https://arxiv.org/abs/2411.00972

Classical mechanics as the high-entropy limit of quantum mechanics

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Assumptions of Physics

https://assumptionsofphysics.org

Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

Assumptions of Physics

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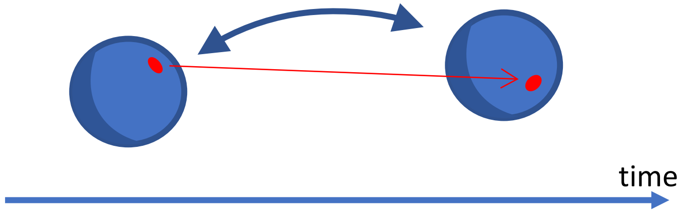
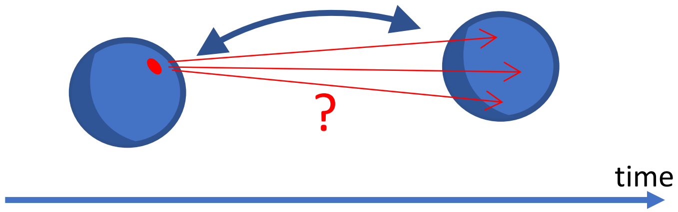
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

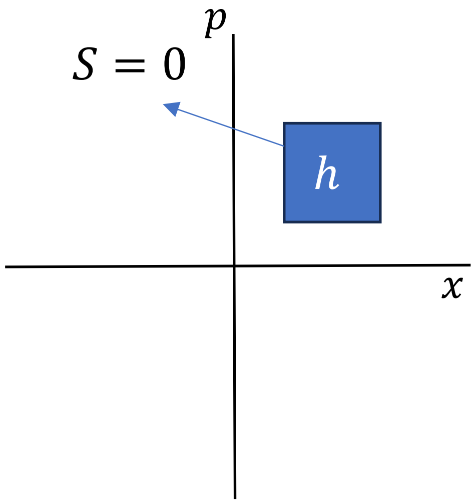
For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras previous results

Third law of thermodynamics

$$S \geq 0$$

Physical entropy is absolute



Statistical mechanics

Define h as the support of a uniform distribution of zero entropy

$$S(\rho_U) = \log \frac{\Delta x \Delta p}{h}$$

$$S(\rho) = -\int \rho \log h\rho \, dx dp$$

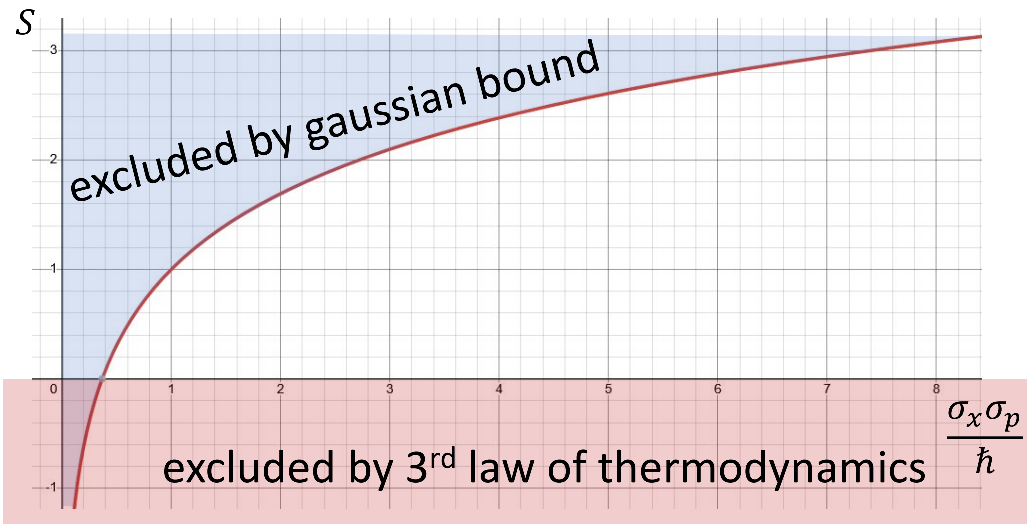
Fixes units (i.e. log argument is a pure number) and zero of entropy

Let's plot entropy against uncertainty

Gaussian maximizes entropy for a given uncertainty

$$S(\rho) \leq \log \frac{2\pi e \sigma_x \sigma_p}{h}$$

$$\sigma_x \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$



$$S \geq 0 \;\;\Rightarrow\;\; \sigma_x \sigma_p \geq \frac{\hbar}{e}$$

Classical uncertainty principle

Carcassi and Aidala,
Found. Phys. 52:40, 2022

Quantum mechanics incorporates the third law while classical mechanics does not

Is this the only difference?

