The Assumptions of Physics project

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Introduction

- There has been extensive focus and effort for the last half century on developing new theories within physics, searching for beyond-the-Standard-Model physics

- We believe that in order to search most effectively for new physical theories, *first we should better understand our various established physical theories and their mathematical structures*

Lead a project called Assumptions of Physics

https://assumptionsofphysics.org/

Find a set of minimal physical assumptions from which the laws can be rederived

Reverse Physics: Start with the equations, reverse engineer physical assumptions/principles

What are the basic concepts/idealizations behind the different physical theories?

Physical Mathematics: Start from scratch and rederive everything from physical requirements

Which mathematical structures (or which parts) are physical?

Outline

- “elevate” the discussion from mathematical constructs to physical principles, assumptions and requirements (**reverse physics**)

- Construct a perfect map between mathematical and physical objects - Understand which mathematical structures are physical and which aren’t –that the current mathematical foundations are not quite what we need for physical theories (need for **physical mathematics**)

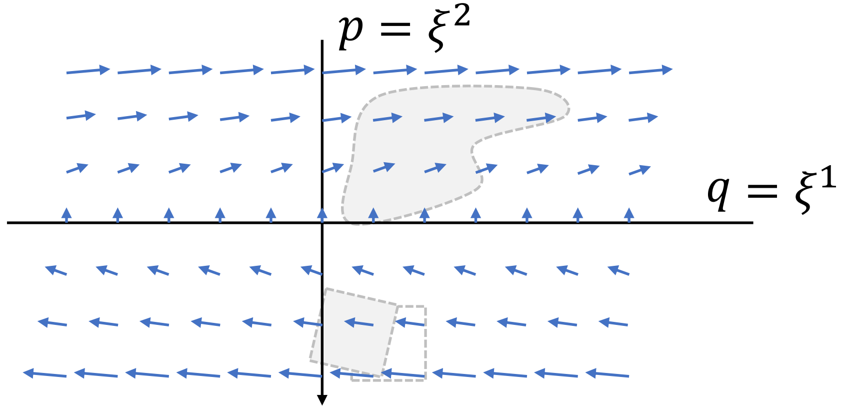
Reverse physics: from laws to physical assumptions

Reverse Physics: From Laws to Physical Assumptions

Gabriele Carcassi, Christine A. Aidala
Foundations of Physics (2022) 52:40
https://arxiv.org/abs/2111.09107

Reversing classical Hamiltonian mechanics

one d.o.f.



Mathematically, fully characterized by an incompressible flow

$div(S^a) = 0$ $|J| = 1$

Physically, equivalent to

(1) Deterministic and reversible evolution

Count of states (volume) is conserved

(2) Thermodynamically reversible evolution

Thermodynamic entropy (log of volume) is conserved

(3) Conservation of information

(4) Conservation of uncertainty

Determinant of covariance matrix is conserved

For generalization, independence of d.o.f.'s is the only additional requirement

Reversing the principle of least action

$$\nabla \cdot \vec{S} = 0$$

No state is "lost" or "created" as time evolves

$$\vec{S} = -\nabla \times \vec{\theta}$$

Minus sign to match convention

$$\mathcal{S}[\gamma] = \int_\gamma L dt = \int_\gamma \vec{\theta} \cdot d\vec{\gamma}$$

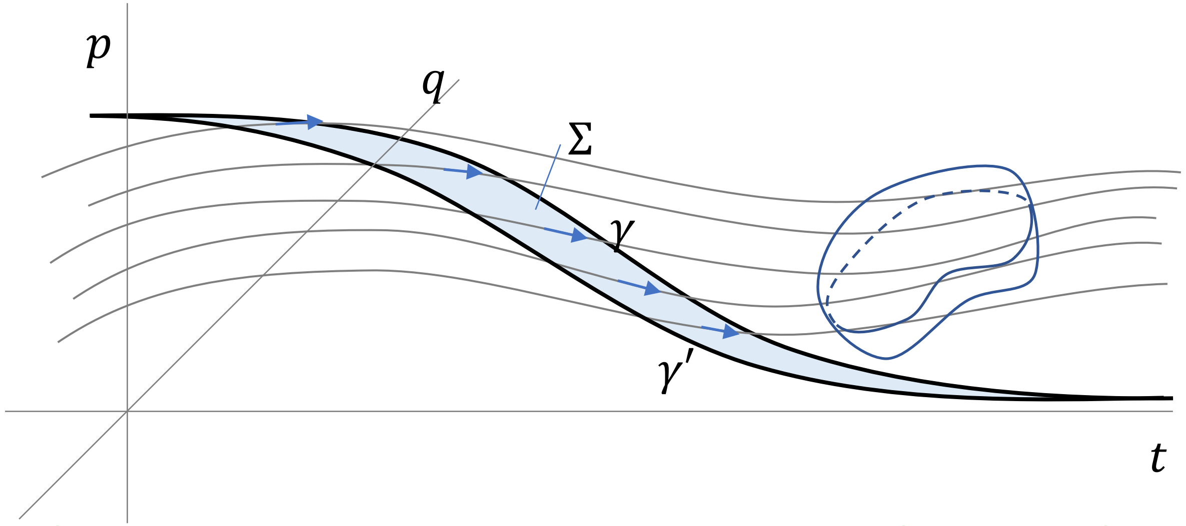
The action is the line integral of the vector potential (unphysical)

Variation of the action

$$\delta\mathcal{S}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$

$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent, physical!



https://arxiv.org/abs/2208.06428
So far rejected without review by five journals because it is not of interest…

Variation of the action measures the flow of states (physical). Variation = 0 → flow of states tangent to the path.

