Dialogue on a new approach to the foundations of physics

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A theory of scientific theories as the starting point for a general theory of science

Foreword

We want to write an introduction to our project, “Assumptions of Physics”, that is light, engaging and entertaining. This is a sample of the text we are working on. Please, send any feedback to dialog@assumptionsofphysics.org.

Boarding (full chapter)

A: Excuse me… I think I am there, next to the window.

B: Ah, yes. Let me get up.

A: Thanks. Let me put this under the seat.

B: I think this is yours.

A: Yes. Sorry about that. All done.

B: Are you traveling or going back home?

A: Me? Oh, I am going back home. Back from a math conference.

B: Are you at the University of Michigan?

A: Yes…

B: I am going there as well! My son is a junior at the University of Michigan. He studies engineering. You teach math? Maybe he had you for one of his classes…

A: Ah, no… I am not a mathematician. I am actually a researcher in the physics department.

B: Ah, physics! I love physics! I have all sorts of books about physics… I mean the pop-science ones. Not "real" physics books… I wouldn't understand the math. I never really got into all the abstract math one needs for advanced physics. At some point physics basically becomes pure math and that's more or less when I stop understanding. I can't follow. But I like hearing about it… the particle accelerators, the dark matter, the gravitational waves… So, what kind of physics do you do, that makes you go to a math conference?

A: Well, I lead a project on the foundations of physics… It's a very interdisciplinary project. I like to discuss the math with mathematicians, physics with physicists, philosophy with philosophers and so on… So I try to present at different conferences and see if I can collaborate with—

B: Foundations of physics? Wow! That's just… wow! So you work on some kind of string theory?

A: Eh, no… not string theory

B: Supersymmetry then?

A: No, that—

B: Quantum gravity?

A: No…

B: Surely some kind of grand unification theory?

A: No, not really.

B: Dark matter and cosmology?

A: No…

B: Ah, right! Quantum interpretations!

A: No, no… no interpretations.

B: Ok… Hhhhmmm…. Ok… So, what is it that do you do exactly?

A: Well, the goal of our project is to identify a key set of minimal assumptions from which the laws of physics can be rederived, in a way that is physically meaningful, mathematically precise and philosophically consistent.

B: Ah, you rederive the laws from principles… but like new laws? Things that explain dark matter or something?

A: At the moment the focus is to make it work for known theories. Theories we know work. I am sure that a better understanding will point to new ones, but—

B: I am confused here.

A: That's ok. I am often confused myself…

B: I mean: I thought that working on the foundations of physics means solving open problems, like making a quantum theory of gravitation; creating a unified theory that can explain new phenomena; tell us what happens in a black hole; predict new particles; describe what happens during a quantum measurement, … Foundations of physics means studying the fundamental forces and the fundamental constituents of the universe. Isn't that what people who work on the foundations of physics do?

A: Generally, yes.

B: But you don't…

A: That's not my primary interest, no…

B: So, you are a weirdo, huh?

A: Heh, heh, heh… I prefer the term "normally challenged".

B: Ha, ha, ha… great!

A: I mean, we did get an article rejected because "this is not what people discuss in the foundations of physics". Which *I* thought was kind of the problem…

B: Ah! So you *are* weird! I mean, "normally challenged".

A: Yeah… Our work is somewhat unusual, yes. But fortunately there are some who appreciate it.

B: But that's a good thing, right? New ideas? New perspectives?

A: People claim they want new ideas and new perspectives… then you go and say, "Here's a different perspective!" And they say, "Well, not *that* different."

B: Oh, yeah?

A: For example: at the opening of a new quantum center, the presenter stressed that it wasn't just quantum information, but all things quantum were invited. At the end, a professor approached the speaker and said: "I do quantum chromodynamics." He laughed and said, "Not *that* quantum…"

B: Heh, heh, heh… we are so inclusive… we are the most inclusive people… except for you.

A: We are inclusive… with the other guys.

B: I mean, those other guys are so much better than you… Great! So, help me understand. How is what you are doing foundations of physics?

A: Well, suppose someone… do you know sports? Like American football?

B: Yes! Go Blue!!![1]

A: Great! Suppose that I ask you to teach me the "foundations of American football". What do you teach me?

B: Well, I'd start with the goal of the game, what is a touchdown, all the rules… well… not all the different penalties… I guess the most basic things you need to know so you can understand and follow the game. That the Wolverines are the best team… Stuff like that.

A: Perfect! Now suppose I wanted to know the foundations of chemistry. What would you teach me?

B: Oh, I am not a chemist… but yes, I guess… that there are atoms… that they form molecules, with different types of bonds. They are all types of electromagnetic interactions. And that you can break bonds to form new ones, and gain or lose energy, and so on… again, the most basic information to—

A: …to have a general understanding, ok. Now, suppose you work on the foundations of mathematics. What would you work on?

B: Hhhmmm… like numbers and operations?

A: Well, not all mathematics is based on numbers. You can work with sets, shapes, propositions…

B: Ah, hhhmmmm… I guess… I guess I don't really know.

A: The foundations of mathematics typically deals with logic and set theory. These are the most fundamental mathematical structures. Everything else is built on top.[2]

B: Ok. I see the pattern. You are arguing that the foundations of a subject should contain the basic tools that are necessary to everybody who studies or is involved in that subject.

A: Correct. So, research in the foundations of mathematics, for example, means clarifying the basic tools and notions used by mathematicians. Two hundred years ago, they didn't have logic and set theory

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[1] College football is a very big deal at the University of Michigan, which hosts the biggest stadium in the United States and the second biggest in the world by capacity.

