Classical mechanics as the high-entropy limit of quantum mechanics

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Main goal of the program

Identify a handful of physical starting points from which the basic laws can be rigorously derived

Assumptions of Physics

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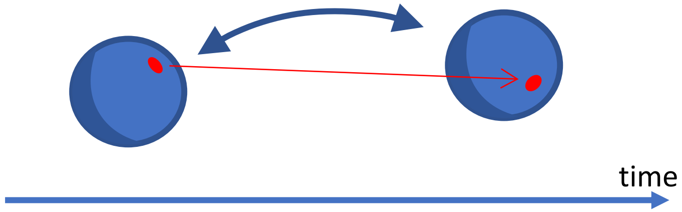
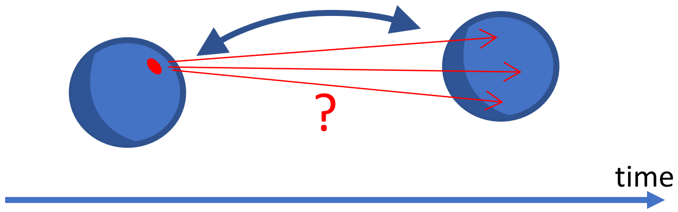
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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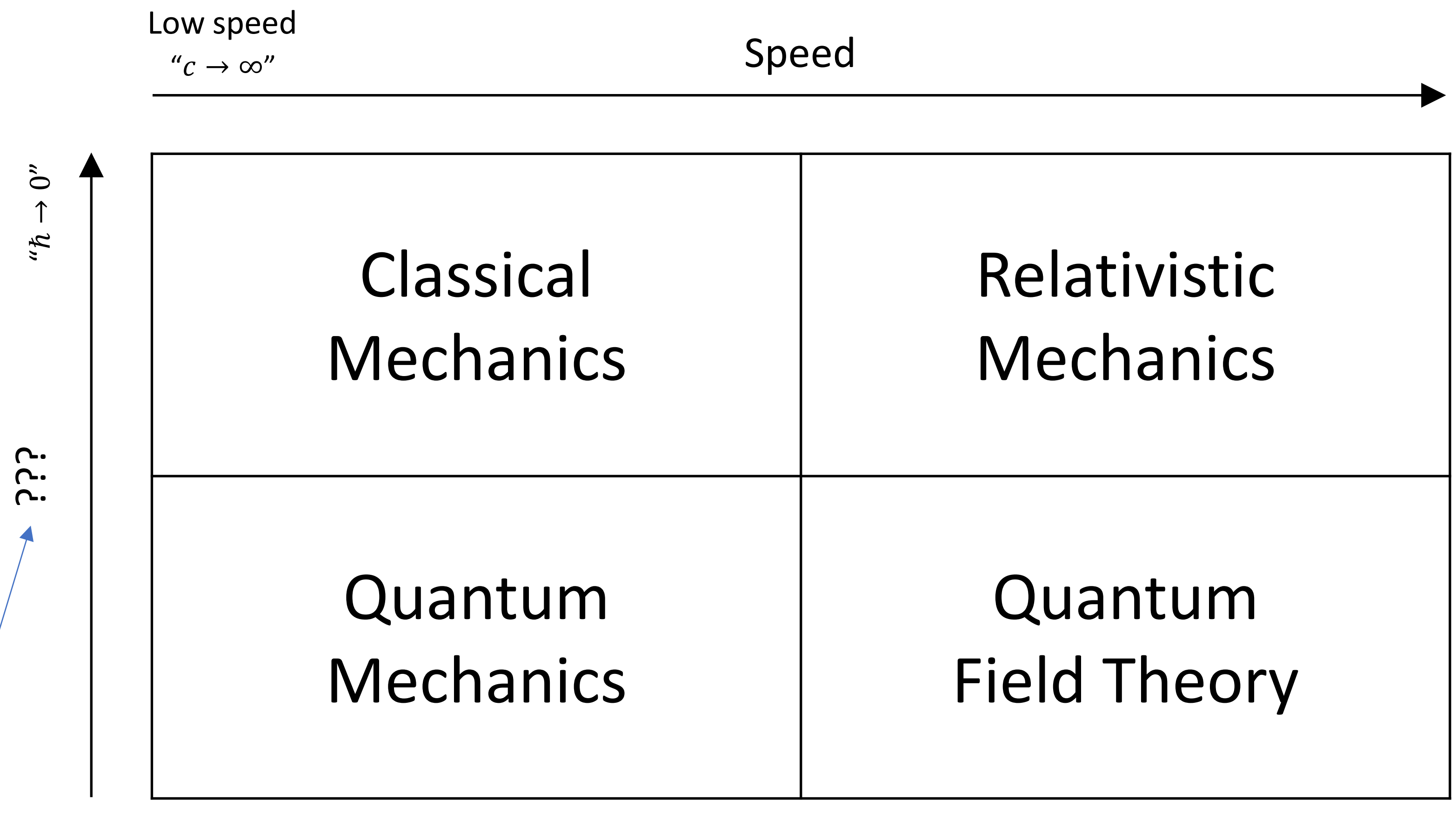

Not about size or number of particles!

