Assumptions of Physics: the role of entropy in reconstructing physical theories

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Assumptions of Physics

Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

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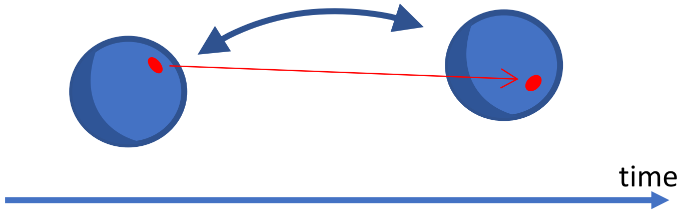
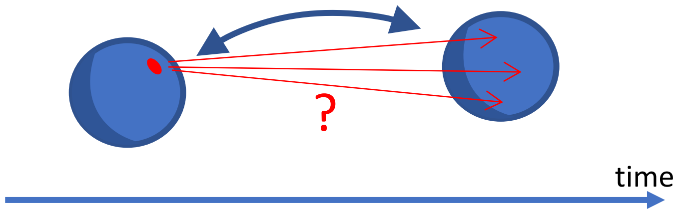
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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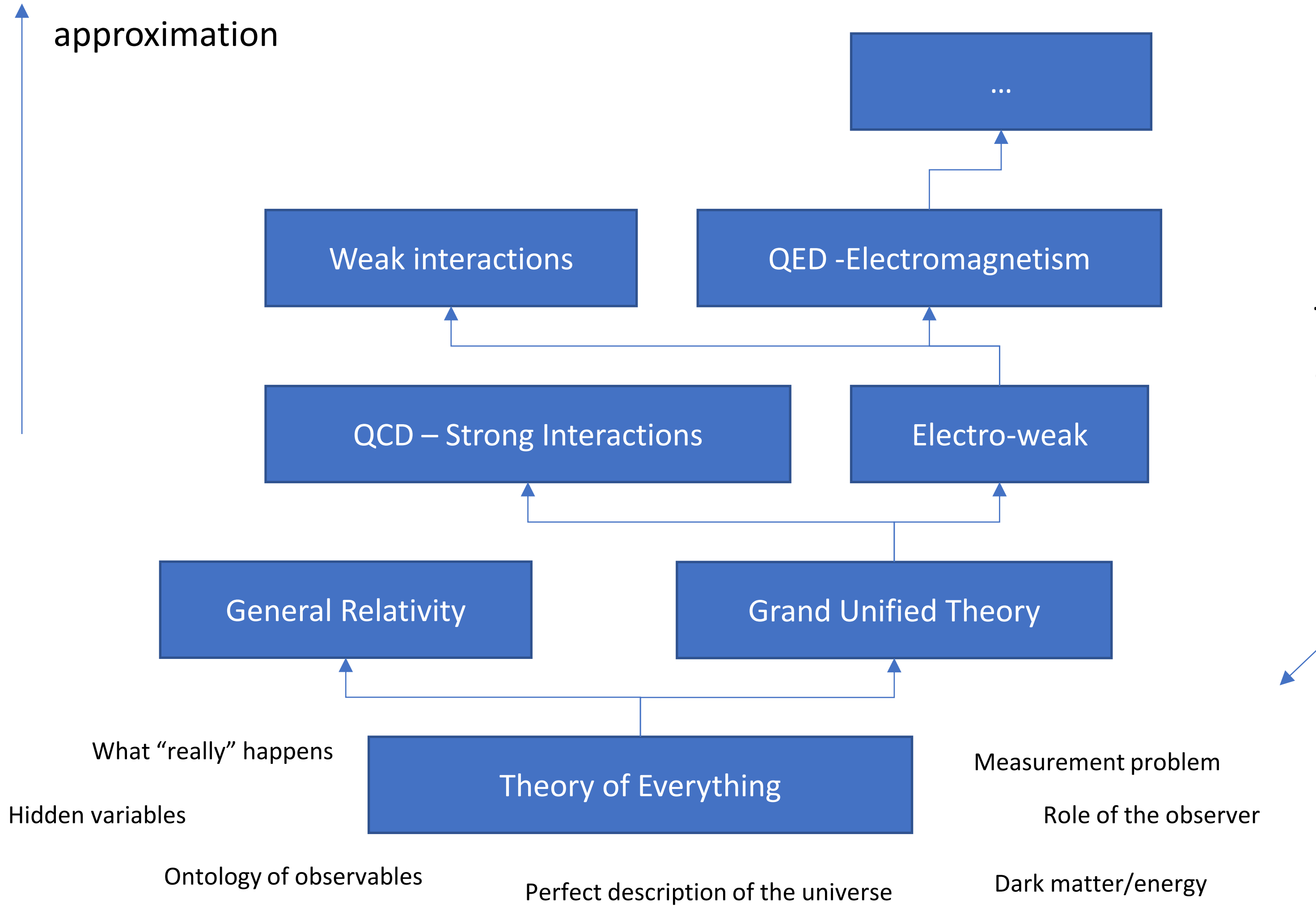
Standard view of the foundations of physics



Goal of physics is to find the true laws of the universe!

The “real” physics!

The foundations of physics!

Everything else is an approximation

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We found:

Experimental verifiability $\Rightarrow$ topologies and σ-algebras

Geometrical structures $\Leftrightarrow$ Entropic structures

Hamiltonian evolution $\Leftrightarrow$ det-rev/isolation + DOF independence

Massive particles and potential forces $\Leftrightarrow$ + Kinematic eq

Physical requirements and assumptions drive most of the theoretical apparatus

~~Goal of physics is to find the true laws of the universe!~~

Less productive point of view

Goal of physics is to find models that can be empirically tested

More productive point of view

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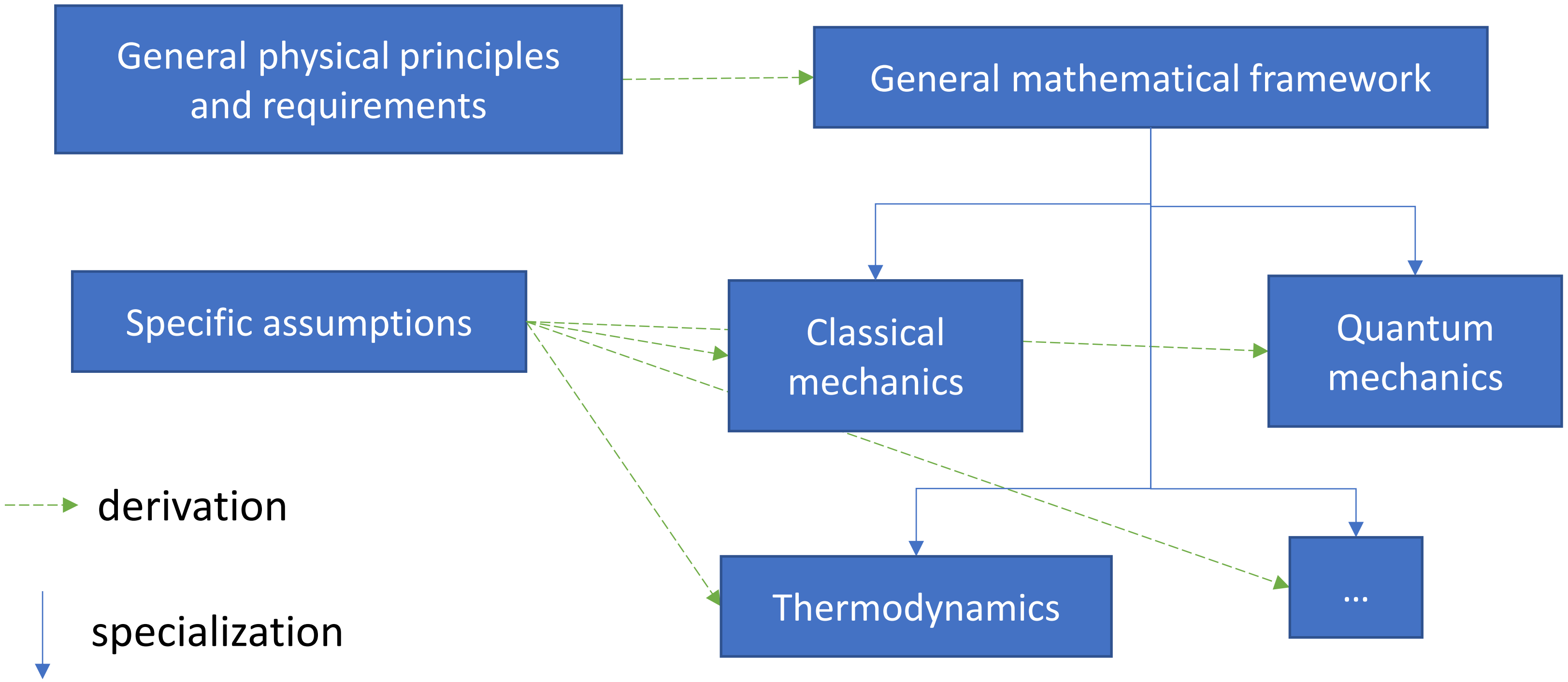
Assumptions of Physics

Our view of the foundations of physics



Foundations of physics
⇕
The theory of physical models

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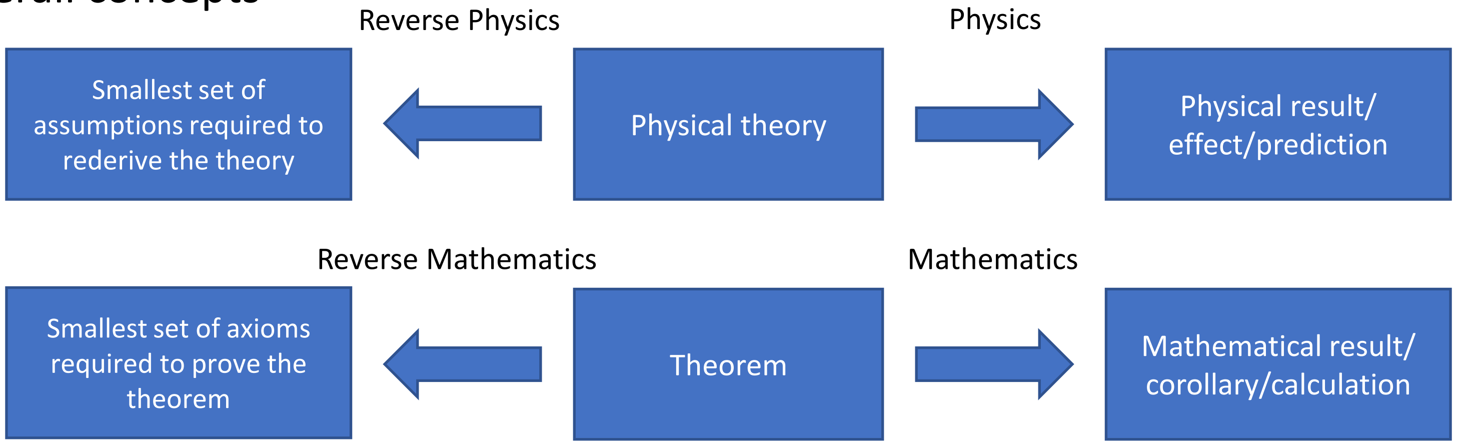
Find the right overall concepts