[2] Some mathematicians may not like set theory, and prefer category theory… Some may not like the axiom of choice because you can only prove an object exists without knowing how to construct it. Even if they may disagree on the details, the foundations of mathematics studies the most basic structures, upon which everything else is constructed. In fact, the fact that people disagree and there are competing views is an indication that the field is healthy.

formalized as we have now. They had more rudimentary tools, which sometimes led to confusion and paradoxes. Research in the foundations of mathematics has provided better tools for mathematicians, clearer understanding of what mathematics can and cannot do, made mathematics more precise, and so on…

B: Ok, but isn't that what is happening in physics as well? The foundations of physics studies the basic constituents and laws, and everything else is built on top of that. So, what's *your* problem?

A: I have *many* problems… but in this specific case: suppose that you are doing solid state physics, studying the property of some nanocrystal material. Some permutations of this element with this other impurity… They make hundreds and hundreds of these things. Someone comes along and says, "Oh, I have found dark matter!" Does that change what you are doing?

B: I guess not? I mean, I have no idea what you are talking about… but that seems the implication.

A: Fine, fine… So, let's try this: suppose you are teaching Newtonian mechanics… with the forces and the velocities and the momenta… You know about those, right? The laws and stuff… Good! Now, someone comes along and says: "I have unified general relativity and quantum mechanics!"

B: During my class? That's awfully rude!

A: Yeah, well… The sciences are full of socially challenged people… Anyway: does the fact that you have a theory of everything change what you are teaching? Is it like "Oh, I am sorry students… I really have to go rewrite the whole course because…" or you keep teaching the forces, the velocities, …

B: I see… Nothing would change. Newtonian mechanics is basically the same and we still teach it as is, even though other theories are technically more correct. It is still sufficient in a lot of cases and it's much simpler. Hhhmm… I see, we study the more correct theories after the simpler ones… I don't know much about string theory or quantum field theory, but I know Newtonian mechanics… at least what I vaguely remember from my days in college. Ok, so your point is that things like grand unified theories, new particles, dark matter are not needed by *everybody* who does physics…

A: Correct. Plenty of other things to do. We study nanomaterials and send probes to Mars without knowing what happens at Planck scale.

B: And we said that the foundations of a subject must include the basic tools, needed by everybody.

A: Right. You can't do any math without logic…

B: Therefore they are not really part of the foundations of physics?

A: That would seem to be the conclusion, wouldn't it?

B: Wouldn't it? Yes. It. Would, my friend. Intriguing. So, what you would call efforts like grand unified theories, resolving dark matter, …?

A: Me? Errr… Frontiers of physics? Aspirations of physics? The point is that they are the *goal* of physics, not the starting point.

B: Ooohhh… You are right! A theory of everything is the ultimate *goal* of physics… It's like… it's the foundation of the *universe*… but it's the *goal* of physics. Ha! Fun, fun, fun, …

A: Glad you are enjoying this.

B: Very much!

A: Look, in the end, I don't care what one calls what. The problem is that by calling those research areas "foundations of physics" the physics community thinks, "Oh, we have people covering the foundations" and we actually don't. And there is a void… which creates a number of problems…

B: Oooohhhh… such as? What problems?

A: Well, for example, are you aware… like the stereotype of a mathematician complaining of how physicists do math?

B: Like that they are fast and loose? And all imprecise about it? Yeah, there was some of that when I studied… But then physicists were impatient with mathematicians worrying about useless details…

A: Right. As we said before, mathematicians did their work on *their* foundations. So they built tools, like basic definitions, standards of proof, explanations, … that help them solve *their* problems, which are not necessarily the same needed to solve *physics'* problems.

B: Right. I always felt that mathematicians should do a better job to explain math with connections to physics, to make it more concrete…

A: That's what I also thought for a long time… until I spoke to a mathematician who made me understand that that's never going to happen, that's not a reasonable expectation. He openly admitted: "I know math, I only know the physics I vaguely remember from high school." It's like… if you are a carpenter, you don't expect a lumberjack to tell you what type of wood you need. That's not his trade. He cuts the tree. It's your job to know what to do with the wood.

B: So, you know about cutting trees, eh?

A: No, not really… it's the first analogy I could think of… A bit stretched?

B: I wouldn't know: I am not a carpenter.

A: Ha!

B: Ok… So, you say: it's not the job of a mathematician to give you the right mathematical tools to do physics.

A: Exactly. It's the job of a physicist working on…

A and B: … the foundations of physics!

A: Heh, heh, heh… exactly! To me it's like this… in the past century fields like mathematics, computer science, information science, probability… they looked inwards and worked on their foundations, their basic tools. Physics had all these cool new discoveries, quantum mechanics, general relativity, … and didn't do that… It focused on the cool new things.

B: Shiny things!!!

A: That drove the interest, and the funding. And we ended up with a void which is currently filled by simply throwing math at the problem, and seeing what sticks. And we have grown to think this is the right way to do it… everybody does it! Everybody has always done it this way!

B: Hhhhmm… You are essentially saying that the current mathematics is not a good foundation for science. Hhhmmm… that seems rather harsh…

A: Maybe… but I don’t think so. Let’s put it like this. Modern mathematics is not what you study in elementary school, or even in high school when you study Euclidean geometry. Modern mathematics studies abstract structures… formal structures. I think it was Hilbert that said, “Mathematics is a game played according to certain simple rules with meaningless marks on paper.” I found that most mathematicians agree with this. Long gone are the days when mathematics studied shapes and numbers from the real world. This is because by not caring what the symbols represent, only about their formal relationships, math has become more precise and more general. Only caring about the abstraction makes math better.

B: Yeah, but that’s where I get lost…

A: We *all* get lost. Only mathematicians don’t get lost. But the net effect is that math knows the rules of everything, but the meaning of nothing. For example, a mathematical equation is not enough to tell you what it is describing.

B: What do you mean?

A: Well, let me write this down. What does it represent?

$$F = m a$$

B: Force equals mass times acceleration: that’s Newton’s second law. See? I know stuff!

