Computer Science and Engineering

Paper - 1

Time Allowed: 3 hours **Maximum Marks: 150**

Name:

Roll No.

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before answering the questions.

There are 33 questions in total.

Candidate has to attempt all the questions.

Marks carried by each question/part is indicated against it.

All parts of the question must be answered at one place.

Unless otherwise mentioned, symbols and notations have usual standard meanings.

Answer must be written in *English* only.

Candidate must write the exam on the assumption that the questions are correct.

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1. (3 points) Compute $[MM^T]^{-1}$ for an orthogonal matrix where

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} & \frac{-2}{\sqrt{2}} \\ \frac{-2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} \end{bmatrix}.$$

2. (5 points) Calculate the eigenvalues of matrix M, M^{-1}, M^2 and $M + 2I$ where

$$M = \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}.$$

3. (5 points) With no unique solution, solve for n with the following system of equations

$$a + b + 2c = 3$$
$$a + 2b + 3c = 4$$
$$a + 4b + nc = 6$$

4. (5 points) Let the function $f(x)$ be defined as follows.

$$f(x) = \begin{cases} x^2 & x \leq 1 \\ 2ax^2 + bx + c & 1 < x \leq 2 \\ x + 2d & x > 1 \end{cases}$$

Find the values of a, b, c and d such that f is continuous and differentiable everywhere.

5. (3 points) Consider rolling of a dice experiment. Let A be the event of getting an even number and B be the event of getting a prime number. Write the sets representing the following events.

   (a) (1 point) A or B

   (b) (1 point) A and B

   (c) (1 point) not B

6. (7 points) Let $\oplus$ sign denote bitwise addition modulo 2. Let n and m be integers.

Consider the set of m equations on n variables as follows.

$$
\begin{aligned}
a_{1,1}x_1 \oplus a_{1,2}x_2 \oplus \ldots \oplus a_{1,n}x_n &= b_1 \\
a_{2,1}x_1 \oplus a_{2,2}x_2 \oplus \ldots \oplus a_{2,n}x_n &= b_2 \\
&\vdots \\
a_{i,1}x_1 \oplus a_{i,2}x_2 \oplus \ldots \oplus a_{i,n}x_n &= b_i \\
&\vdots \\
a_{m,1}x_1 \oplus a_{m,2}x_2 \oplus \ldots \oplus a_{m,n}x_n &= b_i
\end{aligned}
$$

where $a_{i,j}$'s (for $1 \leq i \leq m, 1 \leq j \leq n$, b_i's (for all $1 \leq i \leq m$, and x_k's (for all $1 \leq k \leq n$) take values in $\{0, 1\}$.

(a) (3.5 points) What is the probability of satisfying equation $a_{1,1}x_1 \oplus a_{1,2}x_2 \oplus \ldots \oplus a_{1,n}x_n = b_1$ if all x_i's are picked uniformly and independently at random from $\{0, 1\}$.

(b) (3.5 points) What is the expected number of equations that can be satisfied if x_i's are picked uniformly and independently at random.

7 (6 points) Let T be a subset of $\{1, 2, \ldots, n\}$ of size $\frac{n}{2}$, sampled uniformly at random from all subsets of size $\frac{n}{2}$ of the same set $\{1, 2, \ldots, n\}$. Let $S \subseteq \{1, 2, \ldots, n\}$ be an arbitrary subset of size $\frac{n}{2}$. What is the probability that $|S \cap T| = \frac{n}{4}$.

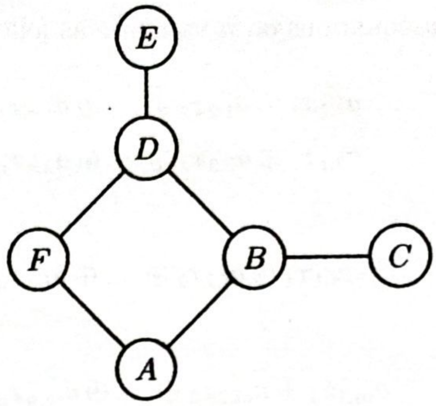
8. (4 points) Given a powerset S of $\{1, 2, 3\}$, its partial order $\leq$ is given by set inclusion. That is, for any subsets $T_1 \neq T_2$ of $\{1, 2, 3\}$ we have $T_1 \leq T_2$ if and only if $T_1 \subset T_2$. Construct the Hasse diagram on S under this partial order definition.

9. (4 points) Count the number of perfect matchings in the bipartite graph whose adjacency matrix A is as follows.

$$
\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.
$$

10. (4 points) Given a tree T and $\lambda \geq 2$ colours $(c_1, c_2, \ldots, c_\lambda)$, how many proper colourings of the tree T are possible?

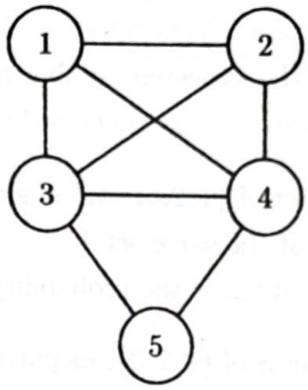
11. (5 points) Consider the following graph.



Find closeness centrality of 'A' node.

12. (2 points) Given two propositions P and Q. "if P then Q" denoted by $P \to Q$ is called Implication. Such implication and its ______ are logically equivalent.

13. (5 points) Consider the following graph.



