Homework Assignment 4

1. Area under normal curve (-1.03, 1.85), in Z ~ N(0,1)

This requires the Z-tables and the normal probability distribution
z = -1.03 –> 0.8485 (on the positive side of the curve), requires reflection along y-axis
z = 1.85 -> 0.9678

Area = z(-1.03) <= Z <= z(1.85)
z(1.85) - z(0) = 0.9678 - 0.5000 = 0.4678
z(-1.03) - z(0) = 0.8485 - 0.5000 = 0.3485

Area = 0.4678 + 0.3485 = 0.8163
Assuming z-bar follows a normal distribution..
P(-1.03 < Z < 1.85) = Area under the curve = 0.8163 = probability of Z

2. Serum cholesterol follow normal distribution.
   mean = 175 mg per 100 ml
   SD = 30 mg per 100 ml

Proportion of kids c- cholesterol within 2 SD x mean?

X ~ N(u, sigmaˆ2/n)

First need to z-normalize the scores
Z = x - u / SD
Zupper = 235 - 175 / 30 = 2
Zlower = 115 - 175 / 30 = -2

P(-2 < Z < 2) = (0.9772 - 0.500) + (0.9772 - 0.500) = 0.9544

3. 20% of a US city believe Nicaragua is in Africa. What is the probability that >= 200 of next 1000 people randomly sample (same city) believes that Nicaragua is located in Africa?

This is a question of probability. The potential events are BELIEVE or NOT BELIEVE, within each individual. The individual events are independent, with the same probability (0.2) per trial. This follows a binomial discrete variable distribution, however the numbers are so large that we cannot use a calculator easily.

n = 1000
k = 200
p = 0.2

u = n x p = 200
sigma = sqrt(n x p x (1-p)) = 12.6491

This problem asks for probability AT LEAST 200 believe in Africa

z = k - u / sigma = 200 - 200 / 12.6491 = 0

P(X >= 200) = 50%

4. Describe distribution of sample mean, unless it can't be determined. Give mean and SD of sample mean as well.

a. Cholesterol measured in CCU, normally distributed
   n = 9
   u = 250
   sigma = 21

X-bar ~ N(u, sigmaˆ2/n) (as it is normal distributed)
X-bar ~ N(250, 49)

b. Cholesterol of random patients in CCU, unknown distribution
   n = 9 ... small population, cannot assume normal distribution
   u = 250
   sigma = 21

Cannot determine sample distribution due to small size, CTL doesn't apply.
ux = u = 250
sigmax = sigma/sqrt(n) = 21/3 = 7

c. Cholesterol for patients in CCU, unknown distribution
   n = 49 (large enough for normal distribution by CTL)
   u = 250
   sigma = 21

X-bar ~ N(250, 9)

d. Cholesterol for random sample of CCU, non-normal distribution
   n = 49
   u = 250
   sigma = 21

We can apply CTL anyways, because it doesn't care about original distribution.

X-bar ~ N(250, 9)

5. Redo question number 4, and calculate probabilities that sample mean of cholesterol will be between 250 and 260 (when possible).

a. Normal distribution, use Z scores

Zlower = 250 - u / sigma = 0.0
Zupper = 260 - u / sigma = 0.4762

P(250 < X < 260) = 0.6808 - 0.5 = 0.1808

b. Non-normal distribution, small size, cannot use either Z or T scores

c. Normal distribution by CTL, and sigma known

Zlower = 0.0
Zupper = 0.4762
P(250 < X < 260) = 0.1808

d. Normal distribution by CTL, sigma known

P(250 < X < 260) = 0.1808

6. Researchers studies sugar in third trimester. Find 95% CI.
   n = 52
   x-bar = 70.12
   sigma = 10

CI = X-bar ± zalpha x sigma/sqrt(n)
= 70.12 ± z(95/2) x 10/sqrt(52)
= 70.12 ± 0.6808 x 1.3868
= 70.12 ± 0.9441

95% CI = (69.1759, 71.0641)

In this sample of pregnant women, using a normal distribution as the population mean glucose is known, we are 95% confident that the mean of sugars lies within our reported CI.

7. Investigators looked at morning plasma citrate concentrations. Known to follow normal distribution. Find 95% CI.
   n = 10
   x-bar = 118.5
   sigma = 20.8

95% CI = (114.022, 122.978)

8. Maternity ward records for low SES. True SD known. Unknown distribution. Find 99% CI for true mean.
   n = 900 babies in low SES
   x-bar = 115
   sigma = 24

99% CI = (114.4497, 115.5503)

9. Pregnant women with sugars, but unknown sigma. Will need T distribution instead.
   n = 52
   x-bar = 70.12
   s = 10
   df = 51

95% CI = (67.3354, 72.9046)

10. Investigators looked at morning plasma citrate concentrations. Sample follows normal distribution.
    n = 10
    x-bar = 118.5
    s = 20.8
    df = 9

95% CI = (103.6216, 133.3784)

11. Maternity ward records for low SES. 99% CI?
    n = 900 babies in low SES
    x-bar = 115
    s = 24
    df = 899

99% CI = (112.9352, 117.0648)