

# MSCR 500 Homework Assignment 5

Anish Shah

11/05/19

## Q1.

Random sample of 10 diabetic women.

$\bar{x} = 84$  mmHg

$s = 9.1$  mmHg

Test if true DBP for female is  $> \mu$  ( $\mu$  = true mean DBP 74.4 mmHg)

$\alpha = 0.05$

$N(0,1)$

### Hypothesis:

$H_0 \rightarrow \mu = \bar{x}$

$H_1 \rightarrow \mu < \bar{x}$  (left tailed hypothesis)

### Parameters:

population mean, sample mean

$\alpha = 0.05$

### Name of hypothesis test:

one-sample, left-tailed t-test

### Justification:

The population follows a normal distribution but population SD is unknown (thus t is correct), and this is talking about a single sample population, with knowledge of the population mean given, but not the population SD. Also,  $n = 10$ , which is  $n < 15$ , but can use normal distribution since that is given.

### Test Statistic Computation:

$$\begin{aligned} T &= (\bar{x} - \mu) / (s / \sqrt{n}) \\ &= 84 - 74.4 / (9.1 / \sqrt{10}) \\ &= 3.336 \end{aligned}$$

Distribution of test statistic under  $H_0$ :

$T \sim t(9)$

**P-value computation:**

$t = 3.336$ , which with  $df = 9$ , gets us to a  $t(0.005) = 3.250$ . This suggests that  $P < 0.005$  when  $T \geq 3.3$

Decision to *REJECT*  $H_0$  at  $\alpha = 0.05$

*Conclusion:*

$\alpha = 0.05$ , and  $P < 0.005$ . Since  $\alpha > P$ , we can reject the  $H_0$ . This means that the sample mean of DBP for diabetics is likely greater than 74.4.

## Q2.

$\bar{x} = 747.3$  mg of calcium intake

$s = 262.2$  mg

$\alpha = 0.05$

$\mu = 800$  mg

$n = 40$

**Hypothesis:**

$H_0 \rightarrow \mu = \bar{x}$

$H_1 \rightarrow \mu > \bar{x}$  (right-tailed)

**Parameters:**

$\alpha = 0.05$

**Name of hypothesis test:**

one-sided, right-tailed t-test

**Justification:**

We have a  $n=40$  population, allowing us to use normal distribution pattern, but population SD is unknown, thus must use t-distribution. Only single sample for comparison to population mean.

**Test Statistic Computation:**

$$\begin{aligned} T &= (\bar{x} - \mu) / (s / \sqrt{n}) \\ &= (747.3 - 800) / (262.2 / \sqrt{40}) \\ &= -1.2712 \end{aligned}$$

**Distribution of test statistic under  $H_0$ :**

$T \sim t(39)$

**P-value computation:**

$t = -1.2712$  with  $df = 39$ .

$t(0.10) = -1.304$

$p(t = -1.271) \leq 0.10$

Decision to *FAIL TO REJECT*  $H_0$  at  $\alpha = 0.05$

**Conclusion:**

We fail to reject the null,  $H_0$ , at an  $\alpha$  of 0.05. The mean calcium of the poor is not significantly less than the population mean of 800 mg calcium.

### Q3.

Random sample of 10 diabetic women.

$\bar{x} = 84$  mmHg

$\sigma = 9.1$  mmHg

Test if true DBP for female is  $> \mu$  ( $\mu$  = true mean DBP 74.4 mmHg)

$\alpha = 0.05$

$N(0,1)$

**Hypothesis:**

$H_0 \rightarrow \mu = \bar{x}$

$H_1 \rightarrow \mu < \bar{x}$  (left tailed hypothesis)

**Parameters:**

population mean, sample mean

$\alpha = 0.05$

**Name of hypothesis test:**

one-sample, left-tailed z-test

**Justification:**

The population follows a normal distribution and population SD is known (thus z is correct), and this is talking about a single sample population, with knowledge of the population mean given. We also know its a normal distribution, can use z-distribution.

**Test Statistic Computation:**

$Z = (\bar{x} - \mu) / (\sigma / \sqrt{n})$

$= 84 - 74.4 / (9.1 / \sqrt{10})$

$= 3.336 \rightarrow \text{area under curve} = 0.9996$

Distribution of test statistic under  $H_0$ :

$X \sim N(\mu, (\sigma/\sqrt{n})^2)$

$X \sim N(74.4, 8.281)$

**P-value computation:**

$$P(Z \geq 3.336) \text{ for left sided test} = 1 - 0.9996 = 0.0004$$

Decision to *REJECT*  $H_0$  at  $\alpha = 0.05$

*Conclusion:*

$\alpha = 0.05$ , and  $P = 0.0004$ . Since  $p < \alpha$ , we can reject the  $H_0$ . This means that the sample mean of DBP for diabetics is likely greater than 74.4.

**Q4.**

$\bar{x} = 747.3$  mg of calcium intake

$\sigma = 262.2$  mg

$\alpha = 0.05$

$\mu = 800$  mg

$n = 40$

**Hypothesis:**

$H_0 \rightarrow \mu = \bar{x}$

$H_1 \rightarrow \mu > \bar{x}$  (right-tailed)

**Parameters:**

$\alpha = 0.05$

**Name of hypothesis test:**

one-sided, right-tailed z-test

**Justification:**

We have a  $n=40$  population, allowing us to use normal distribution pattern, and population SD is known, thus can use z-test.

**Test Statistic Computation:**

$$\begin{aligned} Z &= (\bar{x} - \mu) / (\sigma / \sqrt{n}) \\ &= (747.3 - 800) / (262.2 / \sqrt{40}) \\ &= -1.2712 \end{aligned}$$

**Distribution of test statistic under  $H_0$ :**

$$X \sim N(800, 1718.721)$$

**P-value computation:**

$$P(Z = -1.2712) = 0.1020$$

Decision to *FAIL TO REJECT*  $H_0$  at  $\alpha = 0.05$

**Conclusion:**

The P value associated with the Z score is 0.1020.  $\alpha$  is  $< p$ , thus we cannot reject the null hypothesis,  $H_0$ . We cannot exclude that the sample mean of calcium in the poor is less than the population mean of calcium.