

Mathematical Tools

Exercise 01 :

Two points A and B, have for Cartesian coordinates in space: A (2, 3, -3), B (5, 7, 2)

Determine the components of the vector AB as well as its module, its direction and its direction.

Exercise 02 :

Consider two vectors: $\vec{u} = \begin{pmatrix} -8 \\ 2 \\ 5 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 3 \\ -9 \\ -1 \end{pmatrix}$

Calculate:

- 1) The magnitudes of the two vectors.
- 2) Their scalar product.
- 3) Their vector product.
- 4) The cosine directions of their unit vectors.

Exercise 03 :

Be the vectors $\vec{V}_1, \vec{V}_2, \vec{V}_3$ and \vec{V}_4 such as:

$$\vec{V}_1 = \vec{i} + 4\vec{k}, \vec{V}_2 = 2\vec{i} + y\vec{j} + z\vec{k}, \vec{V}_3 = \vec{i} - 2\vec{j} + 4\vec{k},$$

$$\text{And } \vec{V}_4 = 4\vec{i} + y\vec{j} + 2\vec{k}.$$

1) Determine y and z for vectors \vec{V}_1 and \vec{V}_2 to be collinear,

2) Determine y for vectors \vec{V}_3 and \vec{V}_4 to be perpendicular,

Exercise 04 :

Let points $A(3, 7, -2), B(0, 5, 1)$ and $C(-2, 2, 4)$ be given.

Determine the point $D(x, y, z)$ such that \overrightarrow{CD} is the unit vector of \overrightarrow{AB}

Exercise 05 :

Let $\vec{P} = 2t\vec{i} + 5t^2\vec{j} - 7t^3\vec{k}$ and $\vec{Q} = -4t^3\vec{i} + 10t^2\vec{j} - 2t\vec{k}$

- 1) Check the following relationships.

$$\frac{d}{dt}(\vec{P} \cdot \vec{Q}) = \frac{d\vec{P}}{dt} \cdot \vec{Q} + \vec{P} \cdot \frac{d\vec{Q}}{dt}$$

$$\frac{d}{dt}(\vec{P} \wedge \vec{Q}) = \frac{d\vec{P}}{dt} \wedge \vec{Q} + \vec{P} \wedge \frac{d\vec{Q}}{dt}$$