Task 1

Prove that if a language L is accepted by FA M and a function F is computed by a deterministic finite-state transducer T, then the language $f(L)$ is regular

$M = (Q, \Sigma, \delta, q_0, F)$

$f : \Sigma^* \to \Gamma^*$ is computed by a DFST $T = (Q', \Sigma, \Gamma, \delta', q_0', h)$

where Γ - output alphabet, $h : Q' \times \Sigma \to \Gamma^*$ - output function

For $w \in \Sigma^*$, T computes $f(w)$ as the concatenation of all output symbols generated during the transitions.

Output language $f(L)$: $f(L) = \{ f(w) \mid w \in L \}$

Construction of the combined Automation:

Combine M and T into a single machine M_T, where

$M_T = (Q \times Q', \Sigma, \Gamma, \delta_T, (q_0, q_0'), F_T)$

$\delta_T((q, q'), a) = (\delta(q, a), \delta'(q', a))$ for $a \in \Sigma$

At each transition, M_T produces $h(q', a)$

$F_T : F \times Q'$

Since M and T are finite-state devices, the combined machine M_T is also finite-state device

M_T accepts $f(L)$, implying $f(L)$ is regular, thus,

$\boxed{f(L) \in REG}$