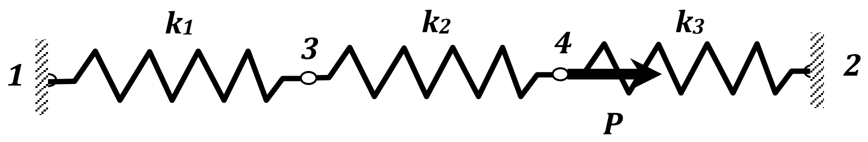
EXERCICE N°01

1. Matrice de Rigidité Globale



➢ ***Matrices de Rigidité Élémentaire :***

$$\left[k_e^{(1)}\right] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \quad ; \quad \left[k_e^{(2)}\right] = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \quad ; \quad \left[k_e^{(3)}\right] = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix}$$

Avec : $k_1 = k$; $k_2 = 2k$, $k_3 = 3k$

$$\left[\boldsymbol{k_e^{(1)}}\right] = \begin{bmatrix} \boldsymbol{k} & \boldsymbol{-k} \\ \boldsymbol{-k} & \boldsymbol{k} \end{bmatrix} \quad ; \quad \left[\boldsymbol{k_e^{(2)}}\right] = \begin{bmatrix} \boldsymbol{2k} & \boldsymbol{-2k} \\ \boldsymbol{-2k} & \boldsymbol{2k} \end{bmatrix} \quad ; \quad \left[\boldsymbol{k_e^{(3)}}\right] = \begin{bmatrix} \boldsymbol{3k} & \boldsymbol{-3k} \\ \boldsymbol{-3k} & \boldsymbol{3k} \end{bmatrix}$$

➢ ***Assemblage des Matrices Élémentaires en Matrice Globale***

$$[K] = \sum_{i=1}^{n} \left[k^{(i)}\right] \qquad \text{Soit :} \qquad [K] = \left[k^{(1)}\right] + \left[k^{(2)}\right] + \left[k^{(3)}\right]$$

✓ ***Concept de la Superposition (Sommation des Matrices Élémentaires exprimés dans le Repère Global)***

$$\left[K_{1-3}^{(1)}\right] = \begin{array}{cc} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\ \begin{bmatrix} k & 0 & -k & 0 \\ 0 & 0 & 0 & 0 \\ -k & 0 & k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \end{array} \qquad \left[K_{3-4}^{(2)}\right] = \begin{array}{cc} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2k & -2k \\ 0 & 0 & -2k & 2k \end{bmatrix} & \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \end{array}$$

$$\left[K_{4-2}^{(3)}\right] = \begin{array}{cc} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3k & 0 & -3k \\ 0 & 0 & 0 & 0 \\ 0 & -3k & 0 & 3k \end{bmatrix} & \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \end{array}$$

$$[K] = \left[k^{(1)}\right] + \left[k^{(2)}\right] + \left[k^{(3)}\right]$$

$$[\boldsymbol{K}] = \begin{array}{cc} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\ \begin{bmatrix} \boldsymbol{k} & \boldsymbol{0} & \boldsymbol{-k} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{3k} & \boldsymbol{0} & \boldsymbol{-3k} \\ \boldsymbol{-k} & \boldsymbol{0} & \boldsymbol{3k} & \boldsymbol{-2k} \\ \boldsymbol{0} & \boldsymbol{-3k} & \boldsymbol{-2k} & \boldsymbol{5k} \end{bmatrix} & \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \end{array}$$

✓ ***Concept de la Superposition (Méthode de Rigidité Directe)***

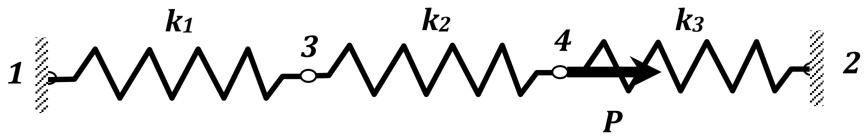
$$\left[k_{e\,1-3}^{(1)}\right] = \begin{array}{cc} \begin{array}{cc} u_1 & u_3 \end{array} & \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} & \begin{array}{c} u_1 \\ u_3 \end{array} \end{array} \qquad \left[k_{e\,3-4}^{(2)}\right] = \begin{array}{cc} \begin{array}{cc} u_3 & u_4 \end{array} & \\ \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix} & \begin{array}{c} u_3 \\ u_4 \end{array} \end{array}$$

$$\left[k_{e\,4-2}^{(3)}\right] = \begin{array}{cc} \begin{array}{cc} u_4 & u_2 \end{array} & \\ \begin{bmatrix} 3k & -3k \\ -3k & 3k \end{bmatrix} & \begin{array}{c} u_4 \\ u_2 \end{array} \end{array}$$

$$
\begin{array}{c}
\begin{array}{cccc} \color{red}{u_1} & \color{red}{u_2} & \color{red}{u_3} & \color{red}{u_4} \end{array} \\
[K] = \begin{bmatrix} k & 0 & -k & 0 \\ 0 & 3k & 0 & -3k \\ -k & 0 & 3k & -2k \\ 0 & -3k & -2k & 5k \end{bmatrix}
\end{array}
\begin{array}{c} \\ \color{red}{u_1} \\ \color{red}{u_2} \\ \color{red}{u_3} \\ \color{red}{u_4} \end{array}
$$