Which nodes form the cliques of size 3?

14. (7 points) Derangements are permutations π of the set $\{1, 2, \ldots, n\}$ such that $\pi(i) \neq i$. Compute the number of derangements on the set $1, 2, \ldots, n$.

15. (6 points) What is the generating function corresponding to Fibonacci series.

$$F_n = F_{n-1} + F_{n-2}.$$

Note that $F_0 = F_1 = 1$.

16. (5 points) Let us say we have a supply of 1 rupee and 2 rupee coins in large quantities. What is the generating function for the number of ways of giving change with 1 rupee and 2 rupee coins.

17. (3 points) Total number of functions from set B to set A with n and m elements, respectively are ______.

18. (5 points) A gardener wants to buy 3 neem plants, 5 rose plants and 1 banyan plant from a nursery having 7 neem, 10 rose and 6 banyan plants. How many choices does a gardener have? .

19. (5 points) How many seven digit numbers are possible with exactly four 4s?

20. (4 points) A partially ordered set $S = (\{3, 4, 12, 24, 48, 72\}, /)$ is a ____ with ____ cycle(s).

21. (3 points) For decimal number -30, the 16-bit 2's complement representation is ________

22. (5 points) How many minimum number of NOR gates are required to implement the function $F = A'B'C' + ABC'$

23. (6 points) For two flip-flops F_1 and F_2 and input X, the next state and output is given as

$$F_1(t+1) = \sum(1, 2, 5, 6)$$
$$F_2(t+1) = \sum(3, 7)$$
$$Y(F_1, F_2, X) = \sum(4, 6)$$

Write functions for the next state and output of the circuit.

24. (7 points) Given a truth table of the full adder for three inputs. Draw a full adder circuit with a decoder and two OR gates.

X	Y	Z	Carry	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

25. (4 points) Simplify the Boolean function $F = W'X'Y' + WX'Y' + W'XYZ' + X'YZ'$

26. (2 points) Provide the correct data structures for the following:

(a) (1 point) ________ is used for delimiter checking and recursion.

(b) (1 point) Asynchronous data transfer and accessing shared resource involve ______.

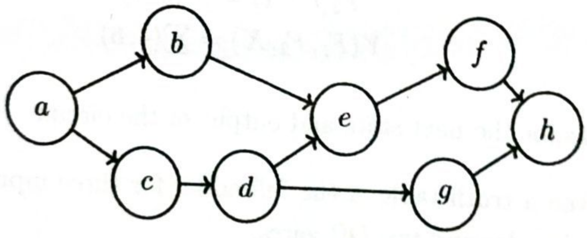
27. (5 points) Find the number of binary tree(s) with 3 nodes (i.e., A, B, and C) which when traversed by pre-order gives the sequence **C B A**. You also need to draw each such tree.

28. (4 points) If the maximum height of a binary tree is N, then how many number of nodes will there be?

29. (5 points) Five items A, B, C, D, E are pushed onto a stack, one after other starting from item A. The stack is then popped by three items, and each item is inserted into a queue. Next, two items are deleted from the queue, and the deleted items are pushed back onto the stack. Now, one item is popped from the stack. Which item is at the top of the stack.

30. (3 points) If the prefix traversal order of a tree is *** + a b - c d**. Then, find the equivalent postfix order of the that tree.

31. (4 points) How many topological sortings (or topological orderings) does the given directed graph have?



32. (5 points) Let us suppose we are given an integer N in binary representation. Let us consider the following algorithm to check if N is a prime.

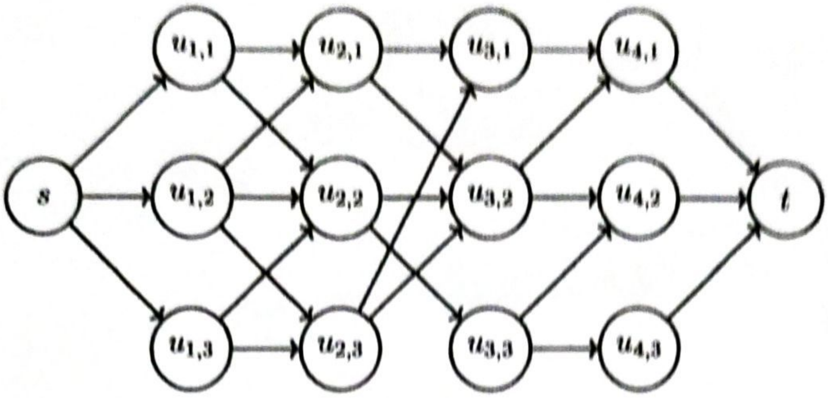
- For every i such that $2 \leq i \leq \lceil\sqrt{N}\rceil$, check if i divides N.
- If any of the i's divides N, return that N is not a prime.
- Else, return that i is a prime.

Based on this information, answer the following.

(a) (2 points) Explain why iterating till $\lceil\sqrt{N}\rceil$ is sufficient.

(b) (3 points) What is the order of magnitude of the running time in terms of the input size? (Assume hypothetically that division can be done in $O(1)$ time).

33. (4 points) Consider the following graph.



(a) (2 points) How many nodes (apart from s) does the Breadth First Search algorithm discover before discovering t when starting from s.

(b) (2 points) How many nodes (apart from s) does the Depth First Search algorithm discover before discovering t when starting from s.