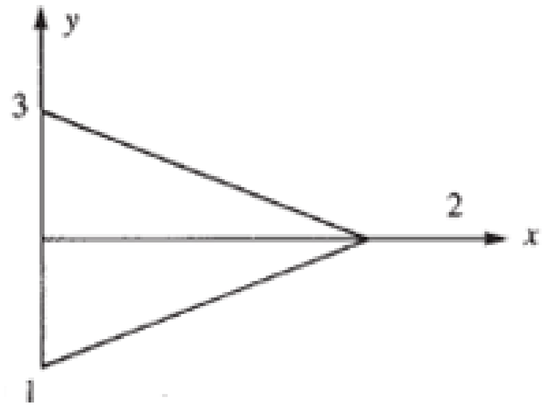
EXERCICE 1

Soit l'élément représenté illustré à la figure 1 :

- Coordonnées *(m)* : *1 (0, -1), 2(2,0), 3(0,1)*
- $E = 200\ GPa\ ,\ \upsilon = 0.25$
- Epaisseur : $e = 2.5\ cm$



- Evaluer la matrice de rigidité pour l'élément.
- Déterminer les contraintes dans l'élément.

A.N : $u_1 = 0\ ,\ v_1 = 0.63\ mm\ ,\ u_2 = 0.3\ mm\ ,\ v_2 = 0\ ,\ u_3 = 0\ \ et\ \ v_3 = 0.63\ mm$

SOLUTION

1- **Type de l'élément : *élément triangulaire à 03 nœuds (02 DDL/nœud*)**

2- **Fonction du déplacement :**

$$u(x,y) = a_1 + a_2 x + a_3 y$$
$$v(x,y) = a_4 + a_5 x + a_6 y$$

$$N_1 = \frac{1}{2} - \frac{1}{4}x - \frac{1}{2}y \quad , \quad N_2 = \frac{1}{2}x \quad , \quad N_3 = \frac{1}{2} - \frac{1}{4}x + \frac{1}{2}y$$

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

3- **Relation : contraintes / déformations / déplacements :**

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\delta u}{\delta x} \\ \frac{\delta v}{\delta y} \\ \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \end{Bmatrix} = \begin{bmatrix} \frac{\delta}{\delta x} & 0 \\ 0 & \frac{\delta}{\delta y} \\ \frac{\delta}{\delta y} & \frac{\delta}{\delta x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} \frac{\delta}{\delta x} & 0 \\ 0 & \frac{\delta}{\delta y} \\ \frac{\delta}{\delta y} & \frac{\delta}{\delta x} \end{bmatrix} \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$[B] = \begin{bmatrix} \frac{\delta}{\delta x} & 0 \\ 0 & \frac{\delta}{\delta y} \\ \frac{\delta}{\delta y} & \frac{\delta}{\delta x} \end{bmatrix} \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} = \begin{bmatrix} \frac{\delta N_1}{\delta x} & 0 & \frac{\delta N_2}{\delta x} & 0 & \frac{\delta N_3}{\delta x} & 0 \\ 0 & \frac{\delta N_1}{\delta y} & 0 & \frac{\delta N_2}{\delta y} & 0 & \frac{\delta N_3}{\delta y} \\ \frac{\delta N_1}{\delta y} & \frac{\delta N_1}{\delta x} & \frac{\delta N_2}{\delta y} & \frac{\delta N_2}{\delta x} & \frac{\delta N_3}{\delta y} & \frac{\delta N_3}{\delta x} \end{bmatrix}$$

$$[B]=\begin{bmatrix} -1/4 & 0 & 1/2 & 0 & -1/4 & 0 \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 \\ -1/2 & -1/4 & 0 & 1/2 & 1/2 & -1/4 \end{bmatrix}$$

$$[B]=\frac{1}{4}\begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix}$$

$$[D]=\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}=\frac{200\ 10^6}{1-0.25^2}\begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix}$$