Reverse physics: Understanding links between theories

Deterministic and reversible evolution

$\Rightarrow$ existence and conservation of energy (Hamiltonian)

Why?

Stronger version of the first law of thermodynamics

Deterministic and reversible evolution

$\Rightarrow$ past and future depend only on the state of the system

$\Rightarrow$ the evolution does not depend on anything else

$\Rightarrow$ the system is isolated

$\Rightarrow$ the system conserves energy

First law of thermodynamics!

Reversing the uncertainty principle

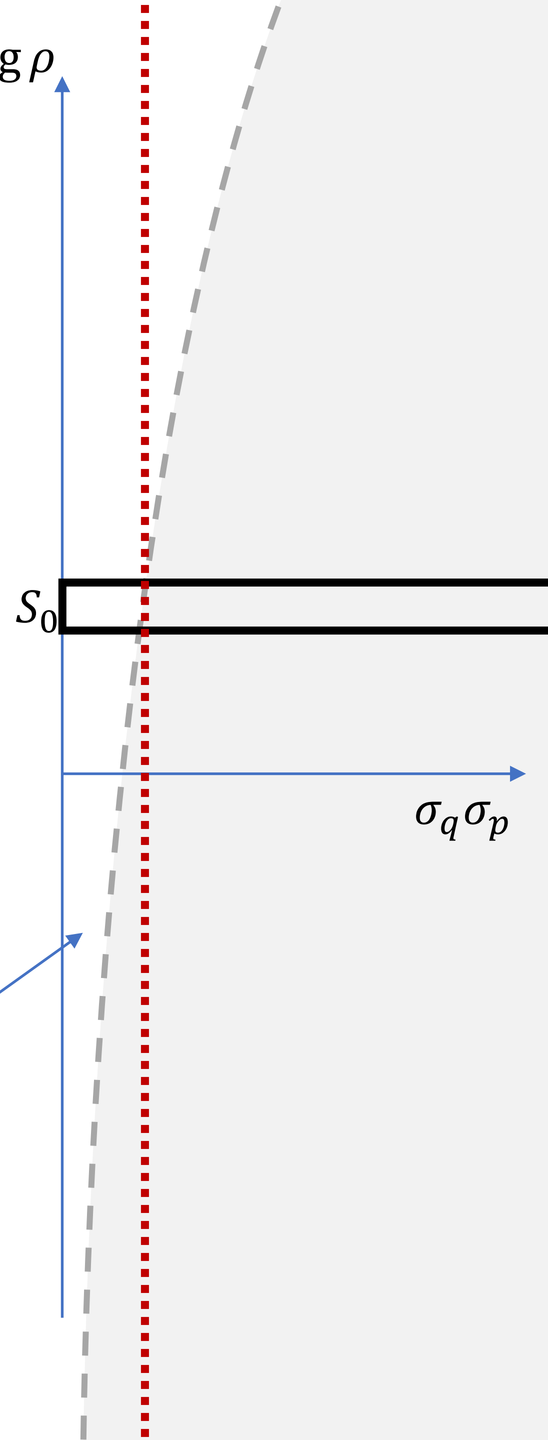
Quantum mechanics has a lower bound on entropy:
for a pure state, $S[|\psi\rangle\langle\psi|] = 0$.
For a density matrix, $S[\rho] = -tr(\rho \log \rho)$.

Take the space of all possible distributions $\rho(q, p)$ and order them by information/Gibbs entropy

Fix the entropy to a constant S_0 and consider all distributions with that entropy

They satisfy $$\sigma_q \sigma_p \geq \frac{e^{S_0}}{2\pi e}$$

equality for independent Gaussians



Lower bound on entropy ⇒ lower bound on uncertainty

Inverse does not work: lower bound on uncertainty does not give a lower bound on entropy

Reversing the uncertainty principle

Lower bound for information entropy (Gibbs/von Neumann)

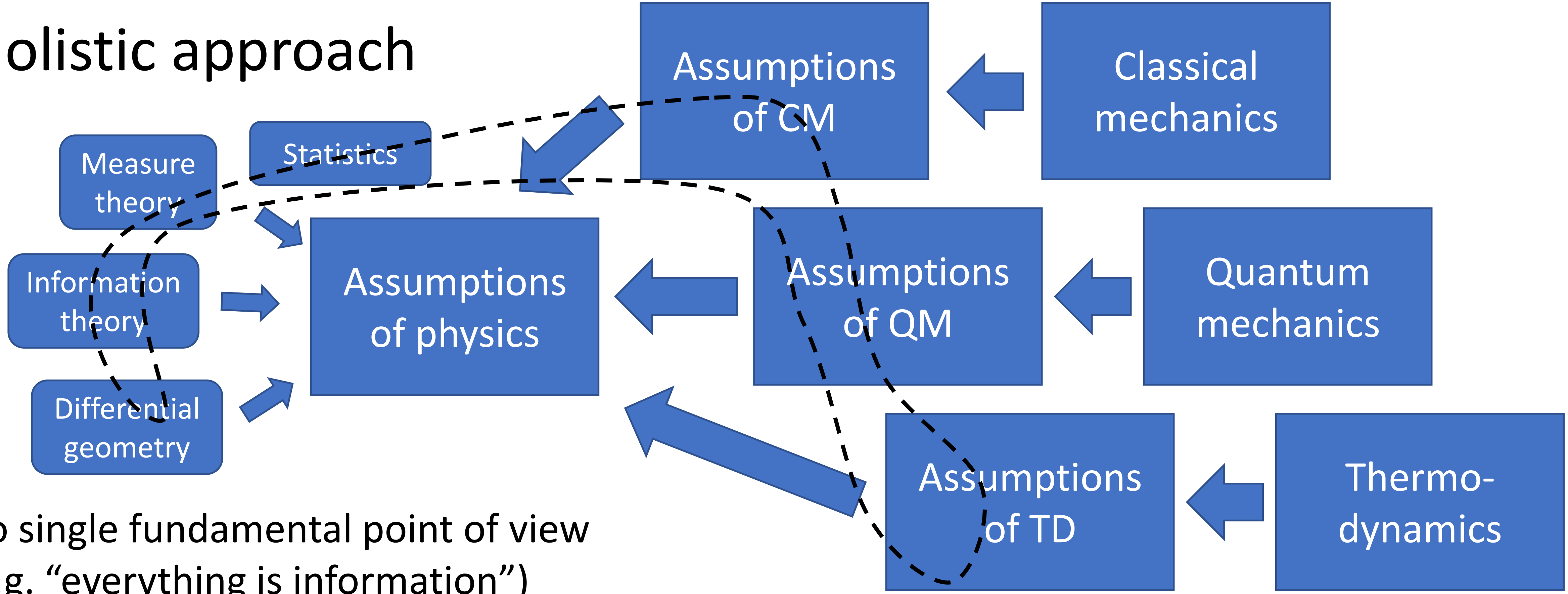
$\Rightarrow$ uncertainty principle (classical/quantum)

