Reverse Physics: uncovering the Assumptions of Physics from its laws

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https://assumptionsofphysics.org

Assumptions of Physics

To identify a handful of physical principles from which the basic laws can be rigorously derived

Can we create a precise and meaningful map between mathematical tools and physical ideas, instead of abstract definitions and analogies?

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What exactly is the action, and

How do we turn physical requirements into precise mathematics?

Which philosophical questions are useful and which are a distraction?

What fundamental physical ideas hide behind t

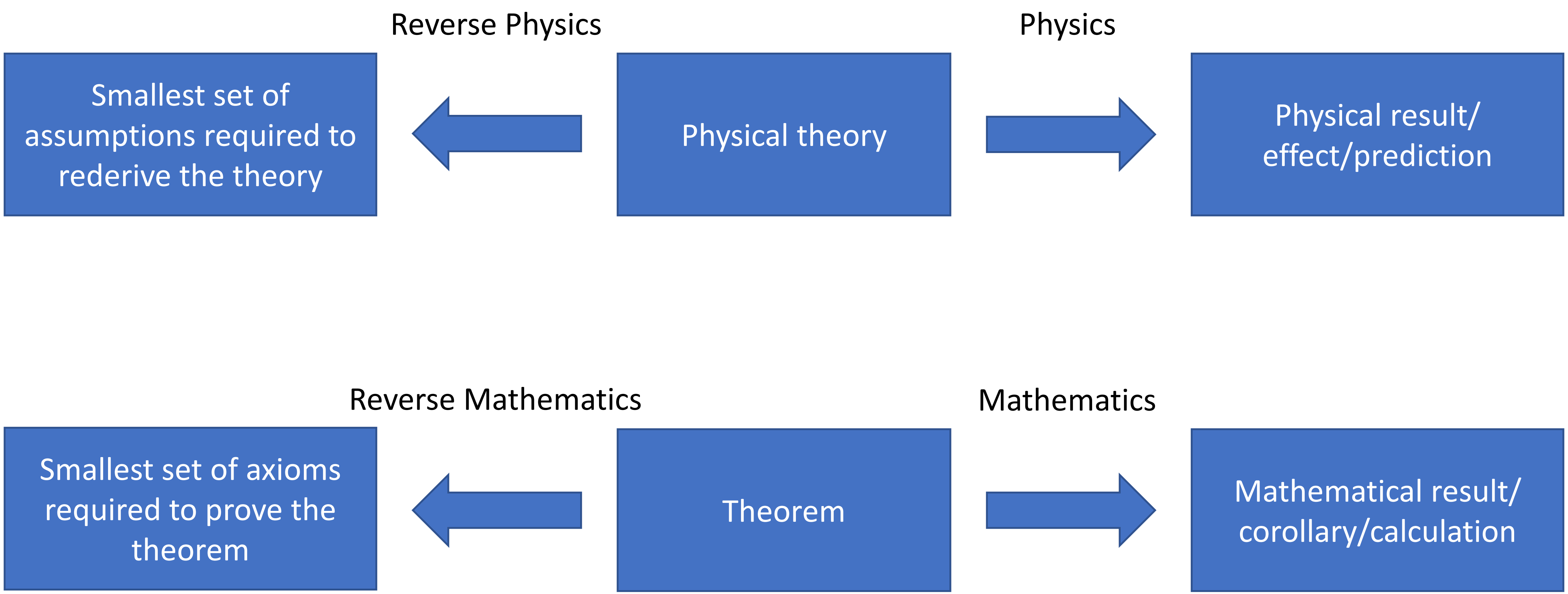
What are mathema

Is the ideal of mathematical precision useful in

Are the current mathematical foundations and tools suitable for physics?

What are the most fundamental notions in physics?

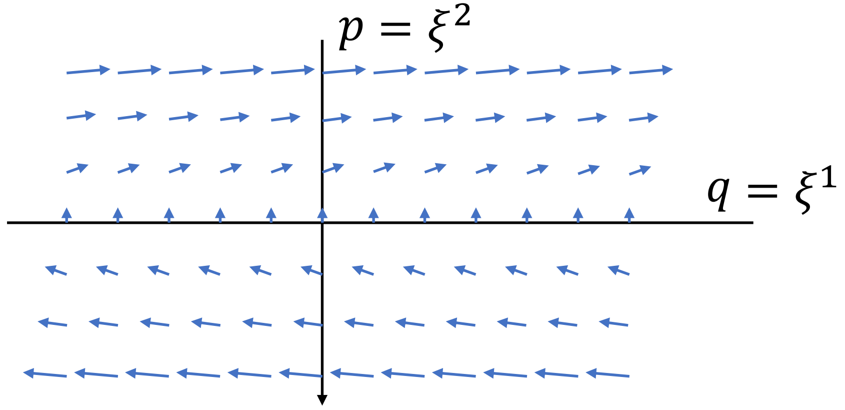
Why position/momentum and pressure/volume, and not position/volume?