4- **Matrice de rigidité**

$$[K]=\iiint [B]^T[D][B]dv$$

$$[B]^T[D][B]=\frac{200\ 10^6}{1-0.25^2}\times\frac{1}{4}\times\frac{1}{4}\begin{bmatrix} -1 & 0 & -2 \\ 0 & -2 & -1 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \\ -1 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix}\begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix}$$

$$[B]^T[D][B]=\frac{200\ 10^6}{1-0.25^2}\times\frac{1}{4}\times\frac{1}{4}\begin{bmatrix} -1 & -1/4 & -3/4 \\ -1/2 & -2 & -3/8 \\ 2 & 1/2 & 0 \\ 0 & 0 & 3/4 \\ -1 & -1/4 & 3/4 \\ 1/2 & 2 & -3/8 \end{bmatrix}\begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix}$$

$$[B]^T[D][B]=\frac{200\ 10^6}{15}\begin{bmatrix} 5/2 & 5/4 & -2 & -3/2 & -1/2 & 1/4 \\ 5/4 & 35/8 & -1 & -3/4 & -1/4 & -29/8 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -3/2 & -3/4 & 0 & 3/2 & 3/2 & -3/4 \\ -1/2 & -1/4 & -2 & 3/2 & 5/2 & -5/4 \\ 1/4 & -29/8 & 1 & -3/4 & -5/4 & 35/8 \end{bmatrix}$$

$$[K]=\iiint [B]^T[D][B]dv = e\ A\ [B]^T[D][B]$$

$$[K]=\frac{10}{15}10^6\begin{bmatrix} 5/2 & 5/4 & -2 & -3/2 & -1/2 & 1/4 \\ 5/4 & 35/8 & -1 & -3/4 & -1/4 & -29/8 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -3/2 & -3/4 & 0 & 3/2 & 3/2 & -3/4 \\ -1/2 & -1/4 & -2 & 3/2 & 5/2 & -5/4 \\ 1/4 & -29/8 & 1 & -3/4 & -5/4 & 35/8 \end{bmatrix}(KN/m)$$

5- **Contraintes :**

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D][B] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{200\ 10^3}{1-0.25^2} \times \frac{1}{4} \times \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.63\ 10^{-3} \\ 0.3\ 10^{-3} \\ 0 \\ 0 \\ 0.63\ 10^{-3} \end{Bmatrix}$$

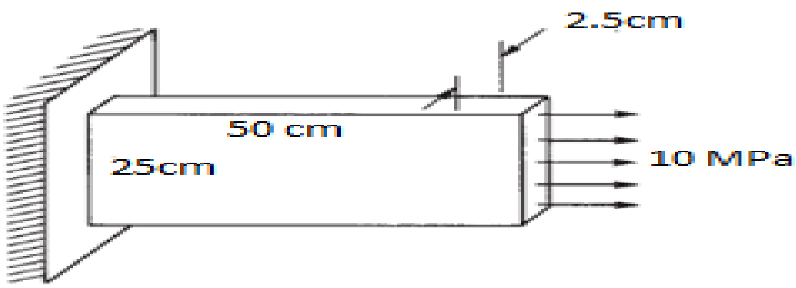
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{200\ 10^3}{1-0.25^2} \times \frac{1}{4} \times \begin{bmatrix} -1 & -1/2 & 2 & 0 & -1 & 1/2 \\ -1/4 & -2 & 1/2 & 0 & -1/4 & 2 \\ -3/4 & -3/8 & 0 & 3/4 & 3/4 & -3/8 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.63\ 10^{-3} \\ 0.3\ 10^{-3} \\ 0 \\ 0 \\ 0.63\ 10^{-3} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{200\ 10^3}{1-0.25^2} \times \frac{1}{4} \times \begin{Bmatrix} 0.0006 \\ 0.00015 \\ -0.0004725 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 32 \\ 8 \\ -25.20 \end{Bmatrix} (MPA)$$

