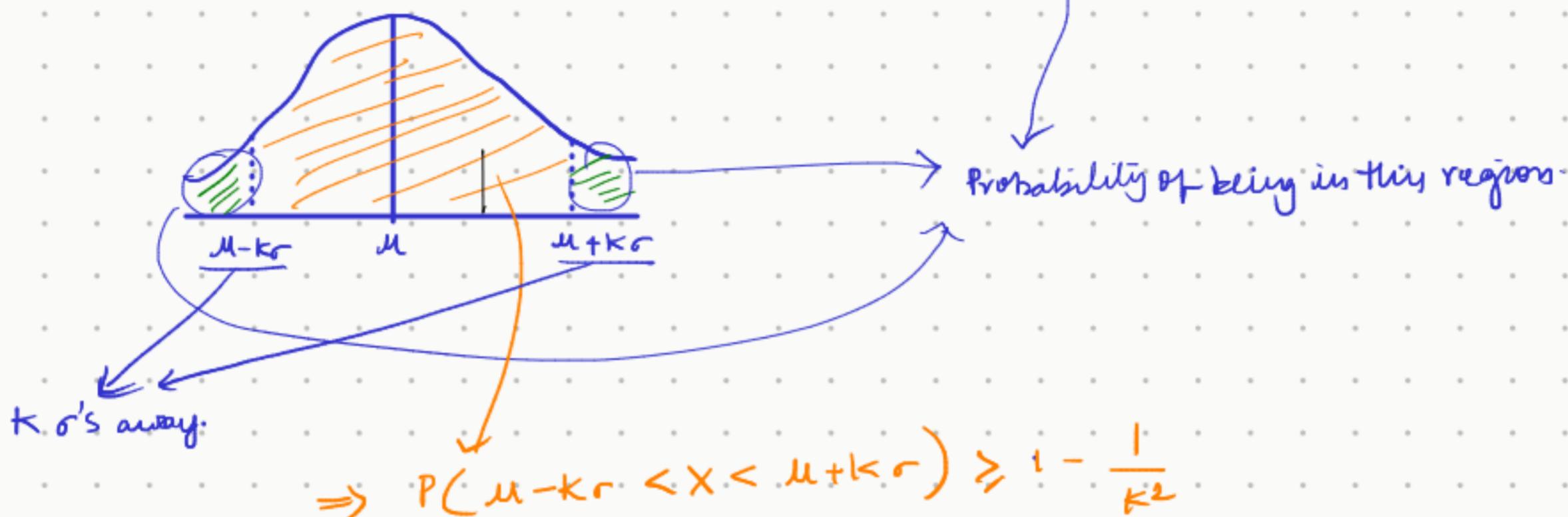
→ Chebyshew's inequality states that

if  $X$  is a random variable with finite mean  $\mu$  and non-zero & finite  $\sigma$

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

$k$  std deviations

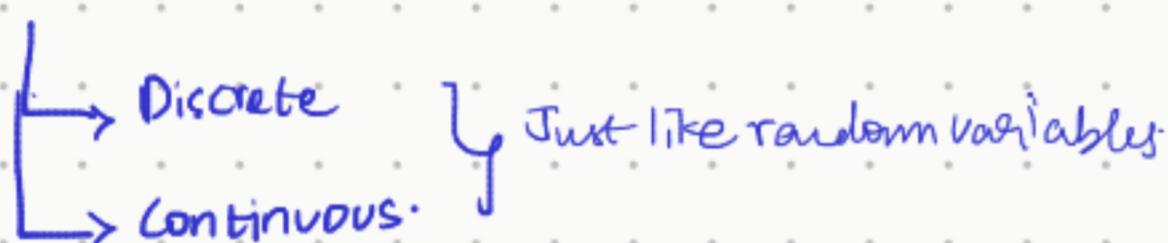
- 68 → 1 std dev away
- 95 → 2 std dev away
- 99.7 → 3 std dev away



→ From Chebyshew's inequality we can conclude that 75% of any distribution lies within  $(\mu - 2\sigma)$  &  $(\mu + 2\sigma)$  and 90% within  $(\mu - 3\sigma)$  and  $(\mu + 3\sigma)$

→ In case the range is not symmetric, there are asymmetric variations of Chebyshew as well.

## Uniform Distributions



PDF (Probability Density Functions) are drawn for continuous random variables.

PMF (Probability Mass Functions) are drawn for discrete random variables.

→ Probability of getting a value in dice is same for all values. This is called equiprobable.

→ CRV has  $N(\mu, \sigma)$  as parameters

DRV has  $(a, b)$  as parameters

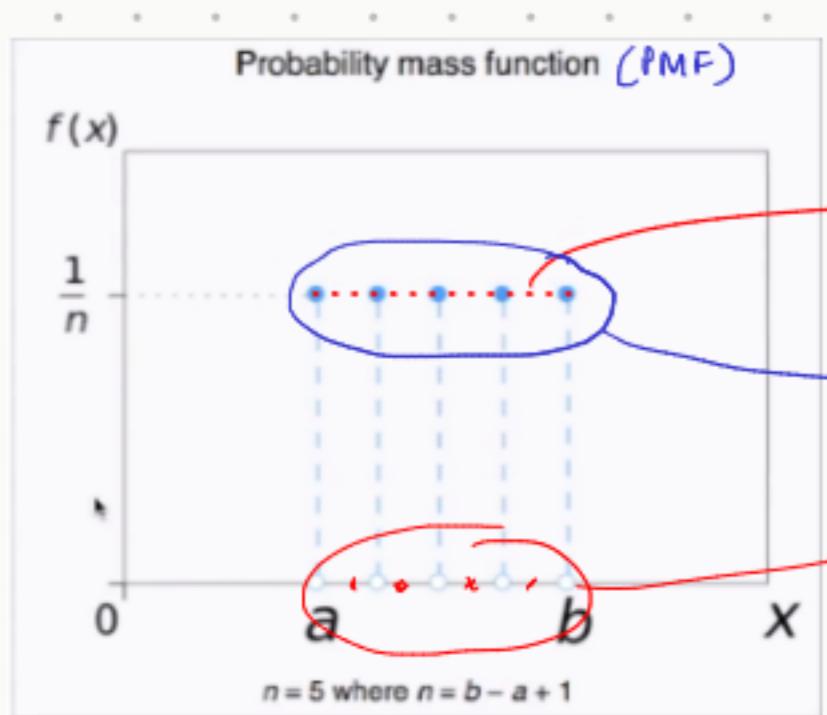
↳ both are numbers.

and

$n = b - a + 1$  . Another formula for calculating  $n = \frac{(b-a+1)}{k}$  where  $k$  is the common difference.