Reverse physics:

Start with the equations, reverse engineer physical assumptions/principles

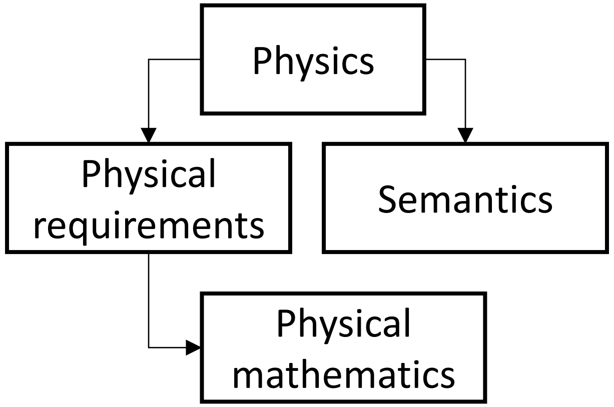
Found Phys **52**, 40 (2022)



Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

Physical mathematics:

Start from scratch and rederive all mathematical structures from physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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Assumptions of Physics

Reverse Physics

Assumptions of Physics,
Michigan Publishing (v2 2023)

J. Phys. Commun. **2** 045026 (2018)

Assumption DR (Determinism and Reversibility)**.** *The system undergoes deterministic and reversible evolution. That is, specifying the state of the system at a particular time is equivalent to specifying the state at a future (determinism) or past (reversibility) time.*

The displacement field is divergenceless: $\partial_a S^a = 0$ (DR-DIV)

The Jacobian of time evolution is unitary: $\left|\partial_b \hat{\xi}^a\right| = 1$ (DR-JAC)

Densities are conserved through the evolution: $\hat{\rho}(\hat{\xi}^a) = \rho(\xi^b)$ (DR-DEN)

Volumes are conserved through the evolution: $d\hat{\xi}^1 \cdots d\hat{\xi}^n = d\xi^1 \cdots d\xi^n$ (DR-VOL)

The evolution is deterministic and reversible. (DR-EV)

The evolution is deterministic and thermodynamically reversible (DR-THER)

The evolution conserves information entropy (DR-INFO)

The evolution conserves the uncertainty of peaked distributions (DR-UNC)

Assumption IND (Independent DOFs)**.** *The system is decomposable into independent degrees of freedom. That is, the variables that describe the state can be divided into groups that have independent definition, units and count of states.*

The system is decomposable into independent DOFs (IND-DOF)

The system allows statistically independent distributions over each DOF (IND-STAT)

The system allows informationally independent distributions over each DOF (IND-INFO)

The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF (IND-UNC)

The evolution leaves ω_{ab} invariant: $\hat{\omega}_{ab} = \omega_{ab}$ (DI-SYMP)

The evolution leaves the Poisson brackets invariant (DI-POI)

The rotated displacement field is curl free: $\partial_a S_b - \partial_b S_a = 0$ (DI-CURL)

$$\Updownarrow$$

$$d_t q^i = \partial_{p_i} H$$

$$d_t p_i = -\partial_{q^i} H$$

$$S_a = S^b \omega_{ba} = \partial_a H$$

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Reversing the principle of least action

DR

$$\nabla \cdot \vec{S} = 0$$

No state is "lost" or "created" as time evolves

$[p, 0, -H(q,p)]$

$$\vec{S} = -\nabla \times \vec{\theta}$$

(Minus sign to recover Ham eq)

KE

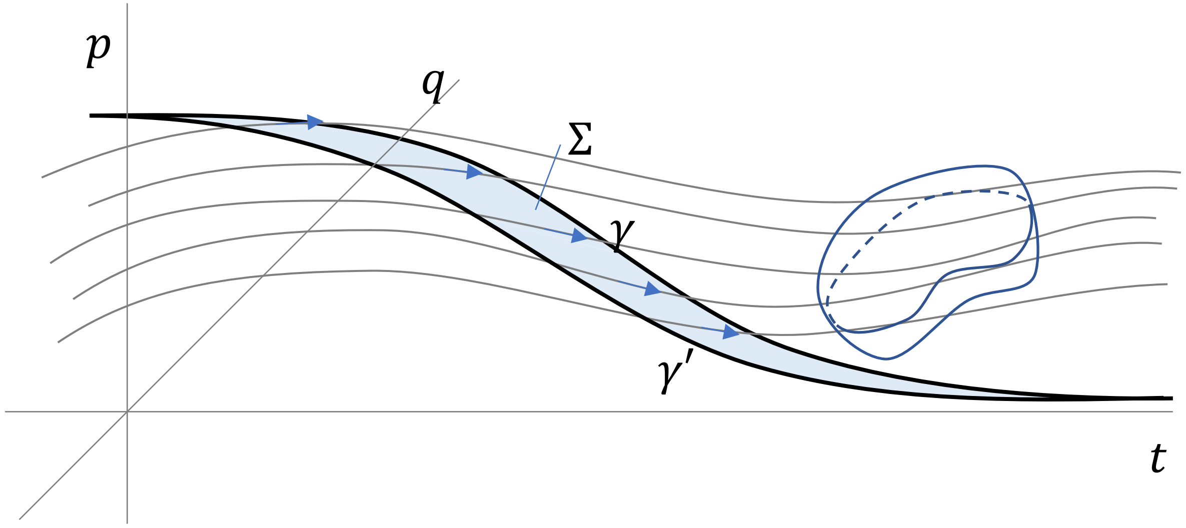
$p\frac{dq}{dt} + 0\frac{dp}{dt} - H\frac{dt}{dt}$

$$\mathcal{A}[\gamma] = \int_\gamma L dt = \int_\gamma \vec{\theta} \cdot d\vec{\gamma}$$

Sci Rep **13**, 12138 (2023)

unphysical

The action is the line integral of the vector potential of the flow of states



Variation of the action

$$\delta\mathcal{A}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$

$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent, physical!

Variation of the action measures the flow of states (physical).
Variation = 0 ⇒ flow of states tangent to the path.

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Assumptions of Physics

Is the uncertainty principle really a feature of quantum mechanics alone?

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Assumptions of Physics

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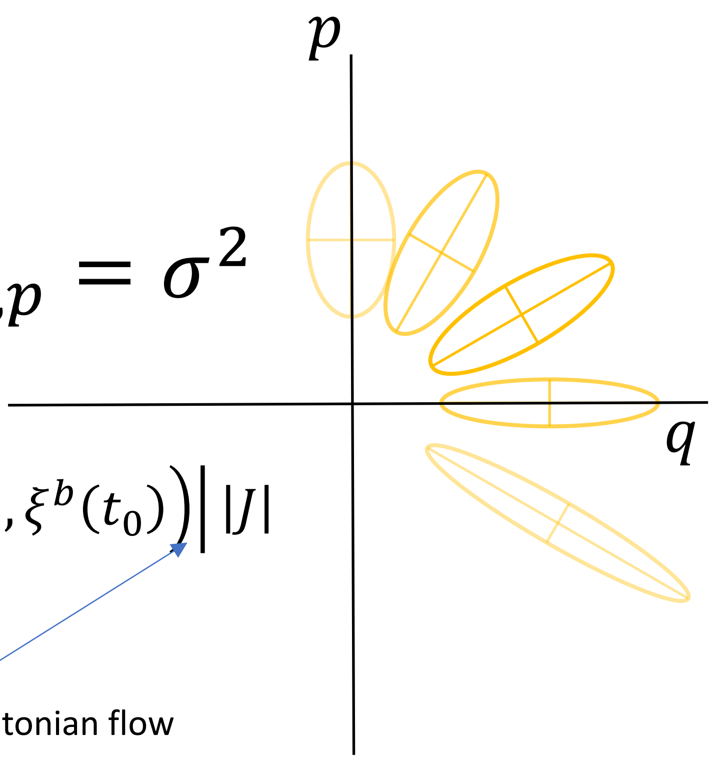
Determinant of covariance matrix:

$$\left|cov(\xi^a,\xi^b)\right| = \begin{vmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{vmatrix} = \sigma_q^2\sigma_p^2 - cov_{q,p}^2 = \sigma^2$$

evolution of covariance matrix



Peaked distribution
⇒ flow is almost linear
⇒ covariance matrix transforms linearly

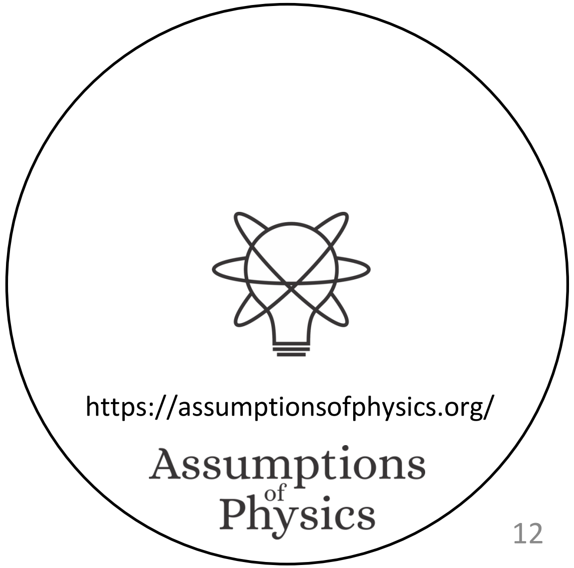
$$\left|cov\left(\xi^a(t),\xi^b(t)\right)\right| = |J| \left|cov\left(\xi^a(t_0),\xi^b(t_0)\right)\right| |J|$$

1 under Hamiltonian flow

$$\sigma_q^2(t)\sigma_p^2(t) - cov_{q,p}^2(t) = \sigma^2(t_0)$$

$$\sigma_q(t)\sigma_p(t) \geq \sigma(t_0)$$

Uncertainty is bounded during classical evolution



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Let's plot entropy against uncertainty

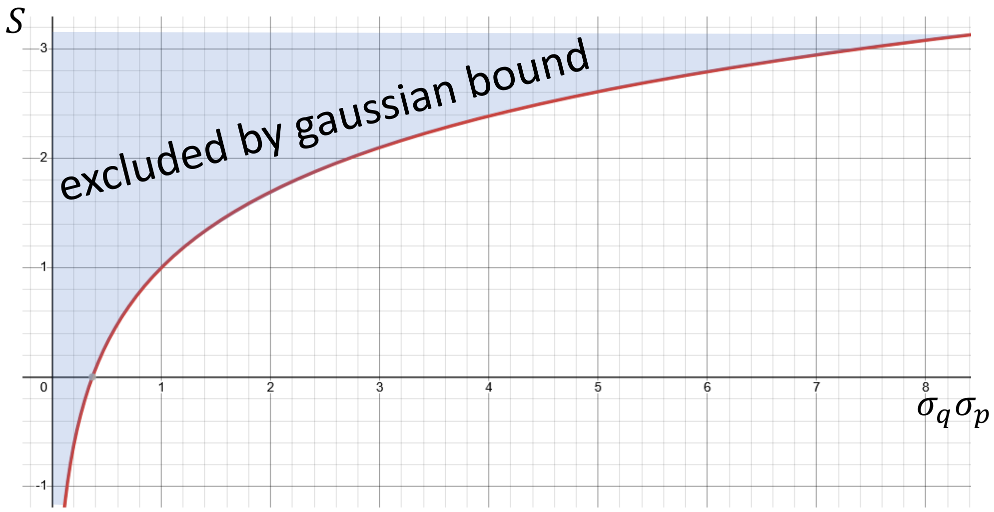
$$S(\rho) = -\int \rho \log h\rho \, dqdp$$

Fixes units

Uniform distribution over volume h has zero entropy

$$S(\rho) \leq \log 2\pi e \frac{\sigma_q \sigma_p}{h}$$

Gaussian maximizes entropy for a given uncertainty



$$\sigma_q \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$

Hamiltonian evolution conserves entropy

Entropy puts a lower bound on the uncertainty

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Assumptions of Physics

Is there anything that puts a lower bound on the entropy?

Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.

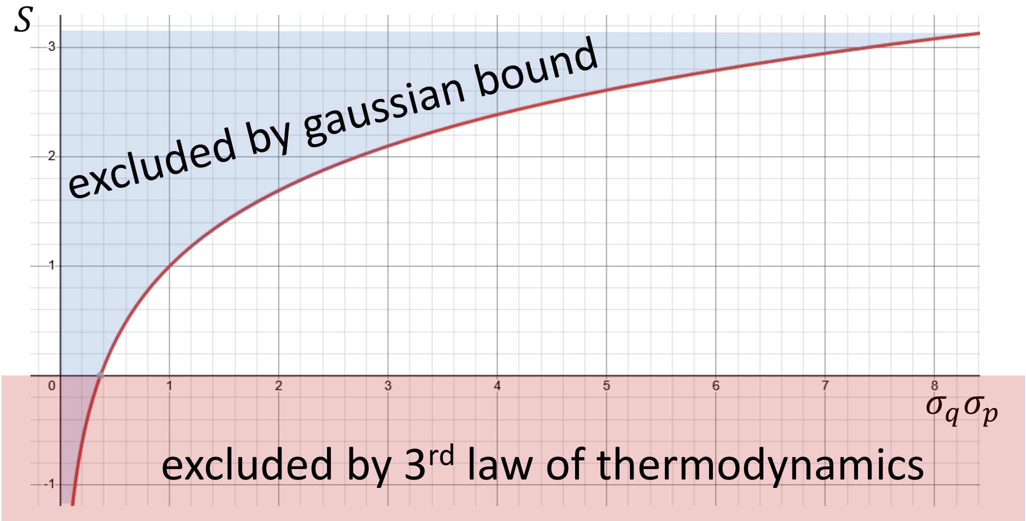
G. N. Lewis and M. Randall, Thermodynamics and the free energy of chemical substances (McGraw-Hill, 1923)

The third law of thermodynamics!

Third law puts a lower bound on the entropy which puts a lower bound on the uncertainty

$$\sigma_q \sigma_p \geq \frac{\hbar}{e} e^0 = \frac{\hbar}{e}$$

Classical uncertainty principle!



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Comparing theories

classical

$$\sigma_q \sigma_p \geq \frac{\hbar}{e}$$

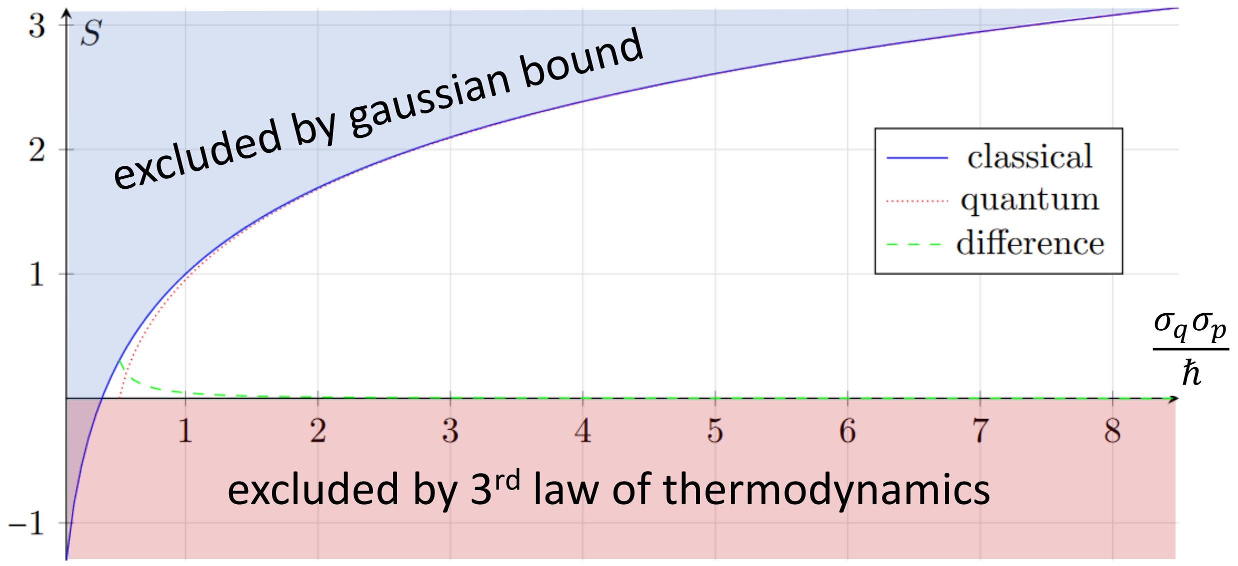
2.71828…

quantum

$$\sigma_q \sigma_p \geq \frac{\hbar}{2}$$

Entropy of quantum states is already non-negative

The gaussian bound quickly becomes very similar across theories



$$S_C = \ln e\sigma$$

$$S_Q = \left(\sigma + \tfrac{1}{2}\right)\ln\left(\sigma + \tfrac{1}{2}\right) - \left(\sigma - \tfrac{1}{2}\right)\ln\left(\sigma - \tfrac{1}{2}\right)$$

Quantum mechanics incorporates the third law

Classical mechanics does not

Is this the only difference?

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Suppose the lower bound on the entropy is the only difference, then in the limit of high entropy of quantum mechanics we should recover classical mechanics

Can we?

Classical mechanics as high entropy limit?

606 views • 10 months ago 06/01/2024

greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

Reply | Reply All | Forward | ...

Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

[Submitted on 1 Nov 2024 (v1), last revised 3 Dec 2024 (this version, v2)]

Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

Comments: 14 pages, 3 figures
Subjects: **Quantum Physics (quant-ph)**
Cite as: arXiv:2411.00972 **[quant-ph]**
(or arXiv:2411.00972v2 **[quant-ph]** for this version)
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Recovering classical mechanics from quantum mechanics

To simplify

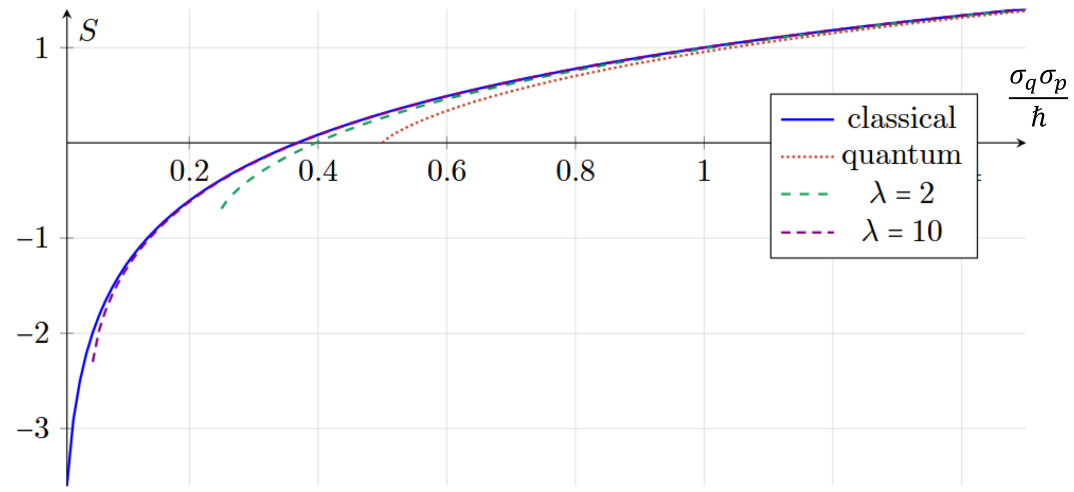
We are looking for a continuous entropy increasing process that “preserves” unitary evolution

$$\left[T_Q(X), T_Q(P)\right] = \lambda[X, P]$$

$$\frac{dX}{dt} = \frac{\imath}{\hbar}[H, X] + \gamma\left(L^\dagger X L - \frac{1}{2}\{L^\dagger L, X\}\right)$$

Lindblad eq (open quantum system)

$$L = a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\imath}{m\omega}P\right) \qquad \gamma = \lambda$$



Mathematically equivalent to lowering the entropy of a pure state to $-\infty$, or $\hbar \to 0$ (group contraction)

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Assumptions of Physics

Speed

"$c \to \infty$"

Entropy

"$\hbar \to 0$"

Classical Mechanics	Relativistic Mechanics
Quantum Mechanics	Quantum Field Theory

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Assumptions of Physics

Geometry is entropy!