We don't need the full quantum theory to derive the uncertainty principle: only the lower bound on entropy

The difference is that in classical mechanics we can prepare ensembles with arbitrarily low entropy…

… which is actually in contradiction with the third law of thermodynamics!!!

Holistic approach



No single fundamental point of view (e.g. "everything is information")

Foundations of different theories are not disconnected

Find those "conceptual clusters" that span multiple areas of physics, math, …

Reverse physics: Summary

- "Reverse physics" is an approach to the foundations of physics that starts from the physical laws and aims to "go back" to a suitable minimum number of physical assumptions

- The goal is to fully map conceptual relationships and dependencies between different theories, different aspects of the theories, and to help foster higher level physical reasoning

- It is, by its nature, an interdisciplinary endeavor, and it can allow us to think more deeply about physical ideas and their relationships

Physical mathematics: from physical requirements to mathematical structures

Physical mathematics

In modern physics, mathematics is used as the foundation of our physical theories

From Hossenfelder's *Lost in Math*: "[…] finding a neat set of assumptions from which the whole theory can be derived, is often left to our colleagues in mathematical physics […]"

But mathematics only deals with formal systems, without any connection to or concern about physical reality. Formal definitions are neither necessary nor sufficient to do physics.

Not useful in a lab

David Hilbert: "Mathematics is a game played according to certain simple rules with meaningless marks on paper."

Bertrand Russell: "It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is, of which it is supposed to be true."

Physics is defined in terms of physical objects and operational definitions. Using assumptions and approximations, physical objects and their properties are idealized. The idealized model can then be expressed in the formal system.

The idealization step is the most important part of this process, and it happens outside the formal system!

Are Hilbert spaces physical?

Hilbert space: complete inner product vector space

Redundant on finite dimensional spaces. For infinite dimensional spaces, it allows us to construct states with infinite expectation values from states with finite expectation values

Exactly captures measurement probability/entropy of mixtures

Physically required

Exactly captures superposition/ statistical mixing

Physically required

⇒ Thus requires us to include unitary transformations (i.e. change of representations and finite time evolution) that change finite expectations into infinite

Extremely physically suspect!!!

Suppose we require all polynomial of position and momentum to have finite expectation

Maybe more physically appropriate?

⇒ Schwartz space

Only space closed under Fourier transform
Used as starting point of theories of distributions

Physical mathematics: differential forms

Differential forms increasing important tool in theoretical physics, but mathematically abstract

Vector defined as derivation

$$v: C^\infty(\mathbb{R}^n) \to C^\infty(\mathbb{R}^n)$$

$$v = v^i \partial_i$$

vector basis

Differential forms are fully anti-symmetric function of vectors

$$dx: V \to \mathbb{R} \qquad B: V \times V \to \mathbb{R}$$

$$dx(v) = dx(v^i \partial_i) = v^x \qquad B(v, w) = B_{ij} v^i w^j$$

Define integral on top of forms

$$\int_\gamma dx = \Delta x \qquad \int_\Sigma B = \Phi$$

Abstract definitions at points, construct finite from infinitesimal

Concrete definitions on finite, infinitesimal as a limit

Start with finite quantities over finite regions

Assume quantity is additive on disjoint regions

$$m(V) = \textstyle\sum_i m(V_i) = \int_V \rho(dV)$$

$$\Phi(\Sigma) = \textstyle\sum_i \Phi(\Sigma_i) = \int_\Sigma B(d\Sigma)$$

$$W(\gamma) = \textstyle\sum_i W(\gamma_i) = \int_\gamma f(d\gamma)$$

Differential forms: infinitesimal limit

Thinking about finite regions/values leads to better physical intuition

Makes it clear that the mathematics is contingent upon the assumptions of additivity (if this fails, differential forms are inapplicable)

Physical mathematics: Experimental verifiability as the 1st basic requirement

Science deals with assertions whose truth can be defined/ascertained experimentally
⇒ **Verifiable statements**: assertions that can be experimentally verified in a finite time

The mass of the photon is less than 10^{-18} eV → Verifiable
The mass of the photon is exactly 0 eV → Not verifiable due to infinite precision, but falsifiable

T	SUCCESS (in finite time)
F	UNDEFINED
	FAILURE (in finite time)

Different logic of verifiable statements:

- Finite conjunction/logical AND (all tests must succeed in finite time)
- Countable disjunction/logical OR (once one test succeeds, we can stop)
- No negation/NOT (FALSE ≠ FAILURE)

