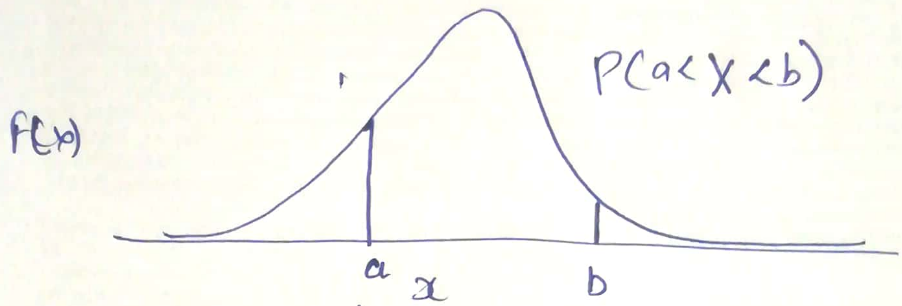
Continuous Probability Distributions

For continuous random variables probabilities are areas under the curve,

$P(a<X<b)$



$P(X=a)=0$

Since a is infinitely small area so it makes sense for calculating probability in an interval.

$F(x) \rightarrow$ represents the height of the curve at point x

For any continuous probability distribution:-

(1) $F(x) \geq 0$ for all x

(2) The area under the entire curve should be equal to one.

(Q) For a random variable X: $F(x) = cx^3$ for $2 \leq x \leq 4$ & 0 otherwise. What value of c makes this a legitimate probability distribution?

$$\Rightarrow \int_{-\infty}^{\infty} F(x)\,dx = 1 \Rightarrow \int_{2}^{4} cx^3\,dx = 1.$$

$$c\int_2^4 x^3\,dx = \frac{c}{4}\left[x^4\right]_2^4 = \frac{c}{4}\left[4^4 - 2^4\right]$$

$$= \frac{c}{\bullet}\left[4^3 - \frac{2^4}{4}\right] = c\left[64 - 4\right] \Rightarrow 60c = 1$$

$$c = \frac{1}{60}$$

Mean $\rightarrow E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$= \int_2^4 x \cdot \frac{1}{60} x^3\, dx = \frac{1}{60}\int_2^4 x^4\, dx$$

$$= \frac{1}{60}\left[\frac{x^5}{5}\right]_2^4 = \frac{1}{60}\left[\frac{4^5}{5} - \frac{2^5}{5}\right] = \cancel{\frac{992}{300}}\ 3.3$$

$$E(x-\mu)^2 = E(x^2) - \left[E(x)\right]^2$$

$\downarrow$

variance

$=$

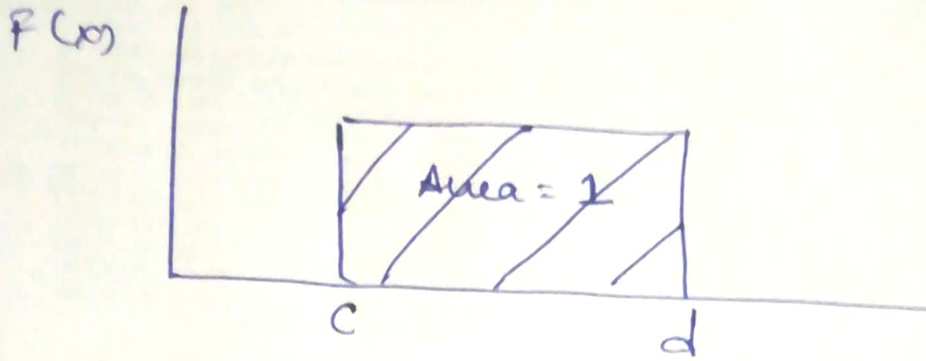
$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_2^4 x^2 \frac{1}{60} x^3\, dx$$

$$= \frac{1}{60}\left[\frac{x^6}{6}\right]_2^4 = \left[\frac{4^6}{6} - \frac{2^6}{6}\right] \times \frac{1}{60} = \frac{4^6 - 2^6}{360}$$

$\rightarrow 11.2$

Var $= 11.2 - (3.3)^2 = \boxed{0.31}$

Continuous uniform distribution



$$\text{Area} = (d-c) \cdot F(x)$$

$$1 = (d-c) \cdot F(x) \Rightarrow \boxed{F(x) = \frac{1}{d-c}}$$

The PDF of uniform distribution

$$F(x) = \begin{cases} \frac{1}{d-c} & c \le x \le d \\ 0 & \text{else} \end{cases}$$

Median is the point where area is 0.5
so it would be $\frac{c+d}{2}$

$$\mu = \frac{c+d}{2}, \quad \sigma^2 = \frac{1}{12}(d-c)^2$$

Normal Distribution

The pdf of normal distribution is

$$F(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

for $-\infty < x < \infty$

μ is the mean of the distribution
$\sigma > 0$

Normal Approximation to the binomial distribution

we can improve the approximation of discrete binomial distribution by Normal distribution using continuity correction.

Ex:-

Desired Prob		Normal Prob
$P(X \geq 52)$		$P\left(Z > \frac{51.5 - \mu}{\sigma}\right)$
$P(X > 52)$		$P\left(Z > \frac{52.5 - \mu}{\sigma}\right)$
$P(X \leq 52)$	$=$	$P\left(Z < \frac{52.5 - \mu}{\sigma}\right)$
$P(X < 52)$	$=$	$P\left(Z < \frac{51.5 - \mu}{\sigma}\right)$

Chi - Square Distribution

If a random variable Z has the standard normal distribution then Z^2 has a χ^2 distribution with one degree of freedom.

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