Coherent communication over 254 Km

https://phys.org/news/2025-04-quantum-messages-km-infrastructure.html

A billion hydrogen atoms in a single Bose–Einstein Condensate

https://doi.org/10.1016/S0921-4526(99)01415-5

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Assumptions of Physics

Why does classical mechanics fail,
and how is it fixed by quantum mechanics?

When can I use classical mechanics,
and when can't I?

What are the rules?!?

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Assumptions of Physics

It’s Always Sunny in Philadelphia

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Let's review some cases and see if we can find common ground

Sometimes coherence (and therefore quantum effects)
can be lost through interaction with environment (decoherence)

Environment introduces entropy in the system

For massive particles moving in an electromagnetic field,
you often can use classical mechanics

Low precision description means high entropy

Classical mechanics

$\Longleftrightarrow$

High entropy?

Superconductivity can be achieved
at high temperature and high pressure

High pressure means low entropy

For Bose Einstein Condensates
you need high density in phase space

High phase space density means low entropy

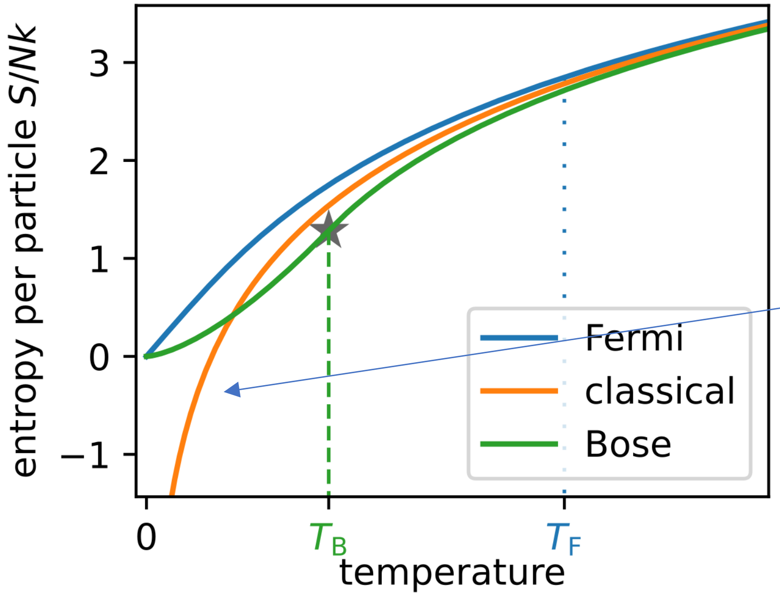
High temperature means high entropy

Many quantum effects are difficult/impossible to achieve at high temperature
(e.g. superconductivity, topological insulators, quantum Hall effect, …)

What’s the problem with classical mechanics and low entropy?

Third law of thermodynamics

Wikipedia



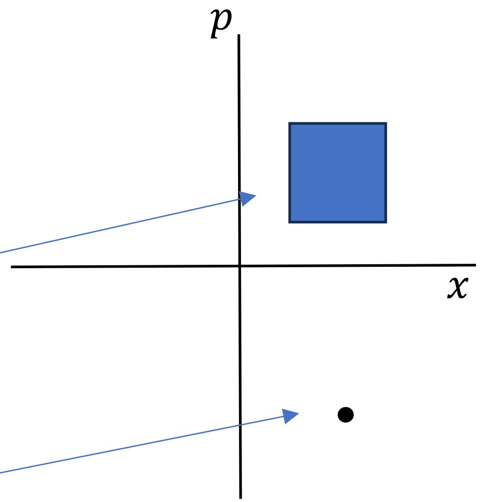
Temperature/entropy curve for ideal gases

to minus infinity!

$$S = \log \Delta x \Delta p$$

For a single microstate

$$S = \log 0 = -\infty$$



Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.

G. N. Lewis and M. Randall, Thermodynamics and the free energy of chemical substances (McGraw-Hill, 1923)

$$S \geq 0$$

Classical physics allows arbitrarily small entropy…

… but thermodynamics doesn't!!!