Note: whether a specific statement is experimentally verifiable or even well defined may depend on context (e.g. premises, idealization, theory, etc…)

The mass of the electron is 511 ± 0.1 KeV

When measuring the mass, it is a verifiable hypothesis

When performing particle identification, it is assumed to be true

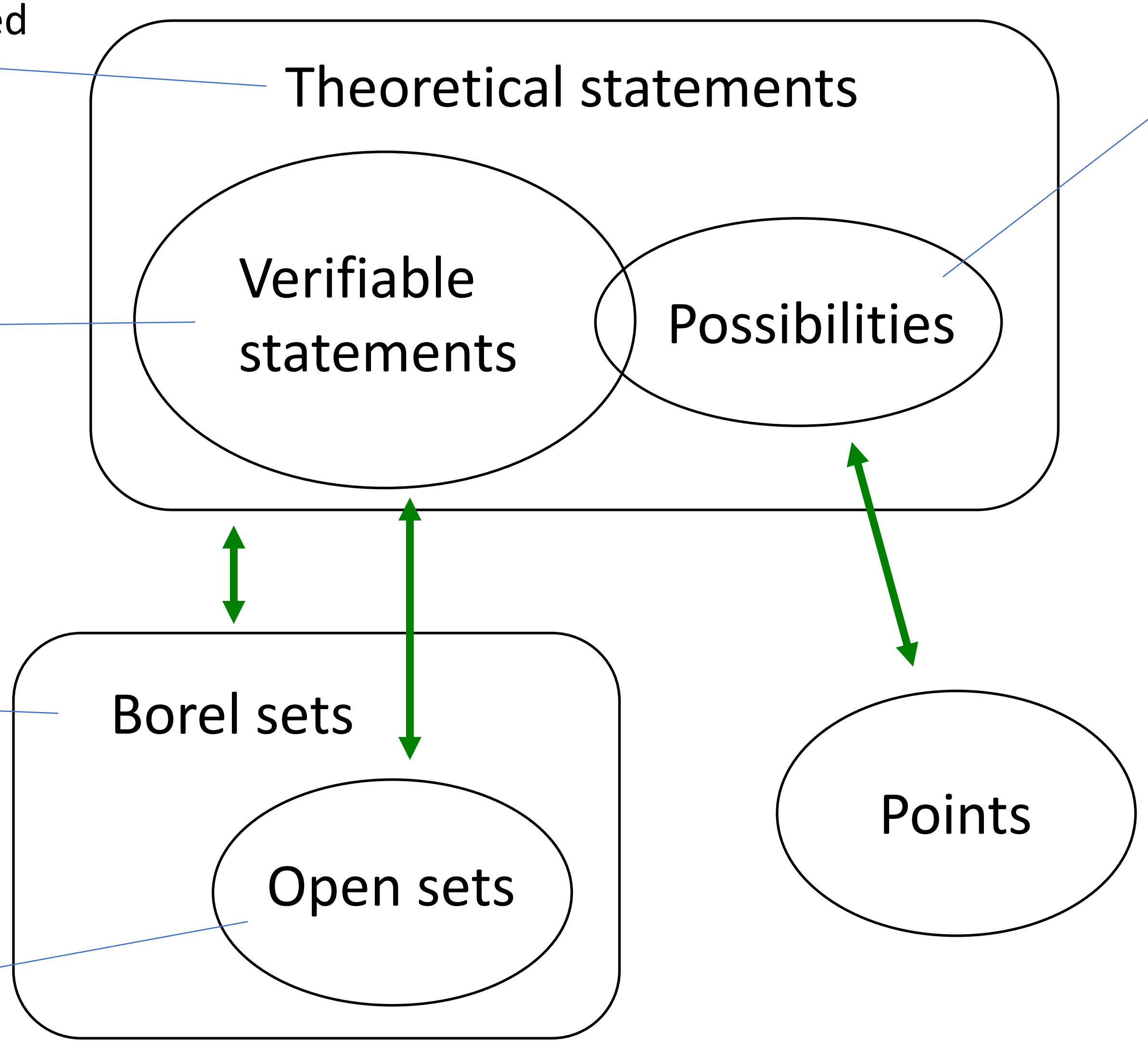
Statements formally associated with an experimental test

Experimentally distinguishable cases

If true, test always succeeds in finite time

σ-algebra

Topology



Precise map between physical concepts and their mathematical representation

All proofs can be "translated" into physically meaningful language

Towards a general mathematical theory of experimental science
https://arxiv.org/abs/1807.07896

The need for physical mathematics

- We can't expect mathematicians to provide the formal structures we need for physics
  - they do not have enough understanding of the practical requirements of physics to create the appropriate abstractions
  - ⇒ the foundations of mathematics are not a good foundation for physics
- The proper foundation for physics is a conceptually consistent formal abstraction of **the practice of experimental science** (not "of the universe")
  - We need to identify the formal structures that are appropriate to encode operational requirements and assumptions: **physically motivated mathematics**
- We can't do this work without a deep understanding of how formal systems work, and how we can bridge the formal and informal parts
  - We need to understand which mathematical details to keep because they are physically relevant and which to "quotient out"
  - ⇒ we need a good understanding of the foundations of mathematics

Practical problems doing a project like this

- High level of specialization of most journals, conferences, and funding programs
  - Even journals that claim to be general. PRL initially told us that our paper reducing the number of postulates of quantum mechanics was "not of general interest." We pointed out that the first paper in the latest issue was "Stochastic interpolation of sparsely sampled time series via multipoint fractional Brownian bridges," and our paper was then accepted (PRL 126, 110402, https://arxiv.org/abs/2003.11007)
- Many journals want to publish articles they expect to be highly cited
  - "This is not what is being discussed in mathematical physics"
- People interested in rigorous mathematics often not interested in the physics. People interested in the physics often want to use "trendy" and "fancy" mathematical tools.
  - Some of our work has been criticized as "not mathematically sophisticated"