A: Did I say what F, m and a meant?

B: Well… no… not really… But what else could they mean? I mean F… Force

A: Who says the letters must be chosen that way? Suppose they mean this:

$$\overset{\text{Voltage}}{F} = \overset{\text{Resistance}}{m}\ \overset{\text{current}}{a}$$

B: Ah… that would be… That’s a dirty trick, though… It’s V=RI: Ohm’s law. Ah, I guess current doesn’t even begin with ‘I’…

A: It’s a trick, yes… but the point is that you can get tricked.

B: Ok, I guess mathematically they are the same equation with different letters. And the math does not really care what the letters mean. The quantity on the left, whatever it is, is the product of the two quantities on the right, whatever they are.

A: Some say it's the same equation with different interpretations. Terminology that I hate, by the way… But yeah: the equation is not enough. So, if I define my theory by giving you the math, like basically is done in most cases in modern physics, what are you going to understand?

B: Well, I wouldn't understand anything anyway because it's too complicated… But I guess you are saying I wouldn't understand anything until someone told me what the symbols meant physically.

A: Right. In practice people get familiar enough with the math to carry out some calculations that, hopefully at the end, are linked to things you can measure. But truth be told, we don't know what the math really means.

B: Ohhhhhhh… This is why we have interpretations of quantum mechanics, right? We have the mathematical structure of the theory, we can describe objects, make calculations, but we need an interpretation to tell us actually what the objects represent, what happens during a measurement, …

A: Right. The math is not a complete physical theory… and the only thing on which we agree is the math… well, and the experiments.

B: Ok, I think I am starting to see what you are saying. If we had a better foundation for physics, we would have math tools that would serve the physics, not the math, and have a precise physical meaning. A physicist would care about the details of those tools because they are the details of his trade, not the ones of a mathematician. But how does that work? What's the starting point?

A: The starting point is a general mathematical theory of experimental science: the theory of scientific theories.

B: A theory of scientific theories… what does that even mean? Is it the same as the theory of everything? Ah, but you said you do NOT work on a theory of everything…

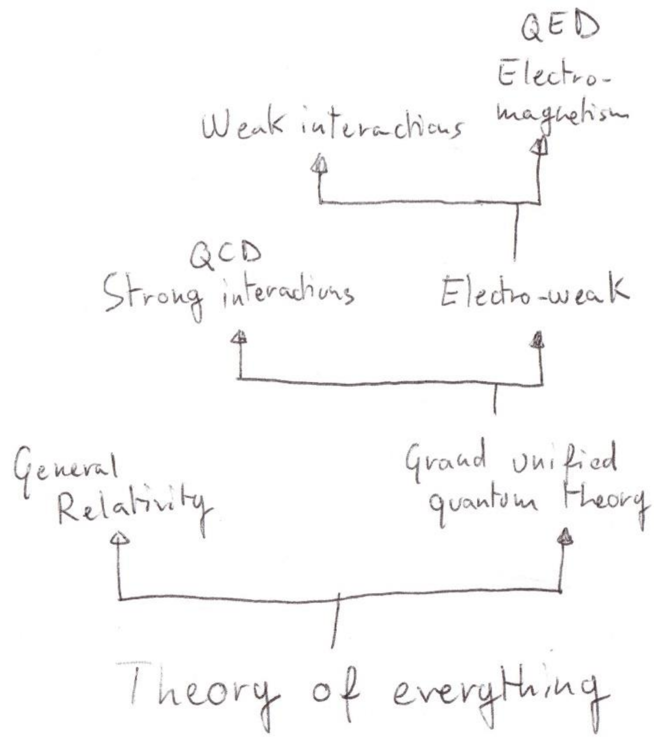
A: Right.

B: Ok, let's *pretend* I do not know what a theory of everything is… and start from the basics…

A: Pretend?

B: Yes, yes. Of course, I know… I just want to make sure *you* know.

A: Heh, heh, heh. Ok, this is how the theory of everything works… or would work since we don't have one… it's not even clear to me that one can be done in the way that we think it… but here. Look at this diagram:



A: You have the theory of everything, at the bottom, that tells you how all the constituents of the universe work: how space-time emerges, what fundamental particles are, if they are particles… maybe they are strings or cats… who knows! This would be very complicated. Like… imagine you had all the laws of the United States, at all level: federal, state, municipality… That would tell you what anybody is allowed or not allowed to do anywhere.

B: Yeah, that would be complicated!

A: So, since it's too complicated and you may not need all the details, you make approximations. Like: the energies are under this threshold or there is equilibrium at small length-scale… whatever… In this simpler case, the theory changes… It's not as correct as before, but it's more manageable. For example, it becomes general relativity here on the left; and with different approximations, on the right, you get some unified quantum field theory.

B: Ok…

A: With the law analogy, you say something like, "In the United States you can't own a cheetah as a pet." Now, this may not be technically true because in Watson, Missouri there is a special ordinance that would allow an ex-marine to own one as an emotional support animal, but, you know, close enough.

B: Is that true?

A: The cheetah thing? I don't know… I made it up.

B: Ah, that was awfully specific.

A: Yeah… well…

B: Please, continue.

A: So, on the right, we have a unified quantum theory. Which is again quite complicated. But see… at lower energy, it really behaves as two different, independent forces. So, in those regimes, we break it up into electroweak interactions and strong interactions. Again, reality is more complicated, but in those cases it's good enough. It would be like saying that there are the laws for crimes against property and laws for crimes against individuals… and that is a good way of thinking about it. But, naturally, there could be laws that apply if you commit crimes against both…

B: Ok, I see… Let me rephrase to see if I understand: you start with the "true" theory, the one that describes everything about everything. The Theory of Everything.