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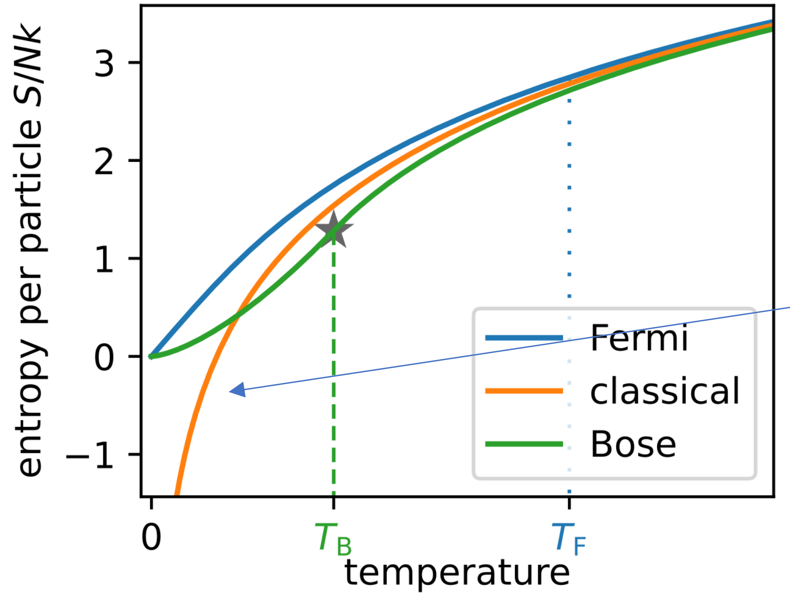
Adam Savage - Mythbusters

Third law of thermodynamics

Wikipedia



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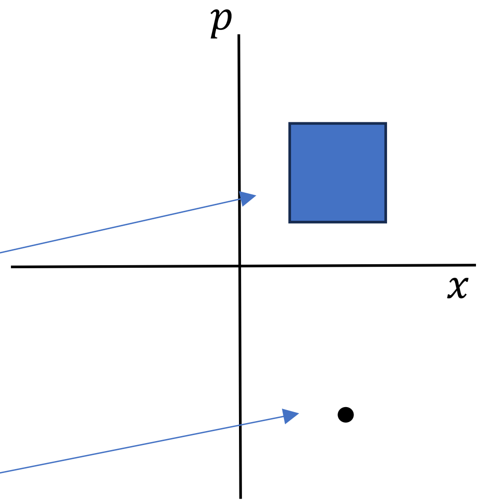

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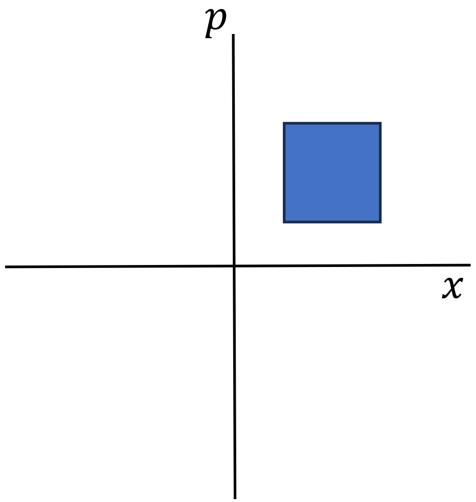
Third law of thermodynamics

$$S \geq 0$$

There is a lower bound on the entropy



Statistical mechanics

$$S(\rho_U) = \log \Delta x \Delta p$$

Dimensionally incorrect!!!

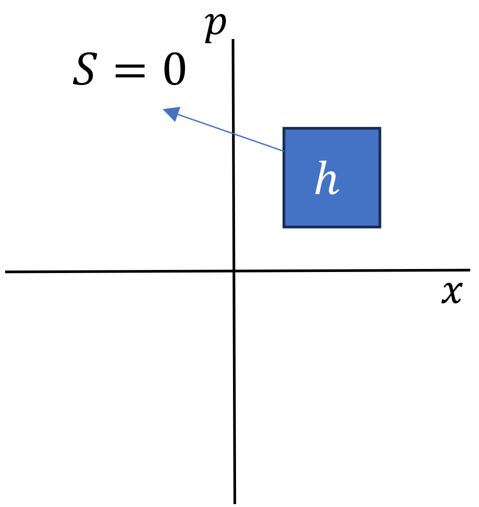
$$S(\rho) = -\int \rho \log \rho \, dxdp$$