Practical problems doing a project like this

- Interdisciplinary research often challenging until a new discipline is formed, e.g. biophysics
  - Some philosophers have a strong technical background and interest in aspects of this project, but philosophy as a discipline has greater pressure for single-author papers. "I don't collaborate."
- Standard physics curriculum doesn't typically cover fundamental mathematical tools underpinning [differential geometry, calculus?], i.e. topologies, sigma-algebras, measure theory, [].
  - People working on "foundations of quantum mechanics" often never studied the mathematical structures of classical mechanics or those underpinning probability and information theory
  - Theoretical physicists working on beyond-the-SM physics not asking questions such as "What conditions allow me to use a metric?" "…to use a manifold?"

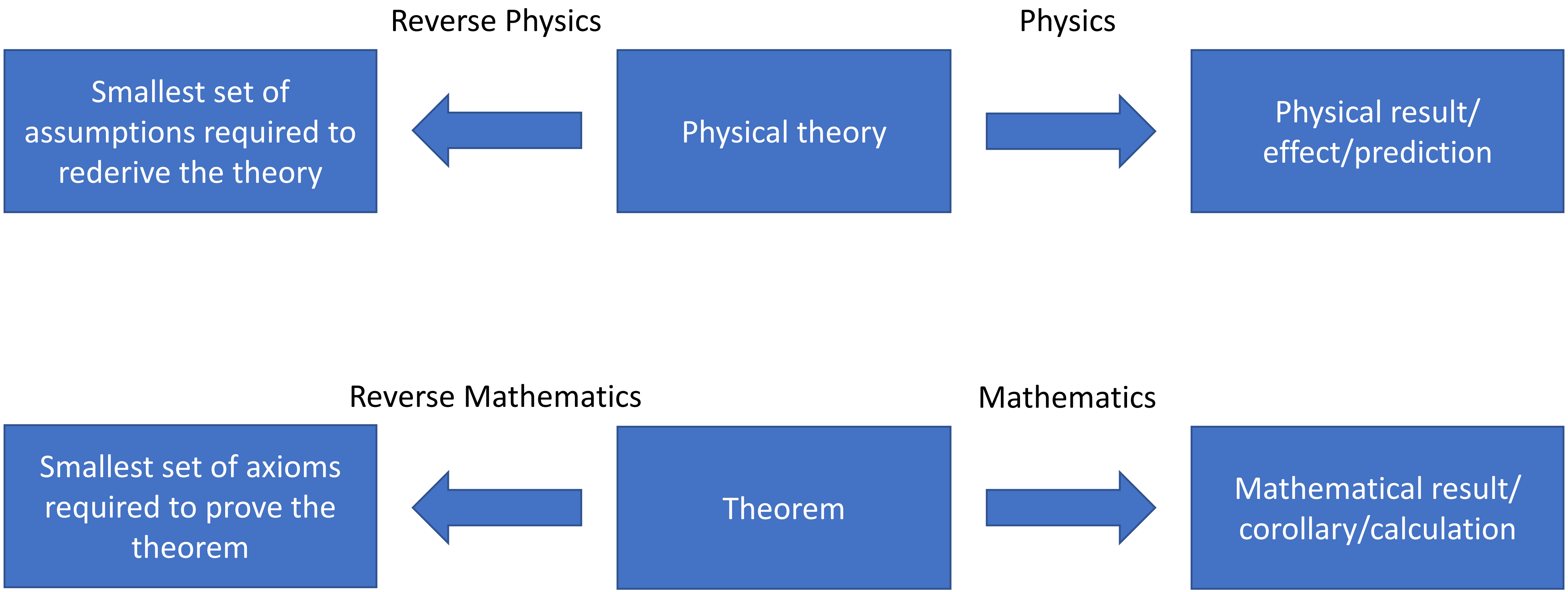
Need space for a small, technical community working on these types of foundational issues, in a spirit similar to the international metrology community

Conclusions

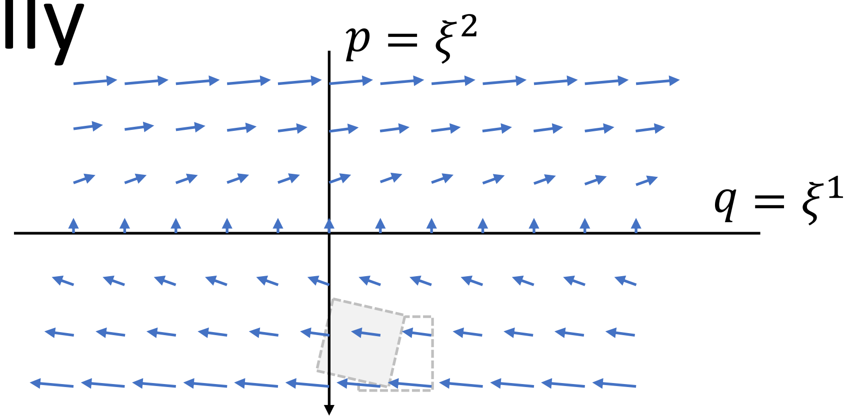
- The solution to many open problems in the foundation of physics lies in a better understanding of the current mathematical tools, their physical meaning and the development of fundamentally new tools
- ***Reverse physics*** helps us reframe the current theories in terms of physical requirements and assumptions, shifting the attention away from math to physical ideas
- ***Physical mathematics*** helps us understand clearly how physical ideas are encoded into the formal systems, and find physically motivated generalizations

We need to leave space within physics for this type of foundational work!

