- Équilibre locale au niveau du nœud
  - la somme égale à zéro
- Équilibre globale la somme (vectoriel) des réactions avec les forces = 0

Série 03.

Exercice 01.

1) Matrice de rigidité.

Tableau de connectivités.

Barre	Nœud	L	α	$\cos\alpha$	$\sin\alpha$
1	1–2	L	0	1	0
2	2–3	L	$120 = \frac{2\pi}{3}$	$-1/2$	$\frac{\sqrt{3}}{2}$
3	1–3	L	$60 = \frac{\pi}{3}$	$1/2$	$\frac{\sqrt{3}}{2}$

$$[K_e] = \frac{EA}{L}\left[\begin{array}{cc|cc} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ \hline -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{array}\right]$$

$$[K^{①}] = \frac{EA}{4L}\begin{array}{c} \begin{array}{cccc} U_1 & V_1 & U_2 & V_2 \end{array} \\ \left[\begin{array}{cc|cc} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} U_1 \\ V_1 \\ U_2 \\ V_2 \end{array}$$

$$[K^{②}] = \frac{EA}{4L}\begin{array}{c} \begin{array}{cccc} U_2 & V_2 & U_3 & V_3 \end{array} \\ \left[\begin{array}{cc|cc} 1 & -\sqrt{3} & -1 & \sqrt{3} \\ -\sqrt{3} & 3 & \sqrt{3} & -3 \\ \hline -1 & \sqrt{3} & 1 & -\sqrt{3} \\ \sqrt{3} & -3 & -\sqrt{3} & 3 \end{array}\right] \end{array} \begin{array}{c} U_2 \\ V_2 \\ U_3 \\ V_3 \end{array}$$

$$[K^{③}] = \frac{EA}{4L}\begin{array}{c} \begin{array}{cccc} U_1 & V_1 & U_3 & V_3 \end{array} \\ \left[\begin{array}{cc|cc} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ \hline -1 & -\sqrt{3} & 1 & \sqrt{3} \\ -\sqrt{3} & -3 & \sqrt{3} & 3 \end{array}\right] \end{array} \begin{array}{c} U_1 \\ V_1 \\ U_3 \\ V_3 \end{array}$$

$$[K] = \frac{EA}{4L}\begin{array}{c} \begin{array}{cccccc} U_1 & V_1 & U_2 & V_2 & U_3 & V_3 \end{array} \\ \begin{bmatrix} 5 & \sqrt{3} & -4 & 0 & -1 & \sqrt{3} \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & 3 \\ -4 & 0 & 5 & -\sqrt{3} & -1 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & 3 & \sqrt{3} & -3 \\ -1 & -\sqrt{3} & -1 & \sqrt{3} & 2 & 0 \\ -\sqrt{3} & 3 & -3 & -3 & 0 & 6 \end{bmatrix} \end{array} \begin{array}{c} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \end{array}$$

2) Les déplacements nodaux

$$[K]\{U\} = \{F\}$$

$$\frac{EA}{4L}\begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}\begin{Bmatrix} U_1 = 0 \\ V_1 \\ U_2 \\ V_2 = 0 \\ U_3 = 0 \\ V_3 = 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} = R_{1x} \\ F_{1y} = 0 \\ F_{2x} = P \\ F_{2y} = R_{2y} \\ F_{3x} = R_{3x} \\ F_{3y} = R_{3y} \end{Bmatrix}$$

CAL.

$U_1 = 0$, $V_2 = 0$, $U_3 = V_3 = 0$

$$\frac{EA}{4L}\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} V_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 \\ P \end{bmatrix}$$

$$\Rightarrow \boxed{V_1 = 0} \text{ et } \boxed{U_2 = \frac{4}{5}\frac{PL}{EA}}$$

3) Les efforts normaux dans la barres.

$$N = \sigma . A = E \varepsilon A = E . \frac{\Delta u}{L} . A$$

$$N = \frac{EA}{L}[c \quad s]\begin{bmatrix} U_j - U_i \\ V_j - V_i \end{bmatrix}$$

$$N^{①}_{1\text{-}2} = \frac{EA}{L}[1 \quad 0]\begin{bmatrix} U_2 - U_1 \\ V_2 - V_1 \end{bmatrix}$$

$$\boxed{N^{①} = \frac{4}{5}P}$$

$$N^{②}_{2-3} = \frac{EA}{L}\left[-\frac{1}{2} \quad \frac{\sqrt{3}}{2}\right]\begin{Bmatrix} U_3 - U_2 \\ V_3 - V_2 \end{Bmatrix}$$

$$\boxed{N^{②} = \frac{2}{5}P}$$

$$N^{③}_{1-3} = \frac{EA}{L}\left[1/2 \quad \frac{\sqrt{3}}{2}\right]\begin{Bmatrix} U_3 - U_1 \\ V_3 - V_1 \end{Bmatrix}$$

$$\boxed{N^{③} = 0}$$

On ne peut pas tirer cette barre à cause de

$3b - n \geq 7$ (isostatique).