Suppose the lower bound on the entropy is the only difference. Then in the limit of high entropy of quantum mechanics we should recover classical mechanics.

Can we?

Classical mechanics as high entropy limit?

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greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

Reply | Reply All | Forward | ...

Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

[Submitted on 1 Nov 2024 (v1), last revised 3 Dec 2024 (this version, v2)]

Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

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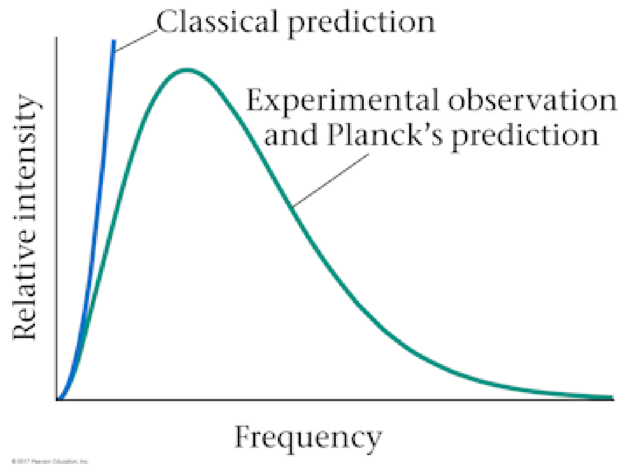
Reinterpreting traditional approaches: Blackbody radiation

- Ultraviolet catastrophe – a crisis for classical physics
- Failure of classical mechanics was a failure of classical mechanics with respect to *statistical* mechanics

Classical: Rayleigh-Jeans
$$\frac{2\nu^2 k_B T}{c^2} = \frac{2\nu^2}{c^2\beta}$$

Quantum: Planck
$$\frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{k_B T}\right)-1} = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\beta\nu}-1}$$



Traditional limit:
Low frequency
$$\frac{2h\nu^3}{c^2}\frac{1}{e^{h\beta\nu}-1} = \frac{2h\nu^3}{c^2}\frac{1}{1+h\beta\nu+O\left(\nu^2\right)-1} \simeq \frac{2\nu^2}{c^2\beta}$$

Alternative limit:
High temperature (low β)
$$\frac{2h\nu^3}{c^2}\frac{1}{e^{h\beta\nu}-1} = \frac{2h\nu^3}{c^2}\frac{1}{1+h\beta\nu+O\left(\beta^2\right)-1} \simeq \frac{2\nu^2}{c^2\beta}$$

Low $\beta \leftrightarrow$ high temperature/high entropy

Reinterpreting traditional approaches:
Wigner's quantum correction for thermodynamic equilibrium

Phys. Rev. 40:749–759, 1932

On the Quantum Correction For Thermodynamic Equilibrium

By E. Wigner
Department of Physics, Princeton University
(Received March 14, 1932)

- Considered Wigner distribution of a system in thermal equilibrium at inverse temperature β: $W(x,p) = \int dy e^{i(x+y)p/\hbar}\langle x+y|e^{-\beta\hat{H}}|x-y\rangle e^{-i(x-y)p/\hbar}$
  - With $\hat{\rho} = e^{-\beta\hat{H}}$ the mixed state that maximizes entropy at given average energy (Boltzmann dist.)
- Transformed Hamiltonian to express it as the sum of classical and quantum terms and then expanded the Wigner distribution in powers of $\hbar$
  - Quantum corrections only come in at second order in $\hbar$, recovering classical mechanics in the limit as $\hbar \to 0$

If we are dealing with a system, the behavior of which in statistical equilibrium is nearly correctly given by the classical theory, we can expand (18) into a power of h and keep the first few terms only. The term with the zero

Reinterpreting traditional approaches: Wigner's quantum correction for thermodynamic equilibrium

$$\tilde{H} = e^{ixp/\hbar}\hat{H}e^{-ixp/\hbar} = \frac{(p + i\hbar\partial/\partial x)^2}{2m} + V(x) = \underbrace{\epsilon(x,p)}_{\text{classical Hamiltonian}} + \underbrace{i\frac{\hbar p}{m}\frac{\partial}{\partial x} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}}_{\text{quantum corrections } \equiv \hat{Q}}$$