Supplemental


Reverse Physics
Physics
Smallest set of assumptions required to rederive the theory
Physical theory
Physical result/ effect/prediction
Reverse Mathematics
Mathematics
Smallest set of axioms required to prove the theorem
Theorem
Mathematical result/ corollary/calculation

(5) Deterministic and thermodynamically reversible evolution



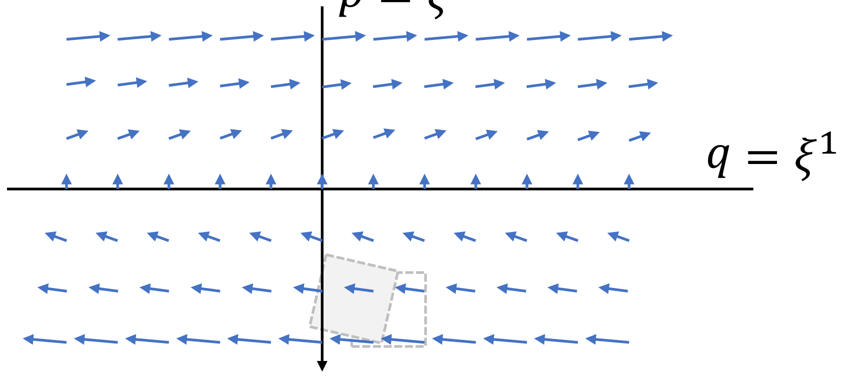
Link between statistical mechanics and thermodynamics

$$S = k_B \log W$$

Area conservation $\iff$ entropy conservation

$\iff$ thermodynamically reversible evolution

(6) Information conservation



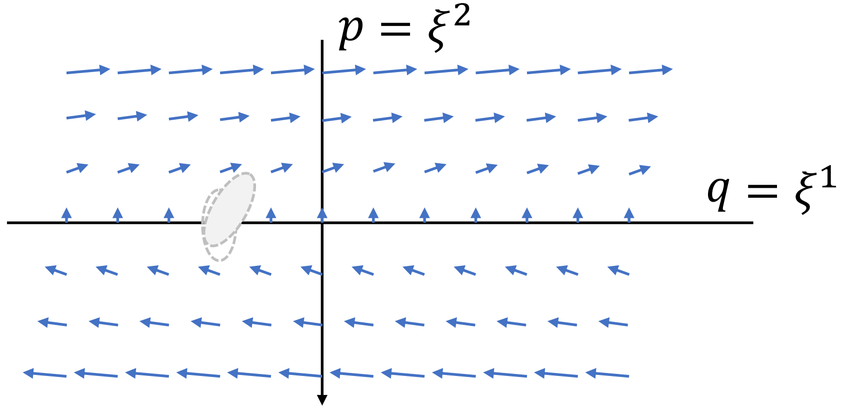
What about information entropy?

$$I[\rho(q,p)] = -\int \rho \log \rho \, dqdp$$

$$I[\rho(t+dt)] = I[\rho(t)] - \int \rho \log |J| \, dqdp$$

Area conservation $\Longleftrightarrow$ information conservation

(7) Uncertainty conservation



What about uncertainty?

covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{bmatrix}$$

Assuming a "very narrow" distribution

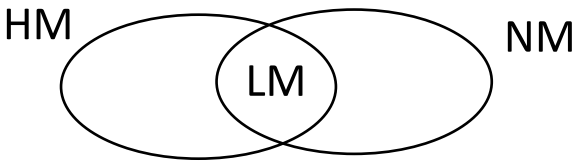
$$|\Sigma(t + dt)| = |J||\Sigma(t)||J|$$

Area conservation $\Leftrightarrow$ uncertainty conservation

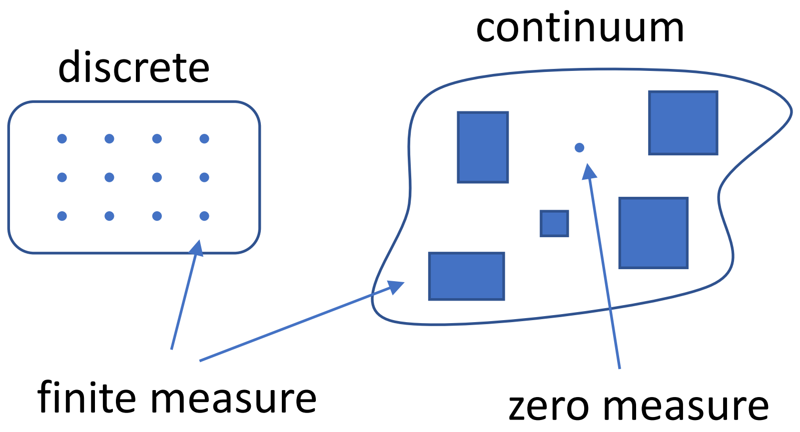
Three fundamental assumptions in Classical Mech

Infinitesimal Reducibility (IR) Determinism/Reversibility (D/R) Kinematic Equivalence (KE)