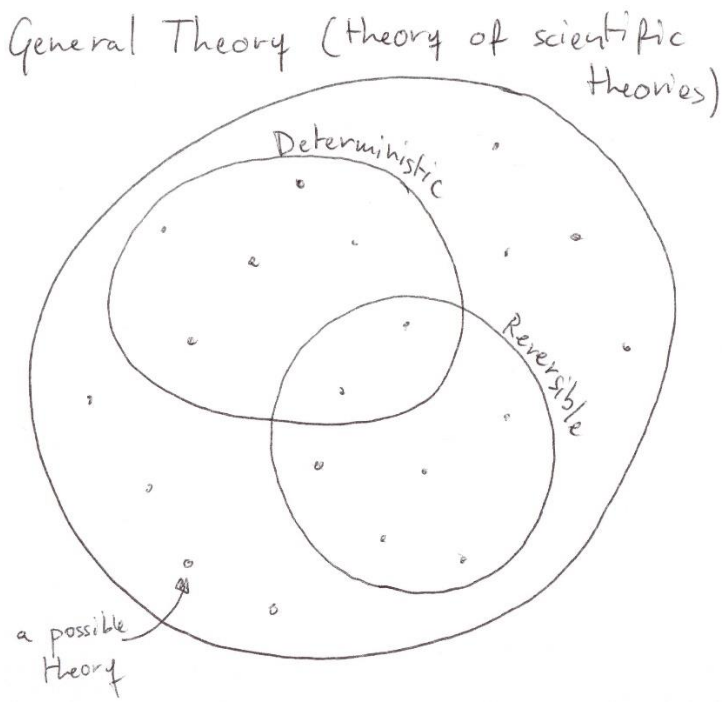
A: Yes.

B: Then you simplify it, make it less precise, by throwing out some detail, and gradually get to the theories we have now, which are less precise versions of the true theory, and only work in limited conditions.

A: Exactly. You start from a theory that is the most complicated and knows everything, and you keep changing it into simpler theories that know less.

B: Ok. So, that was the theory of everything… which is not what YOU do. You do the theory of… of whatchamacallit.

A: The theory of scientific theories… let me… or general theory, I also call it general theory… because it does not give anything specific. Here you go:



A: The outside circle represents the general theory. It says: a scientific theory must have at least these characteristics… must be generated from a set of experimentally verifiable statements, they must describe some physical process, you must be able to define a system and its states, blah, blah, blah… By defining what the fundamental features are, it defines the set of all possible scientific theories. So the general theory is essentially a bag that contains all theories.

B: So, each point in the general theory is a possible scientific theory?

A: Yes. It may be useless… but at least it has these basic requirements. As an analogy, imagine we have a definition for animals, that would be the general theory, which would identify the set of all possible animals.

B: Ok. This is the set of all possible theories… All the theories we know are here, and also the ones we don't yet know. The theory of scientific theories tells you what basic properties each theory must have. Like all animals, I don't know… eat, reproduce, move… do all animals move? I don't know… whatever…

A: Yes. Now you say you pick a feature a theory can have. For example: I am interested in the theories that are deterministic; where the past state can predict the future state. Not all theories have that property, so you have a subset. Like if I chose animals that can fly, or animals that have a beak. They would be a subset of all the animals. You can also have theories that are reversible: the future state is enough to reconstruct the past state…

B: And where the set intersects like a Venn diagram, you have those that are both deterministic and reversible?

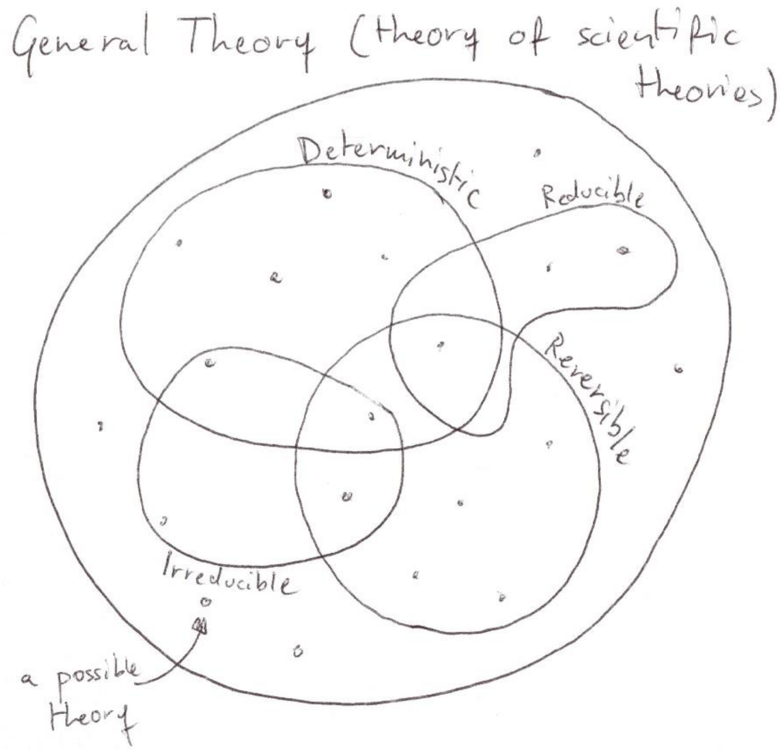
A: Yes: for each past state you have one and only one future state.

B: How do you know which theory is true?

A: Doesn't matter at this point. We are basically cataloguing theories. And then you pick the one you need based on the circumstances.

B: Oh… This is really a taxonomy for theories…

A: Yes. For example, let's add two things:



A: Determinism and reversibility were assumptions about the process. Now we add assumptions about the system. The system could be reducible: the state of the whole tells you everything about all the parts. The system could be irreducible: the state of the whole tells you nothing about the parts. These are incompatible, either you are reducible or irreducible, so the sets are disjoint…

B: And you have all the different cases: non-deterministic, non-reversible and reducible; deterministic, reversible and irreducible. I mean, I don't know exactly what they mean, but I understand you have all these different cases you are cataloguing…

A: Right.