Reverse Physics
Physics
Smallest set of assumptions required to rederive the theory
Physical theory
Physical result/ effect/prediction
Reverse Mathematics
Mathematics
Smallest set of axioms required to prove the theorem
Theorem
Mathematical result/ corollary/calculation

Reversing Hamiltonian mechanics

one degree of freedom

(1) Hamilton’s equations



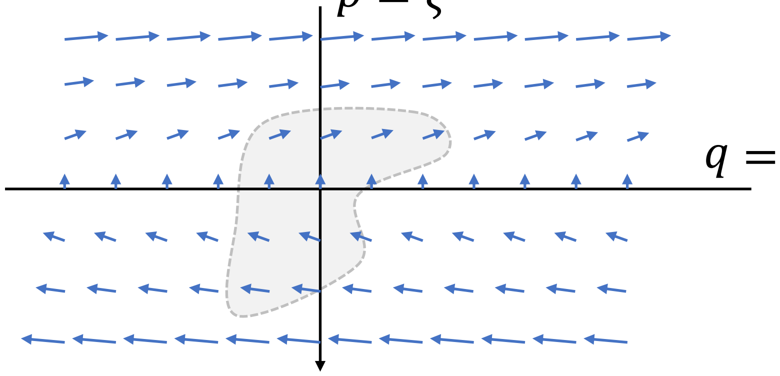
$$\frac{dq}{dt} = \frac{\partial H}{\partial p} \qquad = S^q$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q} \quad = S^p$$

$$\xi^a = \{q, p\}$$

$$S^a = \frac{d\xi^a}{dt} = \left\{\frac{dq}{dt}, \frac{dp}{dt}\right\}$$

(2) Divergenceless displacement



Suppose S^a divergenceless

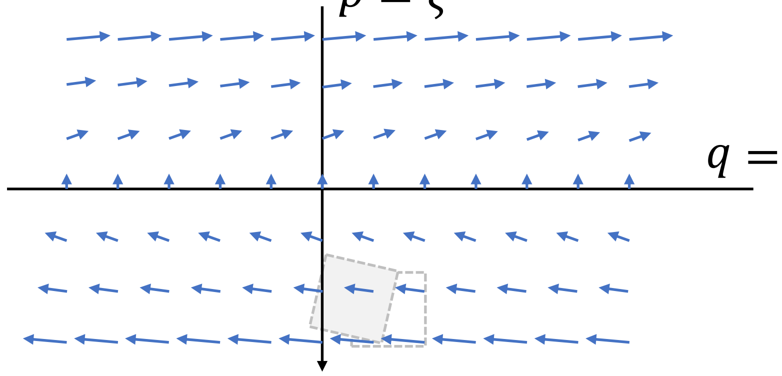
$$div(S^a) = \frac{\partial S^q}{\partial q} + \frac{\partial S^p}{\partial p} = 0$$

Then there exists a stream function H such that

$$\left\{\frac{\partial H}{\partial p}, -\frac{\partial H}{\partial q}\right\} = S^a = \frac{d\xi^a}{dt} = \left\{\frac{dq}{dt}, \frac{dp}{dt}\right\}$$

(3) Area conservation ($|J| = 1$)

Study how the area evolves



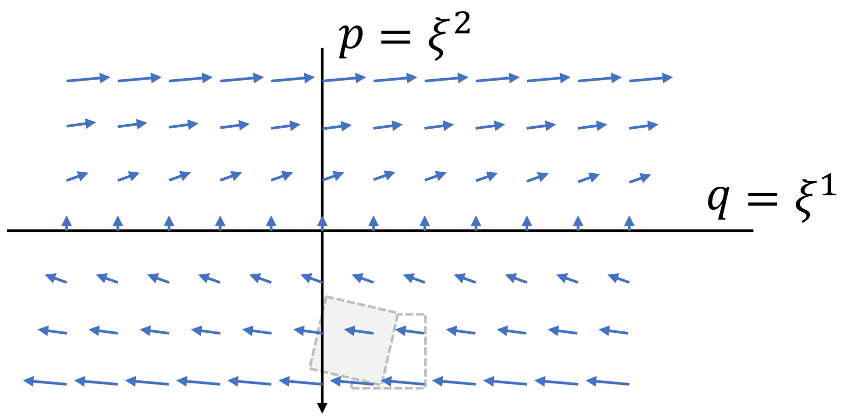
$$dQdP = |J|dqdp$$

$$|J| = \begin{vmatrix} \frac{\partial Q}{\partial q} & \frac{\partial Q}{\partial p} \\ \frac{\partial P}{\partial q} & \frac{\partial P}{\partial p} \end{vmatrix} = \begin{vmatrix} 1 + \frac{\partial S^q}{\partial q} dt & \frac{\partial S^q}{\partial p} dt \\ \frac{\partial S^p}{\partial q} dt & 1 + \frac{\partial S^p}{\partial p} dt \end{vmatrix}$$

$div(S^a) = 0$

$$= 1 + \left(\frac{\partial S^q}{\partial q} + \frac{\partial S^p}{\partial p} \right) dt + O(dt^2)$$

(4) Deterministic and reversible evolution



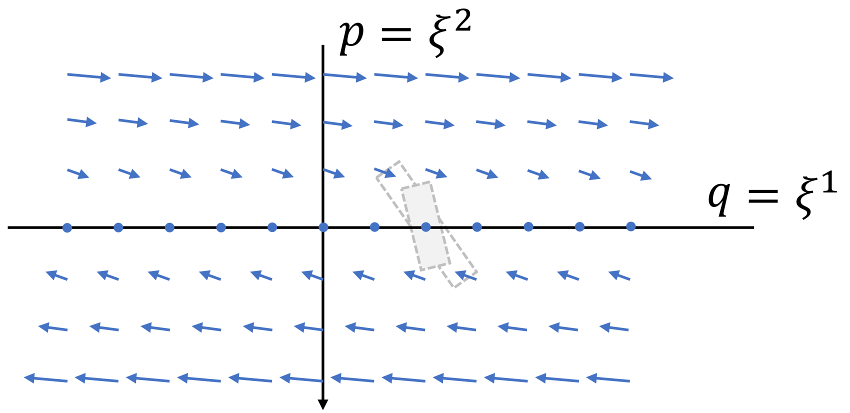
Statistical mechanics $\Rightarrow$ use areas in phase space to count states

Area conservation $\Leftrightarrow$ state count conservation

$\Leftrightarrow$ deterministic and reversible evolution

Key insight: det/rev is not just a bijection!
On continuous spaces, counting points is not enough!