- IR ⇔ Classical phase space (symplectic manifolds ⇔ *unit independent* state count/densities/information entropy/thermodynamic entropy)
- IR+Directional degree of freedom ⇒ Space has three dimensions (2-sphere only symplectic manifold)
- IR+Directional degree of freedom ⇒ Classical analog for non-relativistic spin (open problem: relativistic analog)
- IR+D/R ⇔ Hamiltonian mechanics (Hamiltonian flow ⇔ conservation of state count/density/information entropy/thermodynamic entropy/dof independence)
- IR+D/R ⇒ energy-momentum co-vector, energy/Hamiltonian time component (pre-relativistic aspects w/o proper notion of space-time)
- IR+D/R ⇒ change of time variable changes the effective mass (similar to relativistic mass → rest mass scaled by time dilation)
- IR+D/R ⇒ classical antiparticles (w/o field theory, without quantum theory or full relativity/metric tensor)
- IR+D/R ⇒ classical uncertainty principle (uncertainty bound during evolution)
- IR+D/R ⇒ stationary action principle (with physical/geometrical interpretation, but w/o Lagrangian)
- IR+D/R+KE ⇒ Massive particles under scalar and vector potential forces
- IR+D/R+KE ⇒ $F^{\alpha\beta}$ is Poisson bracket between kinetic momenta; metric tensor as a geometrical feature of the tangent bundle ($dx^{\alpha} g_{\alpha\beta} du^{\beta}$); mass counts states per unit velocity; metric tensor locally flat (open problem: what about curvature?); speed of light converts count of possible time instants into number of possible spatial positions (i.e. ratio of measures, not speed).
- IR+D/R ⇔ Hamiltonian mechanics (HM); IR+KE ⇔ Newtonian mechanics (NM); IR+D/R+KE ⇔ Lagrangian mechanics (LM); LM = HM ∩ NM



Quantifying discrete cases is fundamentally different than quantifying cases over the continuum



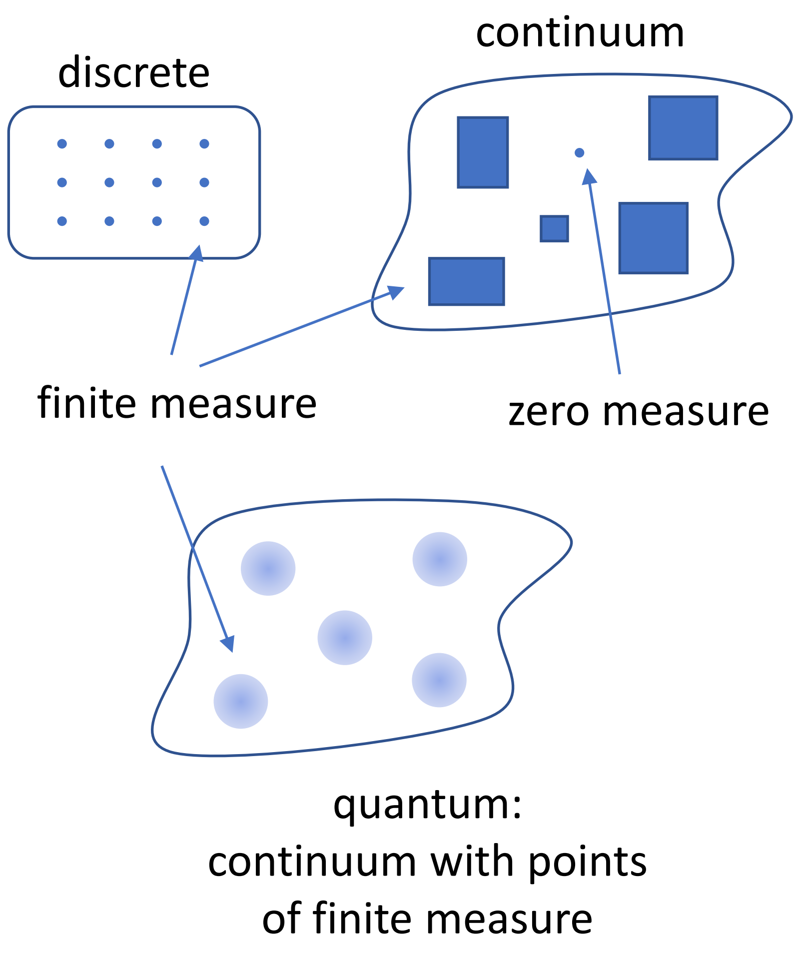
Why? Because fully identifying a discrete case requires finite information (finitely many experimental tests) while identifying a case from a continuum requires infinite information (an infinite sequence of increasingly precise tests)

This is something most physicists haven't yet fully digested

A single classical state in phase space (i.e. a microstate) ⇒ zero volume; minus infinite entropy; infinite information.

“Empty state” ⇒ one discrete case; zero entropy; finite information.

Quantum mechanics “fixes” this, by introducing a fixed lower bound on entropy.



New insights lead to new ideas

Measure theory plays a foundational role for theories of integration (e.g. geometrical sizes), probability and information theory: common physically motivated underpinning?

Consider the following statements:

“The position of the object is between 0 and 1 meters” and “The position of the object is between 2 and 3 kilometers”

“The fair die landed on 1” and “The fair die landed on 3 or 4”

“The first bit is 0 and the second bit is 1” and “The third bit is 0”

In all three cases, the first statement is “more precise”, it is of a finer granularity (noted $\preceq$)

Constraining to a smaller volume gives finer description

Less likely events give more information

Statements with more information give a finer description

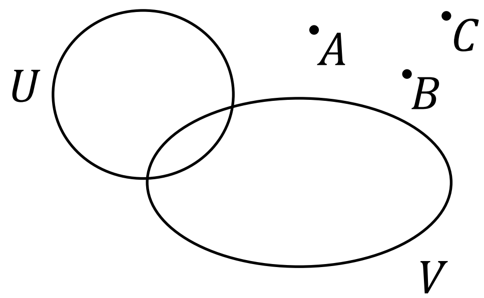
Comparing statements based on their granularity is another fundamental feature a physical theory must have

We need a generalized version of measure theory that covers all cases

Some statements are incomparable:

“The position of the object is between 0 and 1 meters” vs

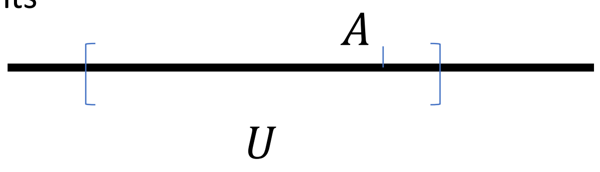
“The velocity of the object is between 2 and 3 meters per seconds”