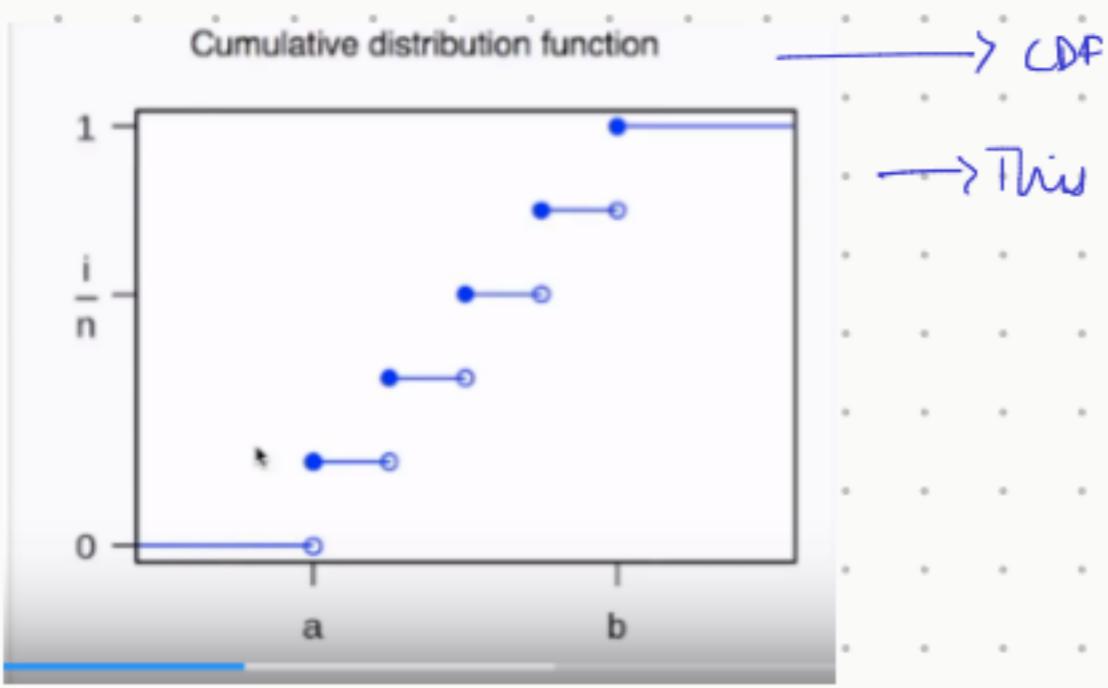
↳ number of outcomes.

→ In uniform distributions all the variables are equiprobable. i.e.,  $\frac{1}{n}$



This line can't be drawn because these points don't exist as uniform distribution

∴ Only these does exist in PMF



→ This is called a non smooth function

Props of URV :-

$$\text{Mean} = \frac{a+b}{2}$$

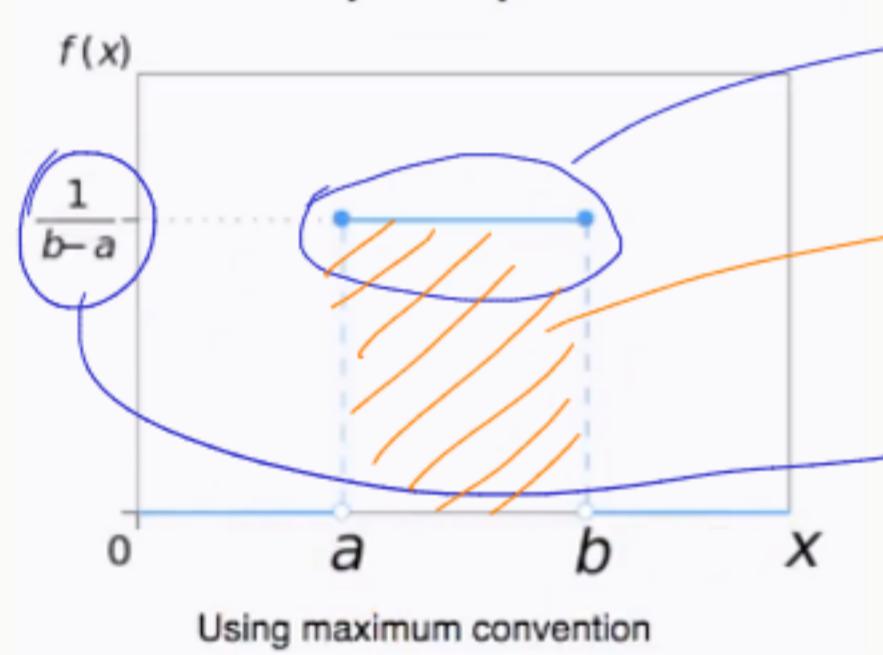
$$\text{Median} = \frac{a+b}{2}$$

$$\text{Variance} = \frac{(b-a+1)^2 - 1}{12}$$

$$\text{Skewness} = 0$$

Continuous Random Variable :-

Probability density function

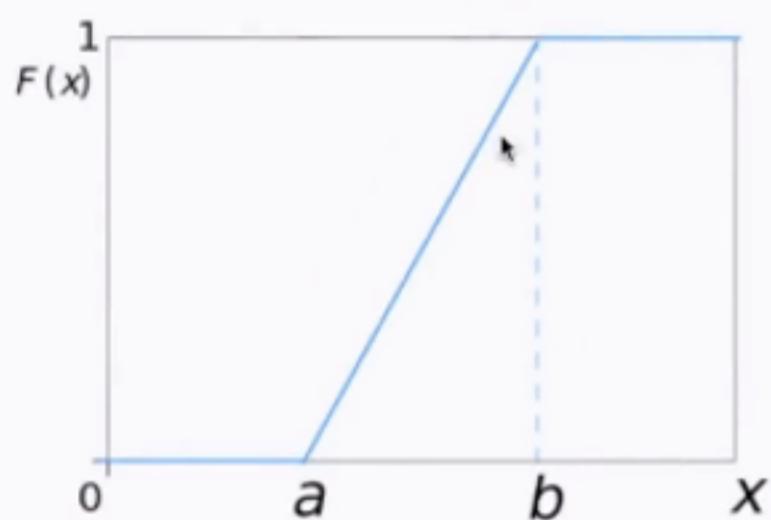


→ Line can exist here because it's continuous.

→  $\frac{1}{b-a}$  because this area has to be 1. The length is  $b-a$ . So the height is  $\frac{1}{b-a}$ .

CDF

Cumulative distribution function



Properties of CURV :-

Parms:  $a, b$

$$\text{PDF} = \begin{cases} \frac{1}{b-a} & \text{for } x \in [a, b] \\ 0 & \text{for everything else.} \end{cases}$$

$$\text{Mean} = \frac{a+b}{2}$$

$$\text{Median} = \frac{a+b}{2}$$

$$\text{Variance} = \frac{(b-a)^2}{12}$$

PDF → Density of data at the point

PMF → Probability of finding that point there

Random Number generators :-

→ Most random number generators generate uniform random variables unless explicitly specified.

→ Python's `random.random()` picks a number b/w 0 & 1 with uniform probability.

on:-

→ Picking 30 random values from iris

```

n=150 // Length of iris
m=30 // Sample size
p = m/n // 0.2
sample_data = []
for i in range(0, n):
    if random.random() <= p:
        sample_data.append(iris[i, :])
len(sample_data)

```

$\text{random.random}()$  picks value b/w  $1 \leq n$  with equal probability.  
 chances of getting val  $\leq 0.2 = 20\%$ .  
 $30 \text{ or } 150 = 20\%$ .

$\downarrow \approx 30$

→ An application of continuous uniform random variables.

### Bernoulli & Binomial Distributions:-

→ Discrete Distributions

→ Bernoulli has only two outcomes. Value 1 has probability  $p$  & value 0 has probability of  $1-p$

ex:-

$$X \sim \text{Bernoulli}(p=0.5)$$

$$\text{PMF} = \begin{cases} q = (1-p) & \text{for } k=0 \\ p & \text{for } k=1 \end{cases}$$

$$\text{Mean} = p$$

$$\text{Variance} = pq$$

### Binomial Random Variable:-

Let  $X$  be taking a coin

$$X \sim \text{Bernoulli}(p=0.5)$$

Let  $Y = \{\text{Number of heads when coin is tossed } n \text{ times (let } n=30)\}$

$$Y \in \{0, 1, 2, 3, \dots, 30\}$$

$Y \sim \text{Binomial}(n, p)$

$n$  → number of trials  
 $p$  → probability of value 1

### Parameters :-

$$\text{PMF} = \binom{n}{k} p^k (1-p)^{n-k} \text{ for } P(Y=k)$$

Not used often in Machine Learning.

→ If in Binomial distribution,  $n=1 \Rightarrow$  it's a Bernoulli distribution.

