Ch3. 'Primer'

Ordinary differential eqns - Linear - first order

$$\frac{dx}{dt} = rx \qquad x(t) = x_0 e^{rt}$$

$r > 0 \Rightarrow$ exponential growth

$r < 0 \Rightarrow$ -"- decay

$r = 0 \rightarrow$ no change

$$\frac{dx}{dt} = \boxed{K(t)}\, x$$

Separation of variables

$$\int_{x(t=0)}^{x(t)} \frac{dx}{x} = \int_0^t K(s)\, ds$$

$$\downarrow$$

$$\ln(x) \Big|_{x(0)}^{x(t)} = \int_0^t K(s)\, ds$$

$$\ln\left(\frac{x(t)}{x(0)}\right) = \int_0^t K(s)\, ds$$

$$x(t) = x(0) \exp\left(\int_0^t K(s)\, ds\right)$$

$$\frac{dx}{dt} = \boxed{\underline{I} - \underline{\gamma x}}$$

$I \downarrow$ production

$\gamma x \rightarrow$ decay

$A \longrightarrow B$

$$\frac{dx}{dt} = rx$$

$$u = I - \gamma x$$

$$\frac{d\underline{x}}{dt} = f(t) + r\underline{x}$$ linear, nonhomogeneous (explicit t dep.)

(i) General solution for the homogeneous ODE

$$\frac{dx}{dt} = rx \qquad x(t) = \underbrace{c}_{\text{const.}} e^{rt}$$

(ii) Find a particular solution to the nonhomogeneous ODE

(a) $f(t) =$ const. particular sol$\underline{n}$ $\underline{\text{const}}$

Gen. sol. to Nonhomogeneous ODE

$$x(t) = x_g(t) + x_p(t)$$

$$x(t) = c_1 e^{rt} + c_2$$

AIDs epidemic

$$\boxed{\frac{dN_I}{dt} = c_1 N_I}$$

$$\boxed{\frac{dN_V}{dt} = KN_V + \gamma e^{K_2 t}}$$

(b) $\underline{f(t) = \exp(kt)}$

$x_p(t) = \alpha e^{\beta t} \rightarrow$ particular soln.

$$\boxed{\frac{dx}{dt} = Ae^{\beta t} + \underline{rx}}$$

$$x(t) = ce^{rt} + \frac{\alpha e^{\beta t}}{\beta - r}$$

$$\boxed{\frac{dx}{dt} = rx} + \alpha e^{\beta t}$$

Particular $x_p(t) = Ae^{\beta t}$

$$\beta A\cancel{e^{\beta t}} = rA\cancel{e^{\beta t}} + \alpha\cancel{e^{\beta t}} \qquad A = \frac{\alpha}{\beta - r}$$

Gen. homogeneous $x(t) = ce^{rt}$

$$\boxed{x(t) = c\exp(rt) + \frac{\alpha}{\beta - r}\exp(\beta t)}$$ assumes $\beta \neq r$

if $\beta = r$ particular $\text{sol}^{\underline{n}}$ is $\underline{Ate^{rt}}$