A dissipative force maps points to points, but areas become smaller.



Determinism and reversibility

$\Rightarrow$ existence and conservation of energy (Hamiltonian)

Why?

Stronger version of the first law of thermodynamics

Determinism and reversibility

$\Rightarrow$ past and future depend only on the state of the system

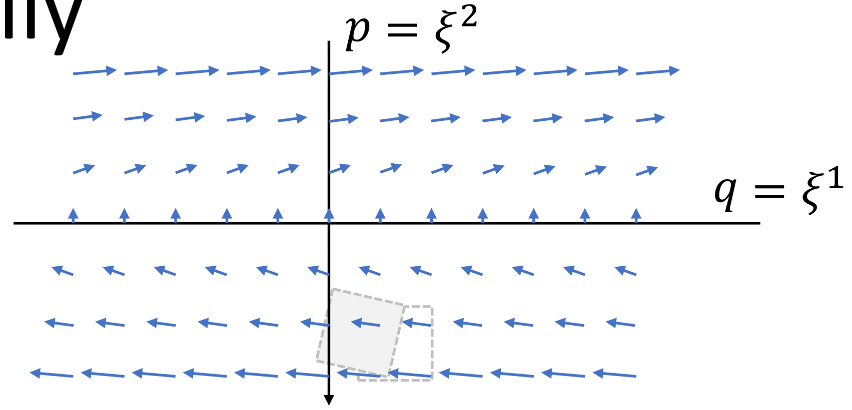
$\Rightarrow$ the evolution does not depend on anything else

$\Rightarrow$ the system is isolated

First law of thermodynamics!

$\Rightarrow$ the system conserves energy

(5) Deterministic and thermodynamically reversible evolution



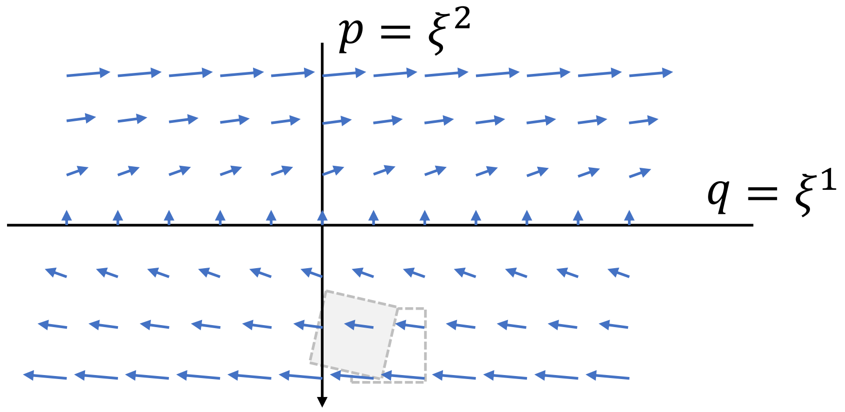
Link between statistical mechanics and thermodynamics

$$S = k_B \log W$$

Area conservation $\Longleftrightarrow$ entropy conservation

$\Longleftrightarrow$ thermodynamically reversible evolution

(6) Information conservation



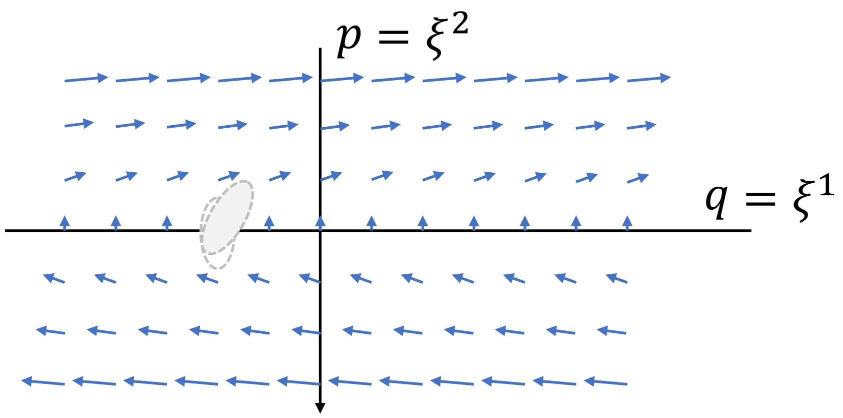
What about information entropy?

$$I[\rho(q,p)] = -\int \rho \log \rho \, dqdp$$

$$I[\rho(t+dt)] = I[\rho(t)] - \int \rho \log |J| \, dqdp$$

Area conservation $\Leftrightarrow$ information conservation

(7) Uncertainty conservation



What about uncertainty?

covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{bmatrix}$$

Assuming a "very narrow" distribution

$$|\Sigma(t + dt)| = |J||\Sigma(t)||J|$$

Area conservation $\Leftrightarrow$ uncertainty conservation

(1) Hamilton's equations

All equivalent!

(2) Divergenceless displacement

(3) Area conservation ($|J| = 1$)

(4) Deterministic and reversible evolution (i.e. isolation)

(5) Deterministic and thermodynamically reversible evolution

(6) Information conservation

(7) Uncertainty conservation

Connections between Hamiltonian mechanics, vector calculus, differential (symplectic) geometry, statistical mechanics, thermodynamics, information theory and plain statistics

We can’t study the foundations of one physical theory without looking at the foundations of all of them

Uncertainty principle in classical and quantum mechanics

one degree of freedom

Quantum uncertainty principle

$$\sigma_q \sigma_p \geq \frac{\hbar}{2}$$

What is its origin? What "causes" it?