The geometric structures of both classical and quantum mechanics are equivalent to the entropic structure

$\mu(U) = \int_U \omega^n$

Symplectic manifold

ρ uniform over U

Standard probability

$\rho(x) = \dfrac{1}{\mu(U)}$

Information theory

$S(\rho) = \log \mu(U)$

Thermodynamics/Statistical mechanics are not built on top of mechanics

Mechanics is the ideal case of thermodynamics/statistical mechanics

$\langle\psi|\phi\rangle$

Projective Hilbert space

$\rho = \dfrac{1}{2}\rho_\psi + \dfrac{1}{2}\rho_\phi$

Quantum probability

$p(\psi|\phi) = |\langle\psi|\phi\rangle|^2$

Quantum information theory

$S(\rho) = S\left(\dfrac{1+\sqrt{p}}{2}, \dfrac{1-\sqrt{p}}{2}\right)$

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Assumptions of Physics

Extracting principles/assumptions behind the laws gives us solid intuition that cuts across fields and leads to new insights/results

Not enough: you can’t truly claim to understand higher-level structures without fully understanding the lower-level structures

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Physical mathematics

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Examples of unphysical mathematics

In differential geometry, tangent vectors are derivations

$$v : C^{\infty}(X) \to C^{\infty}(X)$$

$$v = v^i \partial_i$$

component basis

In polar coordinates

$$\partial_r + \partial_\theta = ???$$

$[\mathrm{m}]$ $[\mathrm{rad}]$

Doesn't work with units

In phase space

$$\partial_q + \partial_p = ???$$

$[\mathrm{m}]$ $[\mathrm{Kg\ m\ s^{-1}}]$

Mathematically precise $\not\Rightarrow$ physically precise

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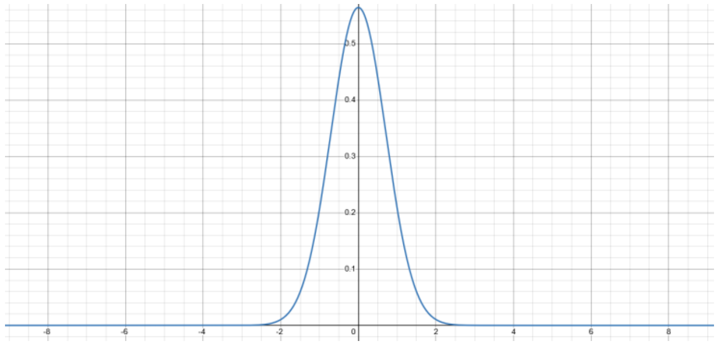
Assumptions of Physics

Quantum states represented by L^2 Hilbert space

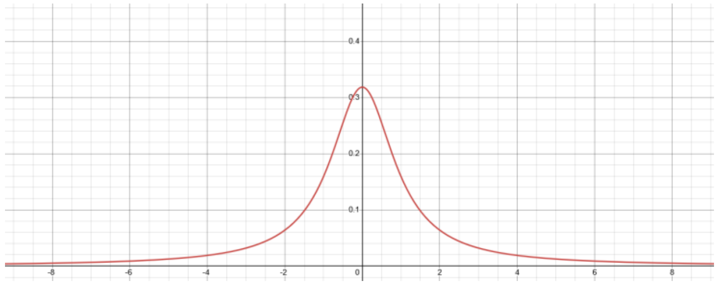
$$\psi(x) = \sqrt{\frac{e^{-x^2}}{\sqrt{\pi}}} \qquad \int |\psi|^2 dx = 1$$

$$\rho_\psi(x) = \frac{e^{-x^2}}{\sqrt{\pi}} \qquad \langle X^2 \rangle_\psi = \frac{1}{2}$$



$$\phi(y) = \sqrt{\frac{1}{\pi(y^2+1)}} \qquad \int |\phi|^2 dx = 1$$

$$\rho_\phi(y) = \frac{1}{\pi(y^2+1)} \qquad \langle Y^2 \rangle_\phi \to \infty$$



Different observers see finite/infinite expectation

$$y = \tan\left(\frac{\pi}{2}\operatorname{erf}(x)\right)$$

$$\psi(y) = \psi(x)\sqrt{\frac{dx}{dy}}$$

Expectation can have finite/infinite oscillations

$$x(x_0, t) = x_0 \cos^2\frac{\pi t}{2} + \tan\left(\frac{\pi}{2}\operatorname{erf}(x_0)\right)\sin^2\frac{\pi t}{2}$$

Every continuous linear operator defined on the whole Hilbert space is bounded ⇒ position/momentum/energy/number of particles are not defined on the whole Hilbert space!!!

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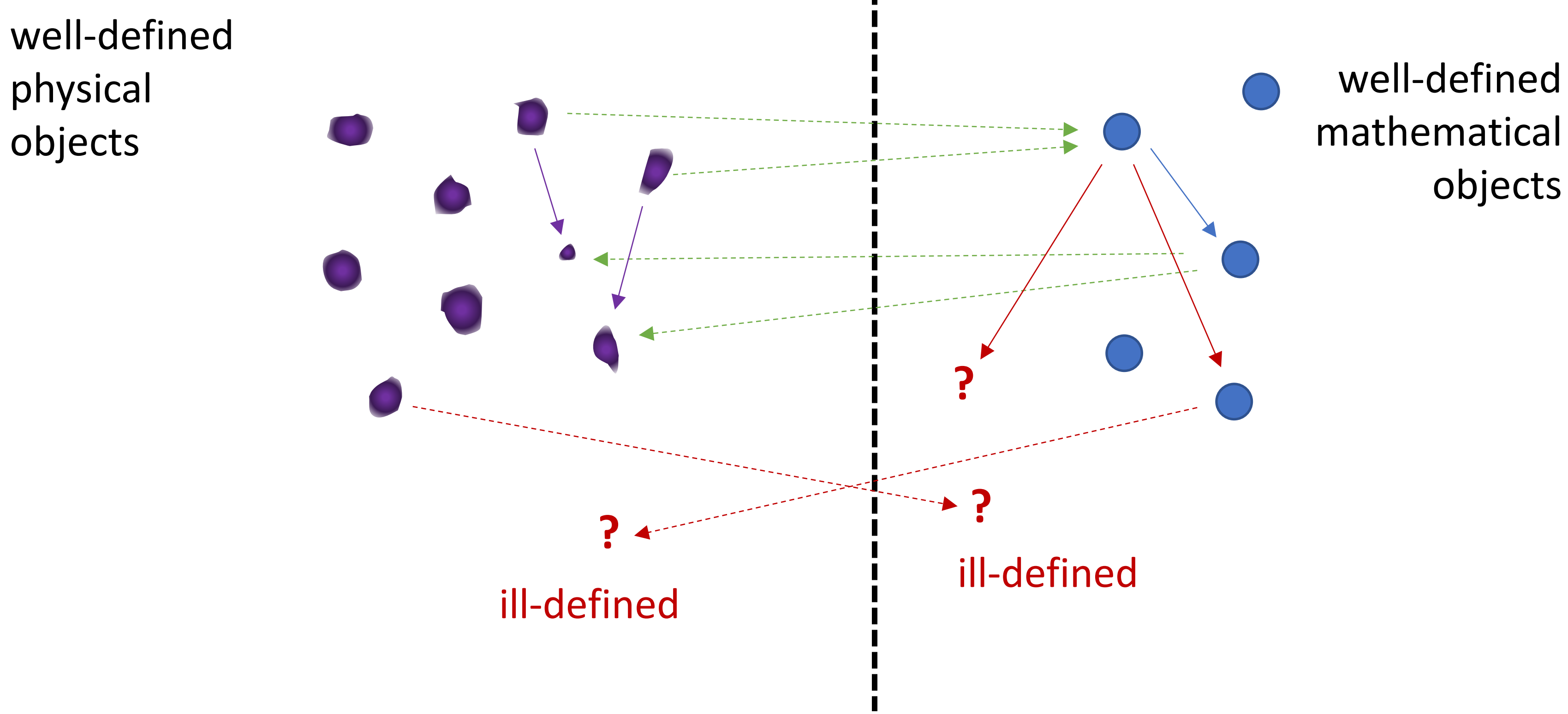
Assumptions of Physics

Physical world

(informal system)

Mathematical representation

(formal system)



Current state of the art in theoretical physics

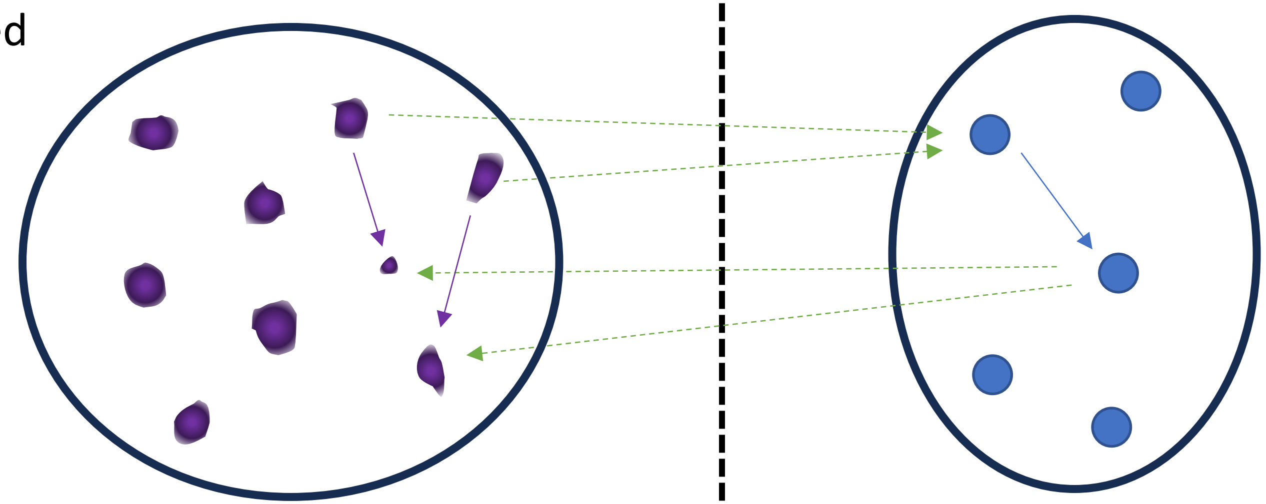
Physical world

(informal system)

Mathematical representation

(formal system)

well-defined physical objects



well-defined mathematical objects

Physical specifications ⟺ Mathematical definition

A mathematical definition is **physical** if it captures and only captures an aspect of the physical system

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Assumptions of Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble.*

Informal intuitive statement

(something that makes sense to a physicist or an engineer)

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Assumptions of Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble.* $\boxed{\textit{Formally,}}$ *an ensemble space $\mathcal{E}$ is equipped with an operation $+ : [0,1] \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ called* ***mixing****, noted with the infix notation $\mathsf{pa} + \bar{\mathsf{p}}\mathsf{b}$, with the following properties:*

- ***Continuity:*** *the map $+(p, \mathsf{a}, \mathsf{b}) \to p\mathsf{a} + \bar{p}\mathsf{b}$ is continuous (with respect to the product topology of $[0,1] \times \mathcal{E} \times \mathcal{E}$)*
- ***Identity:*** $1\mathsf{a} + 0\mathsf{b} = \mathsf{a}$
- ***Idempotence:*** $p\mathsf{a} + \bar{p}\mathsf{a} = \mathsf{a}$ *for all* $p \in [0,1]$
- ***Commutativity:*** $p\mathsf{a} + \bar{p}\mathsf{b} = \bar{p}\mathsf{b} + p\mathsf{a}$ *for all* $p \in [0,1]$
- ***Associativity:*** $p_1\mathsf{e}_1 + \bar{p}_1\left(\overline{\left(\frac{p_3}{\bar{p}_1}\right)}\mathsf{e}_2 + \frac{p_3}{\bar{p}_1}\mathsf{e}_3\right) = \bar{p}_3\left(\frac{p_1}{\bar{p}_3}\mathsf{e}_1 + \overline{\left(\frac{p_1}{\bar{p}_3}\right)}\mathsf{e}_2\right) + p_3\mathsf{e}_3$ *where* $p_1 + p_3 \leq 1$

Informal intuitive statement

(something that makes sense to a physicist or an engineer)

Formal requirement

(something a mathematician will find precise)

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Assumptions of Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble.* *Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+ : [0,1] \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ called **mixing**, noted with the infix notation $p\mathsf{a} + \bar{p}\mathsf{b}$, with the following properties:*