Transformed Hamiltonian

$$W(x,p) = \int dy \langle x + y | e^{-\beta\tilde{H}} | x - y \rangle$$

Wigner distribution in terms of $\tilde{H}$

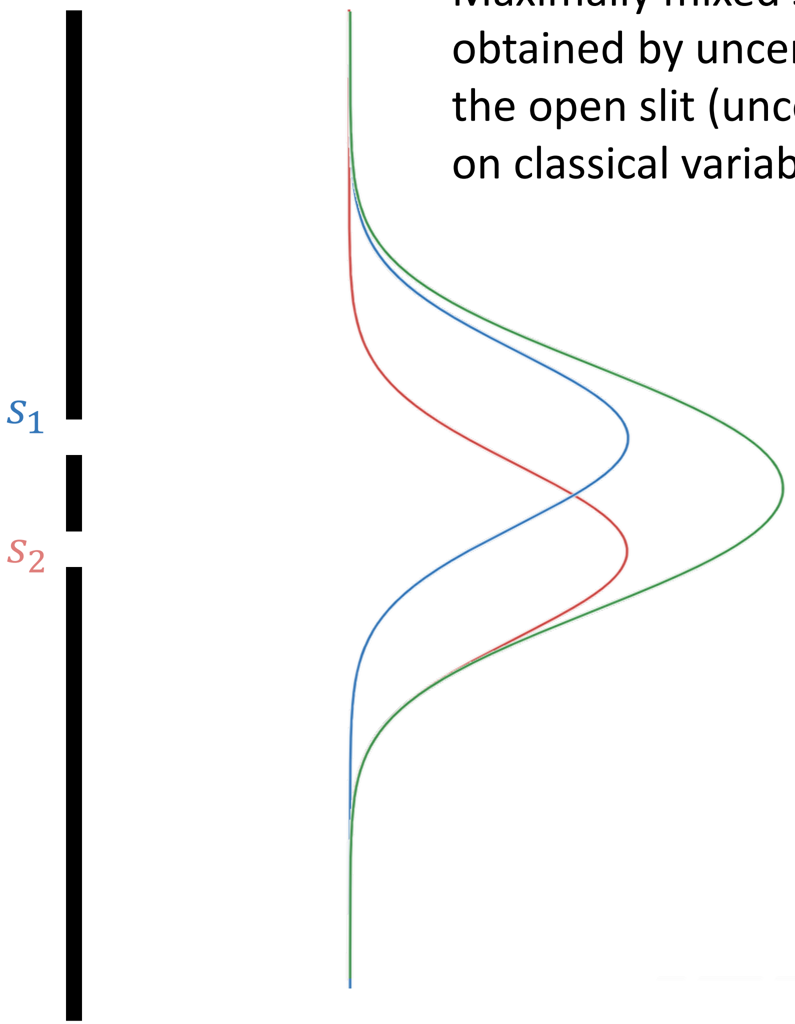
- Can expand this in powers of β for $\beta \ll 1$, rather than powers of $\hbar$
- First order: $e^{-\beta\tilde{H}} \simeq 1 - \beta(\epsilon + \hat{Q}) \quad \Rightarrow \quad W(x,p) \simeq 1 - \underbrace{\beta\epsilon(x,p)}_{\text{classical}} - \underbrace{\beta\int dy \langle x + y|\hat{Q}|x - y\rangle}_{\text{first-order quantum}}$
- Show first-order quantum correction in β is zero by inserting an identity in the momentum eigenbase:

$$\int dy \langle x + y|\hat{Q}|x - y\rangle = \int\int dk dy e^{i2ky}\left(-\frac{\hbar p}{m}k + \frac{\hbar^2}{2m}k^2\right) = \int dk \delta(2k)\left(-\frac{\hbar p}{m}k + \frac{\hbar^2}{2m}k^2\right) = 0$$

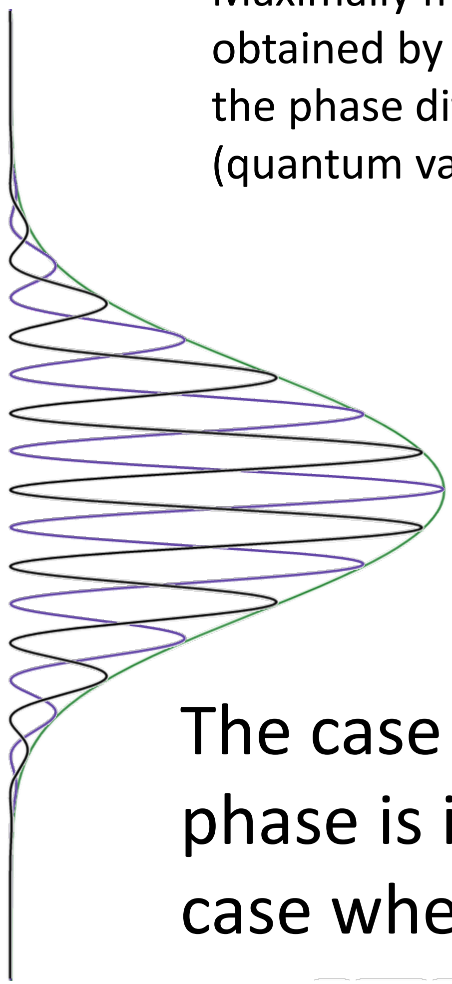
- Thus, *quantum corrections only enter at second order in* β, recovering classical mechanics in the limit as $\beta \to 0$