What explains it?

How does $\sigma_q \sigma_p$ behave in classical mechanics?

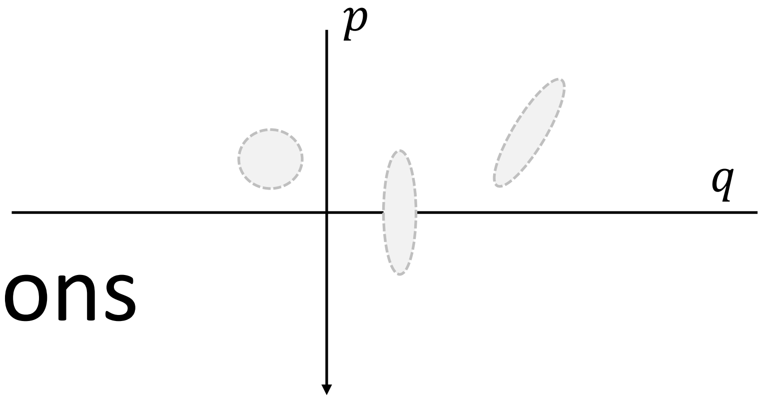
$$|\Sigma| = \begin{vmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{vmatrix} = \sigma_q^2 \sigma_p^2 - cov_{q,p}^2$$

constant of motion

$$\sigma_q^2 \sigma_p^2 = |\Sigma| + cov_{q,p}^2 \geq |\Sigma|$$

Uncertainty is bounded during Hamiltonian evolution

Lowest bound is in absence of correlation



But the bound depends on initial conditions

The bound on quantum uncertainty seems to be intrinsic to the nature of the quantum state

Let's look at the von Neumann entropy

$$I[\rho] = -tr(\rho \log \rho)$$

For a pure state $|\psi\rangle$

lowest possible entropy

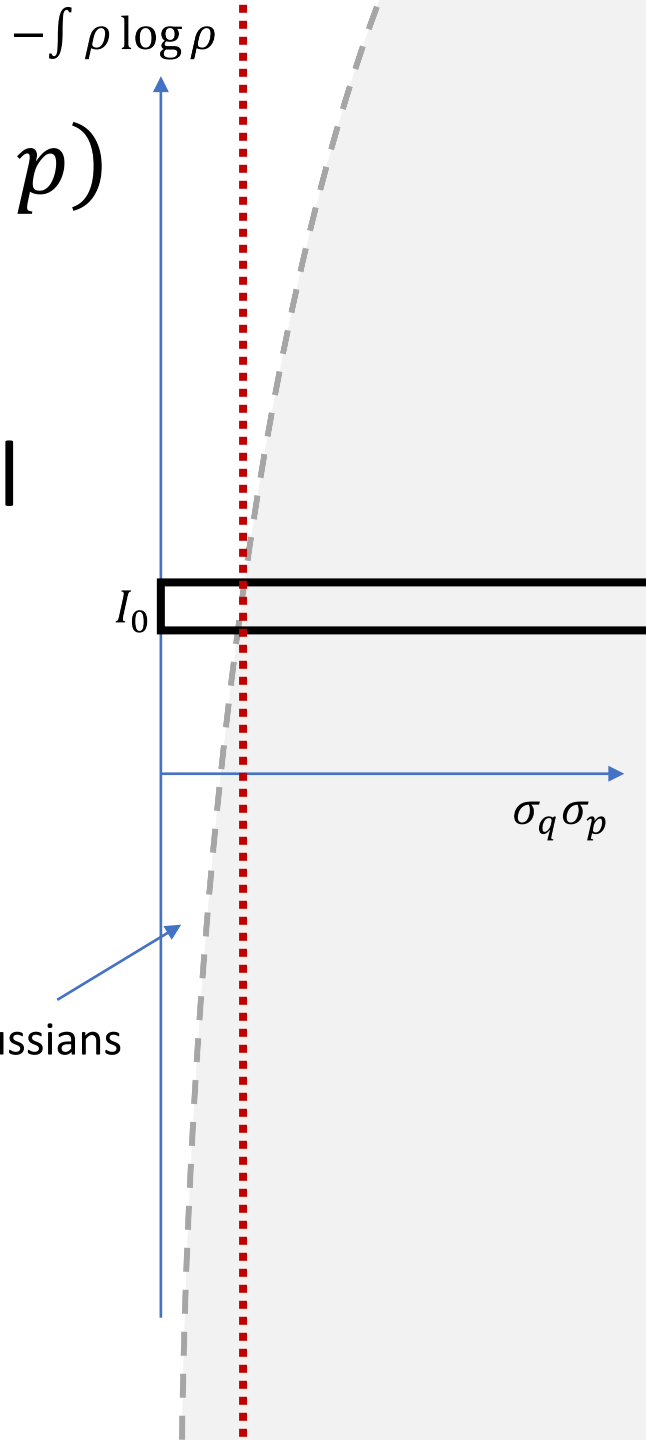
$$I[|\psi\rangle\langle\psi|] = 0$$

Could this bound, by itself, explain everything?

Take the space of all possible distributions $\rho(q,p)$ and order them by information/Gibbs entropy

Fix the entropy to a constant I_0 and consider all distributions with that entropy

They satisfy $$\sigma_q\sigma_p \geq \frac{e^{I_0}}{2\pi e}$$



equality for independent Gaussians

Lower bound on entropy ⇒ lower bound on uncertainty

Inverse does not work: lower bound on uncertainty does not give a lower bound on entropy

Lower bound for information entropy (Gibbs/von Neumann) ⇒ uncertainty principle (classical/quantum)

We don’t need the full quantum theory to derive the uncertainty principle: only the lower bound on entropy

The difference is that in classical mechanics we can prepare ensembles with arbitrarily low entropy…

But can we really?