Exercice 02,

1) La matrice de rigidité globale,

Barre	Noeud	L	θ	$\cos\theta$	$\sin\theta$
1	1-2	$2L$	$60°$	$\frac{1}{2}$	$\sqrt{3}/2$
2	1-3	$\frac{2}{\sqrt{3}}L$	$30°$	$\frac{\sqrt{3}}{2}$	$1/2$
3	1-4	L	0	1	0
4	1-5	$\frac{2}{\sqrt{3}}L$	$-30°$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

$$[K^{①}] = \frac{\sqrt{3}EA}{8L}\begin{array}{c} \begin{matrix} U_1 & V_1 & U_2 & V_2 \end{matrix} \\ \begin{bmatrix} & & - & \\ & & & \\ & & + & \\ - & & & \end{bmatrix} \end{array} \begin{matrix} U_1 \\ V_1 \\ U_2 \\ V_2 \end{matrix}$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \sqrt{3} \\ 1 & \sqrt{3} \end{bmatrix}$$

$$[K^{②}] = \frac{\sqrt{3}EA}{8L}\begin{array}{c} \begin{matrix} U_1 & V_1 & U_3 & V_3 \end{matrix} \\ \begin{bmatrix} 3 & \sqrt{3} & & \\ \sqrt{3} & 1 & & \\ & & & \\ & & & \end{bmatrix} \end{array} \begin{matrix} U_1 \\ V_1 \\ U_3 \\ V_3 \end{matrix}$$

$$[K^{③}] = \frac{\sqrt{3}EA}{8L}\begin{bmatrix} \frac{8}{\sqrt{3}} & & 1 & - \\ & & & - \\ - & & & - \\ & - & & - \end{bmatrix}$$

$$[K^{④}] = \frac{\sqrt{3}EA}{8L}\begin{array}{c} \begin{matrix} U_1 & V_1 & U_5 & V_5 \end{matrix} \\ \begin{bmatrix} 3 & -\sqrt{3} & & \\ -\sqrt{3} & 1 & & \\ & & & \\ & & & \end{bmatrix} \end{array} \begin{matrix} U_1 \\ V_1 \\ U_5 \\ V_5 \end{matrix}$$

$$[K] = \frac{EA}{8L}\begin{bmatrix} 9+6\sqrt{3} & \sqrt{3} & \cdots \\ \sqrt{3} & 3+2\sqrt{3} & \\ -1 & \sqrt{3} & \\ -\sqrt{3} & & \\ -3\sqrt{3} & & \\ -3 & & \\ -1 & & \\ 0 & & \\ -3\sqrt{3} & & \\ 3 & & \end{bmatrix}$$

photo nacer

2) Réaction et déplacement au noeud 1,

$$U_2 = V_2 = U_3 = V_3 = U_4 = V_4 = U_5 = V_5 = 0$$

$\boxed{U_1 = -V_1}$ à partir de $tg45 = \frac{V_1}{U_1} = 1$

U_1, V_1, 45, δ

$$[K]\begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \\ U_4 \\ V_4 \\ U_5 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \\ F_{5x} \\ F_{5y} \end{Bmatrix}$$

$F_{1x} = R_{1x} = R_1 \underset{\sin}{\cancel{\cos}} 45$

$F_{1y} = R_{1y} - F = R\cos 45 - F$

$$\frac{EA}{8L}\begin{bmatrix} 9+6\sqrt{3} & \sqrt{3} \\ \sqrt{3} & 3+2\sqrt{3} \end{bmatrix}\begin{Bmatrix} U_1 \\ -U_1 \end{Bmatrix} = \begin{Bmatrix} R_1\cos 45 \\ R_1\sin 45 - F \end{Bmatrix}$$

$$\begin{cases} u_1 = -v_1 = \dfrac{4(2-\sqrt{3})FL}{3EA} \\ \\ k_{1N} = \sqrt{2}(3+\sqrt{3}) \end{cases}$$