- ***Continuity**: the map $+(p, \mathsf{a}, \mathsf{b}) \to p\mathsf{a} + \bar{p}\mathsf{b}$ is continuous (with respect to the product topology of $[0,1] \times \mathcal{E} \times \mathcal{E}$)*
- ***Identity**: $1\mathsf{a} + 0\mathsf{b} = \mathsf{a}$*
- ***Idempotence**: $p\mathsf{a} + \bar{p}\mathsf{a} = \mathsf{a}$ for all $p \in [0,1]$*
- ***Commutativity**: $p\mathsf{a} + \bar{p}\mathsf{b} = \bar{p}\mathsf{b} + p\mathsf{a}$ for all $p \in [0,1]$*
- ***Associativity**: $p_1\mathsf{e}_1 + \bar{p}_1\left(\left(\overline{\frac{p_3}{\bar{p}_1}}\right)\mathsf{e}_2 + \frac{p_3}{\bar{p}_1}\mathsf{e}_3\right) = \bar{p}_3\left(\frac{p_1}{\bar{p}_3}\mathsf{e}_1 + \left(\overline{\frac{p_1}{\bar{p}_3}}\right)\mathsf{e}_2\right) + p_3\mathsf{e}_3$ where $p_1 + p_3 \leq 1$*

Justification. This axiom captures the ability to create a mixture merely by selecting between the output of different processes. Let e_1 and e_2 be two ensembles that represent the output of two different processes P_1 and P_2. Let a selector S_p be a process that outputs two symbols, the first with probability p and the second with probability $\bar{p}$. Then we can create another process P that, depending on the selector, outputs either the output of P_1 or P_2. All possible preparations of such a procedure will form an ensemble. Therefore we are justified in equipping an ensemble space with a mixing operation that takes a real number from zero to one, and two ensembles.

Given that mixing represents an experimental relationship, and all experimental relationships must be continuous in the natural topology, mixing must be a continuous function. Note that p is a continuously ordered quantity, where no value is perfectly experimentally verifiable, and therefore the natural topology is the one of the reals. This justifies continuity.

If $p = 1$, the output of P will always be the output of P_1. This justifies the identity property. If P_1 and P_2 are the same process, then the output of P will always be the output of P_1. This justifies the idempotence property. The order in which the processes are given does not matter as long as the same probability is matched to the same process. The process P is identical under permutation of P_1 and P_2. This justifies commutativity. If we are mixing three processes P_1, P_2 and P_3, as long as the final probabilities are the same, it does not matter if we mix P_1 and P_2 first or P_2 and P_3. This justifies associativity. □

Informal intuitive statement

(something that makes sense to a physicist or an engineer)

Formal requirement

(something a mathematician will find precise)

Show that the formal requirement follows from the intuitive statement

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Assumptions of Physics

Principle of scientific objectivity. Science is universal, non-contradictory and evidence based.

⇒ Science is about statements that are associated to experimental tests

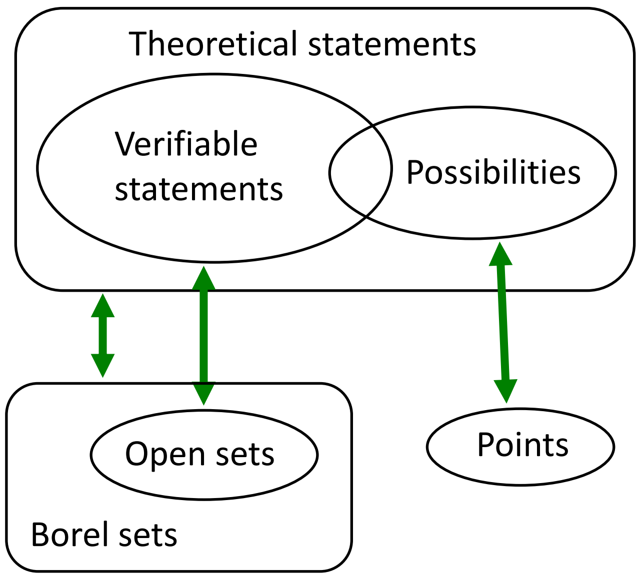
Statements must be either true or false for everybody

Tests may or may not terminate (i.e. be conclusive)

Statement	Test Result
T	SUCCESS (in finite time)
	UNDEFINED
F	UNDEFINED
	FAILURE (in finite time)

Verifiable statement	Test Result
T	SUCCESS (in finite time)
F	UNDEFINED
	FAILURE (in finite time)

Topology and σ-algebra



s_1	Test Result
T	SUCCESS (in finite time)
T	UNDEFINED
F	UNDEFINED
F	FAILURE (in finite time)

$int(A)$ corresponds to the verifiable part of a statement

∂A corresponds to the undecidable part of a statement

$ext(A)$ corresponds to the falsifiable part of a statement

Open set $(509.5, 510.5)$ ⟺ Verifiable "the mass of the electron is 510 ± 0.5 KeV"

Closed set $[510]$ ⟺ Falsifiable "the mass of the electron is exactly 510 KeV"

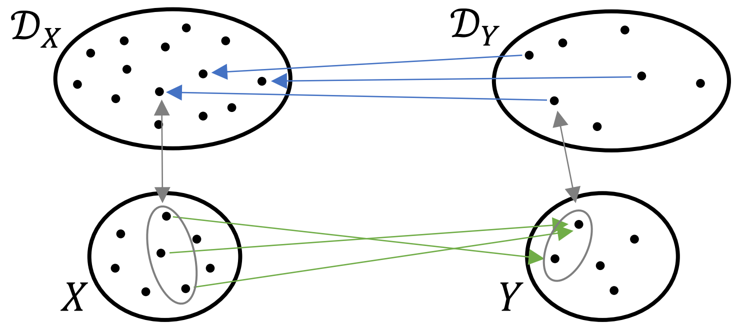
Borel set $\mathbb{Q}$ $(int(\mathbb{Q}) \cup ext(\mathbb{Q}) = \emptyset)$ ⟺ Theoretical "the mass of the electron in KeV is a rational number" (undecidable)

Experimental verifiability ⇒ topology and σ-algebras (foundation of geometry, probability, …)

Perfect map between math and physics

NB: in physics, topology and σ-algebra are parts of the **same** logic structure

Inference relationship $r: \mathcal{D}_Y \to \mathcal{D}_X$ such that $r(s) \equiv s$

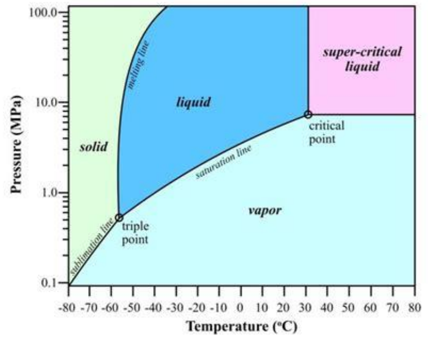


Causal relationship $f: X \to Y$ such that $x \preccurlyeq f(x)$

Inference relationship
⟺
Causal relationship

Relationships must be topologically continuous

Topologically continuous consistent with analytic discontinuity on isolated points



Phase transition ⟺ Topologically isolated regions

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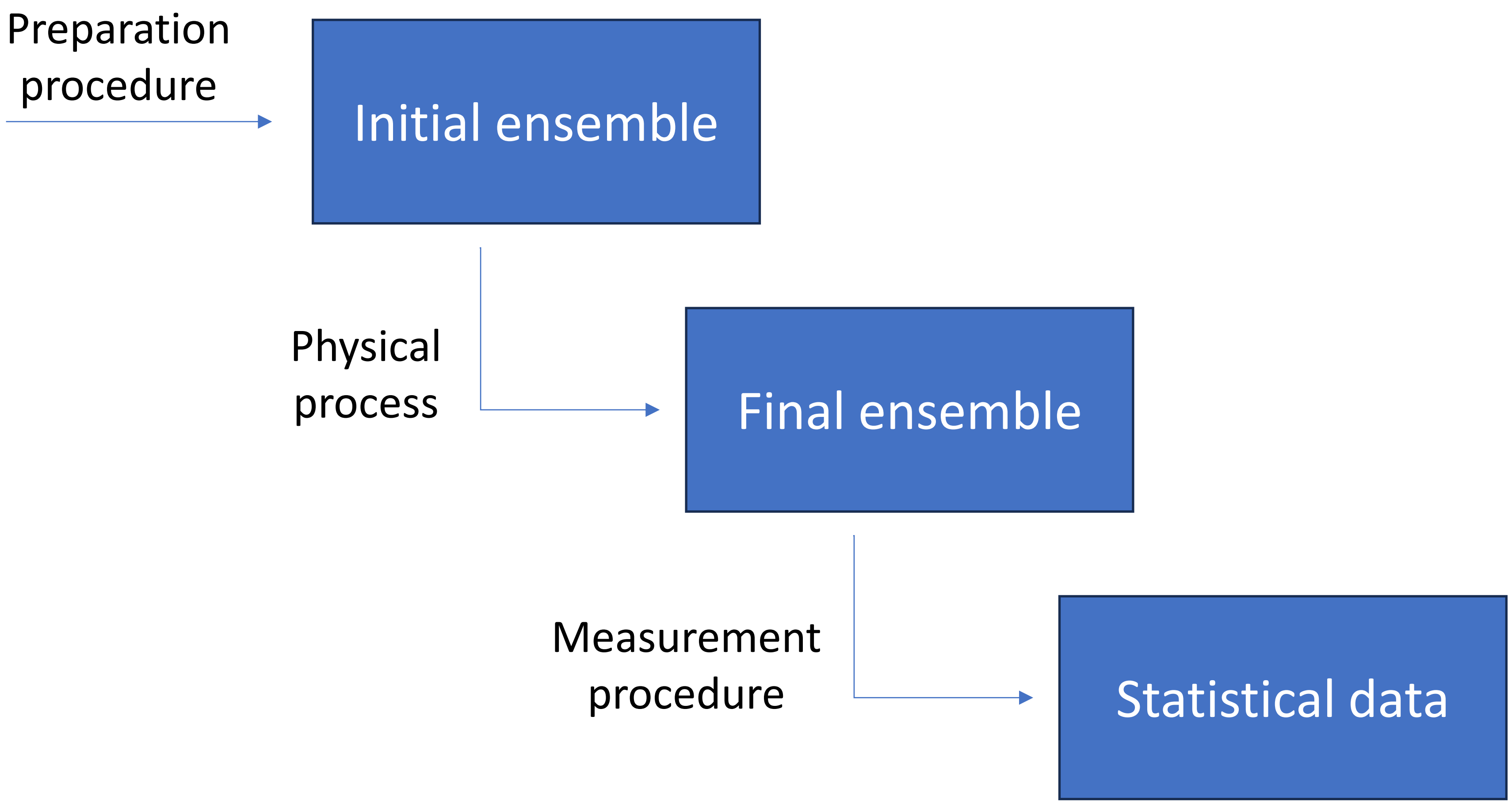
Assumptions of Physics

Principle of scientific reproducibility. Scientific laws describe relationships that can always be experimentally reproduced.