Comparability cannot be captured by a single measure:

$\{A\} \lessdot \{B, C\} \lessdot U \lessdot V$ while $\{A\} \not\gtrdot \{B, C\} \not\gtrdot U \not\gtrdot V$

Entropy in quantum mechanics is consistent with first two requirements



Quantization breaks additivity:

Single point is a single case (i.e. $\mu(\{A\}) = 1$)

Finite range carries finite information (i.e. $\mu(U) < \infty$)

Measure is additive for disjoint sets (i.e. $\mu(\cup\, U_i) = \sum \mu(U_i)$)

From what we understand, this is new mathematics

What could a generalized measure theory be useful for?

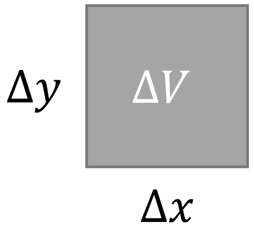
In a field theory, the value at each point is an independent d.o.f.

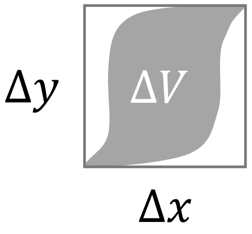
$\Rightarrow$ Measure of the volume “counts” the independent d.o.f.

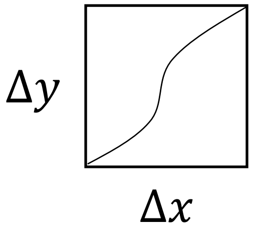
Yet, in a singularity this can’t be the case: value of the field at each point loses meaning;
Information encoded on the surface (holographic principle)

Flat space, zero curvature,
measure factorizes (i.e. $\Delta V = \Delta x \Delta y \Delta z$)

Singularity, infinite curvature,
“volume flattens”







What could a generalized measure theory be useful for?

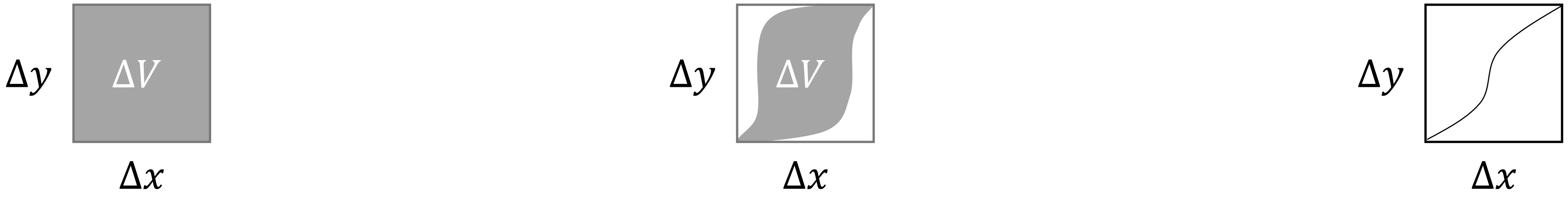
In a field theory, the value at each point is an independent d.o.f.

$\Rightarrow$ Measure of the volume “counts” the independent d.o.f.

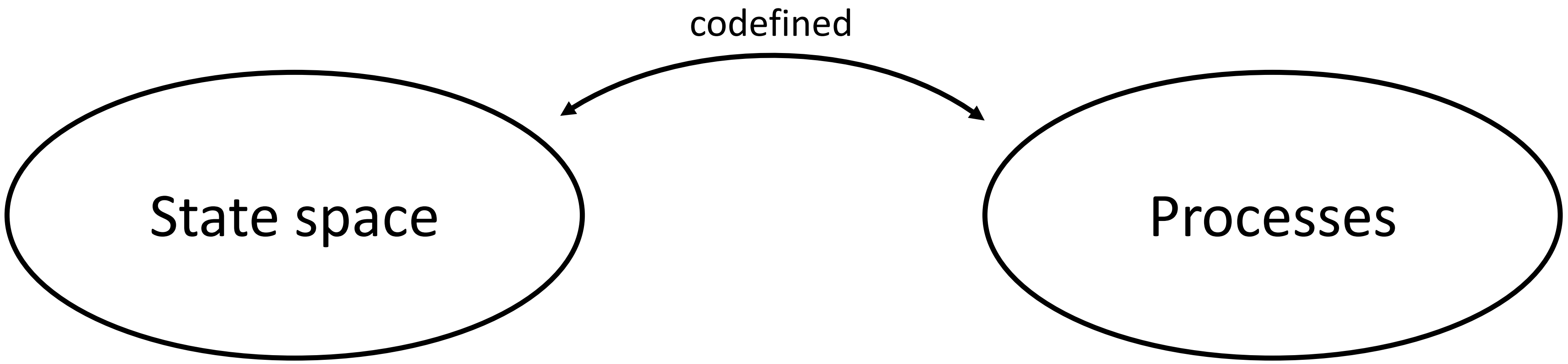
Is the curvature an indicator for how independent the values of the fields are? Does “quantizing” space-time mean using a non-additive measure, so that the count of d.o.f. does not go to zero (but to a finite measure)?

Flat space, zero curvature, measure factorizes (i.e. $\Delta V = \Delta x \Delta y \Delta z$)

Singularity, infinite curvature, “volume flattens”



Need a generalized theory of physical systems



State space must always be equipped with the processes under which the system is defined

Consistency requirements: state symmetries ↔ process symmetries; measurement processes ↔ open sets; system decoupling ↔ measure (and entropy) defined on states; …