B: I think I get the general picture. You start with something simple that has few details. Then you add more detail… Like: I have an animal… Then I say: the animal has four legs, so it rules out birds and insects, but it can still be a mammal or a reptile. Now I know that it has fur, so it can't be a reptile anymore…

A: Exactly. You start from the simplest theory, that only knows the most basic facts, and you keep adding things and get a more and more detailed theory. And you have different choices, different assumptions you can make along the way.

B: It's kind of the opposite of the theory of everything. From simple to complex instead of from complex to simple. In practice, how is this better?

A: First, because the definitions and theorems you have in the general case are always valid. Like for animals: all things that are true for animals are also true for vertebrates; all things that are true for vertebrates are true for mammals and so on. You start with a basic set of equations. And then you add another set of equations… but you don't invalidate the old ones.

B: While, in the theory of everything, you change the equations? I guess because you are making the approximations?

A: Right. In the theory of theories, instead, all equations and theorems proved in the general case apply to any specific case. So, if I prove something for deterministic theories, it is valid for theories that are deterministic and reducible. So when you talk about general concepts, like states and processes… it's the same exact concept in all theories. You can reuse a lot of concepts and the mathematical results that go with them.

B: Reduce, reuse, recycle: very efficient!

A: It's also better because you can find simple characterizations that recover the actual theories. So, classical systems are reducible systems. If you add determinism and reversibility you get classical Hamiltonian mechanics. Quantum systems are irreducible systems. If you add determinism and reversibility you get the Schroedinger equation. So, if you understand what those words mean, you have completely understood the theories… because you can rederive the math.

B: Ah! So, I don't need the math?

A: No. The math is just a way to formally rephrase the physics. Don't get me wrong: you need it to make sure everything checks out. *I* need to know the math. But to understand the ideas? No.

B: Ok, this is pretty different… Never heard of anything like it. It really seems like a different approach. I guess I am really curious… Can you tell me more?

A: Sure! But maybe we should wait until after the airplane safety video.

Takeoff (excerpt)

…

B: There is still one thing that really bothers me.

A: Only one?

B: Well, let's start with this one. From what I gathered, and correct me if I am wrong, but you seem to say: there are these things we need, experimental verification, this and that… and then there are things we assume, like deterministic whatevers and whatnot… and then you crank out the laws, correct?

A: Yes.

B: You are basically deriving the laws from ideas, right?

A: Yeah… at least that's the goal … I mean it was surprising to us how much you can actually—

B: Fine, fine… But here's the bit that puzzles me: aren't the laws of physics found experimentally? I thought that science works by formulating different models, different hypotheses, and then performing experiments to see which model is correct. In thermodynamics, for example, you see that energy is always conserved, and you make it a law. Quantum mechanics was put together from different results that couldn't be explained with classical mechanics. I mean, medicine historically has been trying different things and seeing what works. The way I understand it, science is about induction: you play around with something, you see some patterns, and you generalize them. Experimental data is the starting point. You seem to be saying we can deduce the laws… with a logical argument, right? How is that even possible? Isn't that like the opposite of science? Isn't this philosophy?

A: Well, let's give a concrete example. You know Galileo?

B: Yes, the one dropping things from the tower of Pisa. Watch out! He's up there again!

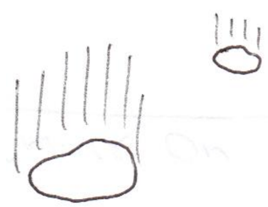
A: Very dangerous man… Do you remember why he did that?

B: To check whether different weights reach the ground at the same time. Before people thought that heavier objects fell faster than lighter objects. Galileo showed, experimentally, that they all reached the ground at the same time. He prepared different balls of the same size but with different materials, dropped them from the top of the tower and saw that they touched the ground together.

A: Right. Aristotelean philosophy said heavier objects fall faster.

B: But that's exactly what I mean: he learnt that experimentally. In principle, we could have had that heavier objects fall faster. It's the experiment that tells us how nature works. How can we reason one way or the other?

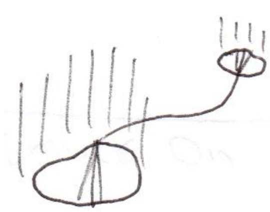
A: Well, suppose we have two objects, one heavier than the over. Like this:



A: If heavier objects fall faster than lighter objects, at some point the lighter will be left behind by the heavier. Right?

B: Right.

A: Now, suppose we tie them together. Like this:



A: Now they are forced to fall at the same speed. Will they fall faster or slower?

B: Ah. Well… That's easy. I imagine the lighter one would slow the heavier one down. In the same way that a slower runner would have to be dragged by a faster one tied to him. So the whole thing would fall somewhere in between: slower than the heavier one but faster than the lighter one.

A: But don't the two objects tied together form a third object?

B: I suppose so…

A: Wouldn't its weight be the sum of the two? Wouldn't it be heavier than both of them?

B: Yes, it would.

A: But didn't we suppose that heavier objects fall faster? So, shouldn't the objects tied together, as a whole, fall at a faster rate than the two individual objects? Because they form one heavier object?

B: Ok…

A: So, which one is it: does it fall faster or slower?

B: Ohhhhhhh… My head hurts… Yeah, this does not make sense.

A: You see: supposing that heavier objects fall faster than lighter objects leads to this double answer. You could both argue that the composite system falls more slowly or faster than the heavier component. This makes no sense. Note that you don't have this problem if all objects fall at the same rate, tied or not.

B: This is really interesting. If I can summarize: the law has to tell us what happens to both the whole and the parts. A part will always be lighter than the whole. If heavier objects fall faster than lighter, we would have that the whole falls faster than the parts. This is contradictory because parts have to fall at the same rate as the whole. Therefore heavier objects cannot fall faster than lighter. Heavier objects must fall at the same speed as lighter ones.