⇒ Scientific laws are relationships between ensembles



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Assumptions of Physics

Axiom 1.4 (Axiom of ensemble)**.** *The state of a system is represented by an* ***ensemble****, which represents all possible preparations of equivalent systems prepared according to the same procedure. The set of all possible ensembles for a particular system is an* ***ensemble space****. Formally, an ensemble space is a T_0 second countable topological space where each element is called an ensemble.*

Experimental verifiability ⇒ topological space

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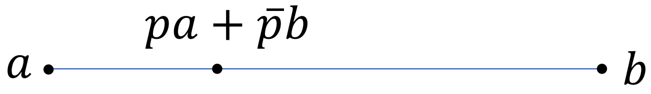
Assumptions of Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+ : [0,1] \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ called **mixing**, noted with the infix notation $\mathsf{pa} + \bar{\mathsf{p}}\mathsf{b}$, with the following properties:*

- *Continuity: the map $+(p, \mathsf{a}, \mathsf{b}) \to \mathsf{pa} + \bar{\mathsf{p}}\mathsf{b}$ is continuous (with respect to the product*

Ensembles can be mixed ⇒ Convex structure



Only finite mixtures $\sum_{i=1}^{n} p_i e_i$ are guaranteed

Topology tells us which infinite mixtures $\sum_{i=1}^{\infty} p_i e_i$ converge

I.e. where experimental verifiability converges

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Assumptions of Physics

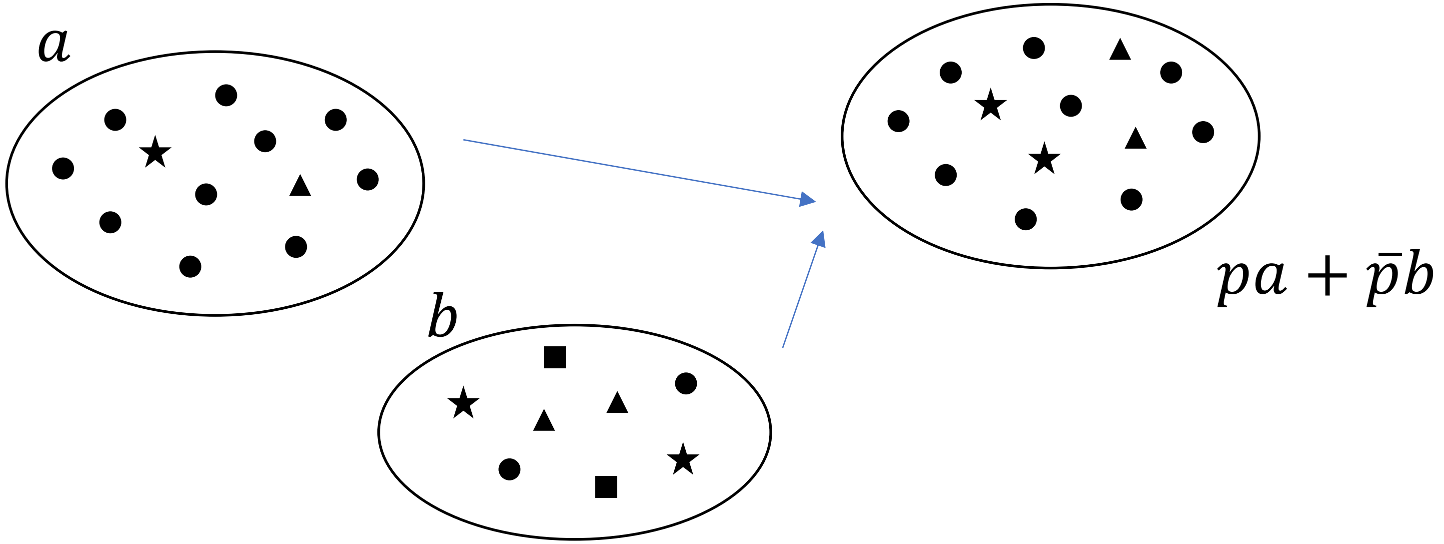
Axiom 1.21 (Axiom of entropy). *Every element of the ensemble is associated with an* ***entropy*** *which quantifies the variability of the preparations of the ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with a function $S : \mathcal{E} \to \mathbb{R}$, defined up to a positive multiplicative constant representing the unit numerical value. The entropy has the following properties:*

Ensemble variability $\Rightarrow$ Entropy

$a = b$ defined as $a \perp b$

$$pS(a) + \bar{p}S(b) \leq S(pa + \bar{p}b) \leq I(p, \bar{p}) + pS(a) + \bar{p}S(b)$$

Maximum entropy increase $\Rightarrow$ orthogonality



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Assumptions of Physics

Separate ensembles

$$\mathsf{a} \barV \mathsf{b}$$

No “common component” $\mathsf{c} \in \mathcal{E}$

such that
$$\mathsf{a} = p_1\mathsf{c} + \bar{p}_1\mathsf{e}_1$$
$$\mathsf{b} = p_2\mathsf{c} + \bar{p}_2\mathsf{e}_2$$

$\Leftarrow$

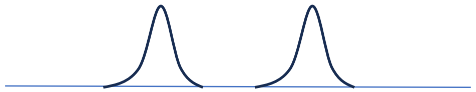
Orthogonal ensembles

$$\mathsf{a} \perp \mathsf{b}$$

Saturate upper entropy bound

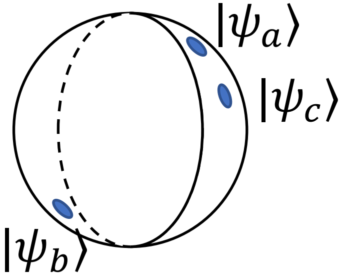
$$S(p\mathsf{a} + \bar{p}\mathsf{b}) = I(p, \bar{p}) + pS(\mathsf{a}) + \bar{p}S(\mathsf{b})$$

Coincide in
classical ensemble spaces



disjoint support

Different in
quantum ensemble spaces



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Assumptions of Physics

And now a series of results
implied by the existence of an entropy

The entropy upper bound $I(p, \bar{p})$ is uniquely determined

Theorem 1.25 (Uniqueness of entropy). *The entropy of the coefficients $I(p, \bar{p})$ is the Shannon entropy. That is, $I(p, \bar{p}) = -\kappa\,(p \log p + \bar{p} \log \bar{p})$ where $\kappa > 0$ is the arbitrary multiplicative constant for the entropy. For a mixture of arbitrarily many elements, $I(\{p_i\}) = -\kappa \sum_i p_i \log p_i$.*

Shannon entropy

Proof “does not know” whether we are dealing with classical ensembles, quantum ensembles, or ensembles for a theory yet to be discovered

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Assumptions of Physics

Definition 1.42. *A convex space X is* ***cancellative*** *if* $p\mathsf{a} + \bar{p}\mathsf{e} = p\mathsf{b} + \bar{p}\mathsf{e}$ *for some* $p \in (0,1)$ *implies* $\mathsf{a} = \mathsf{b}$.

Theorem 1.43 (Ensemble spaces are cancellative)**.** *Let $\mathcal{E}$ be an ensemble space. Let* $\mathsf{a}, \mathsf{b}, \mathsf{e} \in \mathcal{E}$ *such that* $p\mathsf{a} + \bar{p}\mathsf{e} = p\mathsf{b} + \bar{p}\mathsf{e}$ *for some* $p \in (0,1)$. *Then* $\mathsf{a} = \mathsf{b}$.

Entropy bounds force mixing to be “invertible”

Is the inverse continuous?

Definition 1.53 (Affine combinations)**.** *Let* $\{\mathsf{e}_i\}_{i=1}^n \subseteq \mathcal{E}$ *be a finite sequence of ensembles and* $\{r_i\}_{i=1}^n \subseteq \mathbb{R}$ *be a finite sequence of coefficients such that* $\sum_{i=1}^n r_i = 1$. *The* ***affine combination*** $\sum_{i=1}^n r_i \mathsf{e}_i$ *is, if it exists, the ensemble* $\mathsf{a} \in \mathcal{E}$ *such that* $\sum_{i \in I} \frac{r_i}{r} \mathsf{e}_i = \frac{1}{r}\mathsf{a} + \sum_{i \notin I} \frac{-r_i}{r} \mathsf{e}_i$ *where* $I = \{i \in [1,n] \mid r_i \geq 0\}$ *and* $r = \sum_{i \in I} r_i$.

Can define affine combinations (i.e. negative probabilities)

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Assumptions of Physics

Definition 1.55 (Ensemble differences). *Given an ensemble space, a difference between two ensemble represents the change required to transform one ensemble into another. Formally, an* ***ensemble difference****, noted $r(\mathsf{b}-\mathsf{a})$, is a triple formed by a real number $r \in \mathbb{R}$ and an ordered pair of ensembles $\mathsf{a}, \mathsf{b} \in \mathcal{E}$.*

Theorem 1.65 (Differences from a vector space). *Let $\mathsf{a} \in \mathcal{E}$ be an interior point and let $V = \{[r(\mathsf{b}-\mathsf{a})]\}$ be the set of equivalence classes of ensemble differences from a. Then V is a vector space under the scalar multiplication and addition.*

Definition 1.68. *Given an internal point a, the* ***natural embedding*** *of $\mathcal{E}$ into V_{a} is the map $\iota_{\mathsf{a}} : \mathcal{E} \hookrightarrow V_{\mathsf{a}}$ defined as $\iota(\mathsf{e}) \to [(\mathsf{e}-\mathsf{a})]$ is the of the vector space in the space of differences from a.*

Ensemble spaces embed into vector spaces

Connection to analysis

Do they embed continuously in a topological vector space?