previous results

Third law of thermodynamics

$$S \geq 0$$

There is a lower bound on the entropy



Statistical mechanics

Define h as the support of a uniform distribution of zero entropy

$$S(\rho_U) = \log \frac{\Delta x \Delta p}{h}$$

$$S(\rho) = -\int \rho \log h\rho \, dx dp$$

Fixes units (i.e. log argument is a pure number) and zero of entropy

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Entropy vs uncertainty

$$S(\rho) = -\int \rho \log h\rho \, dq dp$$

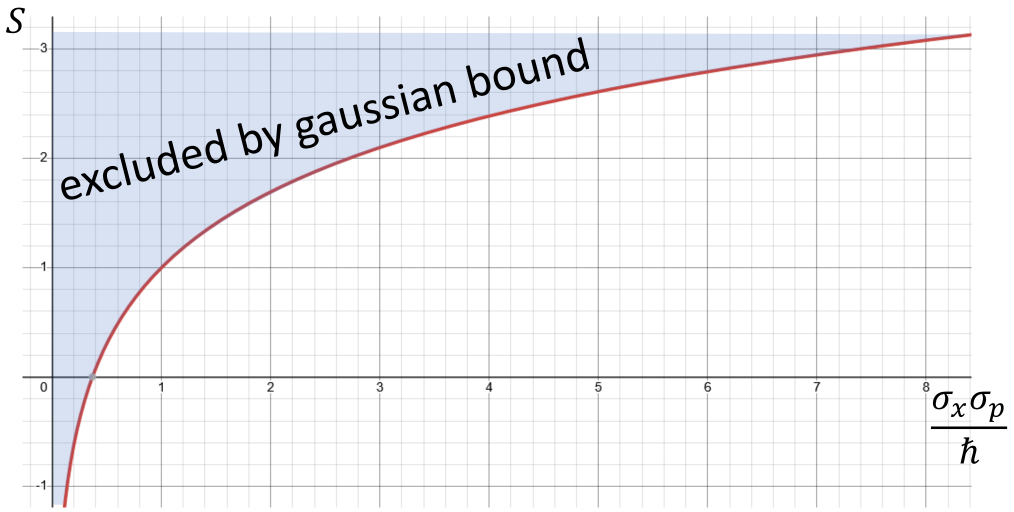
Fixes units

Uniform distribution over volume h has zero entropy

Let's plot one against the other

Gaussian maximizes entropy for a given uncertainty

$$S(\rho) \leq \log 2\pi e \frac{\sigma_x \sigma_p}{h}$$



$$\sigma_x \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$

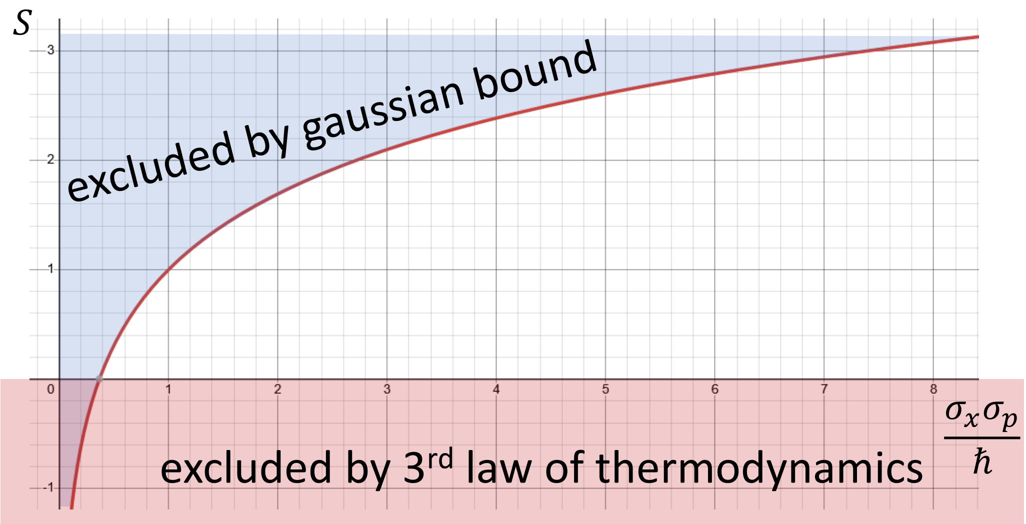
Entropy puts a lower bound on the uncertainty

But the third law imposes a lower bound on the entropy!

With the 3rd law

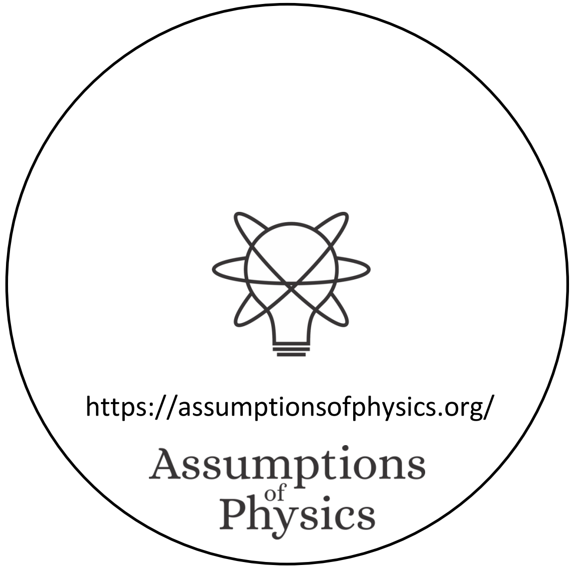
Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.