$$W(x,p) \simeq 1 - \underbrace{\beta\epsilon(x,p) - \beta^2\epsilon^2(x,p)}_{\text{classical}} - \underbrace{\beta^2\int dy \langle x + y|\hat{Q}V(\hat{x}) + V(\hat{x})\hat{Q}|x - y\rangle}_{\text{second-order quantum}}$$

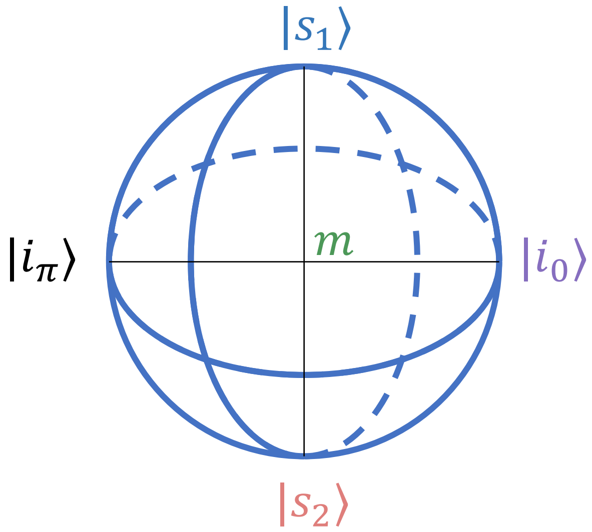
When entropy $S \to \infty$, $\beta \to 0$



Maximally mixed state obtained by uncertainty on the open slit (uncertainty on classical variable)



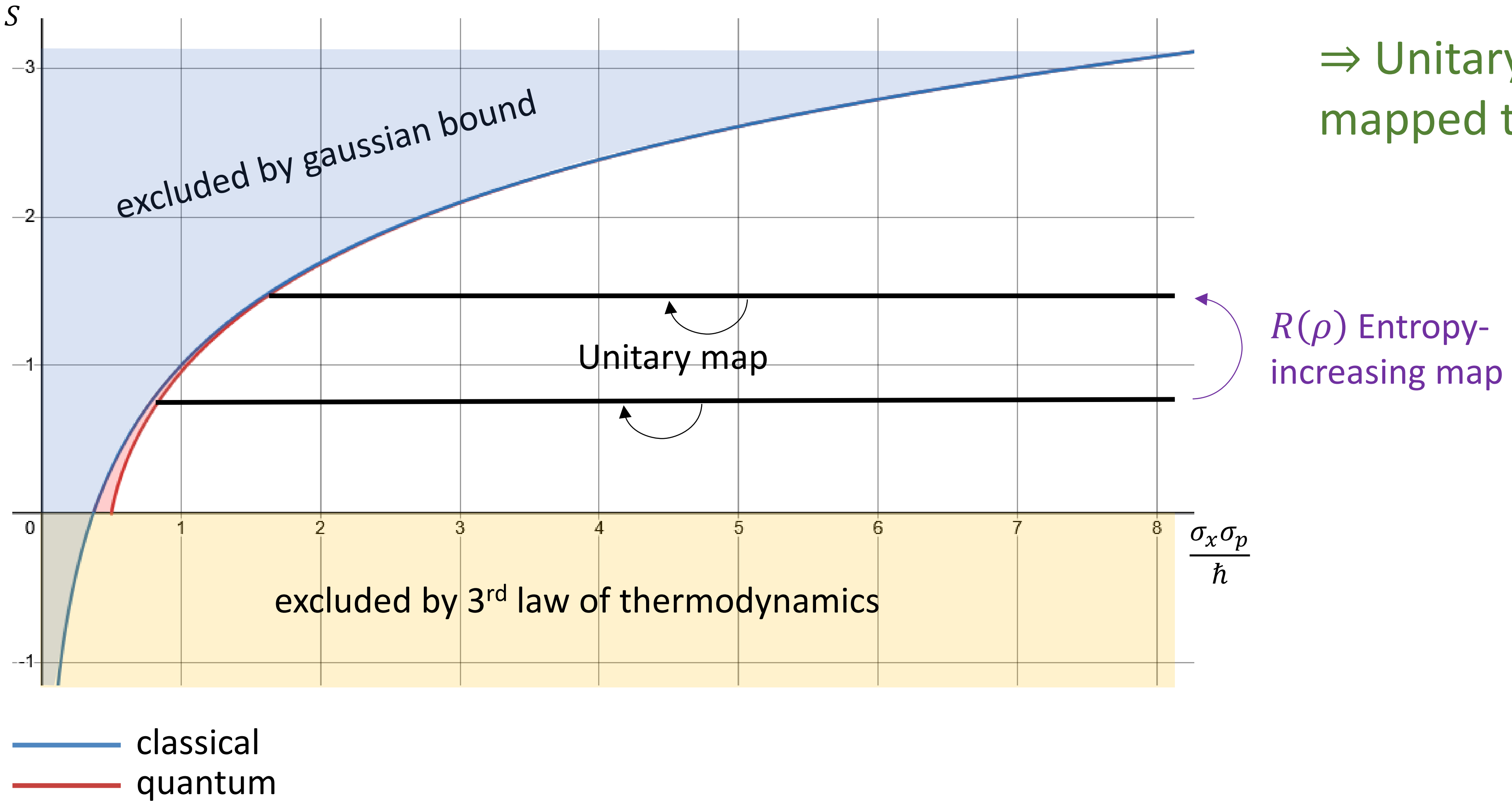
Maximally mixed state obtained by uncertainty on the phase difference (quantum variable)