EXERCICE 2

Considérons la plaque mince soumise à la traction surfacique représentée dans la *figure 2* :

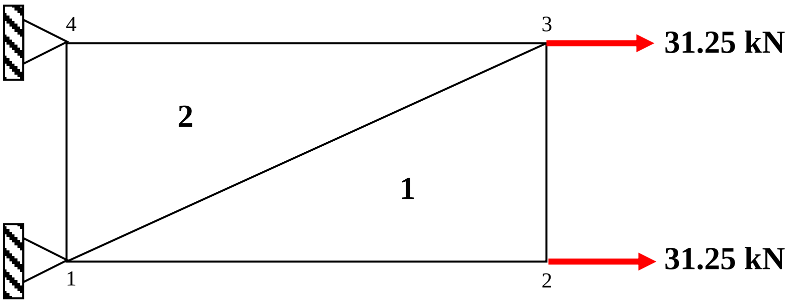


- Déterminer les déplacements nodaux et les contraintes dans l'élément.

A.N : $E = 200\ GPa\ ,\ \ \upsilon = 0.25\ \ , e = 2.5\ cm.$

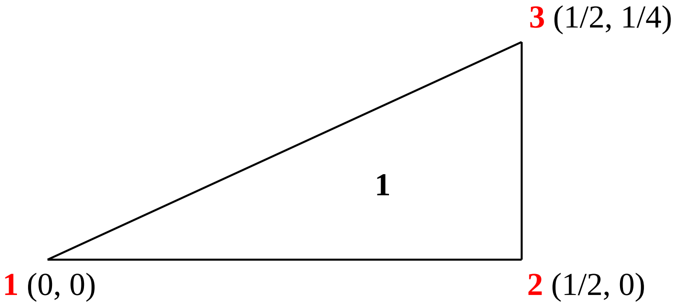
SOLUTION

Modélisation



$$F = \frac{1}{2} T\ A = \frac{1}{2} 10000 * 0.25 * 0.025 = 31.25\ kN$$

Element 1



1- **Type de l'élément : triangle à 3 noeuds (2DDL/noeud)**
2- **Fonction du déplacement**

$$\begin{cases} u(x,y) = a_0 + a_1 x + a_2 y \\ v(x,y) = a_3 + a_4 x + a_5 y \end{cases}$$

$$\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$N_1 = 1 - 2x$$
$$N_2 = 2x - 4y$$
$$N_3 = 4y$$

3- **Relations $\boldsymbol{\sigma/\varepsilon/u}$**

$$[B] = \begin{bmatrix} \frac{\delta N_1}{\delta x} & 0 & \frac{\delta N_2}{\delta x} & 0 & \frac{\delta N_3}{\delta x} & 0 \\ 0 & \frac{\delta N_1}{\delta y} & 0 & \frac{\delta N_2}{\delta y} & 0 & \frac{\delta N_3}{\delta y} \\ \frac{\delta N_1}{\delta y} & \frac{\delta N_1}{\delta x} & \frac{\delta N_2}{\delta y} & \frac{\delta N_2}{\delta x} & \frac{\delta N_3}{\delta y} & \frac{\delta N_3}{\delta x} \end{bmatrix}$$

$$[B] = \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 \\ 0 & -2 & -4 & 2 & 4 & 0 \end{bmatrix}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{200\ 10^6}{1-0.25^2} \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix}$$