G. N. Lewis and M. Randall, Thermodynamics and the free energy of chemical substances (McGraw-Hill, 1923)



$$S \geq 0 \quad \Rightarrow \quad \sigma_x \sigma_p \geq \frac{\hbar}{e}$$

Classical uncertainty principle



Comparing theories

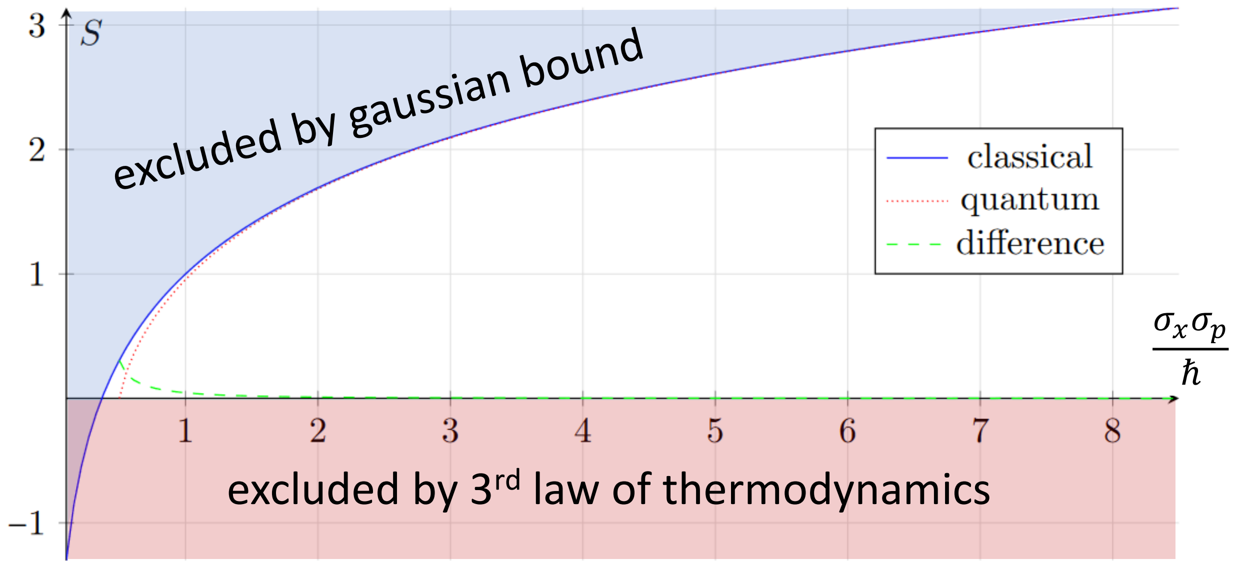
classical

$$\sigma_x \sigma_p \geq \frac{\hbar}{e}$$

2.71828…

quantum

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$



$$S_C = \ln e\sigma$$

$$S_Q = \left(\sigma + \tfrac{1}{2}\right) \ln\left(\sigma + \tfrac{1}{2}\right) - \left(\sigma - \tfrac{1}{2}\right) \ln\left(\sigma - \tfrac{1}{2}\right)$$

Entropy of quantum states is already non-negative

The gaussian bound quickly becomes very similar across theories

Quantum mechanics incorporates the third law
Classical mechanics does not

Looks like: quantum and classical mechanics only differ at low entropy

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Is classical mechanics recovered at high entropy? Why?