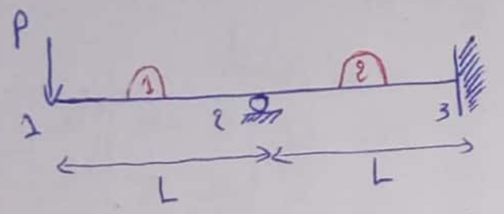
4

$-\frac{9}{2}PL + \frac{5}{2}PL$

MEF

Serie 04

Exercice 01 :



1) La matrice de rigidité globale :

$$[K^{①}] = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} V_1 & \phi_1 & V_2 & \phi_2 \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ \text{syst} & & 12 & -6L \\ & & & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \end{array}$$

$$[K^{②}] = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} V_2 & \phi_2 & V_3 & \phi_3 \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \begin{array}{c} V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{array}$$

$$[K] = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccccc} V_1 & \phi_1 & V_2 & \phi_2 & V_3 & \phi_3 \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{array}$$

2) Les déplacements nodaux :

$$[K] \begin{Bmatrix} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{Bmatrix} = \begin{bmatrix} F_{y_1} \\ M_1 \\ F_{y_2} \\ M_2 \\ F_{y_3} \\ M_3 \end{bmatrix}$$

C.A.L

$V_2 = 0 \; ; V_3 = \phi_3 = 0$

Syst reduit,

$$\frac{EI}{L^3} \begin{bmatrix} 12 & 6L & 6L \\ 6L & 4L^2 & 2L^2 \\ 6L & 2L^2 & 8L^2 \end{bmatrix} \begin{Bmatrix} V_1 \\ \phi_1 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} -P \\ 0 \\ 0 \end{Bmatrix}$$

$\Rightarrow \boxed{V_1 = \frac{-7}{12} \frac{PL^3}{EI}} \; ; \boxed{\phi_1 = \frac{3}{4} \frac{PL^2}{EI}}$

$\boxed{\phi_2 = \frac{1}{4} \frac{PL^2}{EI}}$

- Pour vérifier l'équilibre, on fait la somme des réactions $= 0$, et la somme des moments $= 0$

3) Les forces nodales globales :

$F_{y_1} = -P \; ; M_1 = 0 \; ; F_{y_2} = \frac{5}{2} P$

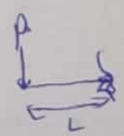
$M_2 = 0 \; ; F_{y_3} = 0 \; ; M_3 = \frac{PL}{2}$

4) Les forces nodales locales :

Element 1.

$$\begin{Bmatrix} F_{y_1}^{①} \\ m_1^{①} \\ F_{y_2}^{①} \\ m_2^{①} \end{Bmatrix} = [K^{①}] \begin{bmatrix} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \end{bmatrix}$$

$$\begin{cases} F_{y_1}^{①} = -P & ; m_1^{①} = 0 \\ F_{y_2}^{①} = +P & ; m_2^{①} = -PL \end{cases}$$