The case where we randomize the phase is indistinguishable from the case where we randomize the path

Entropic aliasing: high-entropy states allow (approximate) decomposition over incompatible observables

Looking for a map $R(\rho)$ that increases entropy of all mixed states, such that every level set of entropy maps to another level set



$\Rightarrow$ Unitary maps must be mapped to unitary maps

In classical mechanics

$$S(R(\rho)) = S(\rho) + \log \lambda \Leftrightarrow \{R(x), R(p)\} = \lambda\{x, p\}$$

Jacobian is a constant:
all volumes rescaled
by the same factor

In quantum mechanics

Stretching map $$[R(X), R(P)] = \lambda[X, P]$$

Pure stretching map $$T(X) = \sqrt{\lambda}\, X \quad T(P) = \sqrt{\lambda}\, P$$

Need to take care of operator ordering!!!

Infinitesimal pure stretching map
$$\frac{dX}{dt} = \frac{\imath}{\hbar}[H, X] + \gamma\left(L^\dagger X L - \frac{1}{2}\{L^\dagger L, X\}\right)$$

Lindblad eq (open quantum system)

$$L = a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\imath}{m\omega}P\right) \qquad \gamma = \lambda$$

Anti-normal ordering and Husimi Q are preferred

Effects of stretching map on phase-space representations

Husimi Q is simply stretched

Wigner function is stretched and convolved with a Gaussian, and coincides with the Husimi Q in the limit

It is interesting to consider what happens to the negative regions of W under the stretching map. We know that W can have negative regions, but their size is limited by the uncertainty principle. In fact, convolving W with a 2D Gaussian with unitary spread, as in the definition of Q, returns a function that is never negative. In the limit $\lambda \gg 1$, the formula for W_λ reduces to

$$W_\lambda(\beta) \to_{\lambda \gg 1} \frac{2}{\pi\lambda^2} \int W_1\left(\frac{\alpha}{\sqrt{\lambda}}\right) e^{-\frac{2}{\lambda}|\alpha-\beta|^2} d^2\alpha = Q_\lambda(\beta). \tag{53}$$

Therefore, while negative regions can be in principle found at any finite λ, W tends to a positive function in the limit. As usual for the W distribution, the phase space size of negative regions is limited to $\hbar$ by the uncertainty principle. The weight of the function in such regions is limited between $\pm 2/\hbar$ for pure states. The effect of the stretching map is to reduce this bound to $\pm 2/(\lambda\hbar)$ for large values of λ. A clear interpretation can be made by working directly on the Fourier transform of W; see Eq. (51). The function $F(W_\lambda)$ is a scaled version of $F(W_1)$ with a Gaussian filter applied to it. Quantum information in W_1 is carried by spectral weights with k-vectors larger than 1. The bandwidth of the Gaussian filter is given by $\lambda/(\lambda - 1)$. This cutoff approaches 1 in the limit, filtering away the interference terms.

