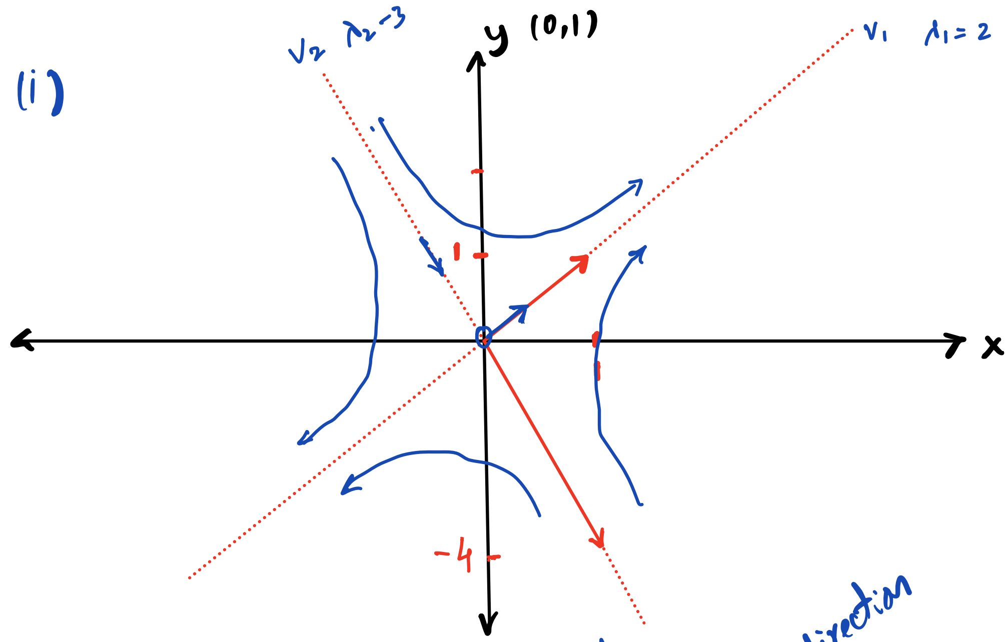
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\qquad \frac{d\bar{x}}{dt} = A\bar{x} \qquad \bar{v}_1 , \bar{v}_2$

$A \to \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

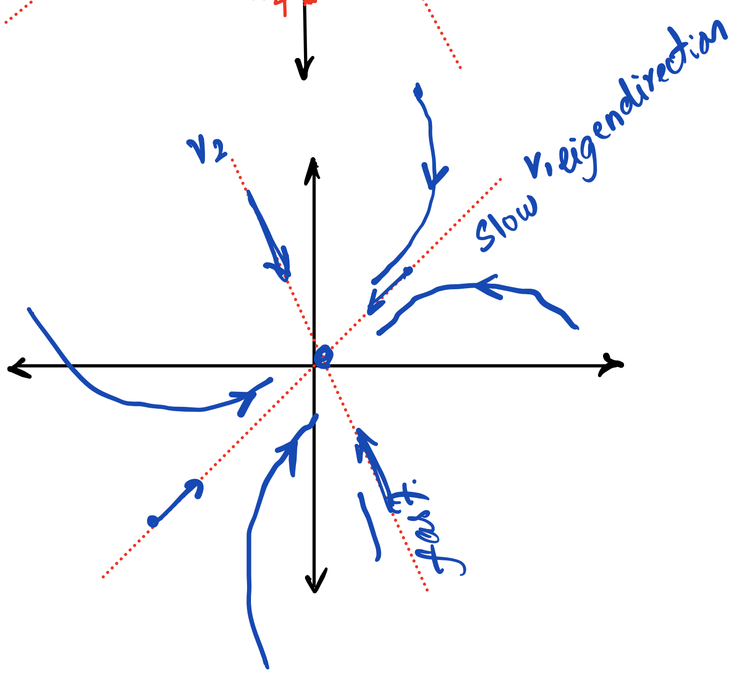
Case (i)



$\lambda_1 > 0, \ \lambda_2 < 0$

Saddle

Case (ii)



$\lambda_1, \lambda_2 < 0 \qquad \lambda_1 \neq \lambda_2$

$|\lambda_1| < |\lambda_2|$

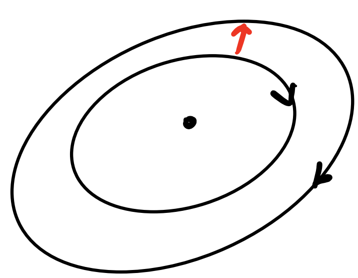
Case (iii) $\quad \tau^2 - 4\Delta < 0$

$\lambda_1 = \underline{\alpha} + i\underline{\omega} \ ; \ \lambda_2 = \alpha - i\omega$

