Classical mechanics as the high-entropy limit of quantum mechanics

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Assumptions of Physics

Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

Assumptions of Physics

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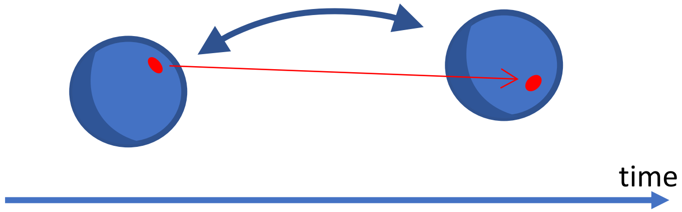
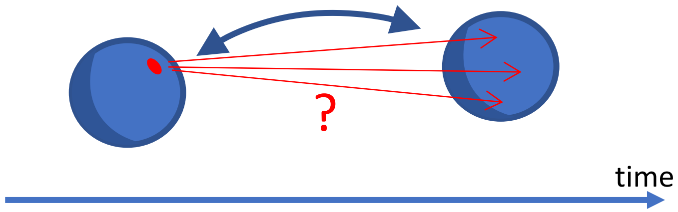
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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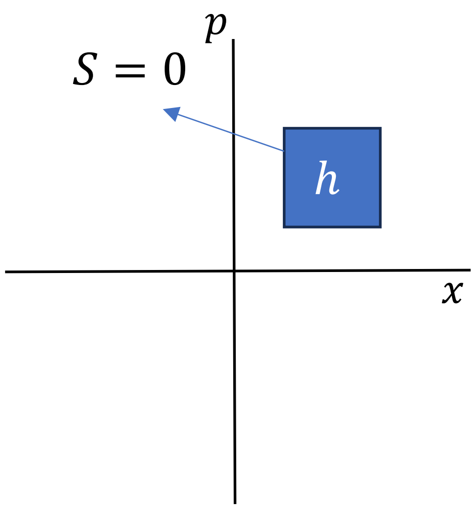
Assumptions of Physics

Third law of thermodynamics

$$S \geq 0$$

Physical entropy is absolute



Statistical mechanics

Define h as the support of a uniform distribution of zero entropy

$$S(\rho_U) = \log \frac{\Delta x \Delta p}{h}$$

$$S(\rho) = -\int \rho \log h\rho \, dxdp$$

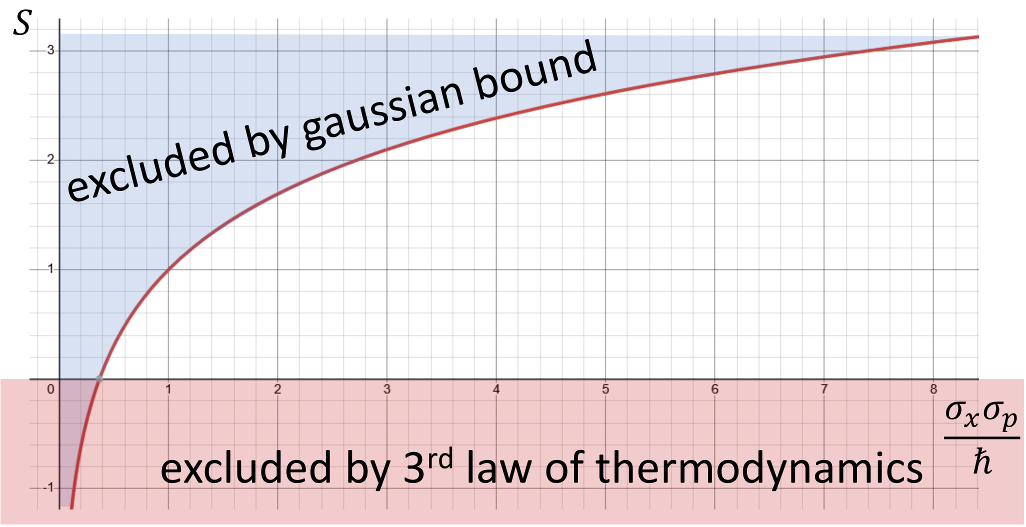
Fixes units (i.e. log argument is a pure number) and zero of entropy

Let’s plot entropy against uncertainty

Gaussian maximizes entropy for a given uncertainty

$$S(\rho) \leq \log \frac{2\pi e \sigma_x \sigma_p}{h}$$

$$\sigma_x \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$



$$S \geq 0 \quad \Rightarrow \quad \sigma_x \sigma_p \geq \frac{\hbar}{e}$$

Classical uncertainty principle

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Assumptions of Physics

Quantum mechanics incorporates the third law while classical mechanics does not

Is this the only difference?

Suppose the lower bound on the entropy is the only difference, then in the limit of high entropy of quantum mechanics we should recover classical mechanics

Can we?