2. Déplacements u_2 et u_3

$$\{F\} = [K]\,\{U\}$$



$$
\begin{Bmatrix} \cancel{F_{1x}} \\ \cancel{F_{2x}} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} \cancel{k} & \cancel{0} & \cancel{-k} & \cancel{0} \\ \cancel{0} & \cancel{3k} & \cancel{0} & \cancel{-3k} \\ \cancel{-k} & \cancel{0} & 3k & -2k \\ \cancel{0} & \cancel{-3k} & -2k & 5k \end{bmatrix} \begin{Bmatrix} \cancel{u_1 = 0} \\ \cancel{u_2 = 0} \\ u_3 \\ u_4 \end{Bmatrix}
$$

$$
\begin{Bmatrix} 0 \\ P \end{Bmatrix} = \begin{bmatrix} 3k & -2k \\ -2k & 5k \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}
$$

$$
\begin{cases} 3k\,u_3 - 2k\,u_4 = 0 \\ -2k\,u_3 + 5k\,u_4 = P \end{cases}
$$

$$
\boldsymbol{u_3 = \frac{2}{11}\,\frac{P}{k} \quad ; \quad u_4 = \frac{3}{11}\,\frac{P}{k}}
$$

3. Réactions aux Nœuds *1* et *2*

$$\{F\} = [K]\,\{U\}$$

$$
\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} k & 0 & -k & 0 \\ 0 & 3k & 0 & -3k \\ -k & 0 & 3k & -2k \\ 0 & -3k & -2k & 5k \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \end{Bmatrix}
$$

$$
\boldsymbol{F_{1x} = -\frac{2}{11}\,P \quad ; \quad F_{2x} = -\frac{9}{11}\,P \quad ; \quad F_{3x} = 0 \quad ; \quad F_{4x} = P}
$$

✓ ***Vérification :*** $F_{4x} = F_{1x} + F_{2x}$ ***OK !!***

4. Effort dans Chaque Ressort

$$\{f_e\} = [k_e]\{u_e\}$$

➢ ***Elément 1***

$$
\begin{Bmatrix} f_{1x}^{(1)} \\ f_{3x}^{(1)} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} 0 \\ u_3 \end{Bmatrix} \Rightarrow \boldsymbol{f_{1x}^{(1)} = -\frac{2}{11}P \quad ; \quad f_{3x}^{(1)} = \frac{2}{11}P}
$$

➢ ***Elément 2***

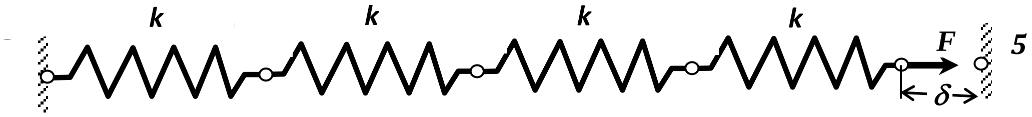
$$
\begin{Bmatrix} f_{3x}^{(2)} \\ f_{4x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} \Rightarrow \boldsymbol{f_{3x}^{(2)} = -\frac{2}{11}P \quad ; \quad f_{4x}^{(2)} = \frac{2}{11}P}
$$

➢ ***Elément 3***

$$
\begin{Bmatrix} f_{4x}^{(3)} \\ f_{2x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 3k & -3k \\ -3k & 3k \end{bmatrix} \begin{Bmatrix} u_4 \\ 0 \end{Bmatrix} \Rightarrow \boldsymbol{f_{4x}^{(3)} = \frac{9}{11}P \quad ; \quad f_{2x}^{(3)} = -\frac{9}{11}P}
$$

EXERCICE N°02

1. Matrice de Rigidité Globale



➢ ***Matrices de Rigidité Élémentaire***

$$[\boldsymbol{k}^{(1)}] = [\boldsymbol{k}^{(2)}] = [\boldsymbol{k}^{(3)}] = [\boldsymbol{k}^{(4)}] = \begin{bmatrix} \boldsymbol{k} & -\boldsymbol{k} \\ -\boldsymbol{k} & \boldsymbol{k} \end{bmatrix}$$

➢ ***Assemblage des Matrices Élémentaires en Matrice Globale***

$$[K] = \sum_{i=1}^{n} [k^{(i)}] \qquad \text{Soit :} \qquad [K] = [k^{(1)}] + [k^{(2)}] + [k^{(3)}] + [k^{(4)}]$$

