Homework 4.

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1) $$\sigma(z)_i = \frac{e^{z_i}}{\sum_{j=1}^{n} e^{z_j}}$$

when $i=j$, $$\frac{d\sigma(z)_i}{dz_j} = \frac{d\sigma(z)_i}{dz_i} = \frac{(e^{z_i})' \cdot \sum_{k=1}^{n} e^{z_k} - e^{z_i} \cdot (\sum_{k=1}^{n} e^{z_k})'}{(\sum_{k=1}^{n} e^{z_k})^2} =$$

$$= \frac{e^{z_i} \cdot \sum_{k=1}^{n} e^{z_k} - e^{z_i} e^{z_i}}{(\sum_{k=1}^{n} e^{z_k})^2} = \frac{e^{z_i}}{\sum_{k=1}^{n} e^{z_k}} \left(\frac{\sum_{k=1}^{n} e^{z_k}}{\sum_{k=1}^{n} e^{z_k}} - \frac{e^{z_i}}{\sum_{k=1}^{n} e^{z_k}} \right) =$$

$$= \sigma(z)_i \cdot (1 - \sigma(z)_i)$$

when $i \neq j$, $$\frac{d\sigma(z)_i}{dz_j} = \frac{(e^{z_i})' \cdot \sum_{k=1}^{n} e^{z_k} - e^{z_i} (\sum_{k=1}^{n} e^{z_k})'}{(\sum_{k=1}^{n} e^{z_k})^2} =$$

$$= \frac{0 \cdot \sum_{k=1}^{n} e^{z_k} - e^{z_i} \cdot e^{z_j}}{(\sum_{k=1}^{n} e^{z_k})^2} = -\frac{e^{z_i}}{\sum_{k=1}^{n} e^{z_k}} \cdot \frac{e^{z_j}}{\sum_{k=1}^{n} e^{z_k}} =$$

$$= -\sigma(z)_i \cdot \sigma(z)_j$$

$$\frac{d\sigma(z)_i}{dz_j} = \begin{cases} \sigma(z)_i \cdot (1 - \sigma(z)_i), & \text{when } i=j \\ -\sigma(z)_i \cdot \sigma(z)_j, & \text{when } i \neq j \end{cases}$$

2) $L1 = c \cdot h \cdot w \cdot p$ (where c - number of channels,
h - height of filter,
w - width of filter,
p - number of channels on previous layer)

$L1 = 100 \cdot 5 \cdot 5 \cdot 1 = 2500$

$L2 = 100 \cdot 5 \cdot 5 \cdot 100 = 250\,000$

$L3 = 0$ (pooling has no parameters)

$L4$ = number of neurons from current layer · pervious layer =
$= 100 \cdot 100 \cdot 50 \cdot 50 = 25\,000\,000$

$L5 = 100 \cdot 100 = 10\,000$

$L6 = 100 \cdot 1 = 100$

Weights $= 2500 + 250\,000 + 0 + 25\,000\,000 + 10\,000 + 100 =$
$= 25\,262\,600$

3) Pooling layer is used for a non-linear transformation that permits to summarize the output of a network at a certain location with a single value. This single value is obtained from the statistics of the neighbouring outputs which makes the feature descriptions more robust and invariant to small shifts and distortions of the input data.

4) 1. $z_1 = w_1 \cdot x \qquad z_2 = w_2 \cdot a_1$

$a_1 = g_1(z_1) \qquad a_2 = g_2(z_2)$

2. $Loss(a_2, y^*) = Loss(g_2(z_2), y^*) = Loss(g_2(w_2 \cdot a_1), y^*) =$

$= Loss(g_2(w_2 \cdot g_1(z_1)), y^*) =$

$= Loss(g_2(w_2 \cdot g_1(w_1 \cdot x)), y^*)$

3. $$\frac{dLoss}{dw_2} = \frac{dLoss}{da_2} \cdot \frac{da_2}{dw_2} = \frac{dLoss}{da_2} \cdot \frac{da_2}{dz_2} \cdot \frac{dz_2}{dw_2}$$

4. $$\frac{dLoss}{da_2} = \left(\frac{1}{2}(a_2 - y^*)^2\right)' = a_2 - y^*$$

$$\frac{da_2}{dz_2} = \frac{dg_2(z_2)}{dz_2} = g_2(z_2) \cdot (1 - \cancel{z_2}\, g_2(z_2)) = a_2(1 - a_2)$$

$$\frac{dz_2}{dw_2} = (a_1 \cdot w_2)' = a_1$$

$$\frac{dLoss}{dw_2} = (a_2 - y^*) \cdot a_2(1 - a_2) \cdot a_1$$

5. $$\frac{dLoss}{dw_1} = \frac{dLoss}{da_2} \cdot \frac{da_2}{dz_2} \cdot \frac{dz_2}{da_1} \cdot \frac{da_1}{dz_1} \cdot \frac{dz_1}{dw_1}$$

6. $$\frac{dz_2}{da_1} = (w_2 \cdot a_1)' = w_2$$

$$\frac{da_1}{dz_1} = \frac{dg_1(z_1)}{dz_1} = g_1(z_1) \cdot (1 - g_1(z_1)) = a_1 \cdot (1 - a_1)$$

$$\frac{dz_1}{dw_1} = (w_1 \cdot x)' = x$$

$$\frac{dLoss}{dw_1} = (a_2 - y^*) \cdot a_2(1 - a_2) \cdot w_2 \cdot a_1 \cdot (1 - a_1) \cdot x$$

7. $$w_1 = w_1 - \alpha \cdot \frac{dLoss}{dw_1} = w_1 - \alpha \cdot ((a_2 - y^*) \cdot a_2(1 - a_2) \cdot w_2 \cdot$$
$$\cdot\, a_1(1 - a_1) \cdot x)$$