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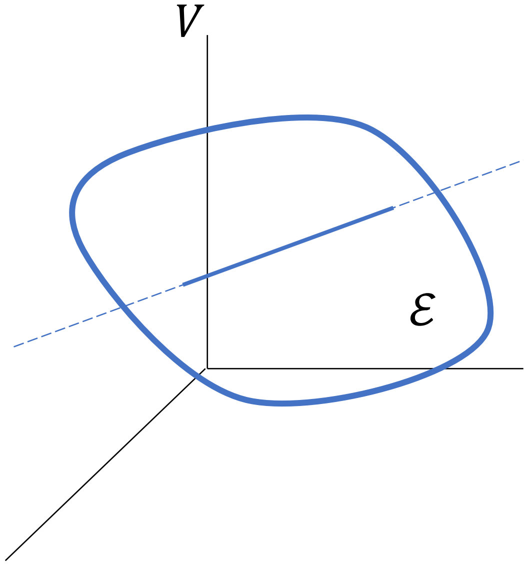
Assumptions of Physics

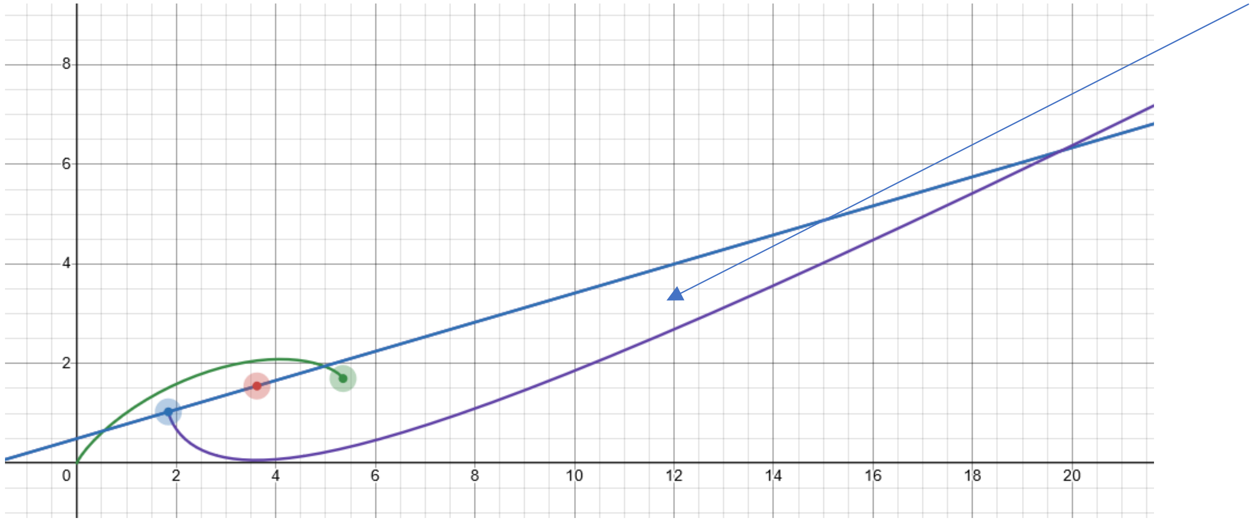
Definition 1.50. *A **line** $A \subseteq \mathcal{E}$ is a convex subset such that for any three elements one can be expressed as a mixture of the other two. That is, for all $\mathsf{e}_1, \mathsf{e}_2, \mathsf{e}_3 \in A$ there exists a permutation $\sigma : \{1,2,3\} \to \{1,2,3\}$ and $p \in [0,1]$ such that $\mathsf{e}_{\sigma(1)} = p\mathsf{e}_{\sigma(2)} + \bar{p}\mathsf{e}_{\sigma(3)}$.*

Theorem 1.52 (Lines are bounded). *Let $A \subseteq \mathcal{E}$ be a line. Then we can find a bounded interval $V \subseteq \mathbb{R}$ and an invertible function $f : A \to V$ such that $f(p\mathsf{a} + \bar{p}\mathsf{b}) = pf(\mathsf{a}) + \bar{p}f(\mathsf{b})$ for all $\mathsf{a}, \mathsf{b} \in A$.*



Ensemble spaces are bounded in all directions

Entropy



Fix three points (i.e. origin, blue and red points)

Ensembles over a line

Entropy bounds ⇒ green point between blue and purple line

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Assumptions of Physics

How much does the entropy increase during mixture?

$$MS(\mathsf{a},\mathsf{b}) = S\left(\frac{1}{2}\mathsf{a} + \frac{1}{2}\mathsf{b}\right) - \left(\frac{1}{2}S(\mathsf{a}) + \frac{1}{2}S(\mathsf{b})\right)$$

Recovers the Jensen-Shannon divergence (JSD)
(both classical and quantum)

1. *non-negativity:* $MS(\mathsf{a},\mathsf{b}) \geq 0$
2. *identity of indiscernibles:* $MS(\mathsf{a},\mathsf{b}) = 0 \iff \mathsf{a} = \mathsf{b}$
3. *unit boundedness:* $MS(\mathsf{a},\mathsf{b}) \leq 1$
4. *maximality of orthogonals:* $MS(\mathsf{a},\mathsf{b}) = 1 \iff \mathsf{a} \perp \mathsf{b}$
5. *symmetry:* $MS(\mathsf{a},\mathsf{b}) = MS(\mathsf{b},\mathsf{a})$

Pseudo-distance from the entropy

Entropy imposes a metric on the affine structure

$$\|\delta \mathrm{e}\|_{\mathrm{e}} = \sqrt{8MS(\mathrm{e}, \mathrm{e} + \delta \mathrm{e})}$$

$$g_{\mathrm{e}}(\delta \mathrm{e}_1, \delta \mathrm{e}_2) = \frac{1}{2}\left(\|\delta \mathrm{e}_1 + \delta \mathrm{e}_2\|_{\mathrm{e}}^2 - \|\delta \mathrm{e}_1\|_{\mathrm{e}}^2 - \|\delta \mathrm{e}_2\|_{\mathrm{e}}^2\right)$$

$$\Rightarrow \qquad g_{\mathrm{e}}(\delta \mathrm{e}_1, \delta \mathrm{e}_2) = -\frac{\partial^2 S}{\partial \mathrm{e}^2}(\delta \mathrm{e}_1, \delta \mathrm{e}_2).$$

Entropy strict concavity means the Hessian is negative definite

Recovers Fisher-Rao information metric
(both classical and quantum)

New structures: non-additive measures for counting states and “mixing probability”

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Assumptions of Physics

Let's generalize $S(\rho_U) = \log \mu(U)$

$$\mu = 2^{S(\sum \lambda_i a_i)}$$

biggest S

Proposition 1.153 (Exponential entropy subadditivity). *Let $\mathsf{e}_1, \mathsf{e}_2 \in \mathcal{E}$. Let $S_1 = S(\mathsf{e}_1)$ and $S_2 = S(\mathsf{e}_2)$. Let $\mathsf{e} = p\mathsf{e}_1 + \bar{p}\mathsf{e}_2$ for some $p \in [0,1]$ and $S = S(\mathsf{e})$. Then $2^S \leq 2^{S_1} + 2^{S_2}$, with the equality if and only if e_1 and e_2 are orthogonal and $p = \frac{2^{S_1}}{2^{S_1}+2^{S_2}}$.*

Definition 1.156. *Let $U \subseteq \mathcal{E}$ be the subset of an ensemble space. The* ***state capacity*** *of U is defined as $\mathrm{scap}(U) = \sup(2^{S(\mathrm{hull}(U))})$ if $U \neq \varnothing$ and $\mathrm{scap}(U) = 0$ otherwise.*

capacity also name
of a non-additive measure

Proposition 1.157. *The state capacity is a set function that is*

1. *non-negative:* $\mathrm{scap}(U) \in [0, +\infty]$
2. *monotone:* $U \subseteq V \implies \mathrm{scap}(U) \leq \mathrm{scap}(V)$
3. *subadditive:* $\mathrm{scap}(U \cup V) \leq \mathrm{scap}(U) + \mathrm{scap}(V)$
4. *additive over orthogonal sets:* $U \perp V \implies \mathrm{scap}(U \cup V) = \mathrm{scap}(U) + \mathrm{scap}(V)$

fuzzy measure

State capacity is a non-additive measure

additive over orthogonal sets

Recovers Liouville measure in classical mechanics and dimensionality of Hilbert subspaces in quantum mechanics

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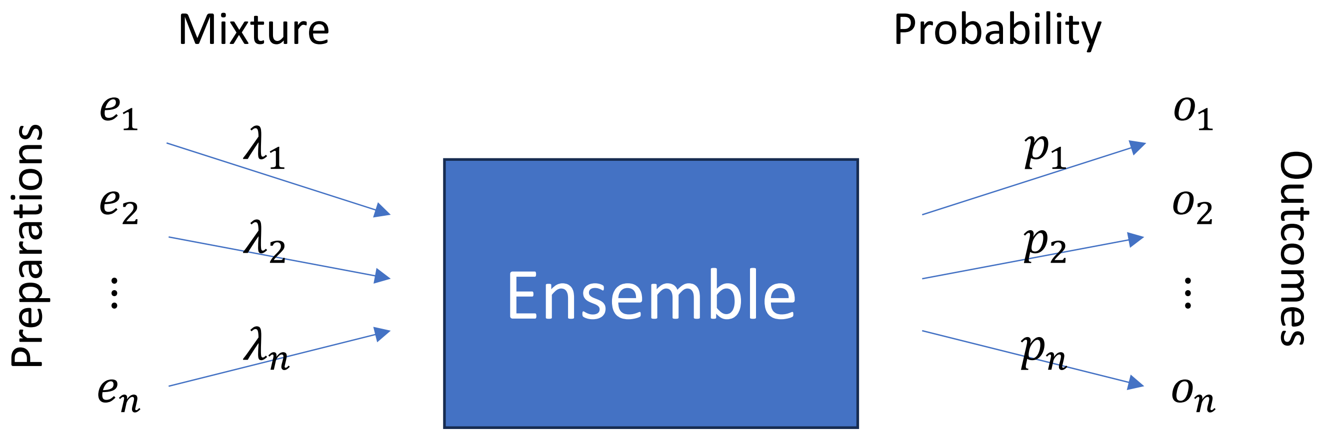
Assumptions of Physics



In classical mechanics, mixtures of preparations and probability of outcomes always coincide

In quantum mechanics, they do not

$\Rightarrow$ ensemble space not a simplex (i.e. classical probability fails)

Can we have common measure theoretic tools on the preparation side?

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Assumptions of Physics

How much of e is a mixture of other ensembles?

$$e = p(\sum \lambda_i a_i) + \bar{p} b$$

biggest p

Definition 1.83. *Let* $\mathsf{e}, \mathsf{a} \in \mathcal{E}$ *be two ensembles. The* ***fraction*** *of* a *in* e *is the greatest mixing coefficient for which* e *can be expressed as a mixture of* a. *That is,* $\mathrm{frac}_{\mathsf{e}}(\mathsf{a}) = \sup(\{p \in [0,1] \mid \exists\, \mathsf{b} \in \mathcal{E} \text{ s.t. } \mathsf{e} = p\mathsf{a} + \bar{p}\mathsf{b}\})$.

Definition 1.85. *Let* $\mathsf{e} \in \mathcal{E}$ *be an ensemble and* $A \subseteq \mathcal{E}$ *a Borel set. The* ***fraction capacity*** *of* A *for* e *is the biggest fraction achievable with convex combinations of* A. *That is,* $\mathrm{fcap}_{\mathsf{e}}(A) = \sup(\mathrm{frac}_{\mathsf{e}}(\mathrm{hull}(A)) \cup \{0\})$.

Proposition 1.87. *The fraction capacity for an ensemble is a set function that is*

1. *non-negative and unit bounded:* $\mathrm{fcap}_{\mathsf{e}}(A) \in [0,1]$
2. *monotone:* $A \subseteq B \implies \mathrm{fcap}_{\mathsf{e}}(A) \leq \mathrm{fcap}_{\mathsf{e}}(B)$
3. *subadditive:* $\mathrm{fcap}_{\mathsf{e}}(A \cup B) \leq \mathrm{fcap}_{\mathsf{e}}(A) + \mathrm{fcap}_{\mathsf{e}}(B)$
4. *continuous from below:* $\mathrm{fcap}_{\mathsf{e}}(\lim_{i\to\infty} A_i) = \lim_{i\to\infty} \mathrm{fcap}_{\mathsf{e}}(A_i)$ *for any increasing sequence* $\{A_i\}$
5. *continuous from above:* $\mathrm{fcap}_{\mathsf{e}}(\lim_{i\to\infty} A_i) = \lim_{i\to\infty} \mathrm{fcap}_{\mathsf{e}}(A_i)$ *for any decreasing sequence* $\{A_i\}$

fuzzy measure

Fraction capacity is a non-additive probability measure

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Assumptions of Physics

Fraction capacity is a non-additive probability measure

State capacity is a non-additive measure

Fuzzy measures!