A: Precisely. I couldn't have said it better myself. This is the type of reasoning we are looking for.

B: I see. So, what you are saying is that Galileo didn't really need to drop objects from the tower of Pisa if he simply reasoned this way?

A: Actually, Galileo *did* reason this way: this example is from one of his books… one of his dialogues.

B: Oh!

A: In fact, many historians think he didn't actually drop—

B: So why have I never heard of this argument? But heard about him dropping things?

A: Beats me! Note that the reasoning is never really enough, though… you may have faulty initial assumptions or reasoning. In fact, Galileo gave other arguments for other things which turned out to be wrong.

B: So you still have to do experiments…

A: Definitely. And many many times the reasoning is driven by new data. But, ask yourself this: what gives you more insight? The actual experiment of dropping things from the tower or the thought experiment of tying the weights together?

B: Well, obviously the thought experiment.

A: Why?

B: Because it points out that there really wasn't another choice. It's not an arbitrary rule: it's necessary. The alternative does not make sense. In your scheme of things: there was no alternative theory in the set of all possible theories.

A: Exactly. Thought experiments are in fact used a lot in physics. And sometimes do lead to actual experiments. Now, suppose we could reorganize the whole of physics like that. Suppose we were sufficiently wise[3] to construct a clever argument that, from simple premises, showed that some law must follow. So, when studying this type of system in these conditions we get classical mechanics. But with this other type in these other conditions we get quantum mechanics. Or thermodynamics. And so on. Wouldn't that be more satisfying?

B: I guess… Still… I don't know, it seems you are relegating experiments to just a final check. They almost seem too unimportant…

A: No, no! Lord, no! That's not at all it! Don't know how I gave you that impression…

B: Ah!

A: The whole object of the law are things we can tell experimentally. The laws are *about* what we can tell experimentally. What I am saying is that once we have fully defined what we are looking at and how, the laws are just a rehashing of what we have already said. We say there are these things we call objects, which we can touch and manipulate; we say each of these things has a weight we can measure; we say we have a way to put objects together to form bigger objects; we see that the weight sums; we say their weight can influence the trajectory, which again we can measure… All of these things are prerequisites. All of these are experimentally defined. The fact that the trajectory is independent from the weight is a consequence.

B: Aaahhh… Ok. Then you say, given all these things that we can experimentally test, they must fall at the same rate regardless of weight…

A: And you go and experimentally test that too. So, experimental verification plays the most central role: it defines the objects of our discourse.

…

Climb (excerpt)

…

B: I want to explore a bit more that difference. The way I was always explained it was that classical mechanics is for large objects, everyday objects… airplanes, seats, pencils… While quantum mechanics is

[3] Richard Hamming wrote: Thus my first answer to the implied question about the unreasonable effectiveness of mathematics is that we approach the situations with an intellectual apparatus so that we can only find what we do in many cases. It is both that simple, and that awful. What we were taught about the basis of science being experiments in the real world is only partially true. Eddington went further than this; he claimed that a sufficiently wise mind could deduce all of physics. I am only suggesting that a surprising amount can be so deduced. Eddington gave a lovely parable to illustrate this point. He said, "Some men went fishing in the sea with a net, and upon examining what they caught they concluded that there was a minimum size to the fish in the sea."

for small objects. I also heard people arguing that we cannot form a good intuition about quantum mechanics because our perception is based on large objects, so quantum mechanics can never be intuitive.

A: Yeah, I've heard those arguments… I hate those arguments… They sound more like arguments to justify one's shortcomings.

B: Ha! Like the fox and the grapes?

A: Yeah! I can't reach them… they are too high! Bah, they weren't ripe anyway.

B: I don't want sour grapes… I see… But you are saying that big and small have nothing to do with classical vs quantum?

A: *I* am not saying this… experiments are saying this. First of all, what do you mean by small?

B: Like, the size.

A: Ok, you can create an entangled pair of photons, which is a purely quantum effect, and use fiber optics to put them kilometers apart. And they are still entangled. So you have a quantum object that spans kilometers. Is that big enough?

B: Fine, but they are only two particles, right? So, maybe it's the number of particles that matters… Or the total mass?

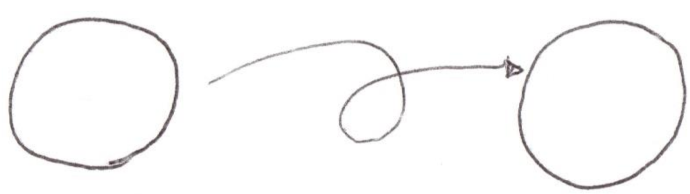
A: Well, you can create Bose-Einstein condensates, which are again quantum systems, with a large number of particles, comparable to Avogadro's number. Superconductivity and superfluidity are macroscopic quantum effects. I am no expert, but don't people build trains with that? So again, they seem big enough to me!

B: Ok. Big vs small does not seem to work. And you are proposing… what is it, reductionality?

A: Reducibility…

B: And what's that?

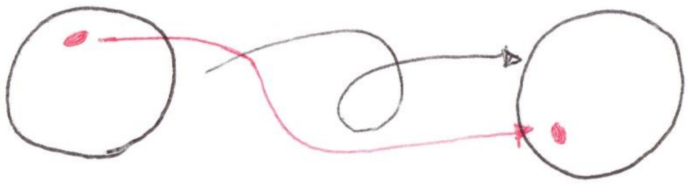
A: Ok, suppose you have a ball:



A: You can throw it, see how it moves, study its motion, write equations for that.

B: Yeeessss…. And?

A: But you can also take a marker and make a red dot on the ball:



A: And you can study the red dot and its motion. The idea is that if you study the motion of the ball it is the same as studying the motion of all the possible red dots you could draw on the ball.