Classical mechanics as high entropy limit?

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greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

Reply | Reply All | Forward | ...

Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

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Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

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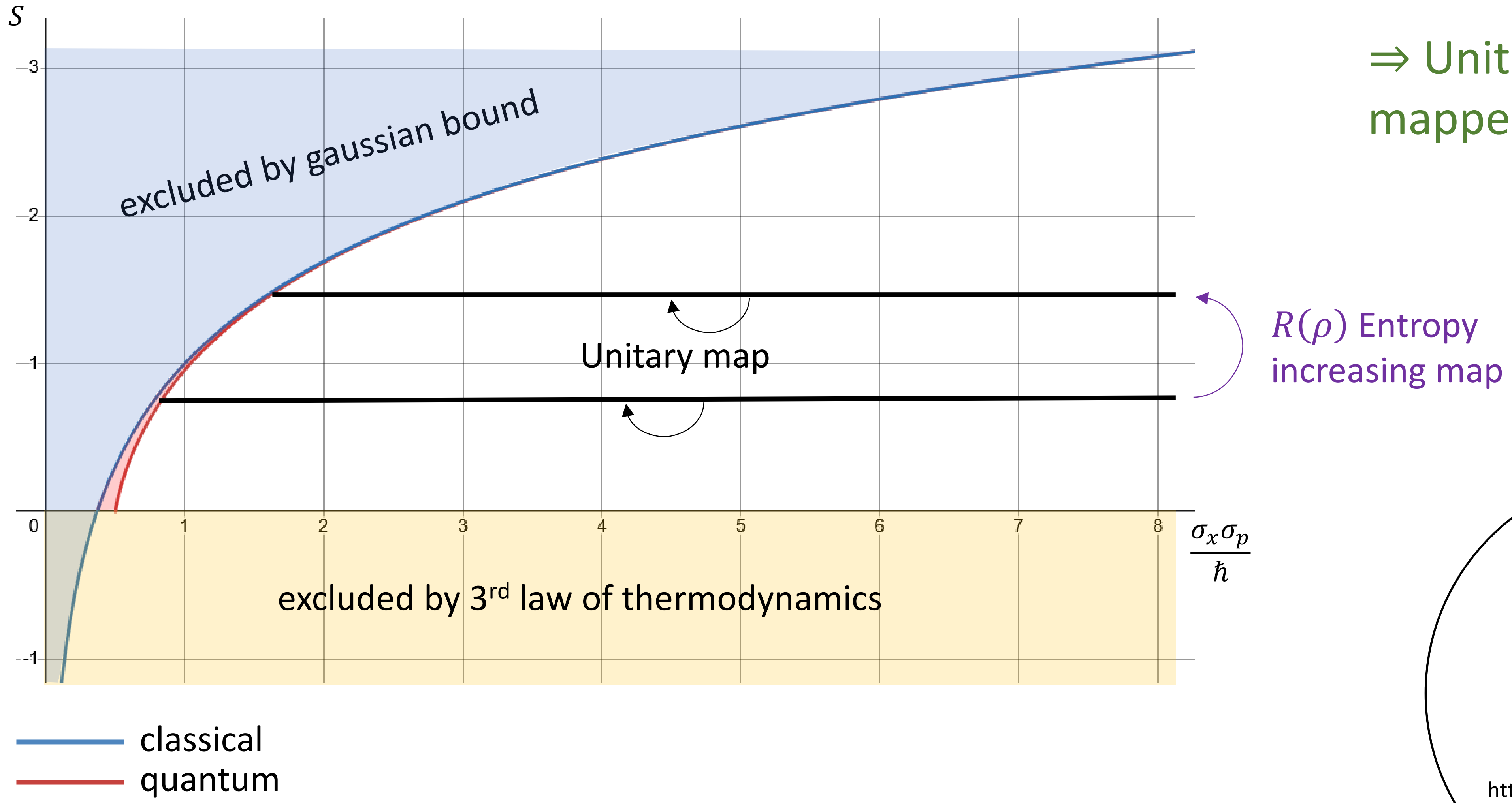
Assumptions of Physics

Looking for a map $R(\rho)$ that increases entropy of all mixed states, such that every level set of entropy maps to another level set



$\Rightarrow$ Unitary must be mapped to unitary

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Assumptions of Physics

In classical mechanics

$$S(R(\rho)) = S(\rho) + \log\lambda \Leftrightarrow \{R(x), R(p)\} = \lambda\{x, p\}$$

Jacobian is a constant:
all volumes rescaled
by the same factor

In quantum mechanics

Stretching map $$[R(X), R(P)] = \lambda[X, P]$$

Pure stretching map $$T(X) = \sqrt{\lambda}\, X \qquad T(P) = \sqrt{\lambda}\, P$$

Need to take care of
operator ordering!!!