The third law of thermodynamics revisited

A lower entropy bound is built into quantum mechanics, but not classical mechanics.

It is also built into thermodynamics in the form of the third law:

> Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.
>
> *G. N. Lewis and M. Randall - Thermodynamics and the free energy of chemical substances*

Where does this lower bound come from?

Thermodynamic entropy is additive for independent systems

$$S_{AB} = S_A + S_B$$

What system acts as a "zero" under system composition? The empty system $\emptyset$: $A \cup \emptyset = A$.

$$S_{\emptyset} = S_{\emptyset\emptyset} = S_{\emptyset} + S_{\emptyset} = 0$$

What do the empty system and a crystalline structure at zero temperature have in common?

They both have only one way to be: saying “the crystalline structure is at zero temperature” or “the system is not present” both fully determine the state of the system

Why can't we have states with entropy less than that of the empty system?

Suppose we had a system in a negative entropy state

⇒ The system is better specified than the empty system

⇒ The system is better specified in that state than saying "the system is not there"

But saying "the system is not there" already tells us everything there is to know about the system! Contradiction!

Let us rephrase the third law as:

Principle of maximal description

No state can describe a system more accurately than stating the system is not there in the first place

In practice, this statement is equivalent to the other one we quoted, but it is more of a logical necessity than a phenomenological assumption

It makes for a better foundational starting point

Third law of thermodynamics

⇒ Lower bound on entropy

⇒ Uncertainty principle

A lot more physical insight without requiring crazy metaphysical ideas, weird interpretations … just basic physical considerations!

Classical ensembles have no lower bound on entropy

⇒ At odds with third law of thermodynamics

⇒ Classical physics is untenable

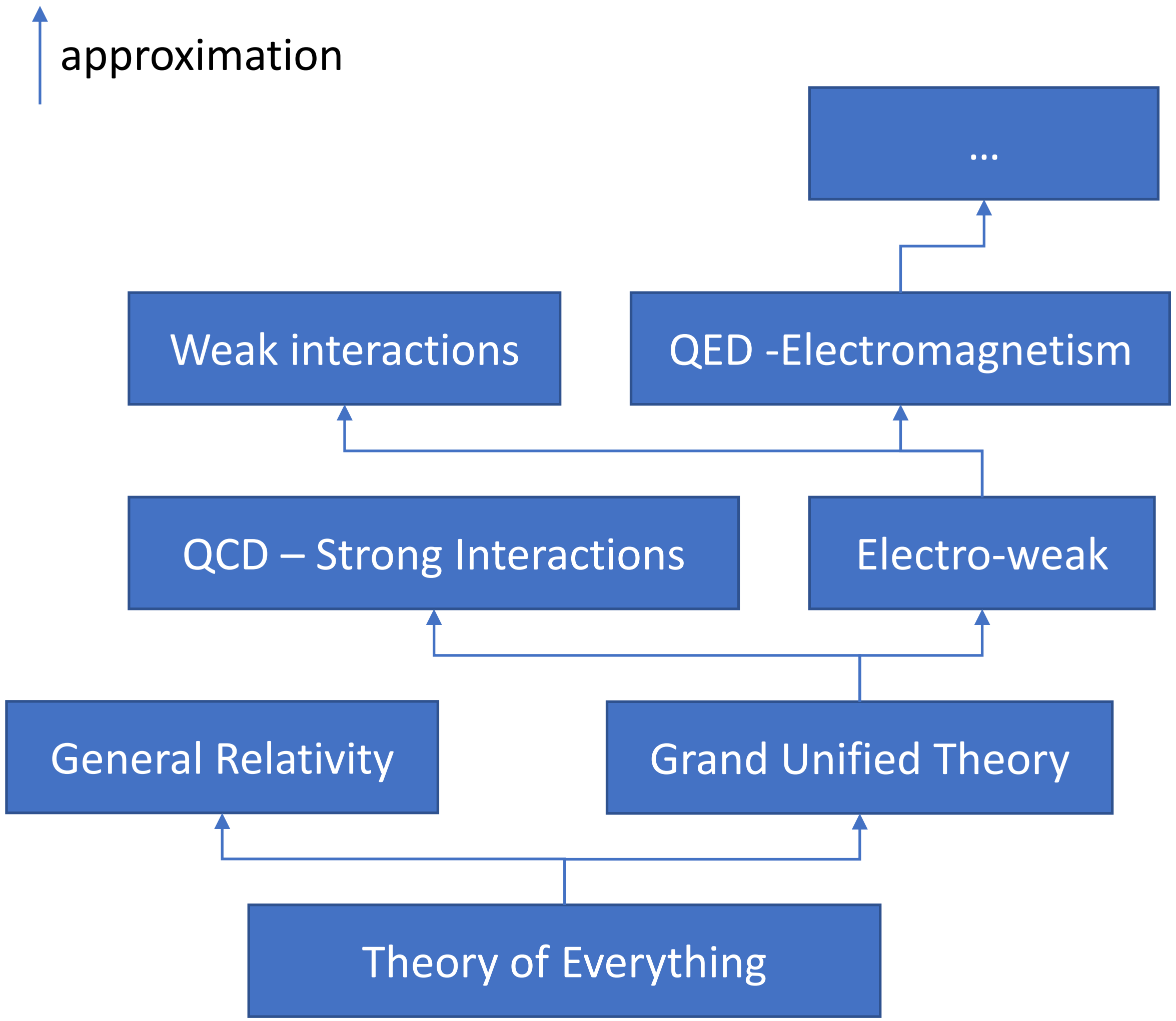
A different approach to the foundations of physics