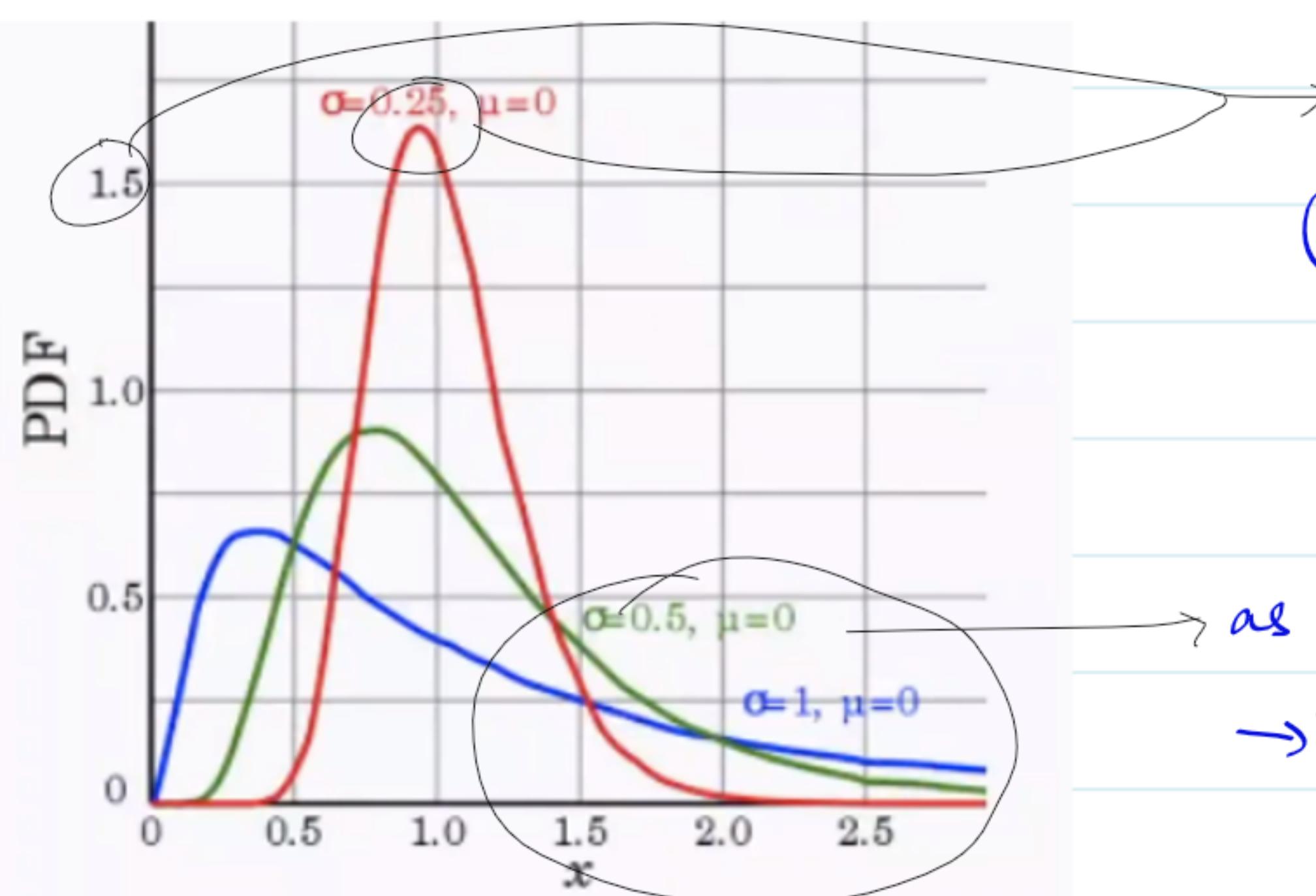
→ Bernoulli is unbiased.

→ Haberman's dataset is not Bernoulli Dist as Attributes of Haberman are continuous random variable & Bernoulli, Binomial is performed on discrete random variable having fixed outcomes.

## Log Normal Distribution :-

$X \sim \text{Log-Normal}(\mu, \sigma)$  if and only if  $\log(X)$  is normally distributed.

↳ Natural log aka ln



The PDF here is  $> 1$ , but its integral, the CDF will be lower than 1 because the regions is narrow  
(The PDF can be  $> 1$  but it can't be  $> 1$  for bigger range)

as  $\sigma$  increases, curve becomes more skewed.

→ For log-normal, the mean is not where the curve peaks.

Some log-normal density functions with identical parameter  $\mu$  but

paper 480p 360p 240p

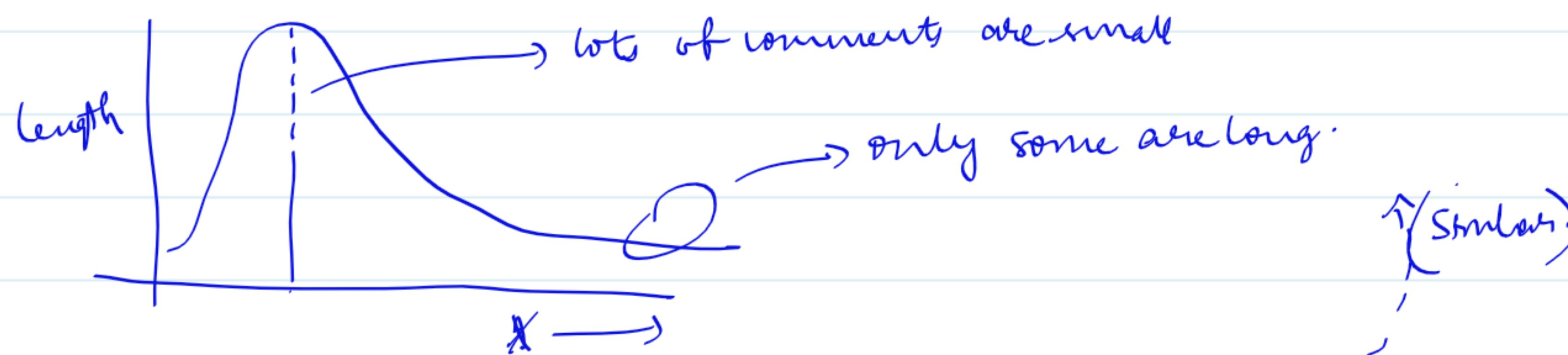
$$\text{PDF} = \frac{1}{\pi \sigma \sqrt{2\pi}} e^{-\frac{(\ln(x-\mu))^2}{2\sigma^2}}$$

$$\text{Skewness} = (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$$

Not 0 like seen in Normal Distribution.

## Applications :-

→ length of comments in internet forum posts follow log-normal distribution.



→ User's dwell time on online articles follows a log-normal distribution.

→ In economics, 97-99% population's income follows log-normal. Distribution of higher income follows Pareto distribution.

→ If we are given a log-normal distribution, we can easily find its normal distribution & apply ML models.

