

Data Structures

Binary Tree Types

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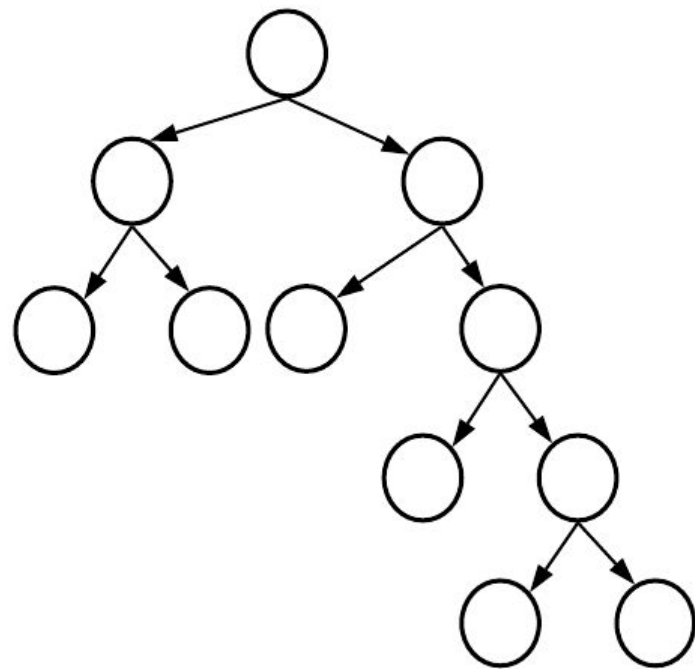
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Full (or strict) Binary Tree

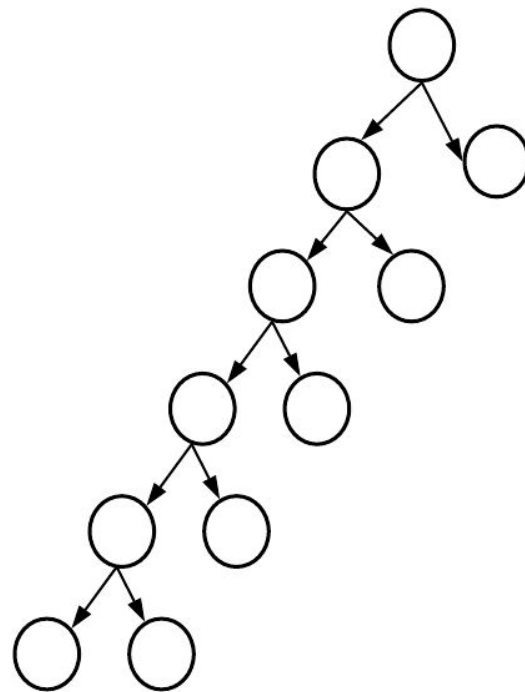
- Condition: Every node has 0 or 2 children
 - Internal node (non-leaf) a node with 1+ child
 - $N - \text{leaf_nodes}$
- In **full** tree, always
$$\text{Leaf_nodes} = \text{Internal_nodes} + 1$$
- The general version is called full **k-ary tree**
 - Each node has either 0 or exactly k children



Full (or strict) Binary Tree

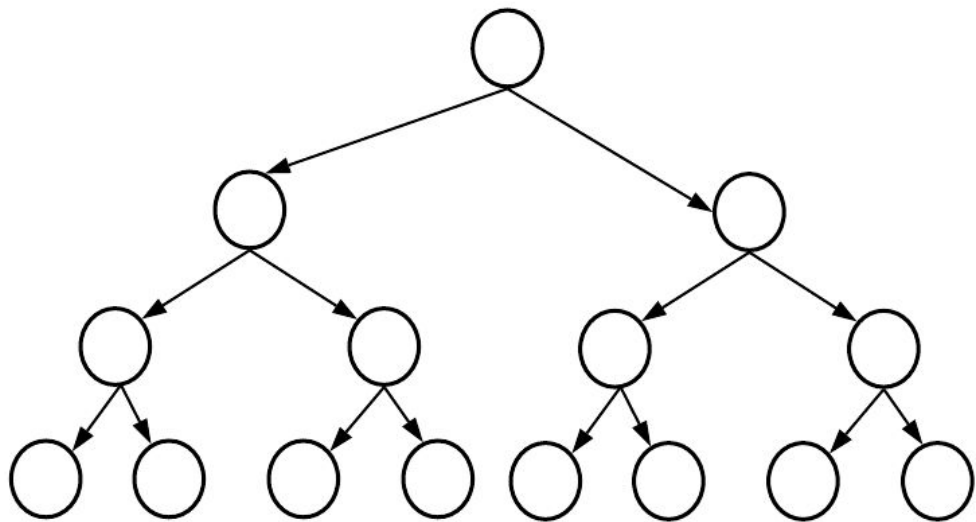
- Given height h , the min # of nodes in a full tree is: **2^{h+1}**
- Can you create such a tree for $h = 5$?
- Take 5 minutes