4- Matrice de rigidité

$$[K^{(1)}] = \iiint [B]^T[D][B]dv$$

$$[B]^T[D][B] = \frac{200\ 10^6}{1-0.25^2}\begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & -2 \\ 2 & 0 & -4 \\ 0 & -4 & 2 \\ 0 & 0 & 4 \\ 0 & 4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix}\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 \\ 0 & -2 & -4 & 2 & 4 & 0 \end{bmatrix}$$

$$[B]^T[D][B] = \frac{200\ 10^6}{1-0.25^2}\begin{bmatrix} -2 & -1/2 & 0 \\ 0 & 0 & -3/4 \\ 2 & 1/2 & -3/2 \\ -1 & -4 & 3/4 \\ 0 & 0 & 3/2 \\ 1 & 4 & 0 \end{bmatrix}\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 \\ 0 & -2 & -4 & 2 & 4 & 0 \end{bmatrix}$$

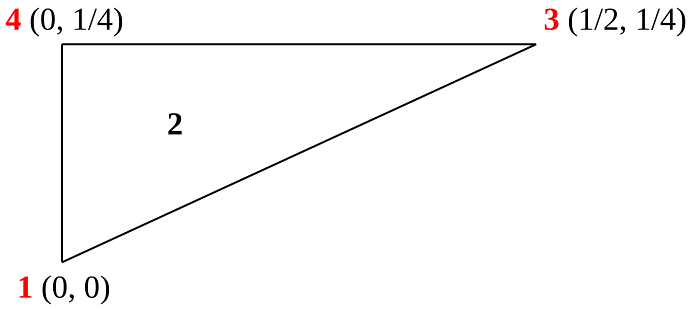
$$[B]^T[D][B] = \frac{200\ 10^6}{1-0.25^2}\begin{array}{cc} \begin{array}{cccccc} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \end{array} & \\ \begin{bmatrix} 4 & 0 & -4 & 2 & 0 & -2 \\ 0 & 3/2 & 3 & -3/2 & -3 & 0 \\ -4 & 3 & 10 & -5 & -6 & 2 \\ 2 & -3/2 & -5 & 35/2 & 3 & -16 \\ 0 & -3 & -6 & 3 & 6 & 0 \\ -2 & 0 & 2 & -16 & 0 & 16 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{array} \end{array}$$

$$[K^{(1)}] = \iiint [B]^T[D][B]dv = e\ A\ [B]^T[D][B]$$

$$[K^{(1)}] = 0.025\ \times 0.25 \times 0.5 \times 1/2 \times \frac{200\ 10^6}{1-0.25^2}\begin{array}{cc} \begin{array}{cccccc} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \end{array} & \\ \begin{bmatrix} 4 & 0 & -4 & 2 & 0 & -2 \\ 0 & 3/2 & 3 & -3/2 & -3 & 0 \\ -4 & 3 & 10 & -5 & -6 & 2 \\ 2 & -3/2 & -5 & 35/2 & 3 & -16 \\ 0 & -3 & -6 & 3 & 6 & 0 \\ -2 & 0 & 2 & -16 & 0 & 16 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{array} \end{array}$$

$$[K^{(1)}](kN/m) = 0.333 \times 10^6\begin{array}{cc} \begin{array}{cccccc} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \end{array} & \\ \begin{bmatrix} 4 & 0 & -4 & 2 & 0 & -2 \\ 0 & 3/2 & 3 & -3/2 & -3 & 0 \\ -4 & 3 & 10 & -5 & -6 & 2 \\ 2 & -3/2 & -5 & 35/2 & 3 & -16 \\ 0 & -3 & -6 & 3 & 6 & 0 \\ -2 & 0 & 2 & -16 & 0 & 16 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{array} \end{array}$$

Element 2



1- **Type de l'élément : triangle à 3 noeuds (2DDL/noeud)**
2- **Fonction du déplacement**

$$\begin{cases} u(x,y) = a_0 + a_1 x + a_2 y \\ v(x,y) = a_3 + a_4 x + a_5 y \end{cases}$$

$$\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

$$N_1 = 1 - 4y$$
$$N_2 = 2x$$
$$N_3 = -2x + 4y$$

3- **Relations $\boldsymbol{\sigma/\varepsilon/u}$**

$$[B] = \begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -4 & 0 & 0 & 0 & 4 \\ -4 & 0 & 0 & 2 & 4 & -2 \end{bmatrix}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} = \frac{200\ 10^6}{1-0.25^2} \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix}$$