B: Right… I know the behavior of the whole, and I know the behavior of the parts.

A: In this case we say the ball in reducible. Studying the whole is the same as studying the parts. We say the whole is REDUCED to its parts.

B: Ok… fair enough… and this would be a classical system?

A: Yes, a classical system is infinitesimally reducible. Not only is studying the parts the same as studying the whole, but we can make the parts as small as we want.[4]

B: So, you say quantum objects do not have this property?

A: Right. Quantum systems are irreducible. Suppose we have an electron.

B: It looks awfully similar to the ball…

A: Yeah, well… Everyone's a critic… Suppose we have an electron: we can shoot it, study its motion. But the only way we can probe an electron is to make it interact with another particle… a photon. And when the photon interacts, it has to interact with the WHOLE electron.

A: We can't take a red marker, and see what a part of an electron is doing. So we can't know what each part is doing.

[4] In this view, classical particles are infinitesimal parts, each concentrated in a point in space with a single momentum. These are abstractions that exist only under the infinitesimal reducibility assumption. The full state of a classical system is a matter distribution $\rho(q, p)$ that tells us where each part of the system is and how it is moving.

B: I see… We can only extract information about the whole.

A: Precisely. So, studying the whole tells us nothing about the parts.[5] This is where the uncertainty comes from. The electron has finite mass. We can describe the state and behavior of the whole mass. We can't describe what this fraction of mass is doing or that fraction. The description stops at the level of the electron. In a way, quantum interpretations miss the point because they try to give a more detailed account. But the whole point of quantum mechanics is that you can't do it experimentally.

B: I see… but if we did find a way to interact with the parts?

A: Then the electron would cease to be a single quantum system. We could probe its internal dynamics and find a substructure. In fact, that is exactly what happened with the proton and the neutron. Initially people thought they were fundamental particles, and in many cases, you can treat them as a single quantum system, and make them interfere, diffract… exactly like a fundamental particle, like an electron. But if you go to higher energy, to smaller length scale… then you start probing the internal structure, you become sensitive to it. And then you can't describe them as a single quantum system. They become a composite system of quarks and gluons.

B: That's— Hhmmm… These are not really fundamental particles— Let me rephrase: you say that, technically, we do not know if the electron is really a fundamental particle. It's a fundamental particle for now, because we don't know how to probe the internal structure? Maybe it is, or maybe it is not?

A: Correct. The statement "the electron is a fundamental particle" is not verifiable… you can only say you didn't find substructure.

B: … but it is falsifiable. Because if you find it…

A: Very good! You got into that stuff!

B: Yeah, I am an expert now! Ok, so the whole thing is that classical systems are divisible, while quantum systems are not…

A: Not divisible! I didn't say divisible! I said reducible! We have to…

A and B: be precise!

B: *Sigh*… Fine, fine… So, what's the difference? Seem the same to me.

A: We say something is divisible into parts if you can take it and break it apart into independent pieces.[6] We say something is reducible to the parts if the description of the whole can be given in terms of the parts.[7]

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[5] The quantum state, then, is given by a function $\psi(q)$ that tells us how much/how frequently the system can be found in a particular place. Saying more (e.g. the distribution given position and momentum; whether part of the system is always at a spot or whether the whole system is at that spot with a given probability) would require knowing more about the dynamics of the parts.

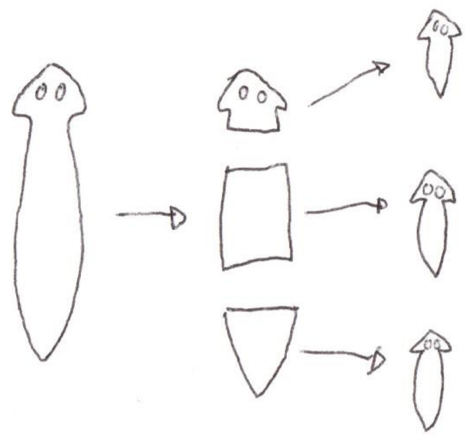
[6] Mathematically, we have a process $\mathcal{T}: \mathcal{S} \to \mathcal{S}_1 \times \mathcal{S}_2$ that takes the initial state $\mathscr{s}$ for the system $\mathcal{S}$ into a final state made by the pair $(\mathscr{s}_1, \mathscr{s}_2)$ of states for the two independent systems $\mathcal{S}_1$ and $\mathcal{S}_2$

[7] Mathematically, we can write $\mathcal{S} = \mathcal{S}_1 \otimes \mathcal{S}_2$ meaning the set of the possible states for the system $\mathcal{S}$ can be constructed through the operation $\otimes$ from the set of the possible states of $\mathcal{S}_1$ and $\mathcal{S}_2$.

B: Definitions? Ooohhh… I thought we agreed we needed…

A: Fine. Examples! You want examples! Do you know what a planarian worm is?

B: A type of worm?



B: Aaahhh… It's the one that if you cut it into parts, each part regrows into a full worm. I vaguely remember that. What was the plenarium worm?

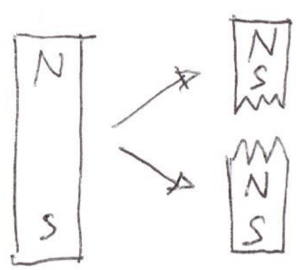
A: Planarian…

B: Planarian… Cool. So, what about it?

A: You can take a planarian worm and divide it into three worms. So, the planarian worm is divisible into three worms. But it's not reducible to three worms. Describing one worm is not the same as describing three worms. It's not that each worm is secretly made of three worms…

B: … which are made of three worms which are made of three worms… I get the point. The worm is DIVISIBLE into three worms but it is NOT REDUCIBLE to three worms.