Checking  $X \sim \text{Log-Normal}(\mu, \sigma)$  or not :-

$$x_1, x_2, x_3, x_4, \dots, x_n$$

↓      ↓      ↓      ↓

$$\ln(x_1), \ln(x_2), \dots, \ln(x_n)$$

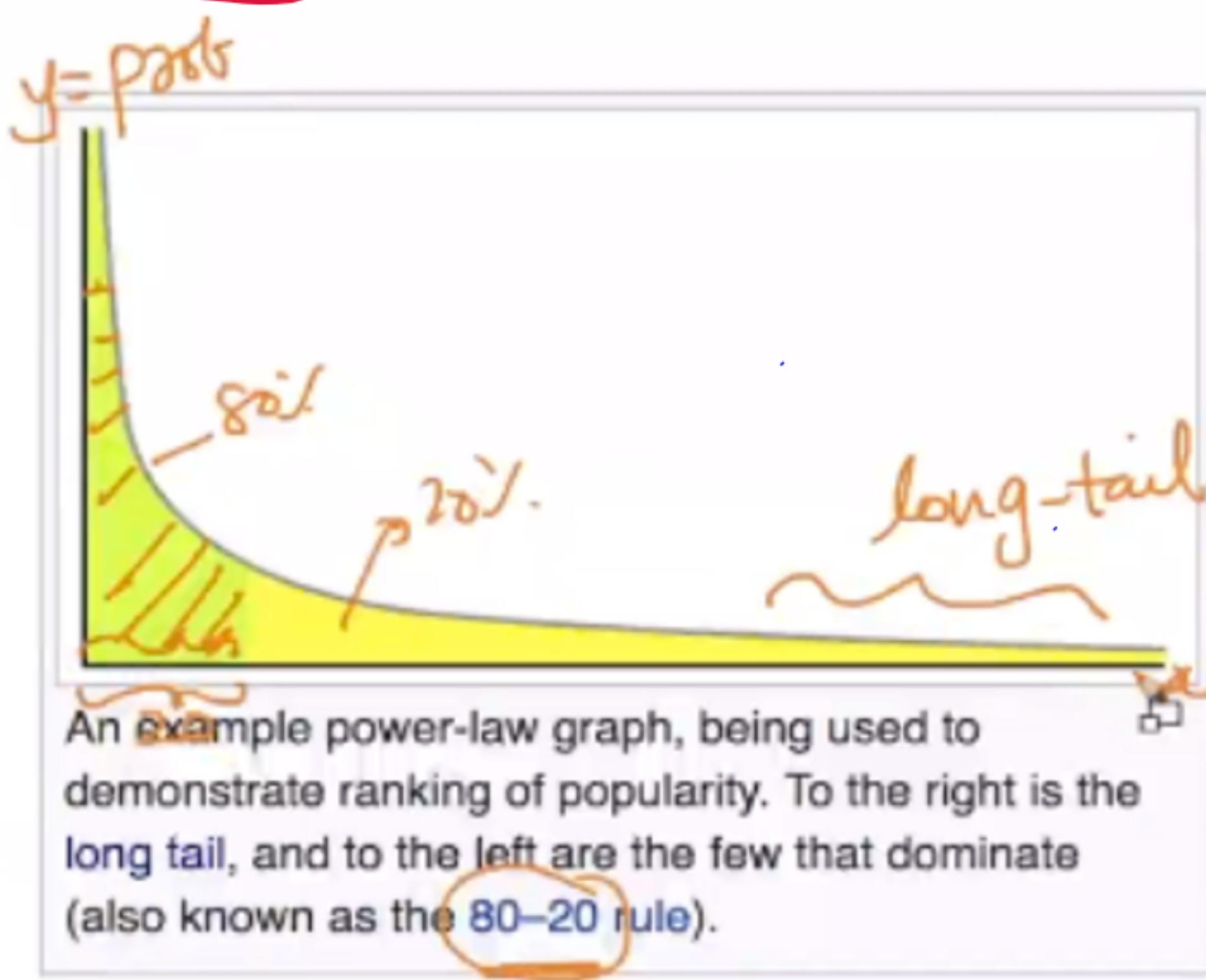
↓      ↓      ↓

$$(y_1), (y_2), \dots, (y_n)$$

→ Q-Q - plot → check if it's gaussian.

→ Log normal more common than normal distributions IRL.

## Power-Law :-



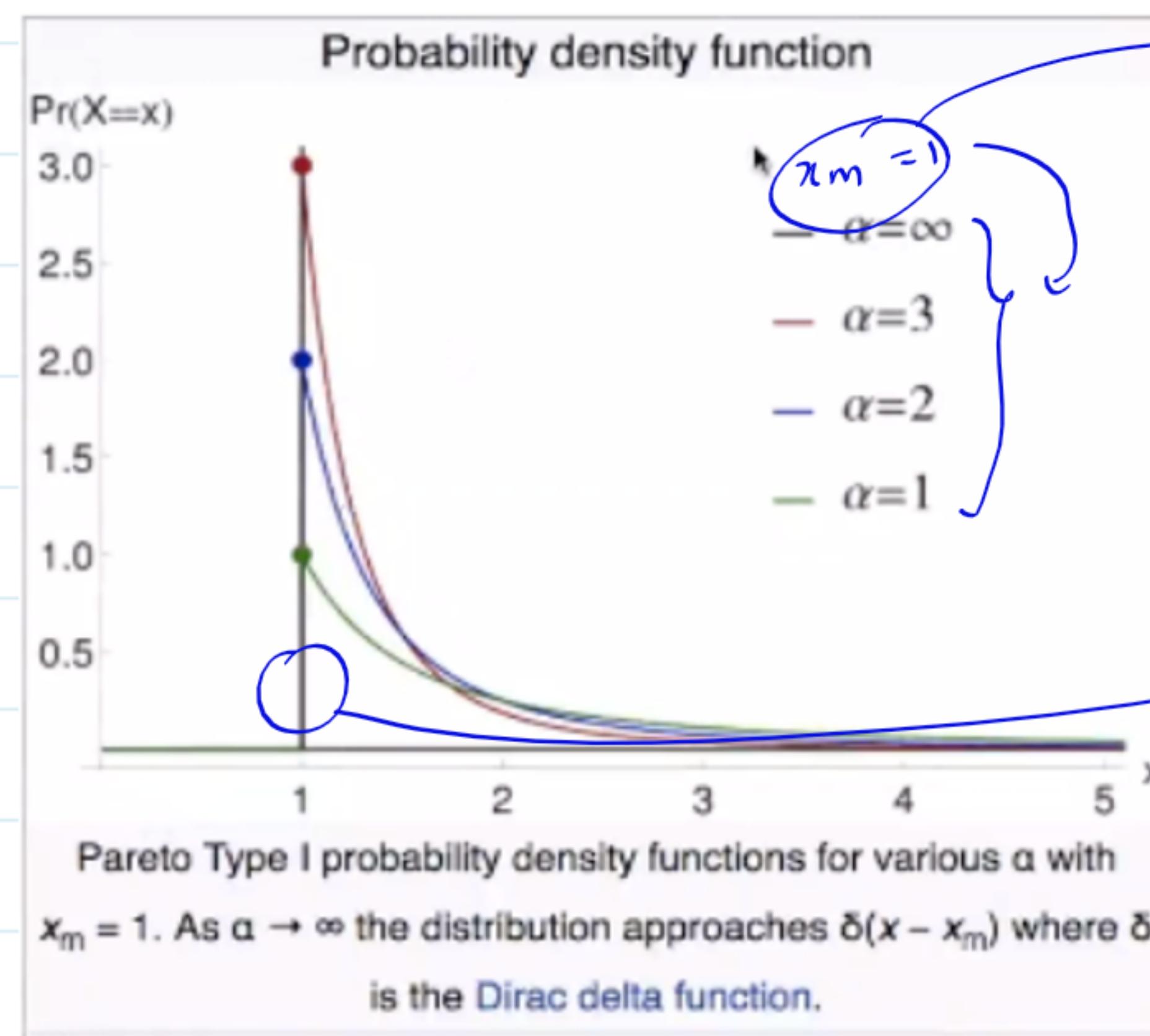
If distribution follows power-law, it's called as a Pareto distribution.

Ocurs a lot in nature.