4- **Matrice de rigidité**

$$[K^{(2)}] = \iiint [B]^T [D][B] dv$$

$$[B]^T[D][B] = \frac{200\ 10^6}{1-0.25^2} \begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \\ -2 & 0 & 4 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -4 & 0 & 0 & 0 & 4 \\ -4 & 0 & 0 & 2 & 4 & -2 \end{bmatrix}$$

$$[B]^T[D][B] = \frac{200\ 10^6}{1-0.25^2}\begin{bmatrix} 0 & 0 & -3/2 \\ -1 & -4 & 0 \\ 2 & 1/2 & 0 \\ 0 & 0 & 3/4 \\ -2 & -1/2 & 3/2 \\ 1 & 4 & -3/4 \end{bmatrix}\begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -4 & 0 & 0 & 0 & 4 \\ -4 & 0 & 0 & 2 & 4 & -2 \end{bmatrix}$$

$$[B]^T[D][B] = \frac{200\ 10^6}{1-0.25^2}\begin{array}{cc} \begin{array}{cccccc} u_1 & v_1 & u_3 & v_3 & u_4 & v_4 \end{array} & \\ \begin{bmatrix} 6 & 0 & 0 & -3 & -6 & 3 \\ 0 & 16 & -2 & 0 & 2 & -16 \\ 0 & -2 & 4 & 0 & -4 & 2 \\ -3 & 0 & 0 & 3/2 & 3 & -3/2 \\ -6 & 2 & -4 & 3 & 10 & -5 \\ 3 & -16 & 2 & -3/2 & -5 & 35/2 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{array} \end{array}$$

$$[K^{(1)}] = \iiint [B]^T[D][B]dv = e\ A\ [B]^T[D][B]$$

$$[K^{(1)}] = 0.025\ \times 0.25 \times 0.5 \times 1/2 \times \frac{200\ 10^6}{1-0.25^2}\begin{array}{cc} \begin{array}{cccccc} u_1 & v_1 & u_3 & v_3 & u_4 & v_4 \end{array} & \\ \begin{bmatrix} 6 & 0 & 0 & -3 & -6 & 3 \\ 0 & 16 & -2 & 0 & 2 & -16 \\ 0 & -2 & 4 & 0 & -4 & 2 \\ -3 & 0 & 0 & 3/2 & 3 & -3/2 \\ -6 & 2 & -4 & 3 & 10 & -5 \\ 3 & -16 & 2 & -3/2 & -5 & 35/2 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{array} \end{array}$$

$$[K^{(1)}](kN/m) = 0.333 \times 10^6\begin{array}{cc} \begin{array}{cccccc} u_1 & v_1 & u_3 & v_3 & u_4 & v_4 \end{array} & \\ \begin{bmatrix} 6 & 0 & 0 & -3 & -6 & 3 \\ 0 & 16 & -2 & 0 & 2 & -16 \\ 0 & -2 & 4 & 0 & -4 & 2 \\ -3 & 0 & 0 & 3/2 & 3 & -3/2 \\ -6 & 2 & -4 & 3 & 10 & -5 \\ 3 & -16 & 2 & -3/2 & -5 & 35/2 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{array} \end{array}$$

