TAE – Lattice tutorial

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- Correlation function for the pion, nucleon and two nucleons, using real lattice data (generated by the NPLQCD collaboration)

- An estimator, which is essentially a function of the *observable* data, is biased if its expectation value does not equal the value of the parameter to be estimated. Resampling techniques especially useful for finding standard error, variance and bias of estimators

  → Process the data: here we will use:
  Jackknife and
  Bootstrap methods.

- Fitting the Effective mass plots: constant, exponentials

- Analyzing the noise (signal-to-noise)

https://github.com/assumpg/TAE-School-LQCD

Observables from a finite number of gauge-field configurations, which show some degree of correlation

Standard procedure:

$$\overline{x} = \frac{1}{N}\sum_{n=1}^{N} x_n \,, \qquad \text{mean}$$

$$\sigma^2_{\overline{x}} = \frac{1}{N(N-1)}\sum_{n=1}^{N} (x_n - \overline{x})^2 \,, \qquad \text{variance}$$

$$\mathcal{C}(x,y) = \frac{1}{N-1}\sum_{n=1}^{N} (x_n - \overline{x})(y_n - \overline{y}) \qquad \text{covariance}$$

The mean value of functions which depend on $\{x_n\}$ is usually calculated as

$$f(\overline{x})$$

with $\sigma_{f(\overline{x})} = \sigma_{\overline{x}} \left| \frac{df}{dx} \right|_{x=\overline{x}}$

$$\overline{f(x)} = \textstyle\sum_n f(x_n)/N \qquad \text{biased estimator}$$

P. Young,
in https://young.physics.ucsc.edu/jackboot.pdf

Error estimation

For large sets of variables and correlated data, one uses resampling techniques.

The most widely used are Jackknife and Bootstrap

You will find the formulas at the github link

For derivation of both estimates, the following references are useful:

- Kackknife Error Estimates, Carleton DeTar

- Jackknife and Bootstrap Resampling Methods in Statistical Analysis to Correct for Bias Peter Young

- Error analysis: jackknife & bootstrap, Kari Rummukainen