Sheet 2
Chapter One

Tautology: A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it.

Example: $p \vee \neg p$

Contradiction: A compound proposition that is always false is called a *contradiction*.

Example : $p \wedge \neg p$
A compound proposition that is neither a tautology nor a contradiction is called a ***contingency***.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The converse of $p \rightarrow q$ is the proposition $q \rightarrow p$

The contrapositive of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$

The inverse of $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$

A conditional statement and its contrapositive are equivalent. The converse and the inverse of a conditional statement are also equivalent, but neither is equivalent to the original conditional statement

BICONDITIONALS: $p \leftrightarrow q$ has exactly the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of Logical Operators:

TABLE 8 **Precedence of Logical Operators.**

Operator	*Precedence*
$\neg$	1
$\land$	2
$\lor$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logical Equivalences

Compound propositions that have the same truth values in all possible cases are called logically equivalent.

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.

Propositional Satisfiability

A compound proposition is **satisfiable** if there is an assignment of truth values to its variables that makes it true.
What is the difference between satisfiability and contingency?

Logical Equivalence Laws:

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.
$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

State the converse, contrapositive, and inverse of each of these conditional statements.

S2

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Show that $[(p\to q)\wedge(q\to r)]\to(p\to r)$ **is a tautology.**

Determine whether each of these compound propositions is satisfiable.

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$

c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

Prove that $(p \wedge \neg q) \vee q \equiv p \vee q$

Prove that $\neg (p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are equivalent.

An island has two kinds of inhabitants, knights, who always tell the truth, and knaves, who always lie.

A says "B is a knight."

B says "The two of us are of opposite types."

What are the types of A and B?

Logic Puzzles

Five friends have access to a chat room:

- **Either Kevin or Heather, or both, are chatting**
- **Either Randy or Vijay, but not both, are chatting**
- **If Abby is chatting, then so is Randy**
- **Vijay and Kevin are either both chatting or neither is**
- **If Heather is chatting, then so are Abby and Kevin**

who is chatting right now?

Best of luck!