The Torsors

Exercise 01 :

Considering the following vectors $\overrightarrow{OA} = 3a\vec{\imath}, \overrightarrow{OB} = 4a\vec{\jmath}$ and $\overrightarrow{OC} = 5a\vec{k}$,

Determine the torsor at O of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CB}$.

Exercise 02 :

Let the torsor $[T_1]_o$ be defined at point O, the origin of a direct orthonormal frame $R(O, \vec{\imath}, \vec{\jmath}, \vec{k})$, by the three following vectors:

$\vec{V}_a = -2\vec{\imath} + 3\vec{\jmath} - 7\vec{k}$ defined at point $A(1,0,0)$

$\vec{V}_b = 3\vec{\imath} - \vec{\jmath} - \vec{k}$ defined at point $B(0,1,0)$

$\vec{V}_c = -\vec{\imath} - 2\vec{\jmath} + 8\vec{k}$ defined at point $C(0,0,1)$

Let $[T_2]_o$ be a torsor defined at point O by its reduction elements $\vec{R}_2$ and $\vec{M}_{2o}$ such that: $[T_2]_o = \begin{cases} \vec{R}_2 = 2\vec{\imath} + \vec{\jmath} + 3\vec{k} \\ \vec{M}_{2o} = -3\vec{\imath} + 2\vec{\jmath} - 7\vec{k} \end{cases}$

1) Determine the reduction elements of $[T_1]$ at point O.

2) Determine the pitch and the central axis of the torsor $[T_2]$

3) Calculate the sum of the two torsors.

4) Calculate the comoment of the two torsors.

5) Calculate the scalar invariant of the sum torsor $[T] = [T_1] + [T_2]$.

Exercise 03 :

Let A be a point in space in $R(O, \vec{\imath}, \vec{\jmath}, \vec{k})$,with

$\overrightarrow{OA} = \frac{21}{9}\vec{\imath} - \frac{4}{9}\vec{\jmath} - \frac{12}{9}\vec{k}$ and a vector $\vec{V}_1 = -3\vec{\imath} + \vec{\jmath} + 3\vec{k}$ whose axis passes through point A.

Let $[T_2]_o$ be a torsor defined at point O by its reduction elements $\vec{R}_2$ and $\vec{M}_{2o}$ such that:

$$[T_2]_o = \begin{cases} \vec{R}_2 = (\alpha - 4)\vec{\imath} + \alpha\vec{\jmath} + 3\alpha\vec{k} \\ \vec{M}_{2o} = (2\alpha + 9)\vec{\jmath} + (-3\alpha - \frac{2}{3})\vec{k} \end{cases}$$

1. Determine the reduction elements of the torsor $[T_1]_O$ whose resultant is the vector $\vec{V}_1$.
2. For what value of α are the two torsors equal;
3. Deduce the pitch and the central axis of the torsor $[T_2]_O$ for this value of α.
4. Calculate the product of the two torsors for $\alpha = 2$.

Exercise 04 :

In the reference frame $R(O,\vec{\imath},\vec{\jmath},\vec{k})$, consider the points $A(3,0,0)$ and $B(-1,2,1)$ in the affine space.

Let $\vec{V}(M)$ be an equiprojective vector field defined as follows:

$$\vec{V}(0) = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \vec{V}(A) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{V}(B) = \begin{pmatrix} 2 \\ -1 \\ 9 \end{pmatrix}$$

1. Determine the resultant vector $\vec{R}$ of the torsor associated with the field $\vec{V}(M)$.
2. Deduce the expression of the vector field $\vec{V}(M)$ at any point $M(x,y,z)$ in the space.
3. Determine the central axis Δ of the torsor.

Exercise 05 :

In a direct orthonormal frame $R(O,\vec{\imath},\vec{\jmath},\vec{k})$ we consider the vector field $\vec{V}(M)$ whose components are defined as a function of the coordinates (x,y,z) of M by:

$$\begin{cases} V_x = 1 + 3y - tz \\ V_y = -3x + 2tz \\ V_z = 2 + tx - t^2 y \end{cases}$$

1. Calculate the vector $\vec{V}(M)$ at point O.
2. For which values of t is this field antisymmetric?
3. For each value of t found, determine the reduction elements of the torsor (resultant and moment at O).
4. Decompose the torsor associated with $\vec{V}(M)$ into a sum of a couple and a slider, and indicate the reduction elements.
5. Determine the position of the central axis of the torsor for $t = 0$ and $t = 2$.