✓ ***Concept de la Superposition (Méthode de Rigidité Directe)***

$$[\boldsymbol{K}] = \begin{bmatrix} \boldsymbol{k} & -\boldsymbol{k} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{k} & \boldsymbol{2k} & -\boldsymbol{k} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{k} & \boldsymbol{2k} & -\boldsymbol{k} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{k} & \boldsymbol{2k} & -\boldsymbol{k} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{k} & \boldsymbol{k} \end{bmatrix}$$

$$[\boldsymbol{K}] = \begin{bmatrix} \boldsymbol{200} & -\boldsymbol{200} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{200} & \boldsymbol{400} & -\boldsymbol{200} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{200} & \boldsymbol{400} & -\boldsymbol{200} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{200} & \boldsymbol{400} & -\boldsymbol{200} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{200} & \boldsymbol{200} \end{bmatrix}$$

2. Déplacements u_2 et u_4

$$\{F\} = [K]\,\{U\}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} = \begin{bmatrix} 200 & -200 & 0 & 0 & 0 \\ -200 & 400 & -200 & 0 & 0 \\ 0 & -200 & 400 & -200 & 0 \\ 0 & 0 & -200 & 400 & -200 \\ 0 & 0 & 0 & -200 & 200 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{1x} = R_1 \\ F_{2x} = 0 \\ F_{3x} = 0 \\ F_{4x} = 0 \\ F_{5x} = F \end{Bmatrix} = \begin{bmatrix} 200 & -200 & 0 & 0 & 0 \\ -200 & 400 & -200 & 0 & 0 \\ 0 & -200 & 400 & -200 & 0 \\ 0 & 0 & -200 & 400 & -200 \\ 0 & 0 & 0 & -200 & 200 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 = \delta = 0.02 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ F \end{Bmatrix} = \begin{bmatrix} 400 & -200 & 0 & 0 \\ -200 & 400 & -200 & 0 \\ 0 & -200 & 400 & -200 \\ 0 & 0 & -200 & 200 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \\ \delta = 0.02 \end{Bmatrix}$$

$$\boldsymbol{u_2 = \frac{\delta}{4}} \quad ; \quad \boldsymbol{u_3 = \frac{\delta}{2}} \quad ; \quad \boldsymbol{u_4 = \frac{3\delta}{4}}$$

$$\boldsymbol{u_2 = 5\,mm} \quad ; \quad \boldsymbol{u_3 = 10\,mm} \quad ; \quad \boldsymbol{u_4 = 15\,mm}$$

3. Forces Nodales Globales

$$\{F\} = [K]\,\{U\}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} = \begin{bmatrix} 200 & -200 & 0 & 0 & 0 \\ -200 & 400 & -200 & 0 & 0 \\ 0 & -200 & 400 & -200 & 0 \\ 0 & 0 & -200 & 400 & -200 \\ 0 & 0 & 0 & -200 & 200 \end{bmatrix} \begin{Bmatrix} 0.000 \\ 0.005 \\ 0.010 \\ 0.015 \\ 0.020 \end{Bmatrix}$$

$$\begin{cases} \boldsymbol{F_{1x} = -1} \\ \boldsymbol{F_{2x} = 0} \\ \boldsymbol{F_{3x} = 0} \\ \boldsymbol{F_{4x} = 0} \\ \boldsymbol{F_{5x} = 1} \end{cases} \qquad \boldsymbol{(kN)}$$

4. Forces Élémentaires Locales

$$\{f_e\} = [k_e]\{u_e\}$$

- ***Elément 1***

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.005 \end{Bmatrix}$$

$$\boldsymbol{f_{1x}^{(1)} = -1\ kN} \quad ; \quad \boldsymbol{f_{2x}^{(1)} = 1\ kN}$$

- ***Elément 2***

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} 0.005 \\ 0.01 \end{Bmatrix}$$

$$\boldsymbol{f_{2x}^{(2)} = -1\ kN} \quad ; \quad \boldsymbol{f_{3x}^{(2)} = 1\ kN}$$

- ***Elément 3***

$$\begin{Bmatrix} f_{3x}^{(3)} \\ f_{4x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} 0.01 \\ 0.015 \end{Bmatrix}$$

$$\boldsymbol{f_{3x}^{(3)} = -1\ kN} \quad ; \quad \boldsymbol{f_{4x}^{(3)} = 1\ kN}$$

- ***Elément 4***

$$\begin{Bmatrix} f_{4x}^{(4)} \\ f_{5x}^{(4)} \end{Bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} 0.015 \\ 0.02 \end{Bmatrix}$$

$$\boldsymbol{f_{4x}^{(4)} = -1\ kN} \quad ; \quad \boldsymbol{f_{5x}^{(4)} = 1\ kN}$$