Sheet 1
Chapter One

Logical operations:

$\mathcal{A}$	$\neg\mathcal{A}$
1	0
0	1

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}\,\&\,\mathcal{B}$	$\mathcal{A}\vee\mathcal{B}$	$\mathcal{A}\to\mathcal{B}$	$\mathcal{A}\leftrightarrow\mathcal{B}$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Which of these sentences are propositions? What are the truth values of those that are propositions?
a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) 2 + 3 = 5.
d) 5 + 7 = 10.
e) x + 2 = 11.
f) Answer this question.
a) This is a true proposition.
b) This is a false proposition (Tallahassee is the capital).
c) This is a true proposition.
d) This is a false proposition.
e) This is not a proposition (it contains a variable; the truth value depends on the value assigned to x).
f) This is not a proposition, since it does not assert anything.
Let p and q be the propositions
p : It is below freezing.
q : It is snowing.
Write these propositions using p and q and logical connectives (including negations).
a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).

e) If it is below freezing, it is also snowing.
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.
a) Here we have the conjunction p A q.
b) Here we have a conjunction of p with the negation of q, namely p A --.q. Note that "but" logically means
the same thing as "and."
c) Again this is conjunction: --.p A --.q.
d) Here we have a disjunction, p V q. Note that V is the inclusive or, so the "(or both)" part of the English sentence is automatically included.
e) This sentence is a conditional statement, p -> q.
f) This is a conjunction of propositions, both of which are compound: (p V q) A (p-> --q.q).
g) This is the biconditional p f-+ q.
For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings
of or do you think is intended?
a) To take discrete mathematics, you must have taken calculus or a course in computer science.
b) When you buy a new car from Acme Motor Company, you get $2000 back in cash or a 2% car loan.
c) Dinner for two includes two items from column A or three items from column B.
d) School is closed if more than 2 feet of snow falls or if the wind chill is below −100.
a) If this is an inclusive *or,* then it is allowable to take discrete mathematics if you have had calculus or computer science or both. If this is an exclusive *or,* then a person who had both courses would not be allowed
to take discrete mathematics-only someone who had taken exactly one of the prerequisites would be allowed
in. Clearly the former interpretation is intended; if anything, the person who has had both calculus and computer science is even better prepared for discrete mathematics.
b) If this is an inclusive *or,* then you can take the rebate, or you can sign up for the low-interest loan, or you
can demand both of these incentives. If this is an exclusive *or,* then you will receive one of the incentives but
not both. Since both of these deals are expensive for the dealer or manufacturer, surely the

exclusive or was intended.
c) If this is an inclusive *or*, you can order two items from column A (and none from B), or three items from
column B (and none from A), or five items (two from A and three from B). If this is an exclusive *or*, which it
surely is here, then you get your choice of the two A items or the three B items, but not both.
d) If this is an inclusive *or*, then lots of snow, or extreme cold, or a combination of the two will close school.
If this is an exclusive *or*, then one form of bad weather would close school but if both of them happened then
school would meet. This latter interpretation is clearly absurd, so the inclusive *or* is intended.
Determine whether each of these conditional statements
is true or false.
a) If 1 + 1 = 2, then 2 + 2 = 5.
b) If 1 + 1 = 3, then 2 + 2 = 4.
c) If 1 + 1 = 3, then 2 + 2 = 5.
d) If monkeys can fly, then 1 + 1 = 3.
a) Since the hypothesis is true and the conclusion is false, this conditional statement is false.
b) Since the hypothesis is false and the conclusion is true, this conditional statement is true.
c) Since the hypothesis is false and the conclusion is false, this conditional statement is true. Note that the conditional statement is false in both parts (b) and part (c); as long as the hypothesis is false, we need to look
no further to conclude that the conditional statement is true.
d) Since the hypothesis is false, this conditional statement is true.

State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Many forms of answers for this exercise are possible.

a) One form of the converse that reads well in English is "I will ski tomorrow only if it snows today." We could state the contrapositive as ''If I don't ski tomorrow, then it will not have snowed today." The inverse is "If it does not snow today, then I will not ski tomorrow."

b) The proposition as stated can be rendered "If there is going to be a quiz, then I will come to class." The converse is "If I come to class, then there will be a quiz." (Or, perhaps even better, "I come to class only if there's going to be a quiz.") The contrapositive is "If I don't come to class, then there won't be a quiz." The inverse is "If there is not going to be a quiz, then I don't come to class.''

c) There is a variable ("a positive integer") in this sentence, so technically it is not a proposition. Nevertheless, we can treat sentences such as this in the same way we treat propositions. Its converse is "A positive integer is prime if it has no divisors other than 1 and itself." (Note that this can be false, since the number 1 satisfies the hypothesis but not the conclusion.) The contrapositive of the original proposition is "If a positive integer has a divisor other than 1 and itself, then it is not prime." (We are simplifying a bit here, replacing ''does have no divisors" with "has a divisor." Note that this is always true, assuming that we are talking about positive divisors.) The inverse is "If a positive integer is not prime, then it has a divisor other than 1 and itself."

Construct a truth table for each of these compound propositions.

a) $p \rightarrow (\neg q \vee r)$

p	q	r	$(p \rightarrow (\neg q \vee r))$
F	F	F	**T**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**T**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

b) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

p	q	r	$((p \rightarrow q) \vee (\neg p \rightarrow r))$
F	F	F	**T**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**T**
T	F	T	**T**
T	T	F	**T**
T	T	T	**T**

c) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

p	q	r	$((p \rightarrow q) \wedge (\neg p \rightarrow r))$
F	F	F	**F**
F	F	T	**T**
F	T	F	**F**
F	T	T	**T**
T	F	F	**F**
T	F	T	**F**
T	T	F	**T**
T	T	T	**T**

d)$(p \leftrightarrow q) \lor (\neg q \leftrightarrow r)$

p	q	r	$((p \leftrightarrow q) \lor (\neg q \leftrightarrow r))$
F	F	F	**T**
F	F	T	**T**
F	T	F	**T**
F	T	T	**F**
T	F	F	**F**
T	F	T	**T**
T	T	F	**T**
T	T	T	**T**

e)$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$((\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r))$
F	F	F	**T**
F	F	T	**F**
F	T	F	**T**
F	T	T	**F**
T	F	F	**F**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

Show that *(p → q) → r* and *p → (q → r)* are not logically equivalent.

p	q	r	$((p \to q) \to r)$
F	F	F	**F**
F	F	T	**T**
F	T	F	**F**
F	T	T	**T**
T	F	F	**T**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

p	q	r	$(p \to (q \to r))$
F	F	F	**T**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**T**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

Show that *(p ∧ q) → r* and *(p → r) ∧ (q → r)* are not logically equivalent.

p	q	r	$((p \land q) \to r)$
F	F	F	**T**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**T**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

p	q	r	$((p \to r) \land (q \to r))$
F	F	F	**T**
F	F	T	**T**
F	T	F	**F**
F	T	T	**T**
T	F	F	**F**
T	F	T	**T**
T	T	F	**F**
T	T	T	**T**

Best of luck!