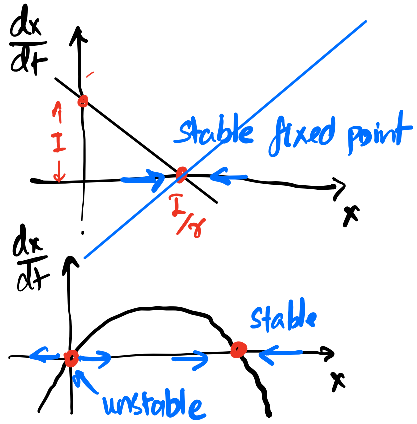
Bifurcations and Stability

$$\frac{dx}{dt} = I - \gamma x$$



$$\frac{dx}{dt} = x(1-x)$$

Stability

Stable ⇒ perturbations decrease in magnitude.

unstable ⇒ " increase "

$$\frac{dx}{dt} = f(x) \qquad x_{ss} \text{ is a steady state.}$$

$$\left.\frac{dx}{dt}\right|_{x=x_{ss}} = f(x = x_{ss}) = 0$$

$$x = x_{ss} + x_p$$

$$\frac{d}{dt}(x_{ss} + x_p) = f(x_{ss} + x_p)$$

$$\cancel{\frac{dx_{ss}}{dt}} + \frac{dx_p}{dt} = \cancel{f(x_{ss})} + x_p \left.\frac{df}{dx}\right|_{x_{ss}} + \underbrace{\frac{x_p^2}{2!}\left.\frac{d^2f}{dx^2}\right|_{x_{ss}} + \cdots}_{\text{higher order terms} \approx 0}$$

$$\frac{dx_p}{dt} = \boxed{\left.\frac{df}{dx}\right|_{x_{ss}}} x_p$$

< 0 = Stable

> 0 ⇒ unstable

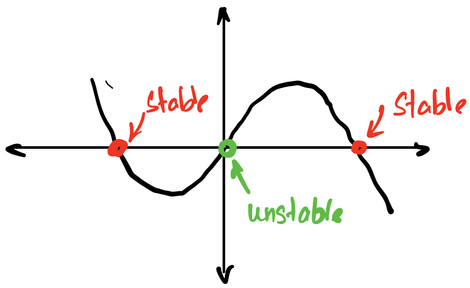
↓

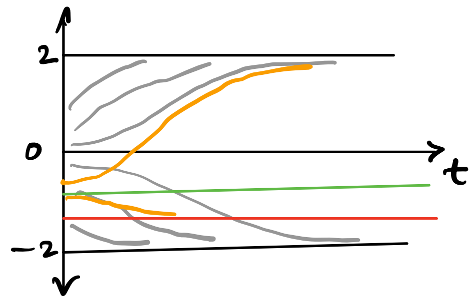
λ

Bifurcation

Cubic kinetics

$$\frac{dx}{dt} = c\left(x - \frac{x^3}{3}\right) \qquad c>0$$





$$\frac{dx}{dt} = c\left(x - \frac{x^3}{3} + A\right)$$