$\alpha = \frac{\tau}{2} \ ; \quad \omega = \frac{1}{2}\sqrt{4\Delta - \tau^2}$

$\lambda_{1,2} = \frac{1}{2}\left(\tau \pm \sqrt{\tau^2 - 4\Delta}\right)$

$\alpha = 0 \qquad$ Center — neutrally stable

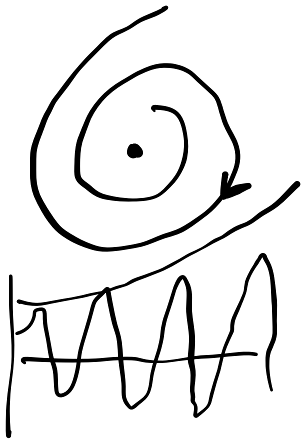


$\alpha \neq 0$ $\quad \vec{x}(t) = c_1 e^{\lambda_1 t}\vec{v}_1 + c_2 e^{\lambda_2 t}\vec{v}_2$

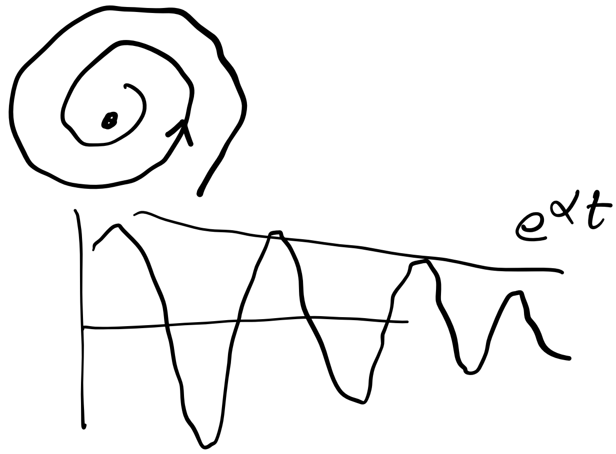
$$e^{\lambda t} = e^{(\alpha + i\omega)t} = e^{\alpha t} e^{i\omega t}$$

$$e^{\alpha t}(\cos \omega t + i \sin \omega t)$$

$\alpha > 0$



$\alpha < 0$



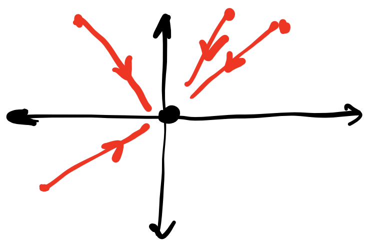
Case (iv) Repeated eigenvalues

$\lambda_1 = \lambda_2 = \lambda$

(iv) a Assume independent eigenvectors.

$$A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \qquad \lambda_{1,2} = \lambda$$

every vector is an eigenvector !!



$\bar{v}_1, \bar{v}_2$

$$\bar{x}_0 = c_1 \vec{v}_1 + c_2 \vec{v}_2$$

$$A\bar{x}_0 = A[c_1 \vec{v}_1 + c_2 \bar{v}_2]$$

$$= c_1 \underbrace{A\bar{v}_1}_{\lambda v_1} + c_2 \underbrace{A\bar{v}_2}_{\lambda v_2}$$

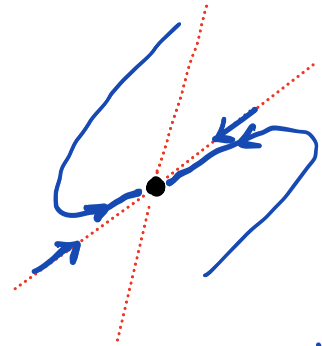
$$A\bar{x}_0 = \lambda \underbrace{\boxed{(c_1 \bar{v}_1 + c_2 \bar{v}_2)}}_{\bar{x}_0}$$

$$A\bar{x}_0 = \lambda \bar{x}_0$$

Case iv b

$\lambda_1, \lambda_2 = \lambda \rightarrow$ repeated eigenvalues, 1-D eigenspace

$$A = \begin{bmatrix} \lambda & b \\ 0 & \lambda \end{bmatrix}$$



$$\frac{dN_1}{dt} = \alpha N_1$$

$$\frac{dN_2}{dt} = \beta N_1 + \gamma N_2$$

$$N = e^{\lambda t} + te^{\lambda t}$$

$$\lambda_{1,2} = \frac{\tau \pm \sqrt{\tau^2 - 4\Delta}}{2}$$