Sheet 1
Chapter One

Logical operations:

$\mathcal{A}$	$\neg\mathcal{A}$
1	0
0	1

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}\,\&\,\mathcal{B}$	$\mathcal{A}\vee\mathcal{B}$	$\mathcal{A}\rightarrow\mathcal{B}$	$\mathcal{A}\leftrightarrow\mathcal{B}$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Which of these sentences are propositions? What are the truth values of those that are propositions?
a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) 2 + 3 = 5.
d) 5 + 7 = 10.
e) x + 2 = 11.
f) Answer this question.

Let p and q be the propositions
p : It is below freezing.
q : It is snowing.
Write these propositions using p and q and logical connectives (including negations).
a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).

e) If it is below freezing, it is also snowing.
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?
a) To take discrete mathematics, you must have taken calculus or a course in computer science.
b) When you buy a new car from Acme Motor Company, you get $2000 back in cash or a 2% car loan.
c) Dinner for two includes two items from column A or three items from column B.
d) School is closed if more than 2 feet of snow falls or if the wind chill is below −100.

Determine whether each of these conditional statements is true or false.
a) If 1 + 1 = 2, then 2 + 2 = 5.
b) If 1 + 1 = 3, then 2 + 2 = 4.
c) If 1 + 1 = 3, then 2 + 2 = 5.
d) If monkeys can fly, then 1 + 1 = 3.

Construct a truth table for each of these compound propositions.

a) $p \rightarrow (\neg q \vee r)$
b) $(p \rightarrow q) \vee (\neg p \rightarrow r)$
c) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$
d) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
e) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

Show that $(p \rightarrow q) \rightarrow r$ **and** $p \rightarrow (q \rightarrow r)$ **are not logically equivalent.**

Show that $(p \wedge q) \rightarrow r$ **and** $(p \rightarrow r) \wedge (q \rightarrow r)$ **are not logically equivalent.**

Best of luck!