Parameters of pareto distribution:-

$x_m > 0$  (scale)  $\rightarrow (\mu)$

$\alpha > 0$  (shape)  $\rightarrow (\sigma)$



peak

as  $\alpha \downarrow \rightarrow$  tail size  $\uparrow$

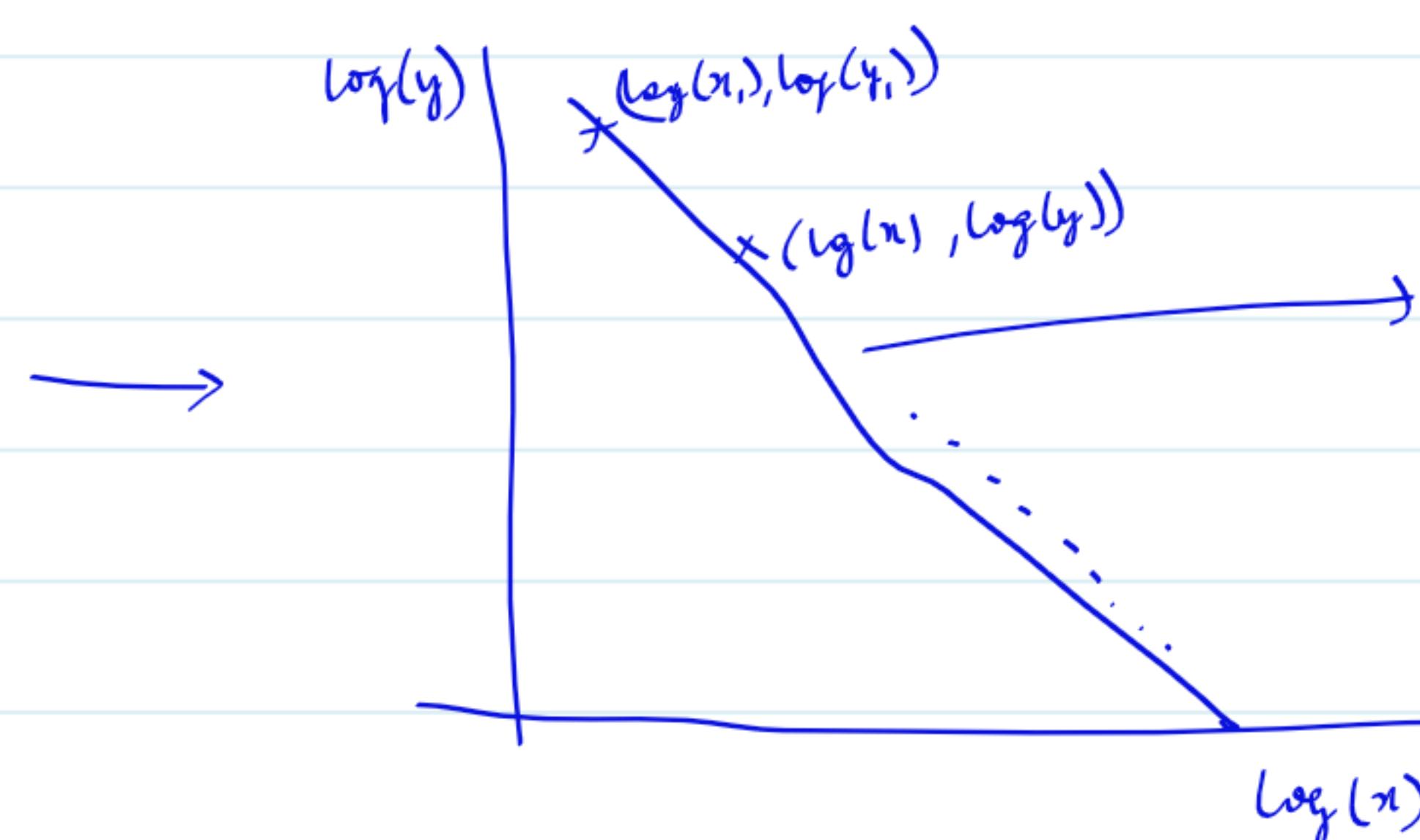
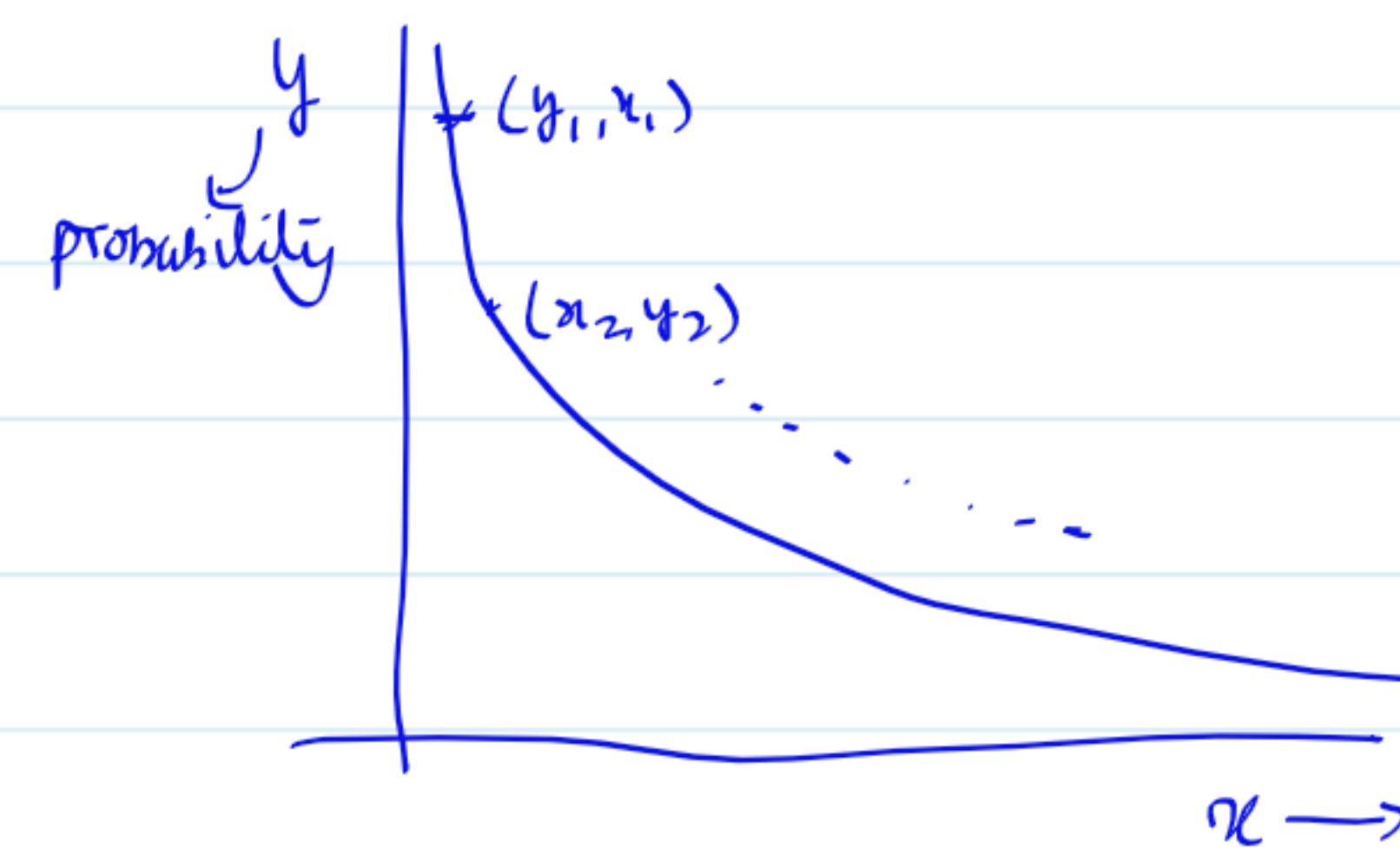
$\alpha = \infty$  (Dirac Delta function)

## Applications :-

- size of human settlements (villages, cities etc.)
- Filesize distribution of Internet
- HTTP error rates

How to check for pareto/power law? :-

→ We use log-log plots.

