**Reverse Physics: from laws to physical assumptions,**
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# Reverse Physics for GR

Gabriele Carcassi and Christine A. Aidala

Physics Department
University of Michigan

Assumptions of Physics

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# Hamiltonian mechanics $\Longleftrightarrow$ det/rev + DOF independence

$$d_t q^i = \partial_{p_i} H$$
$$d_t p_i = \partial_{q^i} H$$

$$S_a = d_t \xi^a \omega_{ab} = \partial_b H$$

$$\omega_{ab}(t + dt) = \omega_{ab}(t) + (\partial_a S_b - \partial_b S_a)dt + O(dt^2)$$

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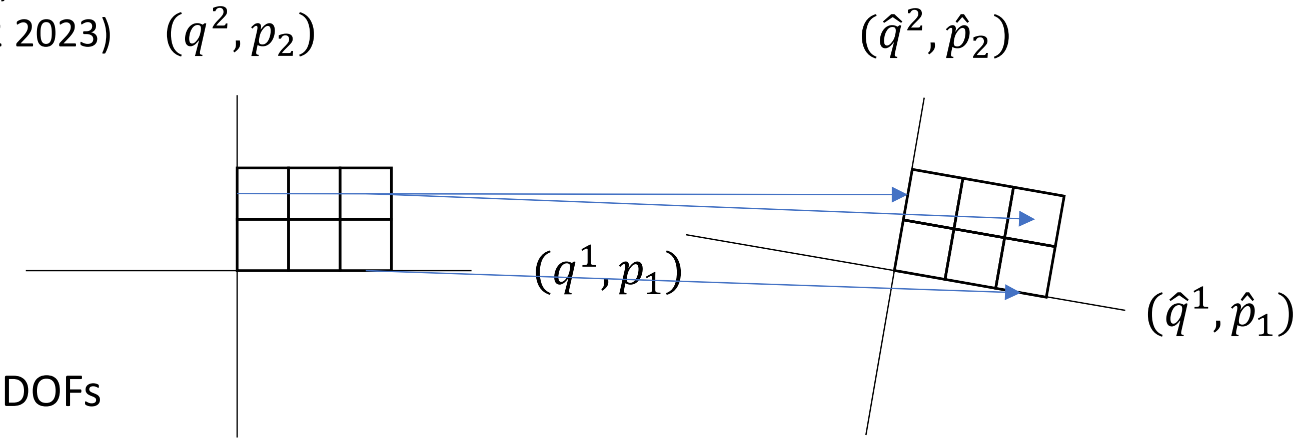
$$\omega_{ab} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \otimes I_n$$

Scalar product across DOFs

Area within each DOF

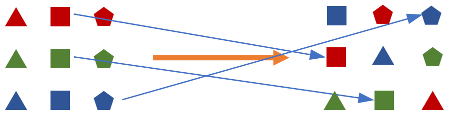



Volume = #states

#states = $\prod$ #confDOF

Areas = #confDOF

Hamiltonian is the continuous version of



Recovers relativistic particle mechanics without additional assumptions

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# Geometry of principle of least action (SDOF)

DR

$$\nabla \cdot \vec{S} = 0$$

No state is "lost" or "created" as time evolves

$[p, 0, -H]$

$$\vec{S} = -\nabla \times \vec{\theta}$$

(Minus sign to match convention)

KE $\quad pd_tq + 0d_tp - Hd_tt$

$$\mathcal{S}[\gamma] = \int_\gamma Ldt = \int_\gamma \vec{\theta} \cdot d_t\xi^a dt$$