Typical approaches in the foundations of physics

- Start with the theory that describes "what really happens"
  - With the most complicated and most complete description
- Gradually derive other theories as approximations

The Assumptions of Physics project does not proceed in this manner

We want to understand each theory as its own set of assumptions

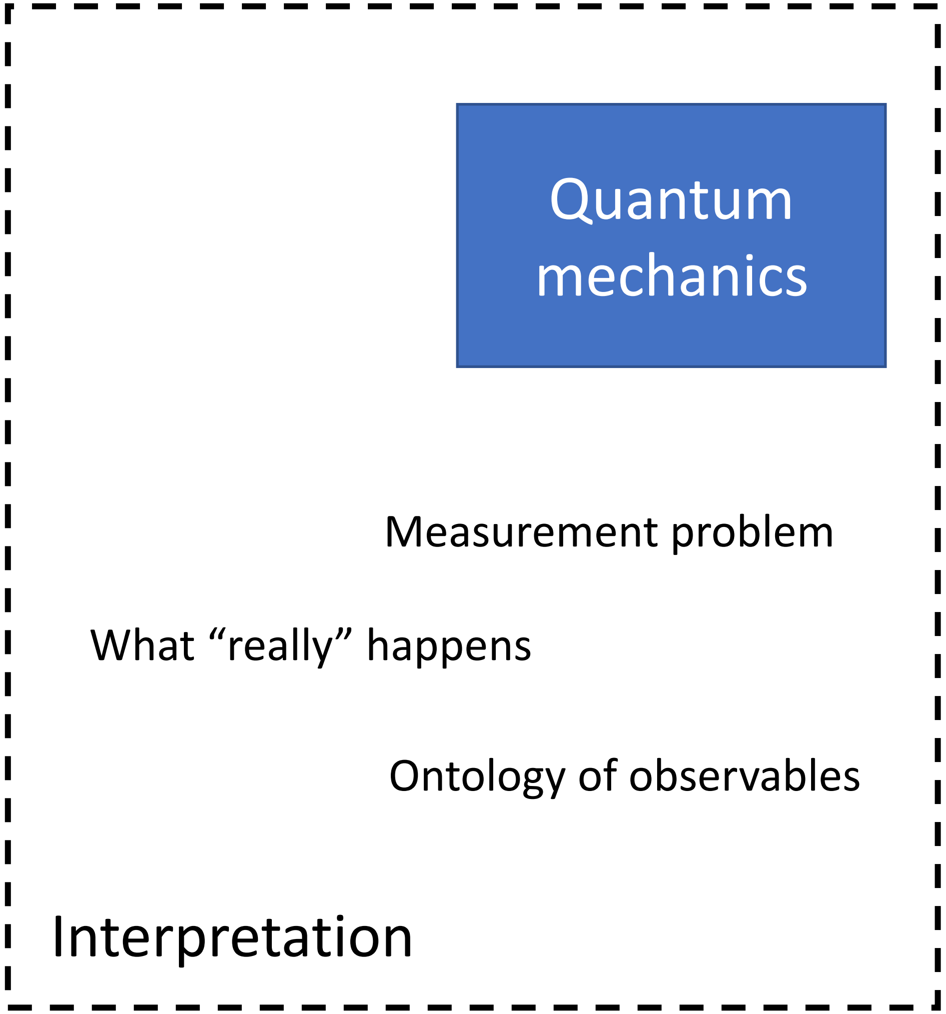


Typical approaches in the foundations of physics

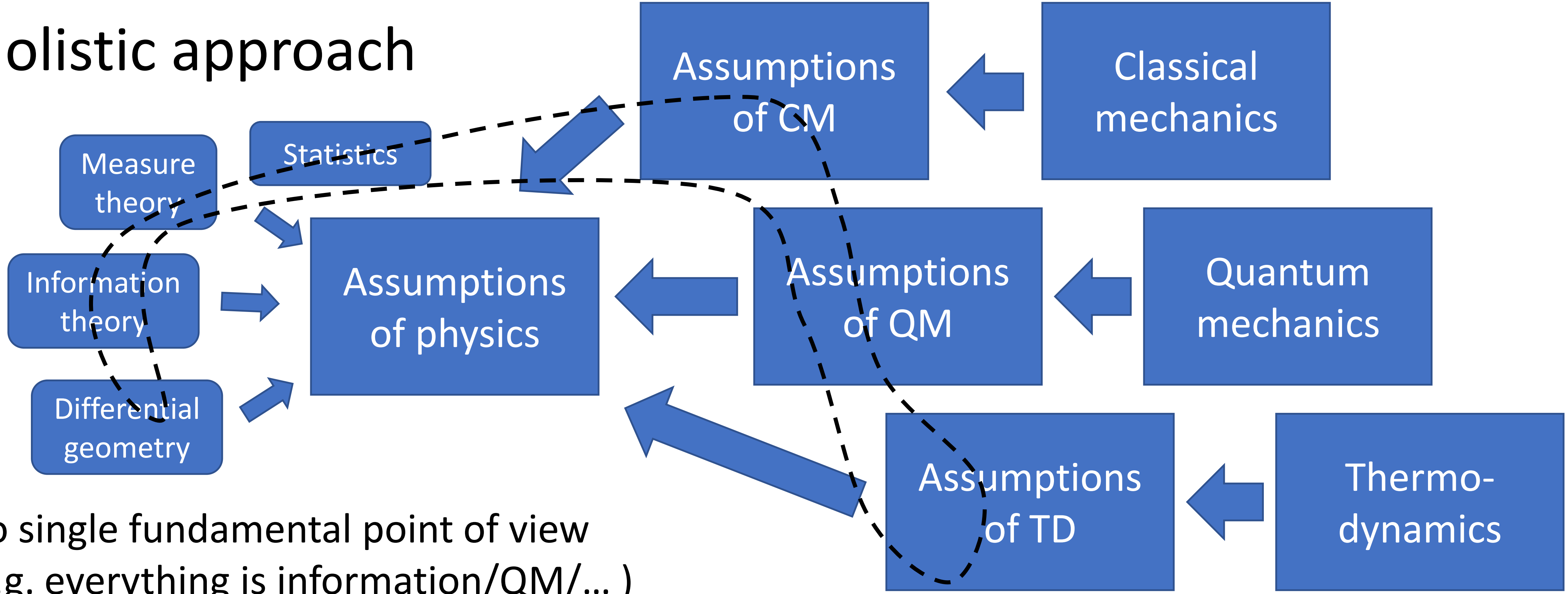
- Focus on a specific theory
  - E.g. Quantum mechanics
- Address ideas and problems related only to that theory
  - E.g. Give an “interpretation” that solves the “measurement problem”