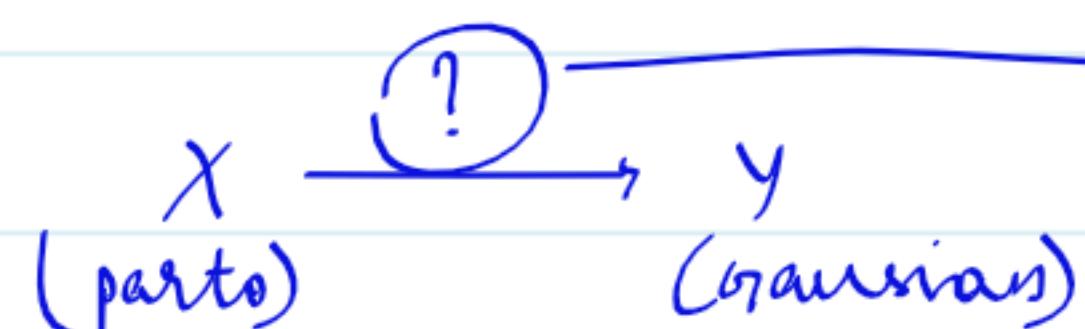


→ To check for pareto, we do log-log plot with power & observation.

## Box Cox Transform:-

In ML we sometimes make assumptions that data is Gaussian Normal Distributed.

We use Power transform / Box Cox transform for this conversion.



$x \rightarrow \text{Pareto} \rightarrow [x_1, x_2, x_3, \dots, x_n] \rightarrow x$

$y \rightarrow \text{Normal} \rightarrow [y_1, y_2, y_3, \dots, y_n] \rightarrow y$

Steps:-

①  $\text{boxcox}(x) \rightarrow \text{return } 'lambda'$  [ $\lambda$  computation is complete and no need to memoize]

②  $y_i = \begin{cases} \frac{x_i - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log(x_i) & \text{if } \lambda = 0 \end{cases}$

$\forall i: 1 \rightarrow n$

$\Rightarrow x_i \text{ is log-normal}$

How?

`scipy.stats.boxcox(x)` → returns boxcox power transformed array and  $\lambda$  value.

Note:- Boxcox transform is not guaranteed to work on all pareto/powerlaw distributions. It only works on some of them. We need to apply boxcox & observe QQ plot to be certain.

### Applications of Gaussian Distributions :-

→ useful in generating random numbers.

→ There are 100s of distributions.

Why are there so many? :- Any **studied** distribution gives a theoretical model on how a random variable behaves. There are 100s of distributions.

### Most commonly used distributions :-

① Bernoulli

② Uniform Distribution

③ Binomial Distributions

④ Normal Distributions

⑤ Poisson Distribution

⑥ Exponential Distribution

### Co-Variance :-

If there are two datasets that are related to find the relationship b/w these, we have 3 types of measurements

① Covariance    ② Pearson Correlation Coefficient    ③ Spearman Rank Correlation Coefficient

ex:- students height & weight

	h	w
s <sub>1</sub>	150.	54
s <sub>2</sub>	162	72
s <sub>3</sub>	135	42
s <sub>4</sub>	180	80

is there a relationship b/w height & weight?

$$\text{cov}(x, y) = \frac{1}{n} \sum (x_i - \mu_x)(y_i - \mu_y)$$

$$\left( \text{similar to } \text{var}(x) = \frac{1}{n} \sum_{i=1}^n (x_i - \mu_x)^2 \right)$$

$$\text{cov}(x, x) = \text{var}(x)$$

$\text{cov}(x, y) = \text{the if } x \uparrow \Rightarrow y \uparrow$  (reasoning)  
 $= -\text{ve if } x \uparrow \Rightarrow y \downarrow$

Drawbacks :-

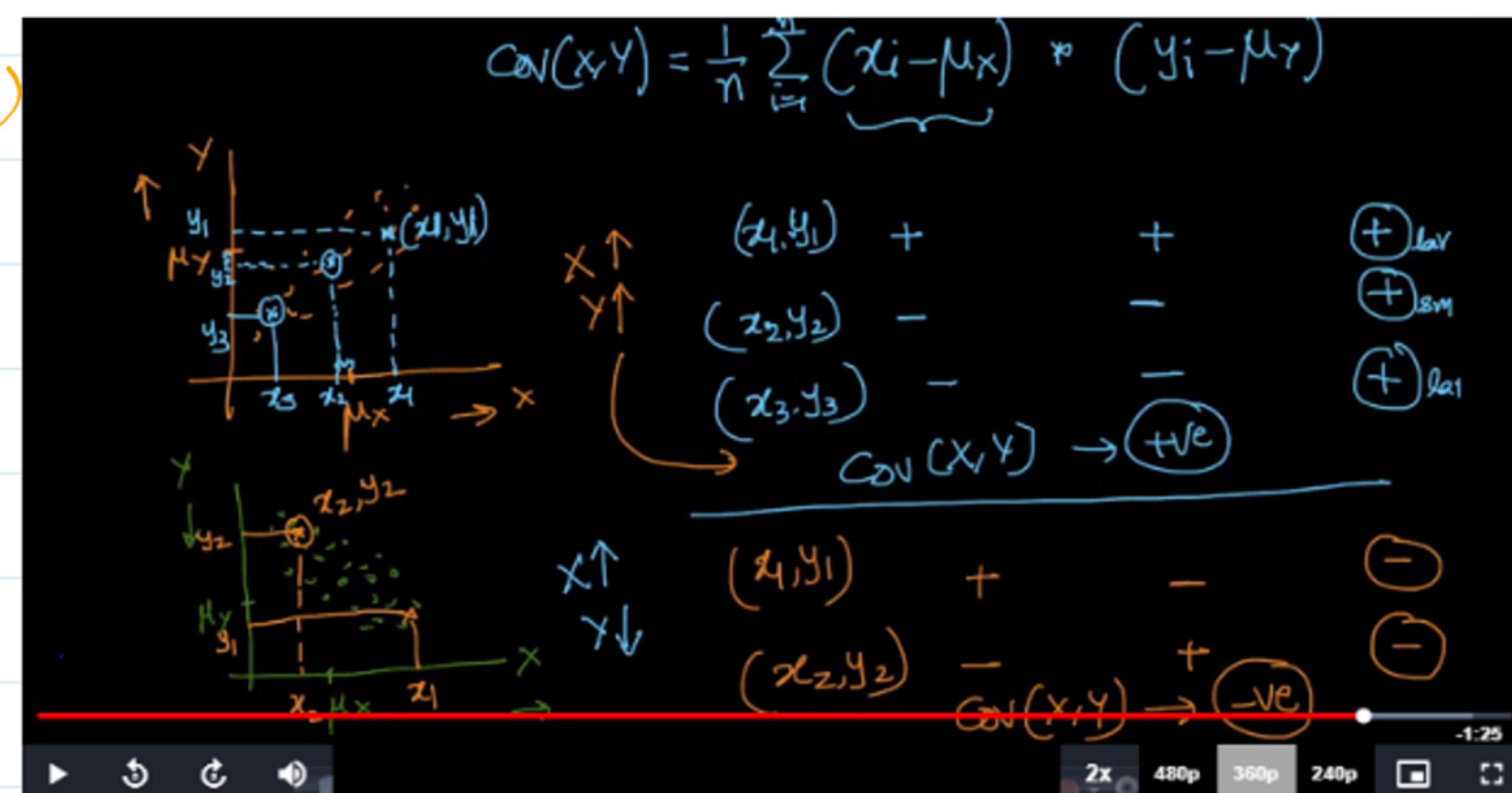
$$\text{cov}(\text{height, weight}) + \text{cov}(\text{height, weight})$$

(cm) (kg) (ft) (lbs)

same data but diff covariances

The sign will be the same

Magnitude of covariance is also important as it tells how strongly related two RVs are.



$$\rightarrow \text{Correlation} = \frac{\text{Covariance}(x, y)}{\sigma_x \sigma_y}$$

$\rightarrow$  Extreme outliers drastically affect covariance. So they need to be removed with the noise & other outliers.