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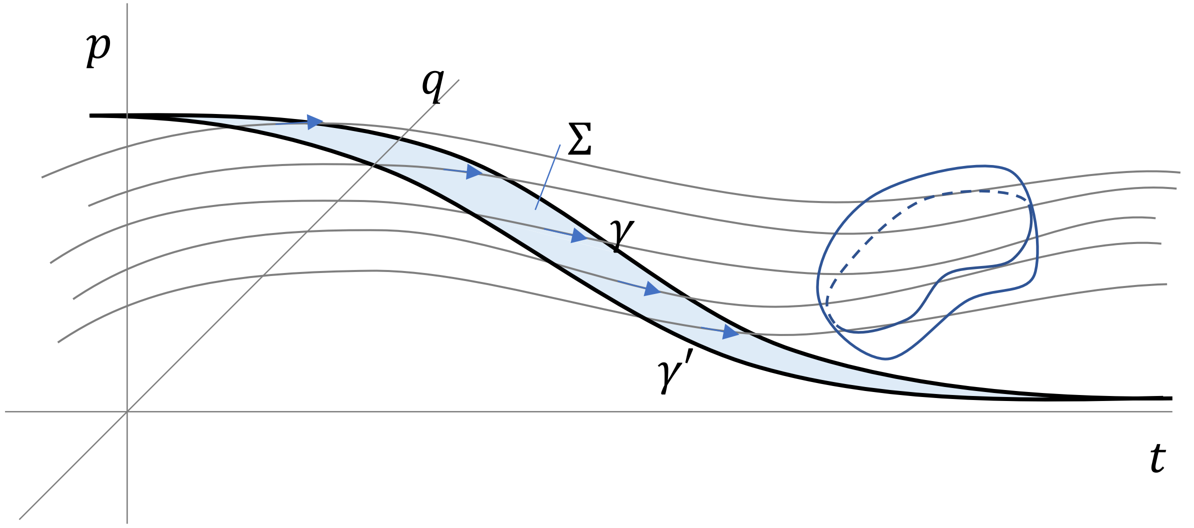
## The action is the line integral of the vector potential (unphysical)




Variation of the action

$$\delta\mathcal{S}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$

$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent, physical!

Variation of the action measures the flow of states (physical).

Variation = 0 ⇒ flow of states tangent to the path.

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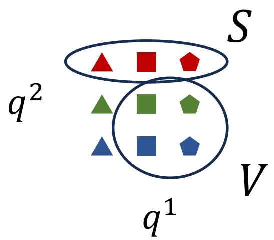
# Counting states and configurations

Discrete case



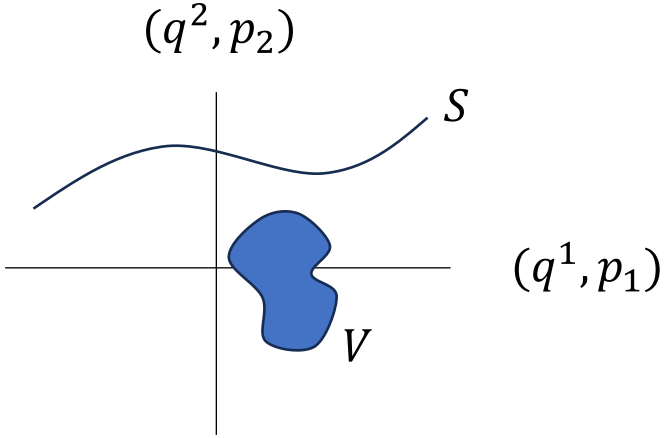

$$\#\mathrm{conf}(S) = \#S \qquad \#\mathrm{states}(V) = \#V \qquad \#\mathrm{DOF}(I) = \#I$$

Continuous case

Configuration density

State density

$$\#\mathrm{conf}(S) = \int_S \omega_{ab}\, d\xi^a d\xi^b \qquad \#\mathrm{states}(V) = \int_V \wedge\, \omega\, d\xi^{a_1} \cdots d\xi^{a_n} \neq \#V = \infty$$




$$\#\mathrm{DOF}(I) = \#I$$

Ham Mech ⇒ Correct count of configurations/states on finitely many dense (i.e. continuous) DOFs

Field theory ⇒ DOFs themselves are dense (i.e. continuous)

$$\#\mathrm{DOF}(I) \neq \#I$$

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# Conjecture: GR $\Longleftrightarrow$ det/rev + DOF independence for infinitely many (dense) DOFs

$$\delta \int_\gamma L dt = \oint_{\partial\Sigma} \theta_a d\gamma^a = \iint_\Sigma \omega_{ab} d\xi^a d\gamma^b$$

$$\delta \int_\gamma \mathcal{L} d^n s = ???$$

Flow of states

$$\frac{\#\text{conf}}{\#\text{DOFs}} = ?$$

Line integral of the vector potential of the flow of state **density**?