5- Assemblage et C-A-L

$$[\boldsymbol{K}] = 0.333 \times 10^6\begin{array}{cc} \begin{array}{cccccccc} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & u_4 & v_4 \end{array} & \\ \begin{bmatrix} 10 & 0 & -4 & 2 & 0 & -5 & -6 & 3 \\ 0 & 35/2 & 3 & -3/2 & -5 & 0 & 2 & -16 \\ -4 & 3 & 10 & -5 & -6 & 2 & 0 & 0 \\ 2 & -3/2 & -5 & 35/2 & 3 & -16 & 0 & 0 \\ 0 & -5 & -6 & 3 & 10 & 0 & -4 & 2 \\ -5 & 0 & 2 & -16 & 0 & 35/2 & 3 & -3/2 \\ -6 & 2 & 0 & 0 & -4 & 3 & 10 & -5 \\ 3 & -16 & 0 & 0 & 2 & -3/2 & -5 & 35/2 \end{bmatrix} & \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{array} \end{array}$$

$$\begin{Bmatrix} \cancel{R_{1x}} \\ \cancel{R_{1y}} \\ \mathbf{31.25} \\ \mathbf{0} \\ \mathbf{31.25} \\ \mathbf{0} \\ \cancel{R_{4x}} \\ \cancel{R_{4y}} \end{Bmatrix} = 0.333 \times 10^6 \begin{bmatrix} \cancel{10} & \cancel{0} & \cancel{-4} & \cancel{2} & \cancel{0} & \cancel{-5} & \cancel{-6} & \cancel{3} \\ \cancel{0} & \cancel{35/2} & \cancel{3} & \cancel{-3/2} & \cancel{-5} & \cancel{0} & \cancel{2} & \cancel{-16} \\ \cancel{-4} & \cancel{3} & \mathbf{10} & \mathbf{-5} & \mathbf{-6} & \mathbf{2} & \cancel{0} & \cancel{0} \\ \cancel{2} & \cancel{-3/2} & \mathbf{-5} & \mathbf{35/2} & \mathbf{3} & \mathbf{-16} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{-5} & \mathbf{-6} & \mathbf{3} & \mathbf{10} & \mathbf{0} & \cancel{-4} & \cancel{2} \\ \cancel{-5} & \cancel{0} & \mathbf{2} & \mathbf{-16} & \mathbf{0} & \mathbf{35/2} & \cancel{3} & \cancel{-3/2} \\ \cancel{-6} & \cancel{2} & \cancel{0} & \cancel{0} & \cancel{-4} & \cancel{3} & \cancel{10} & \cancel{-5} \\ \cancel{3} & \cancel{-16} & \cancel{0} & \cancel{0} & \cancel{2} & \cancel{-3/2} & \cancel{-5} & \cancel{35/2} \end{bmatrix} \begin{Bmatrix} \cancel{0} \\ \cancel{0} \\ \boldsymbol{u_2} \\ \boldsymbol{v_2} \\ \boldsymbol{u_3} \\ \boldsymbol{v_3} \\ \cancel{0} \\ \cancel{0} \end{Bmatrix}$$

$$\begin{Bmatrix} 31.25 \\ 0 \\ 31.25 \\ 0 \end{Bmatrix} = 0.333 \times 10^6 \begin{bmatrix} 10 & -5 & -6 & 2 \\ -5 & 35/2 & 3 & -16 \\ -6 & 3 & 10 & 0 \\ 2 & -16 & 0 & 35/2 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 0.025 \\ 0.0031 \\ 0.023 \\ 0 \end{Bmatrix} (mm)$$

Elément 1

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (\boldsymbol{MPa}) = \frac{200}{1 - 0.25^2} \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 \\ 0 & -2 & -4 & 2 & 4 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.025 \\ 0.0031 \\ 0.023 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (\boldsymbol{MPa}) = \begin{Bmatrix} 10 \\ 0.021 \\ 0.143 \end{Bmatrix}$$

Elément 2

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (\boldsymbol{MPa}) = \frac{200}{1 - 0.25^2} \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 1 & 0 \\ 0 & 0 & 3/8 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -4 & 0 & 0 & 0 & 4 \\ -4 & 0 & 0 & 2 & 4 & -2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.023 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (\boldsymbol{MPa}) = \begin{Bmatrix} 9.81 \\ 2.45 \\ 0 \end{Bmatrix}$$