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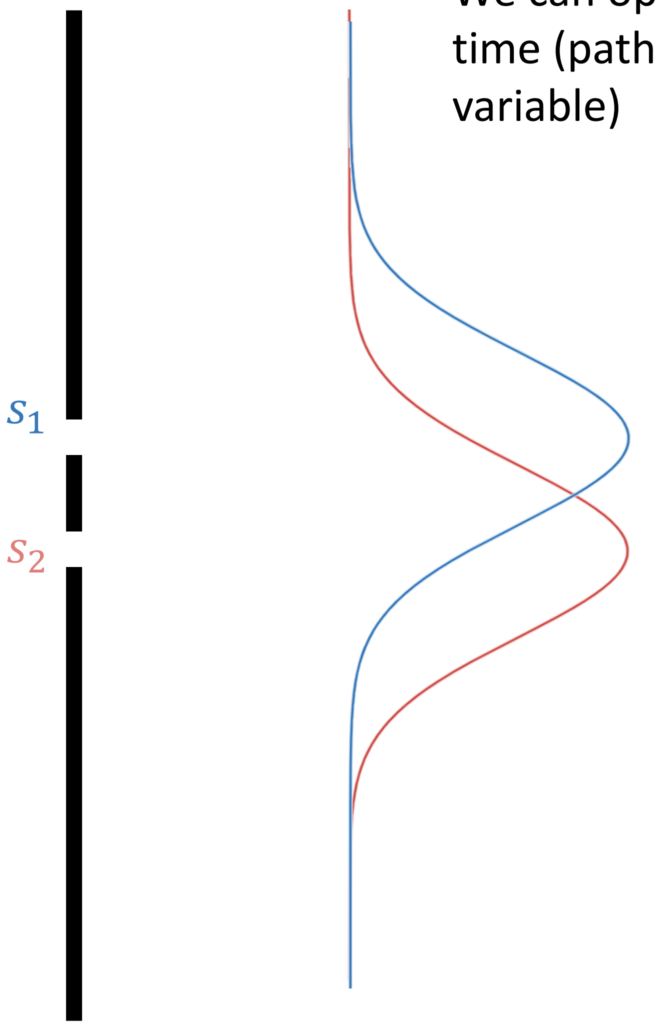
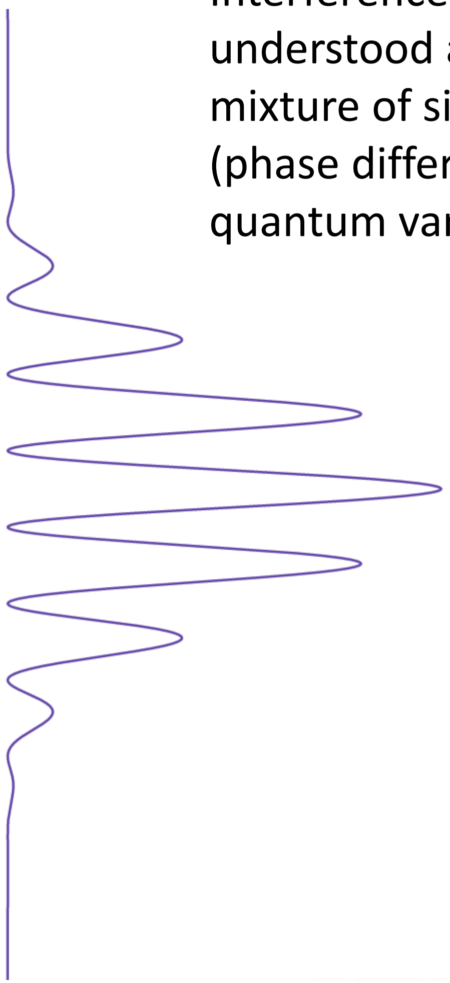
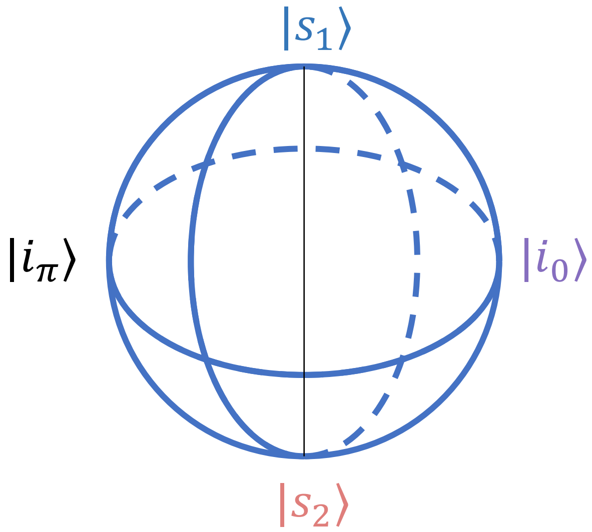


We can open one slit at a time (path is a classical variable)



Interference cannot be understood as statistical mixture of single slit open (phase difference is a quantum variable)



"Quantum variables" cannot be described by "classical variables"