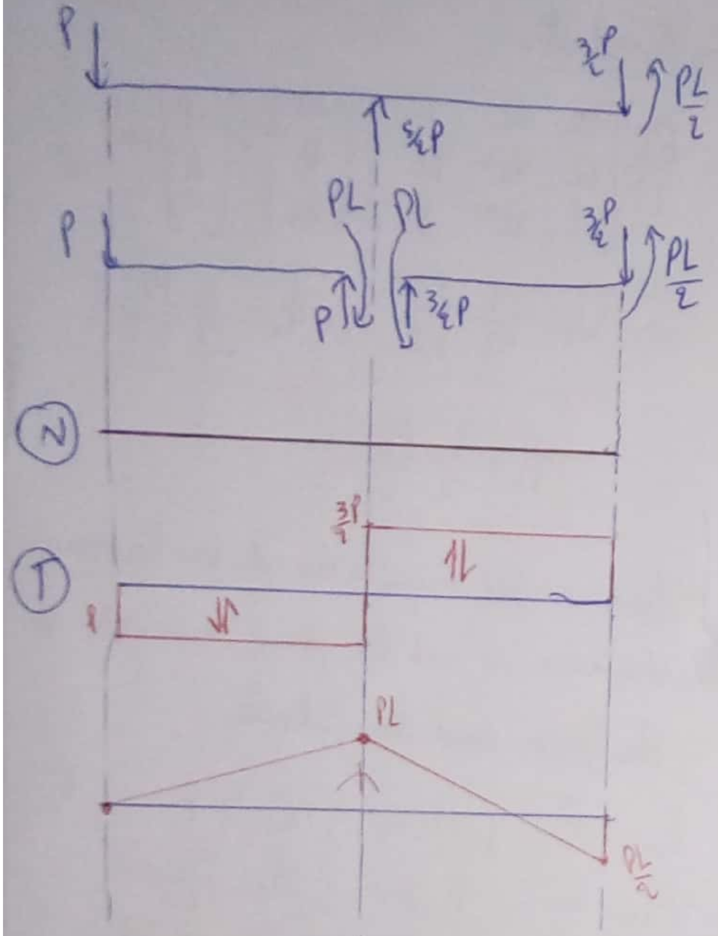


Elmt 2,

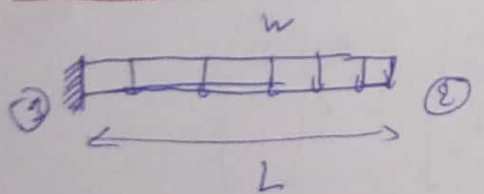
$$\begin{Bmatrix} F_{y_2}^{②} \\ m_2^{②} \\ F_{y_3}^{②} \\ m_3^{②} \end{Bmatrix} = [K^{②}] \begin{bmatrix} V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{bmatrix}$$

$$\begin{cases} F_{y_2}^{②} = \frac{3}{2} P & ; m_2^{②} = -PL \\ F_{y_3}^{②} = -\frac{3}{2} P & ; m_3^{②} = \frac{PL}{2} \end{cases}$$

5) Les diagrammes N, T, M :



Exercice 02.



1) V_2 et ϕ_2 ?

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ \text{Sym}^t & & 12 & -6L \\ & & & 4L^2 \end{bmatrix}$$

$$[K]\{U\} = \{F\}$$

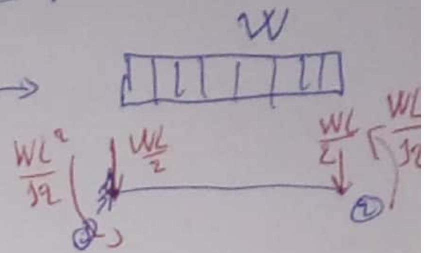
$$[K]\begin{Bmatrix} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} F_{y_1} \\ M_1 \\ F_{y_2} \\ M_2 \end{Bmatrix}$$

C.A.L

$V_1 = \phi_1 = 0$

Vect forces Nod ?

Tableau →



$$\begin{cases} F_{y_1} = -\frac{wL}{2} + R_1 \\ M_1 = -\frac{wL^2}{12} + M_{e_1} \end{cases}$$

$$\begin{cases} F_{y_2} = -\frac{wL}{2} \\ M_2 = \frac{wL^2}{12} \end{cases} \Rightarrow \text{Syst réduit, } \frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} V_2 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{wL}{2} \\ \frac{wL^2}{12} \end{Bmatrix}$$

$$V_2 = -\frac{1}{8}\frac{wL^4}{EI} \; ; \; \phi_2 = -\frac{1}{6}\frac{wL^3}{EI}$$

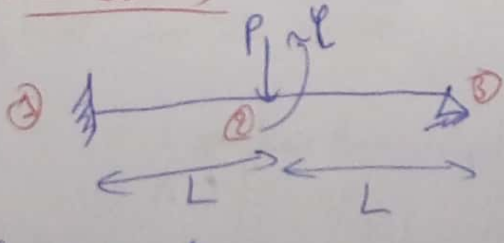
2) Réactions (forces nodales).

$$\{F\} = [K]\{U\}$$

$$\begin{cases} F_{y_1} = \frac{1}{2}wL \Rightarrow R_1 = wL \\ M_1 = \frac{5}{12}wL^2 \Rightarrow M_{e_1} = \frac{wL^2}{2} \end{cases}$$

$$\begin{cases} F_{y_2} = -\frac{wL}{2} \\ M_2 = \frac{wL^2}{12} \end{cases}$$

Exercice 03



les mêmes questions :

⇒ même matrice de rigidité de l'exo 01

2) CAL, $V_1 = \phi_1 = 0$, $V_3 = 0$