Infinitesimal pure
stretching map

$$\frac{dX}{dt} = \frac{\imath}{\hbar}[H, X] + \gamma\left(L^{\dagger}XL - \frac{1}{2}\{L^{\dagger}L, X\}\right)$$

Lindblad eq
(open quantum
system)

$$L = a^{\dagger} = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\imath}{m\omega}P\right) \qquad \gamma = \lambda$$

Anti-normal ordering and Husimi Q are preferred

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Assumptions of Physics

Effects of stretching map on phase space representations

Husimi Q is simply stretched

Wigner function is stretched and convolved with a gaussian, and coincides with the Husimi Q in the limit

It is interesting to consider what happens to the negative regions of W under the stretching map. We know that W can have negative regions, but their size is limited by the uncertainty principle. In fact, convolving W with a 2D Gaussian with unitary spread, as in the definition of Q, returns a function that is never negative. In the limit $\lambda \gg 1$, the formula for W_λ reduces to

$$W_\lambda(\beta) \to_{\lambda \gg 1} \frac{2}{\pi \lambda^2} \int W_1\left(\frac{\alpha}{\sqrt{\lambda}}\right) e^{-\frac{2}{\lambda}|\alpha-\beta|^2} d^2\alpha = Q_\lambda(\beta). \tag{53}$$

Therefore, while negative regions can be in principle found at any finite λ, W tends to a positive function in the limit. As usual for the W distribution, the phase space size of negative regions is limited to $\hbar$ by the uncertainty principle. The weight of the function in such regions is limited between $\pm 2/\hbar$ for pure states. The effect of the stretching map is to reduce this bound to $\pm 2/(\lambda\hbar)$ for large values of λ. A clear interpretation can be made by working directly on the Fourier transform of W; see Eq. (51). The function $F(W_\lambda)$ is a scaled version of $F(W_1)$ with a Gaussian filter applied to it. Quantum information in W_1 is carried by spectral weights with k-vectors larger than 1. The bandwidth of the Gaussian filter is given by $\lambda/(\lambda - 1)$. This cutoff approaches 1 in the limit, filtering away the interference terms.

Another perspective: move the pure states to minus infinite entropy

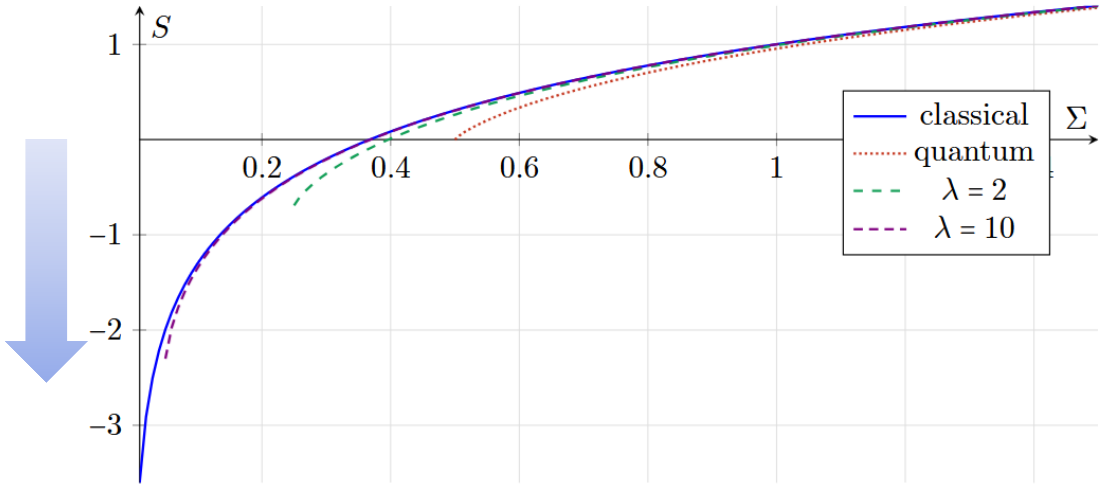
Instead of

$$[X, P] = \imath\hbar \qquad [T(X), T(P)] = \lambda\imath\hbar$$

Redefine original space such that

$$[\hat{X}, \hat{P}] = \frac{\imath\hbar}{\lambda} \qquad [T(\hat{X}), T(\hat{P})] = \imath\hbar$$

$$\lambda \to \infty \;\Rightarrow\; \frac{\hbar}{\lambda} \to 0$$



Mathematically equivalent to lowering the entropy of a pure state to $-\infty$, or $\hbar \to 0$ (group contraction)

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Assumptions of Physics

Low speed
"$c \to \infty$"

Speed

High entropy
"$\hbar \to 0$"

Entropy

Classical Mechanics	Relativistic Mechanics
Quantum Mechanics	Quantum Field Theory

No-mechanism limit
(same as non-relativistic limit)

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