If we replace mean in covariance formula with medians it's called robust covariance.

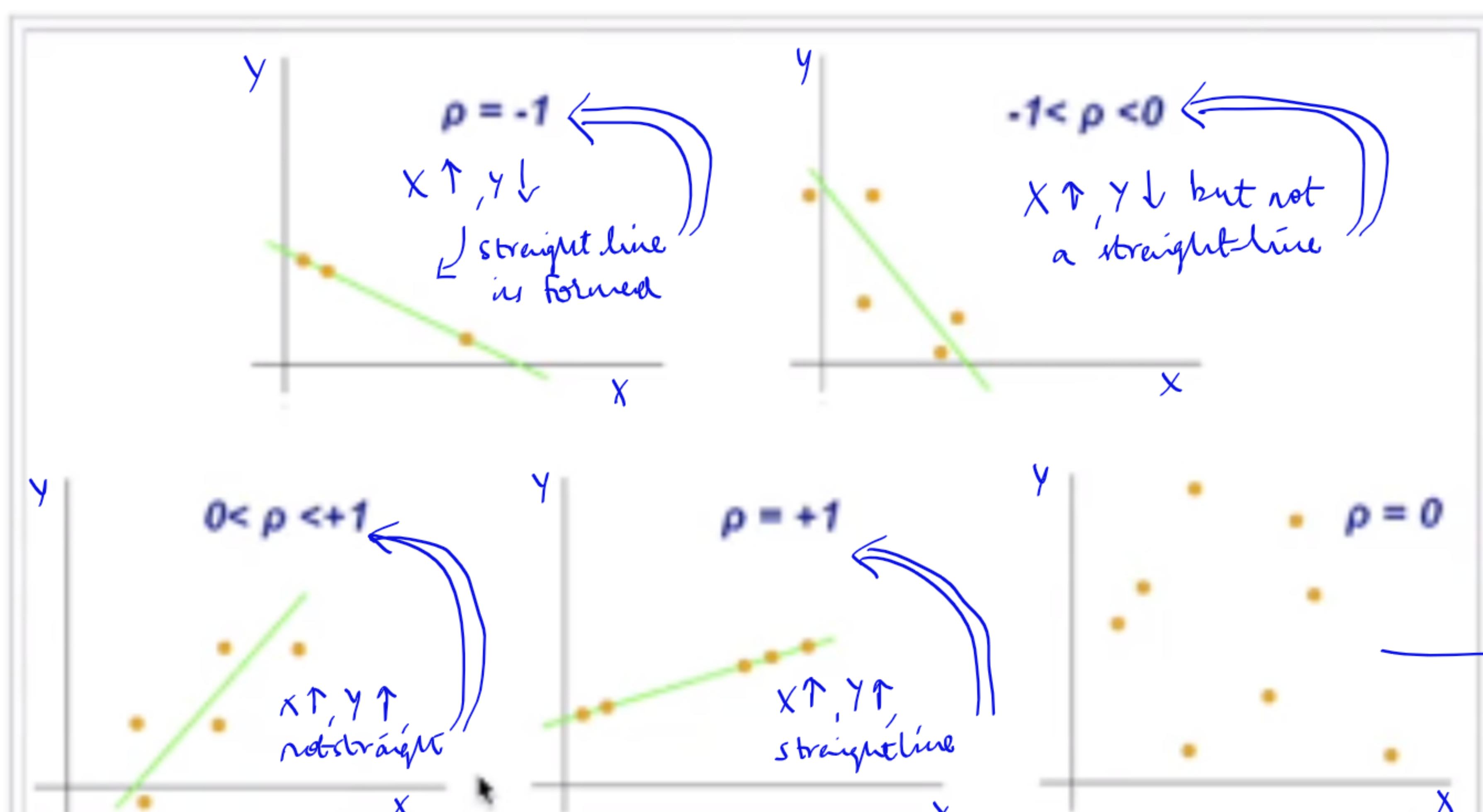
### Pearson Correlation Coefficient (PCC) :-

$$P_{x,y} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

$\rightarrow$  Cov does not take variability into considerations. This does.

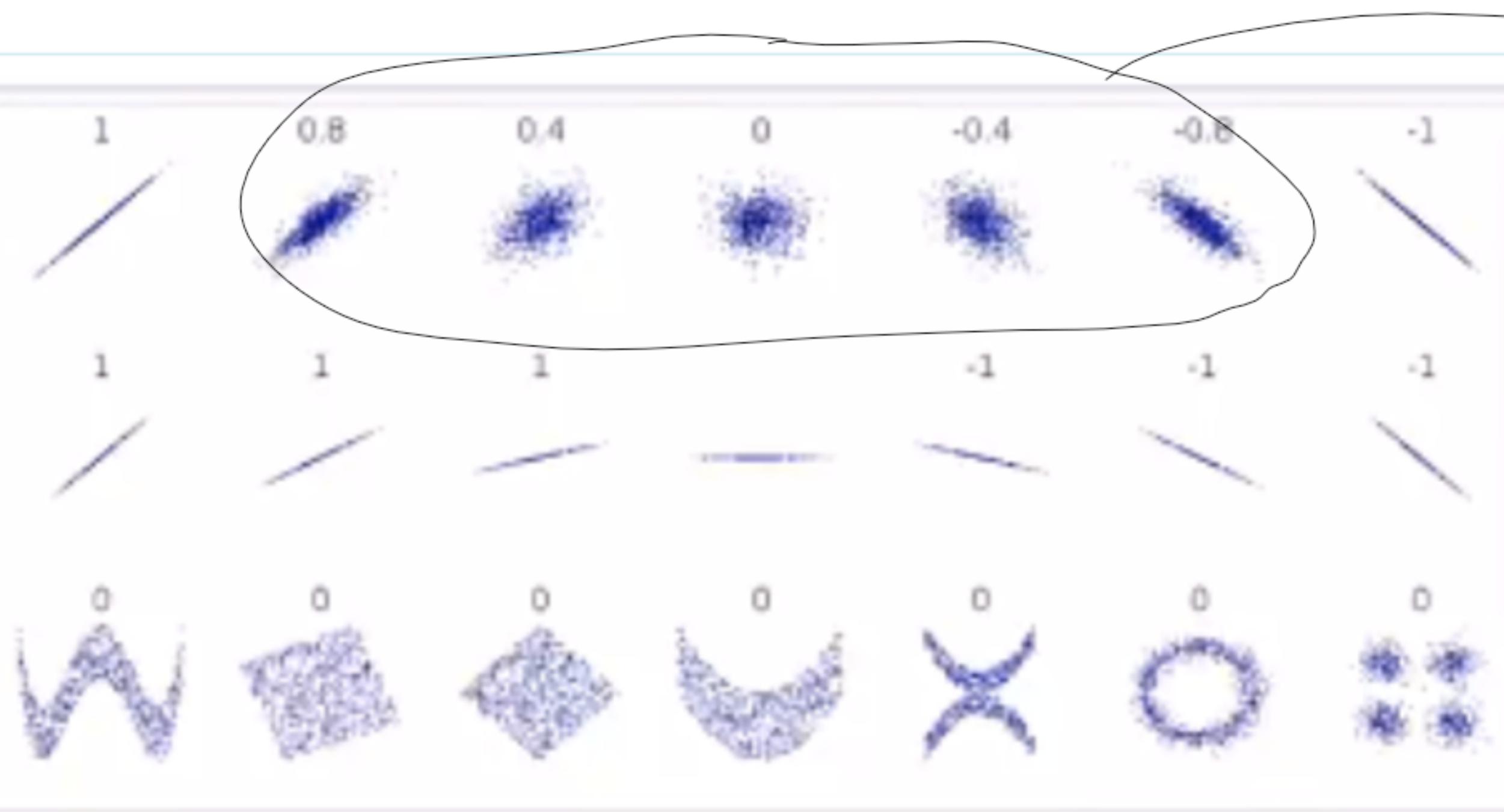
$$-1 \leq p \leq +1$$

$\rightarrow$  PCC is heavily biased towards linearly dependant variables -



Examples of scatter diagrams with different values of correlation coefficient (p)

No relations at all.  
 $\Rightarrow$  No line  
 $\Rightarrow p=0$ .