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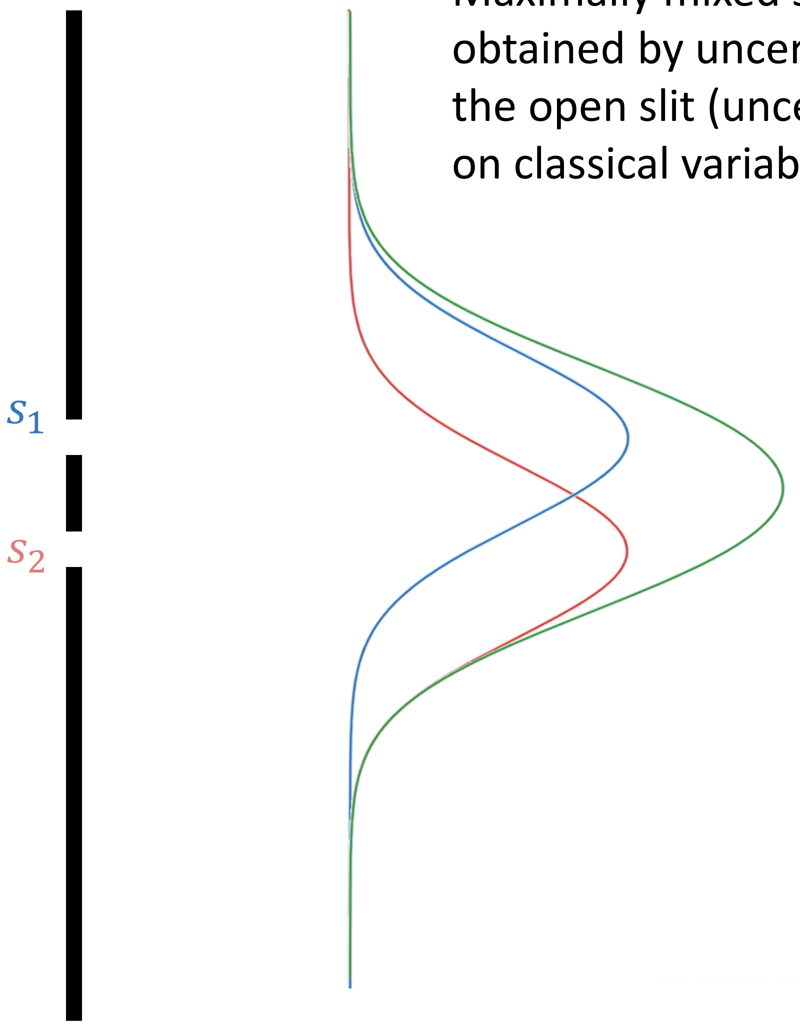
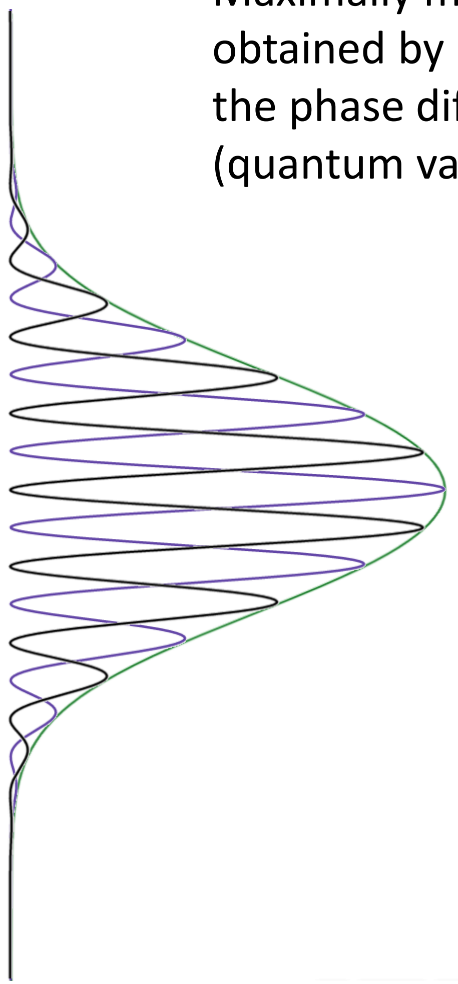
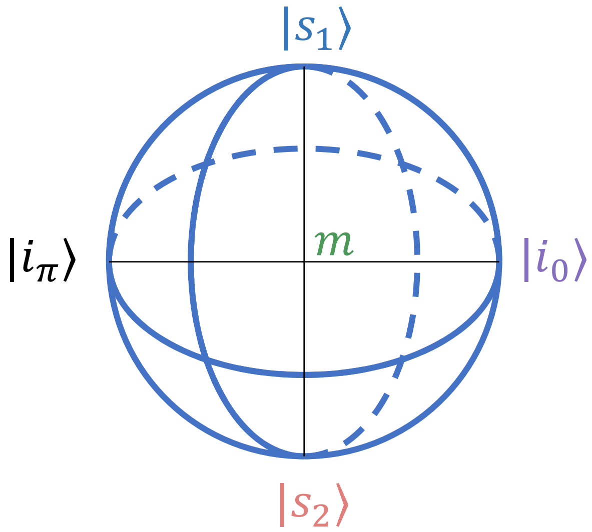


Maximally mixed state obtained by uncertainty on the open slit (uncertainty on classical variable)



Maximally mixed state obtained by uncertainty on the phase difference (quantum variable)



Uncertainty on “quantum variables” can be represented by uncertainty on “classical variables”