A: Now take a magnet:



A: You can describe its arrangement by describing where the north and south poles are. But if you break the magnet, you do not have a separate north and south pole. You get two magnets, with a north and a south pole each. So the magnet is reducible into a north and south pole but not divisible into a north and south pole.

B: Aaahhh… The magnet is REDUCIBLE to north and south poles but it is NOT DIVISIBLE into north and south. So divisibility is about whether you can physically break things apart, while reducibility is about whether you can break the description apart! Why didn't you say so?

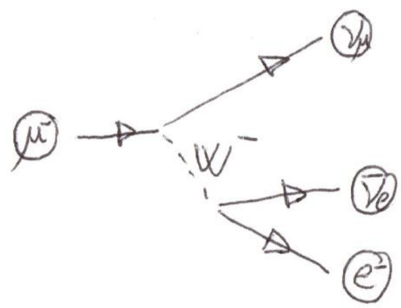
A: But I said—

B: Heh, heh, heh… I am sure you did. Go on…

A: Now, suppose you have a muon.

B: This is like another fundamental particle, right? It's like an electron but heavier?

A: More massive, yes. Now a muon can decay according to this diagram…



A: … in fact it *will* decay given enough time. You start with a muon μ^- on the left of the diagram. It breaks up into two particles: a neutrino ν_μ and a boson W^-. But the W^- is very unstable, so it decays into two other particles: an antineutrino $\bar{\nu}_e$, the bar on top means it's an antiparticle, and the electron, e^-. So, when all things are done, the muon is divided into three particles… but you see: it's not made of those three particles. The state of a muon is not the state of an electron and two neutrinos. A muon is not secretly an electron and two neutrinos in disguise…

B: Yeah! The muon is its own particle!

A: Yes, it's very independent…

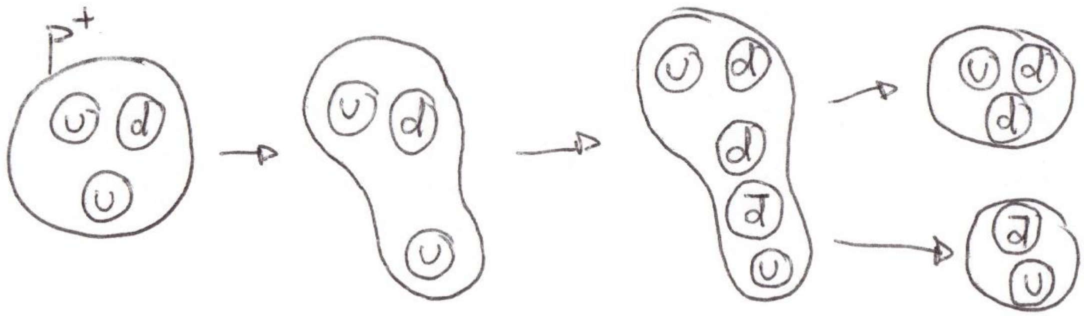
B: So, like the worm, the muon is DIVISIBLE into three particles but it is NOT REDUCIBLE to three particles… Ah, it's really like the worm! Now I see why you chose that example!

A: Yeah, took me a long time to find it… I have been using it for years!

B: Nice! Go on!

A: On the other hand. A proton is made of quarks and gluons. The proton is reducible to quarks and gluons.[8] But you can't divide it. Suppose you start with a proton…

[8] We actually still do not know exactly how.



A: … a proton is made of two u quarks and one d quark. If you try taking one quark out, you have to put energy in and you have an excited proton.

B: I love how particles get excited when they get energy!

A: And the proton gets so excited that the energy itself can't remain energy, but becomes a quark-antiquark pair, for example d and $\bar{d}$. And then it splits into two particles. You can't have a quark by itself.

B: It's very gregarious…

A: A true party animal…

B: So, like the magnet, the proton is REDUCIBLE to the quarks but NOT DIVISIBLE into the quarks. If I try to take one quark apart, I create a quark-antiquark pair… this is like for the magnet, where I create a north-south pair.

A: Exactly!

B: Ok, so divisibility and reducibility are really different. If I divide something, I first have something, break it apart, and at a later time I have multiple things. While reducibility—see, I got it right—reducibility is at the same moment in time. The state of the magnet is the state of the north and south at the same time. The state of the proton is the state of the quarks and gluons. And there is nothing being broken. So… why do you care exactly about this distinction?

A: Because quantum mechanics is about irreducibility, not indivisibility.

B: Ah!

A: A particle is not fundamental because it cannot be divided. Fundamental particles decay, fuse into each other, transfer energy/mass to each other… In this sense, they are divisible: part of their mass/energy can be taken out, converted to other particles, and so on. But they are irreducible: we cannot give a description of its internals. If an atom absorbs and then re-emits the same amount of energy, we can't tell whether the re-emitted energy is the same as the one that left. This is a crucial difference and, unfortunately, is missed by most people.

B: Why is it crucial?

A: Because it tells us how and why the game ends. You see: physics does not stop when we said everything about everything. Physics stops when there is nothing else we can say. Physics stops when reductionism stops, when we can't describe the substructure. That does not mean there is no substructure…

B: Could be like the proton… when you go to higher energies, you find other things.

A: Exactly. There may be substructure, but our measuring devices, all processes at our disposal, are not sensitive to it.

B: I see… This is again the same point over and over: physics is about what we can reach experimentally, not about the ultimate nature of the universe. Quantum mechanics is basically setting a bound to what can be said.[9]

[9] Consistently with this picture, we can look at how entropy works. In classical mechanics, the entropy of a probability distribution $\rho(q, p)$ can be made arbitrarily small, meaning we can create states whose description is as precise as we want. In quantum mechanics, the entropy of a state ρ is minimal and is the same for all pure states, meaning a more precise description is not possible. The entropy of the system is really what is being quantized.