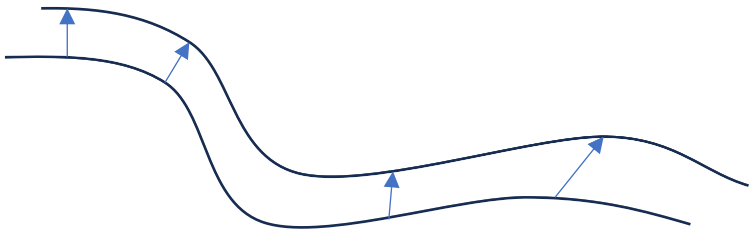
$$\int_U \sqrt{-g}\, d^3x$$

#DOFs?

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_{matter}$$

Flow of DOFs?

Flow of configurations?



We are mapping values between Cauchy surfaces,
#DOFs are the points on the Cauchy surface,
#conf are the possible field values at each point

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# The problem with counting on the continuum

We'd like:

1. Every state is a single case (i.e. $\mu(\{\psi\}) = 1$)
2. Finite continuous range carries finite information (i.e. $\mu(U) < \infty$)
3. Count is additive for disjoint sets (i.e. $\mu(\cup\, U_i) = \sum \mu(U_i)$)

Incompatible!

**Pick two!**

Discard 1 $\Rightarrow$ Lebesgue measure

Discard 2 $\Rightarrow$ counting measure

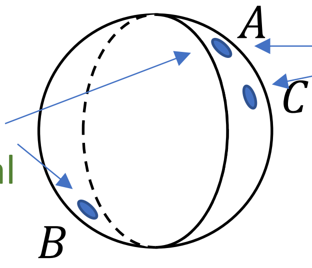
Discard 3 $\Rightarrow$ "Quantum measure"

$$\mu(U) = 2^{\sup\left(S\left(\mathrm{hull}(U)\right)\right)}$$

Exponential of the maximum entropy reachable with convex combinations (statistical mixtures) of $U$ (reduces to counting/Liouville measure)

Orthogonal states: different states all else equal

additive




Non-orthogonal states: different states but in different contexts

sub-additive

$\mu(\{A\}) = 2^0 = 1$

$\mu(\{A, B\}) = 2^1 = 2$

$\mu(\{A, C\}) < 2 = \mu(\{A\}) + \mu(\{C\})$

## Quantum mechanics $\Rightarrow$ lower bound on #conf (entropy) on continuous DOF

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# Conjecture: quantum gravity ⇒ lower bound on DOF count

$$\#\text{conf} = \#\text{DOFs} \quad \frac{\#\text{conf}}{\#\text{DOFs}}$$

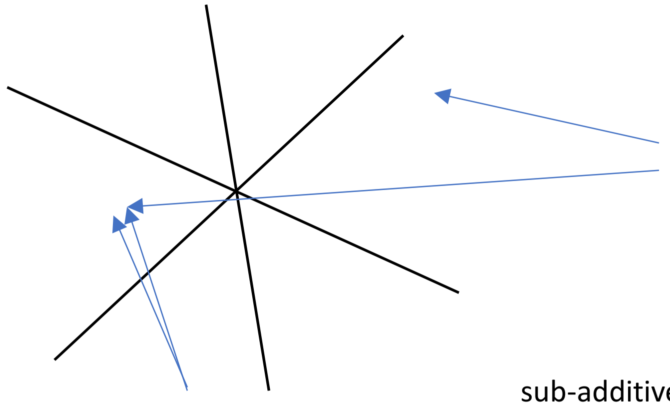
Lower bound on this…

…requires a lower bound on this

From QM: Lower bound on state count requires a severe revisitation of particle state space

Same problem!

1. Every point is a single DOF (i.e. $\mu(\{x\}) = 1$)
2. Finite volume carries finitely many DOFs (i.e. $\mu(U) < \infty$)
3. Count is additive for disjoint regions (i.e. $\mu(\cup U_i) = \sum \mu(U_i)$)




Does lower bound on DOF count require an equally severe revisitation of space-time?

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# Wrapping up

- Classical mechanics is exactly det/rev mapping of configurations over finitely many DOFs
- Conjecture: is general relativity exactly det/rev mapping of configurations over infinitely many (dense) DOFs (i.e. a field theory)?
- Quantum mechanics sets a lower bound on state count
  - Entropy of pure state is zero, pure states count as one state
- Conjecture: is quantum gravity setting a lower bound on the DOF count?
  - No region of space can contain less than one DOF

- Can we generalize the physical/geometric interpretation of the action principle to field theory and to QM?

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