Another perspective: move the pure states to minus infinite entropy

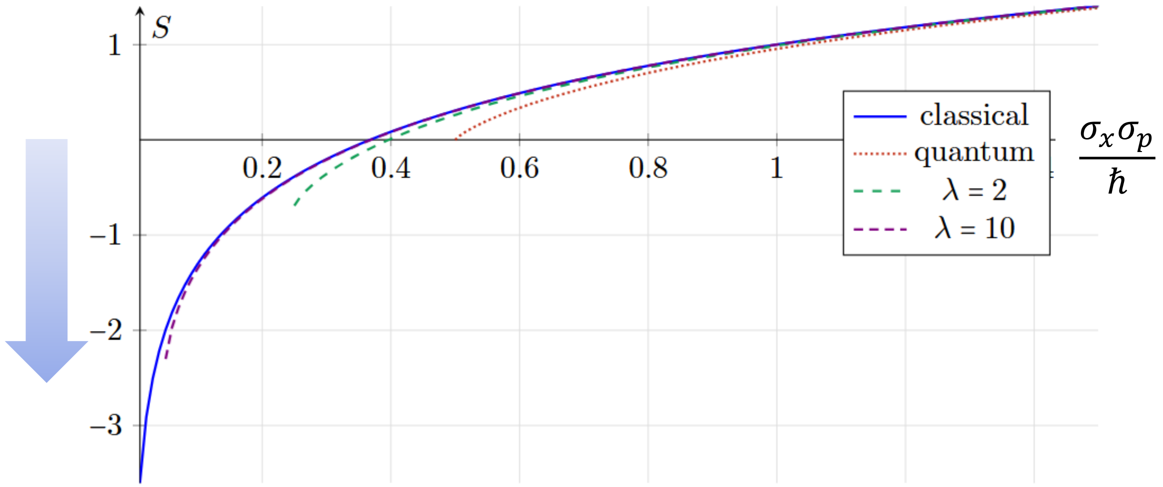
Instead of

$$[X, P] = \imath\hbar \quad [T(X), T(P)] = \lambda\imath\hbar$$

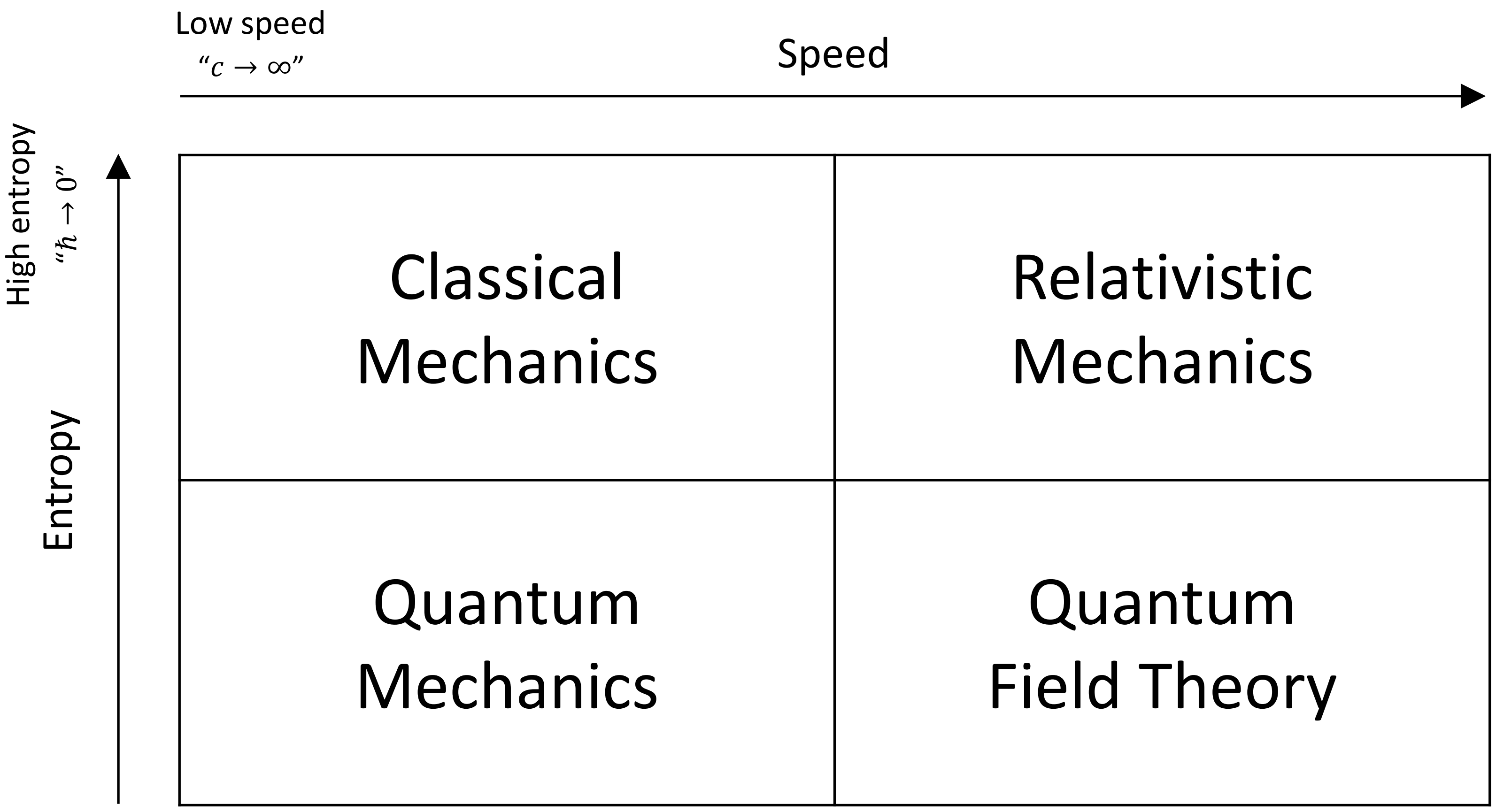
Redefine original space such that

$$[\hat{X}, \hat{P}] = \frac{\imath\hbar}{\lambda} \quad [T(\hat{X}), T(\hat{P})] = \imath\hbar$$

$$\lambda \to \infty \Rightarrow \frac{\hbar}{\lambda} \to 0$$



Mathematically equivalent to lowering the entropy of a pure state to $-\infty$, or $\hbar \to 0$ (group contraction)



No-mechanism limit
(same as non-relativistic limit)

Summary

- The most prominent breakdown of classical mechanics emerged in the thermodynamic predictions from classical statistical mechanics
  - Classical statistical mechanics doesn't follow 3rd law of thermodynamics!
- The classical limit can be recovered by identifying entropy-increasing maps (i.e. stretching maps), as the Wigner function and Husimi Q converge to each other
  - Different operator orderings give the same expectation
  - Wigner function becomes non-negative
- Alternatively, we can take the entropy of pure states to minus infinity, which corresponds to the group contraction ($\hbar \to 0$) that morphs the Moyal bracket to the Poisson bracket
- Dirac's correspondence principle was effectively looking for a theory that would recover classical mechanics at high entropy
  - Moyal bracket is the ***unique*** *one-parameter Lie-algebraic deformation* of the Poisson bracket
  - **If you are looking for a theory with a lower bound on the entropy that recovers classical mechanics at high entropy, quantum mechanics is the only one**