Just put the minimum number of nodes per level, which is 2, except root is 1



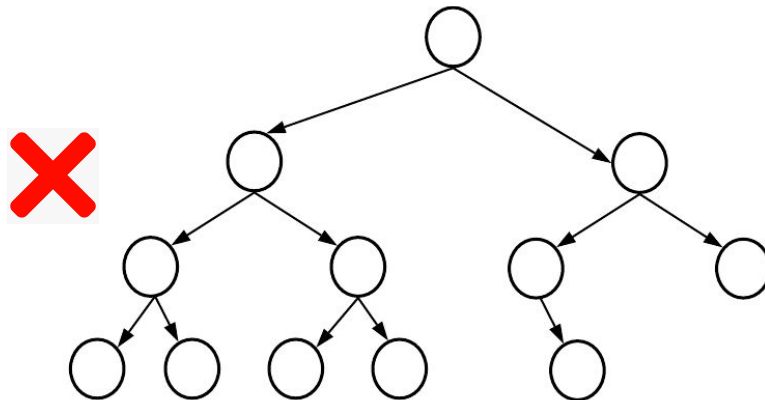
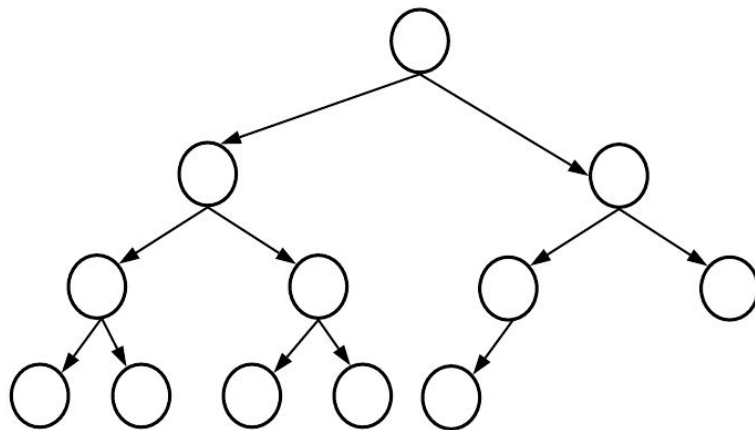
Perfect Binary Tree

- 2 conditions
 - All leaf nodes have the **same level**
 - All other nodes have **2 children**
- This tree has 4 levels
 - All leaf nodes at 4th level
 - First 3 levels has 2 children
- 4 **complete** levels
 - A complete level: **all possible** nodes **exist**
- Recall
 - Levels = 4 but Height = 3



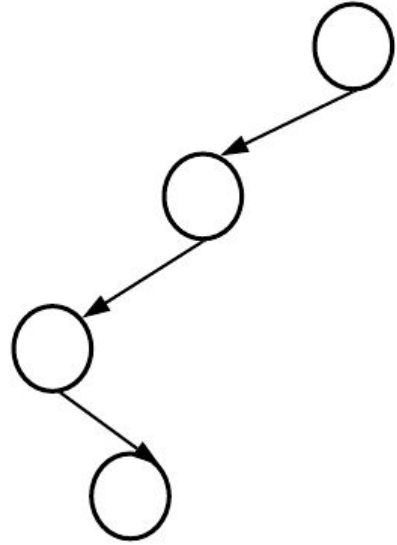
Complete Binary Tree

- All levels are **complete**
 - **except possibly** the last one, which is filled from the **left**.
- Top tree
 - 4 levels
 - First 3 are complete
 - Last one has left nodes
- Bottom tree: NOT complete
 - Has a right node before a left one
- **Perfect** Binary Tree is a **complete** tree / full tree



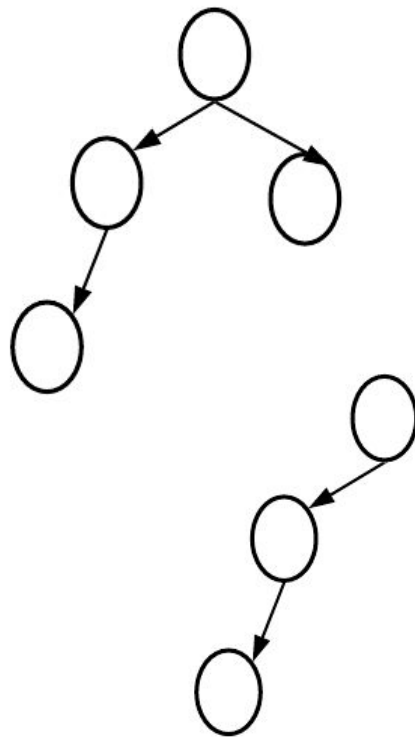
A degenerate tree

- Each node has 1 child
- Given N nodes, we can construct a tree with **largest depth** of N using a degenerate tree
- But what about the **smallest depth**?



A balanced binary tree

- Degenerate subtrees make our code very slow.
- In a balanced tree, we try to make the height as small as possible ($\sim \log n$)
- To achieve that, we build a balanced binary tree which satisfies:
 - Difference between **heights** of left subtree and right subtree **is not more than 1**.
 - E.g. heights of (left, right): (6, 6) or (6, 5) or (5, 6)
 - Left subtree is balanced
 - Right subtree is balanced
- Top tree is balanced, below is not
- Complete/perfect trees are balanced



“Acquire knowledge and impart it to the people.”

“Seek knowledge from the Cradle to the Grave.”