The Assumptions of Physics project does not proceed in this manner

It’s ok that QM is incomplete, we just want to understand the limitation



Holistic approach

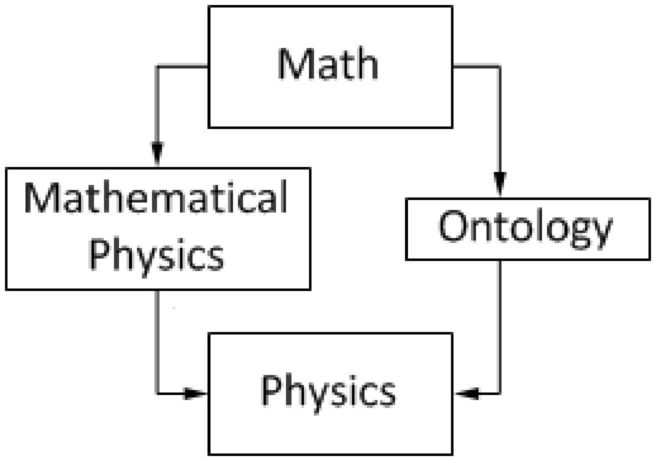


No single fundamental point of view
(e.g. everything is information/QM/…)

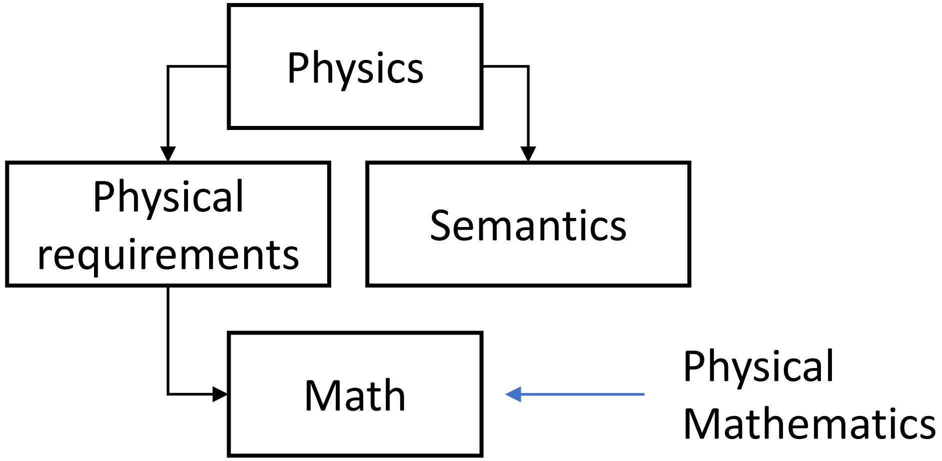
Foundations of different theories are not disconnected

Find those concepts that span multiple areas of physics, math, …

End goal: put physics at the center of physics



From Wikipedia "Mathematical Physics"



David Hilbert: "Mathematics is a game played according to certain simple rules with meaningless marks on paper."

Mathematical content of a theory can never tell us the full physical content

What is a state?

How does the theory map to experiments?

Why is one theory appropriate in one context and not in another?