Is there a suitable generalization of calculus to this non-additive case, which would be valid for all physical theories?

Is there a notion of integral and derivative
so that we can write $e = \int_{\mathcal{E}} \rho_e \, dscap$ and $\rho_e = \frac{dfcap_e}{dscap}$?

Is the type of uncertainty we are characterizing with fuzzy measures compatible with that literature?

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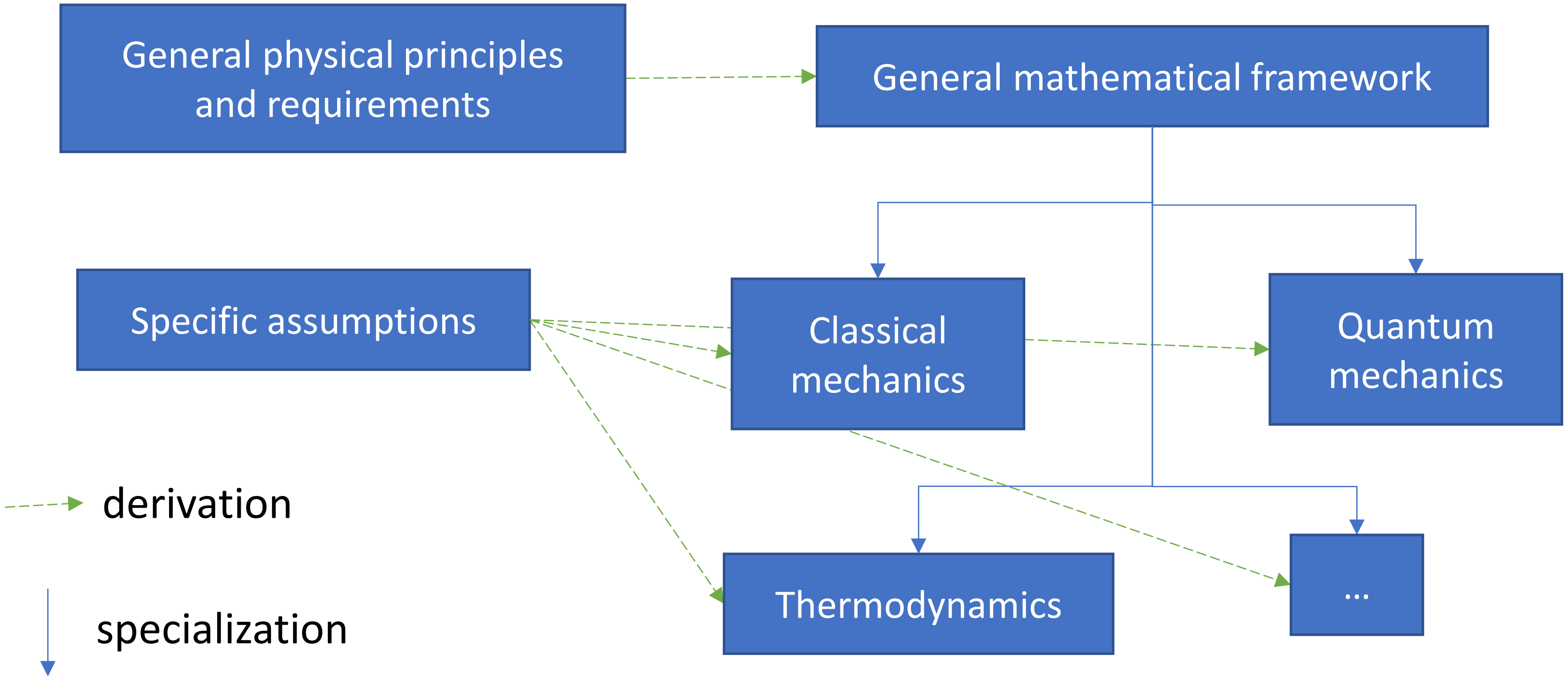
Assumptions of Physics

Experimental verifiability: topologies/σ-algebras
Ensembles: convex space, entropy

Connections to: measure theory, vector spaces, functional analysis, differential geometry, …



Foundations of physics
⇕
The theory of physical models

It must be a concerted effort across physics, math, information theory, philosophy, …

… and I can't know everything!

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Assumptions of Physics

Assumption of Physics is an open project

Our main output is an open access book: https://assumptionsofphysics.org/book/

All our material is developed on GitHub: https://github.com/assumptionsofphysics

One YouTube channel dedicated to publicize results: https://www.youtube.com/user/gcarcassi

Livestream discussions

Another YouTube channel dedicated to research: https://www.youtube.com/@AssumptionsofPhysicsResearch

Activities coordinated through a Discord server (contact me for an invite)

Always looking for experts to gain insights and/or help

Always looking for collaborations

Always looking for editors/journals/conferences that are sympathetic to the mission

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Assumptions of Physics

Wrapping it up

- Different approach to the foundations of physics
  - No interpretations, no theories of everything: physically meaningful starting points from which we can rederive the laws and the mathematical frameworks they need
- Reverse physics (reverse engineer principles from the known laws)
  - Classical mechanics is "completed"; very good ideas for both thermodynamics and quantum mechanics; still do not know how to generalize to field theories
- Physical mathematics (rederive the mathematical structures from scratch)
  - Topology and σ-algebras are derived from experimental verifiability; Good progress on a generic theory of states
- The goal is ambitious and requires a wide collaboration
  - Always looking for people to collaborate with in physics, math, philosophy, …

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Assumptions of Physics

To learn more

- Project website
  - https://assumptionsofphysics.org for papers, presentations, …
  - https://assumptionsofphysics.org/book for our open access book (updated every few years with new results)
- YouTube channels
  - https://www.youtube.com/@gcarcassi
    Videos with results and insights from the research
  - https://www.youtube.com/@AssumptionsofPhysicsResearch
    Research channel, with open questions and livestreamed work sessions
- GitHub
  - https://github.com/assumptionsofphysics
    Book, research papers, slides for videos...

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Assumptions of Physics

> **Principle of scientific objectivity.** Science is universal, non-contradictory and evidence based.

⇒ Science is about statements that are associated to experimental tests

Statements must be either true or false for everybody

Tests may or may not terminate (i.e. be conclusive)

Statement	Test Result
T	SUCCESS (in finite time)
	UNDEFINED
F	UNDEFINED
	FAILURE (in finite time)

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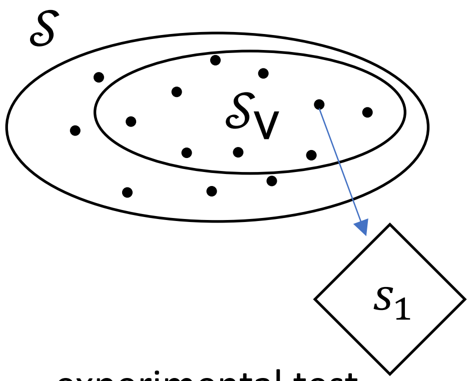
Assumptions of Physics

Axiom 1.27 (Axiom of verifiability). *A **verifiable statement** is a statement that, if true, can be shown to be so experimentally. Formally, each logical context $\mathcal{S}$ contains a set of statements $\mathcal{S}_v \subseteq \mathcal{S}$ whose elements are said to be verifiable. Moreover, we have the following properties:*

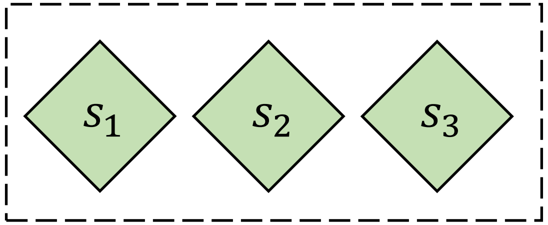
- *every certainty $\top \in \mathcal{S}$ is verifiable*
- *every impossibility $\perp \in \mathcal{S}$ is verifiable*
- *a statement equivalent to a verifiable statement is verifiable*

Remark. The **negation or logical NOT** of a verifiable statement is not necessarily a verifiable statement.



s_1	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED

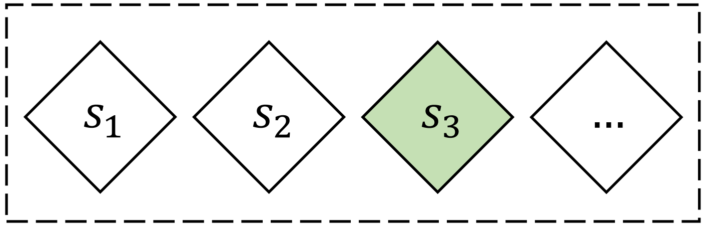
$$\bigwedge_{i=i}^{n} s_i$$



All tests must succeed

Axiom 1.31 (Axiom of finite conjunction verifiability). *The conjunction of a finite collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{n} \subseteq \mathcal{S}_v$ be a finite collection of verifiable statements. Then the conjunction $\bigwedge\limits_{i=1}^{n} s_i \in \mathcal{S}_v$ is a verifiable statement.*

$$\bigvee_{i=1}^{\infty} s_i$$



Axiom 1.32 (Axiom of countable disjunction verifiability). *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_v$ be a countable collection of verifiable statements. Then the disjunction $\bigvee\limits_{i=1}^{\infty} s_i \in \mathcal{S}_v$ is a verifiable statement.*

One successful test is sufficient

⇒ Verifiable statements form a frame/Heyting algebra

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Axiom 1.32 (Axiom of countable disjunction verifiability). *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let* $\{\mathsf{s}_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_\mathsf{v}$ *be a countable collection of verifiable statements. Then the disjunction* $\bigvee_{i=1}^{\infty} \mathsf{s}_i \in \mathcal{S}_\mathsf{v}$ *is a verifiable statement.*

Disjunction (OR) of verifiable statements: check that ONE test terminates successfully

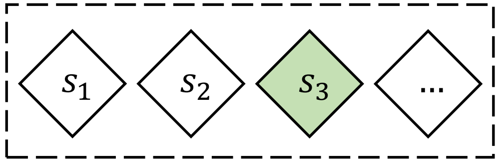
s_1	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED

watch out for non-termination!

$\vee\ (e_i)$:

1. Initialize n to 1
2. For each $i = 1 \dots n$
   a) Run e_i for n seconds
   b) If e_i succeeds, return SUCCESS
3. Increment n and go to 2

$$\bigvee_{i=1}^{\infty} s_i$$



⇒ Only countable disjunction can reach all tests

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