→ As it becomes flatter, value decreases.

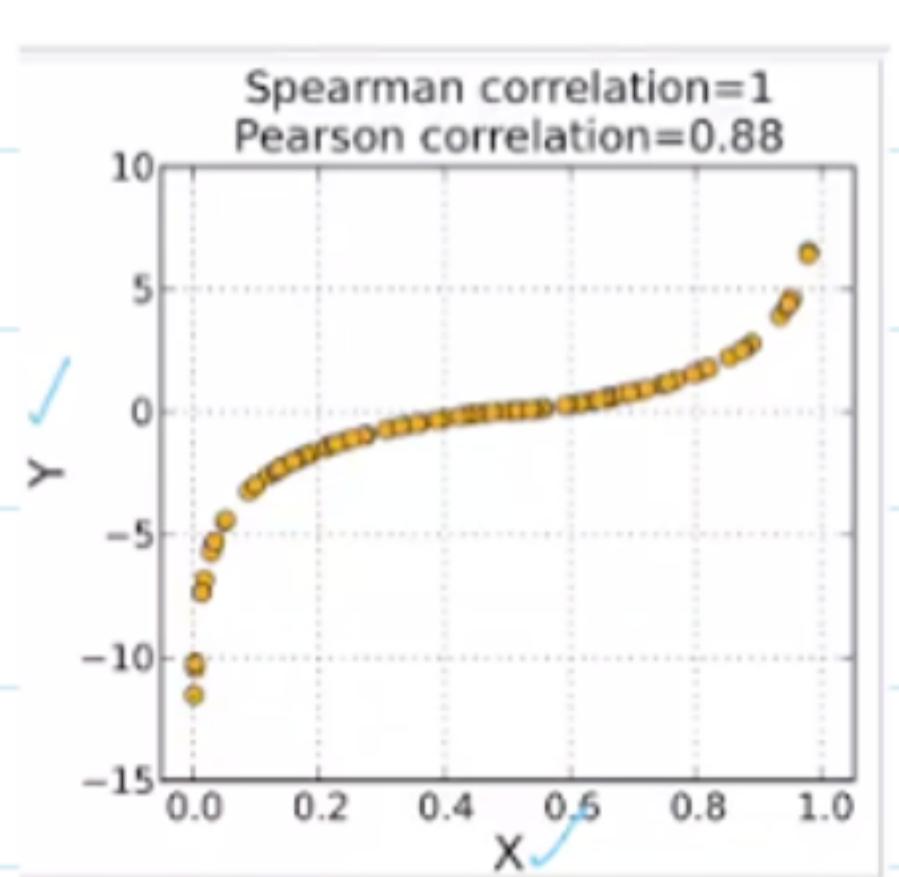
$\downarrow \rightarrow$  PCC does not care about the slope of the line.

$\downarrow \rightarrow$  Does not capture complex/non-linear relationships.

Monotonically non decreasing  $\rightarrow$  if  $x_2 > x_1$  and corresponding  $y_2 \geq y_1$

monotonically increasing  $\rightarrow$  if  $x_2 > x_1$  and corresponding  $y_2 > y_1$

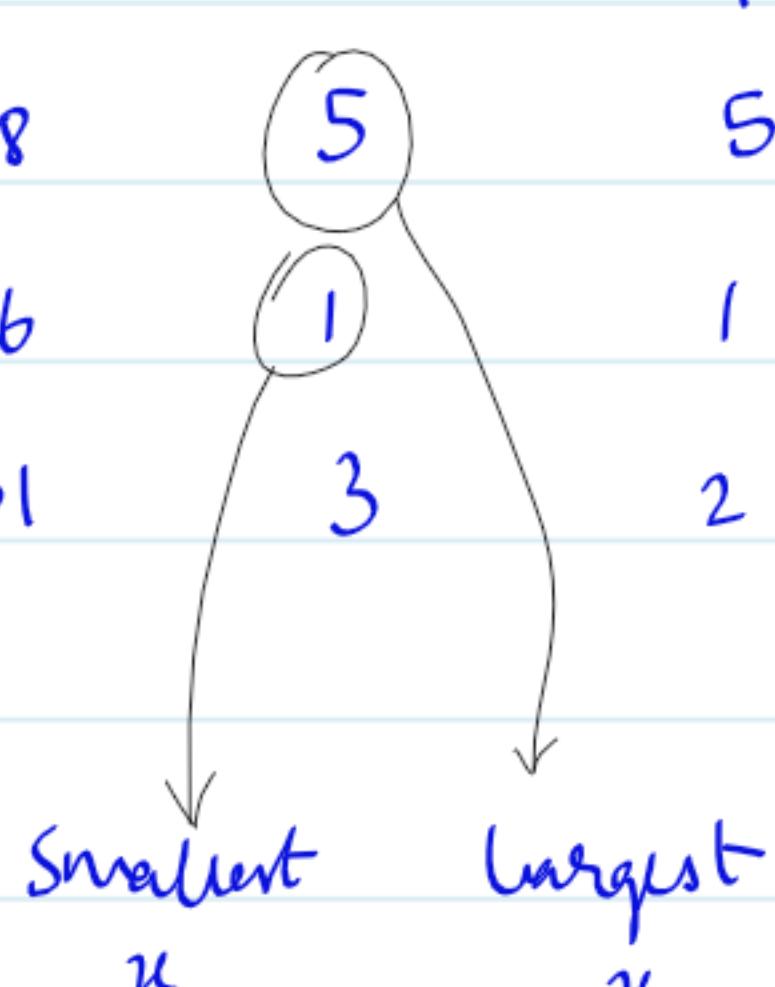
→ Correlation is what defines the change in one item when another item is modified.



### Spearman Correlation Coefficient:-

→ Useful when we have slightly non-linear relationships.

ex:-	X	Y	$r_x$	$r_y$
$s_1$	160	52	4	3
$s_2$	150	66	2	4
$s_3$	170	68	5	5
$s_4$	140	46	1	1
$s_5$	158	51	3	2



During rank calculation, if two vals have same value, then their mean is taken as rank. Fractional rank (0.5)

Same rank depends on the implementation.

$$\text{Spearman coefficient} \quad r = P_{r_x, r_y} \quad \rightarrow \text{Ranks are strictly increasing.}$$

$r = 1 \rightarrow x \uparrow \quad y \uparrow \rightarrow r = 1 \quad \downarrow \rightarrow$  irrespective of whether they are linear or not.  
 $r = -1 \rightarrow x \uparrow \quad y \downarrow \rightarrow r = -1 \quad \downarrow \rightarrow$  As long as there is monotonicity, it will work.

→ Correlations does not imply causations. For causations there are things like causal models.

ex:-