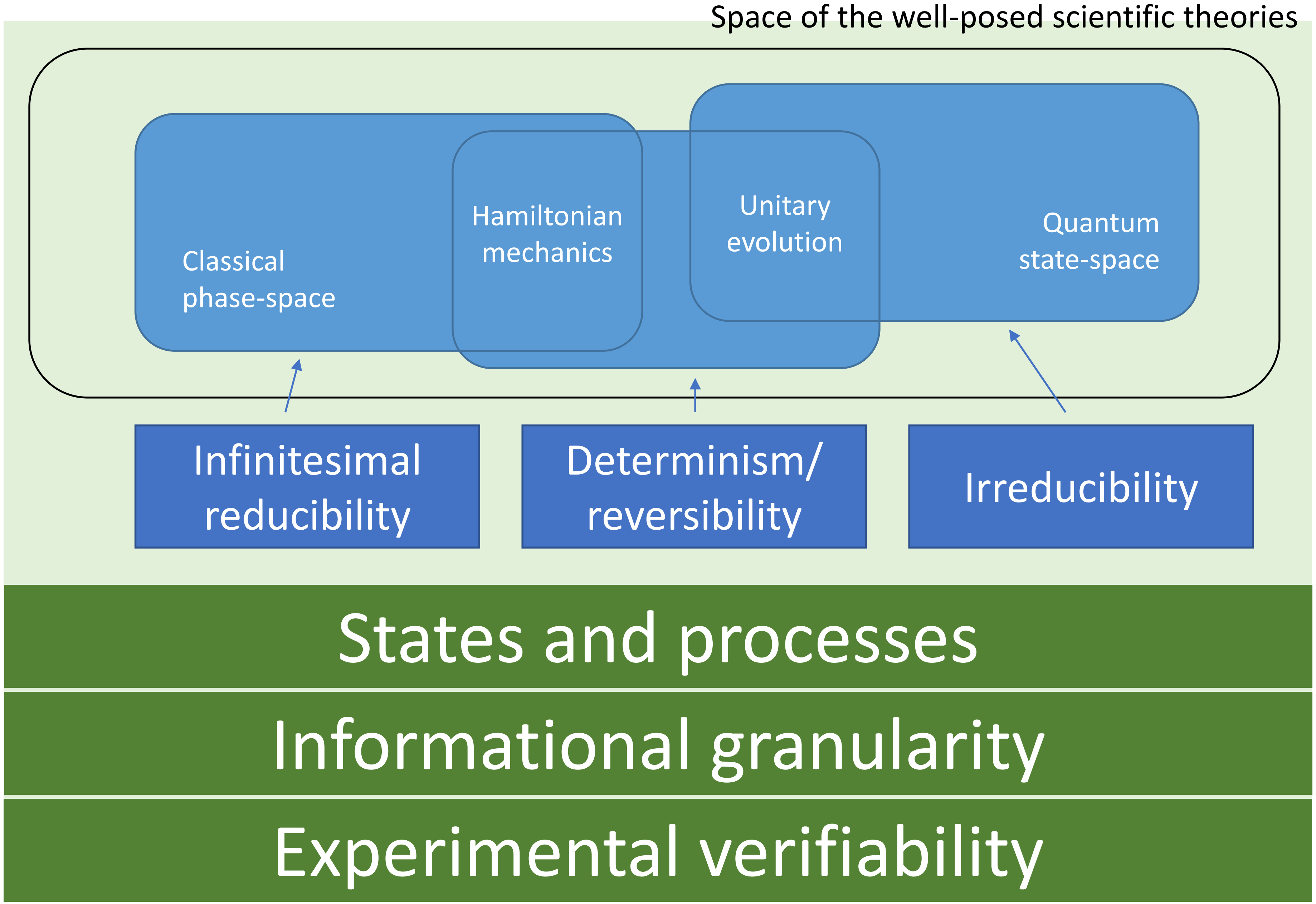
Physical theories

Specializations of the general theory under the different assumptions

Assumptions

General theory

Basic requirements and definitions valid in all theories



Conclusions

- Assumptions of Physics brings a new approach to the foundations of physics
  - We want to understand how fundamental physical assumptions lead to physical theories and the mathematics used to express them
  - As the foundations of mathematics studies the necessary structures for mathematics, we want to understand what are the fundamental features any physical theory must have to be well-posed
- Two main approaches: Reverse Physics …
  - starts from the physical laws and aims to "go back" to a suitable minimum number of assumptions that capture the physics
- … and Physical Mathematics
  - rederives mathematical structures from clearly stated physical requirements
- The end goal is to create a holistic understanding of all physical theories and their mathematical structure

Resources

Project website: https://assumptionsofphysics.org/

Papers, presentation slides, list of open problems, …

YouTube channel: https://www.youtube.com/user/gcarcassi

Popularize results of our research, recorded presentations, …

"Reverse physics: from laws to physical assumptions"

https://arxiv.org/abs/2111.09107

"Geometrical and physical interpretation of the action principle"

https://arxiv.org/abs/2208.06428

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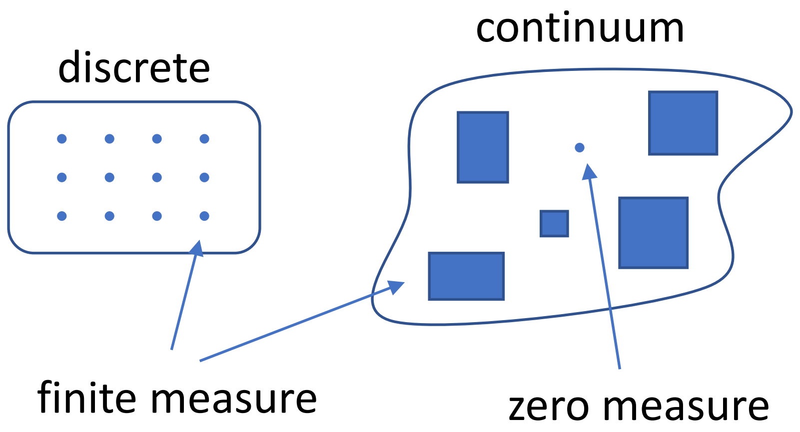
Supplemental

Why does classical mechanics fail?

It lacks a lower bound on the entropy, therefore it violates the third law of thermodynamics

What does that mean in physics terms? How can we understand this better?

Quantifying discrete cases is fundamentally different than quantifying cases over the continuum



Why? Because fully identifying a discrete case requires finite information (finitely many experimental tests) while identifying a case from a continuum requires infinite information (an infinite sequence of increasingly precise tests)

This is something most physicists haven't yet fully digested

A single classical state in phase space (i.e. a microstate) ⇒ zero volume; minus infinite entropy; infinite information.

“Empty state” ⇒ one discrete case; zero entropy; finite information.

Quantum mechanics “fixes” this, by introducing a fixed lower bound on entropy.