More about our project

- Project website
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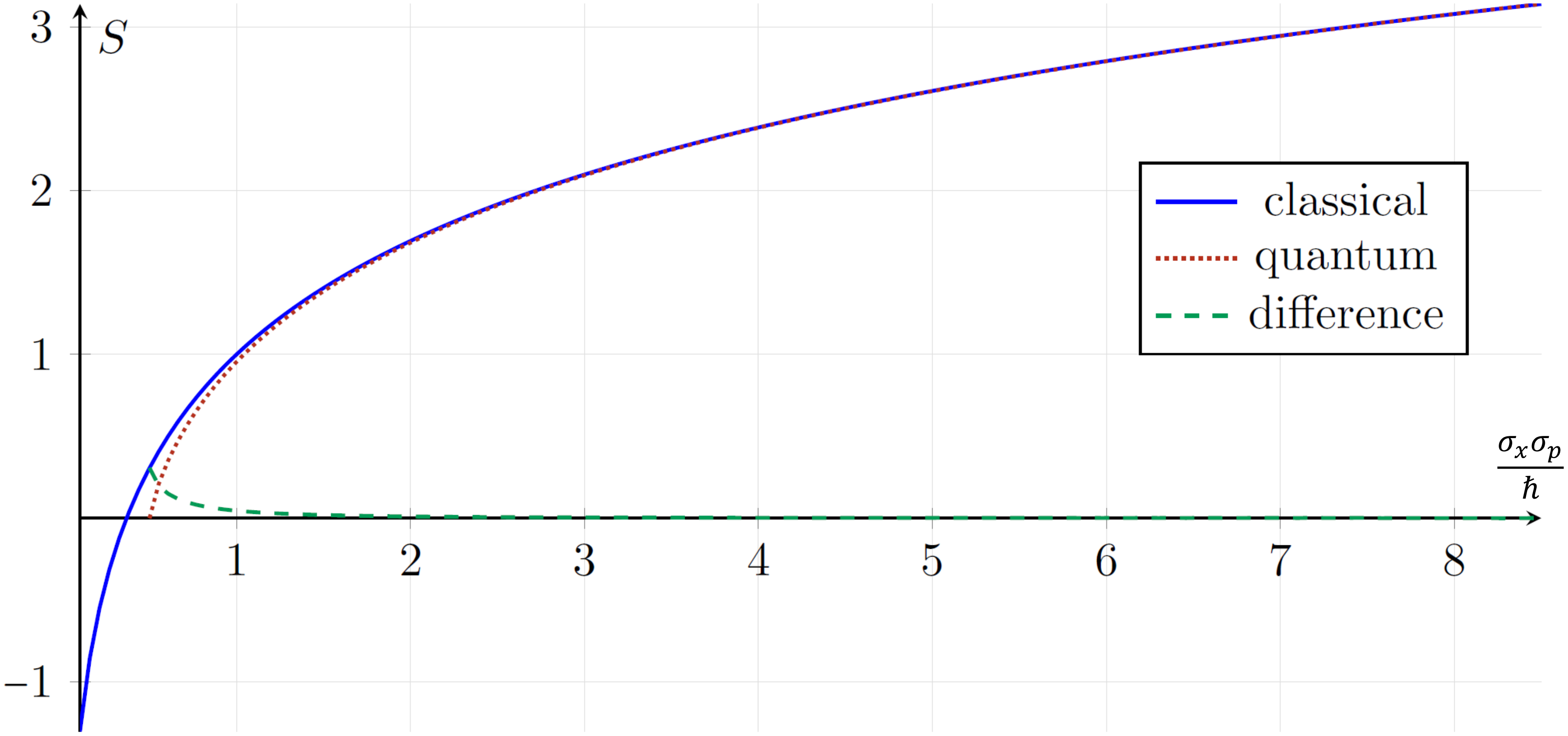
Looking for collaboration with philosophers to fully develop the conceptual implications of different project results

Backup

Entropy vs. uncertainty: Classical and quantum

Entropy in nats



Uncertainty and entropy

- In both classical and quantum mechanics, Gaussian states maximize entropy for a single independent DOF at fixed uncertainty, or equivalently, minimize uncertainty at fixed entropy
- If we fix the entropy S, will have $\Delta x \Delta p \geq \Sigma(S)$
- Specific value of the uncertainty Σ depends on S and on whether we're using classical or quantum mechanics, but relationship always saturated by Gaussian states with no correlation between x and p

See e.g. Pathria + Beale, *Statistical Mechanics*, Academic Press, 2022 or Cover + Thomas, *Elements of Information Theory*, Wiley, 2006.

- Classical mechanics: $S_C(\Sigma) = \ln\left(2\pi e \frac{\Sigma}{h}\right) = \ln\left(\frac{\Sigma}{\hbar}\right) + 1$
- Quantum mechanics: $S_Q(\Sigma) = \left(\frac{\Sigma}{\hbar} + \frac{1}{2}\right) \ln\left(\frac{\Sigma}{\hbar} + \frac{1}{2}\right) - \left(\frac{\Sigma}{\hbar} - \frac{1}{2}\right) \ln\left(\frac{\Sigma}{\hbar} - \frac{1}{2}\right)$

See e.g. Weedbrook et al., *Reviews of Modern Physics* 84:621, 2012.

Entropic aliasing

- States that are essentially mixtures of nonclassical, nonlocal states can be reinterpreted as mixtures of classical, local states
- Quantum effects masked by high entropy

Double-slit experiment:

- Particle can pass through either left or right → two pure states, $|L\rangle$ and $|R\rangle$.
- Equal superposition with phase difference θ: $\frac{1}{\sqrt{2}}|L\rangle + \frac{1}{\sqrt{2}}e^{-i\theta}|R\rangle$
- $\theta = 0$: $\quad |+\rangle = \frac{1}{\sqrt{2}}|L\rangle + \frac{1}{\sqrt{2}}|R\rangle$ with interference peak in middle
- $\theta = \pi$: $\quad |-\rangle = \frac{1}{\sqrt{2}}|L\rangle + \frac{1}{\sqrt{2}}e^{-i\pi}|R\rangle$ with valley in middle
- $|+\rangle$ cannot be understood as a probability distribution (mixture) over $|L\rangle$ and $|R\rangle$. However, an equal mixture of $|+\rangle$ and $|-\rangle$ is the maximally mixed state, which is equal to an equal mixture of $|L\rangle$ and $|R\rangle$.