Mathematically, operators do not commute in general:

$$\mathrm{tr}[AB\rho] \neq \mathrm{tr}[BA\rho]$$

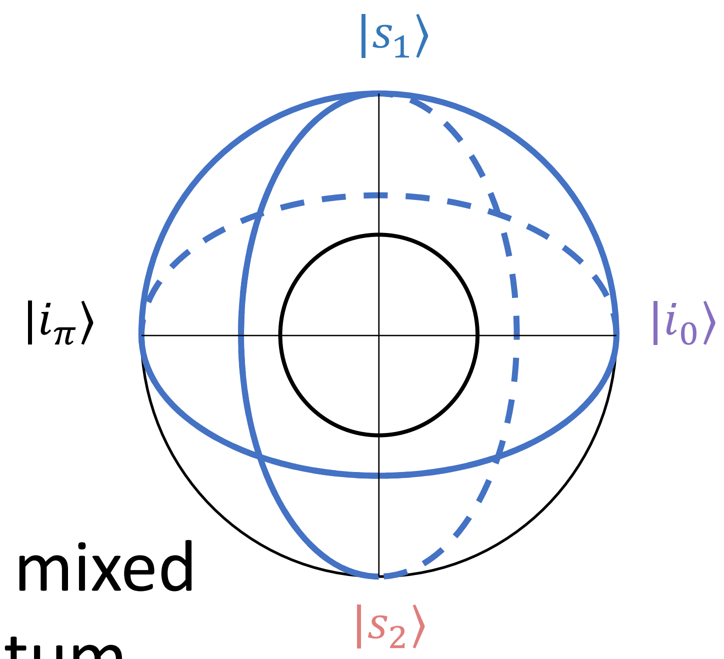
But they "commute" for the maximally mixed state $\rho = I/n$:

$$\mathrm{tr}[AB\, I/n] = \mathrm{tr}[B\, I/n\, A] = \mathrm{tr}[BA\, I/n]$$

ρ commutes with both A and $B \Rightarrow \langle AB \rangle = \langle BA \rangle$

Both observables are well-defined

As you increase entropy, the error you make approximating to a mixture of eigenstates of any observable decreases



Conjecture: this works in infinite dimensions, where there is no maximally mixed state

As you increase entropy, you can approximate your mixed state as commuting with both position and momentum

How do you prove it?

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Classical mechanics as high entropy limit?

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greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

Reply Reply All Forward ...

Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

[Submitted on 1 Nov 2024 (v1), last revised 3 Dec 2024 (this version, v2)]

Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

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Subjects: **Quantum Physics (quant-ph)**
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Assumptions of Physics

Reinterpret old results

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Assumptions of Physics

Classical blackbody radiation
recovered at small frequencies

$$\frac{2h\nu^3}{c^2}\frac{1}{e^{h\beta\nu}-1}=\frac{2h\nu^3}{c^2}\frac{1}{1+h\beta\nu+O\left(\nu^2\right)-1}\simeq\frac{2\nu^2}{c^2\beta}.$$

At thermal equilibrium, high entropy ⇔ high temperature ⇔ low β

Classical blackbody radiation
recovered at small β

$$\frac{2h\nu^3}{c^2}\frac{1}{e^{h\beta\nu}-1}=\frac{2h\nu^3}{c^2}\frac{1}{1+h\beta\nu+O\left(\beta^2\right)-1}\simeq\frac{2\nu^2}{c^2\beta}.$$

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Assumptions of Physics

On the Quantum Correction For Thermodynamic Equilibrium

By E. Wigner
Department of Physics, Princeton University
(Received March 14, 1932)

The probability of a configuration is given in classical theory by the Boltzmann formula $\exp\left[-V/kT\right]$ where V is the potential energy of this configuration. For high temperatures this of course also holds in quantum theory. For lower temperatures, however, a correction term has to be introduced, which can be developed into a power series of h. The formula is developed for this correction by means of a probability function and the result discussed.

Wigner function of a thermal state

$$W(x,p) = \int dy e^{i(x+y)p/\hbar} \langle x+y | e^{-\beta \hat{H}} | x-y \rangle e^{-i(x-y)p/\hbar}$$

Classical distribution is the leading term of expansion in $\hbar$

At thermal equilibrium, high entropy ⇔ high temperature ⇔ low β

We recover the same limit and correction with a more meaningful expansion in β

The first-order quantum correction vanishes, showing that the classical approximation is correct to first order in β, which corresponds to the limit of large entropy. We can consider the second-order correction

$$W(x,p) \simeq 1 - \beta\epsilon(x,p) - \beta^2\epsilon^2(x,p) - \beta^2 \int dy \langle x+y | \hat{Q} V(\hat{x}) + V(\hat{x}) \hat{Q} | x-y \rangle, \qquad (13)$$

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High entropy limit

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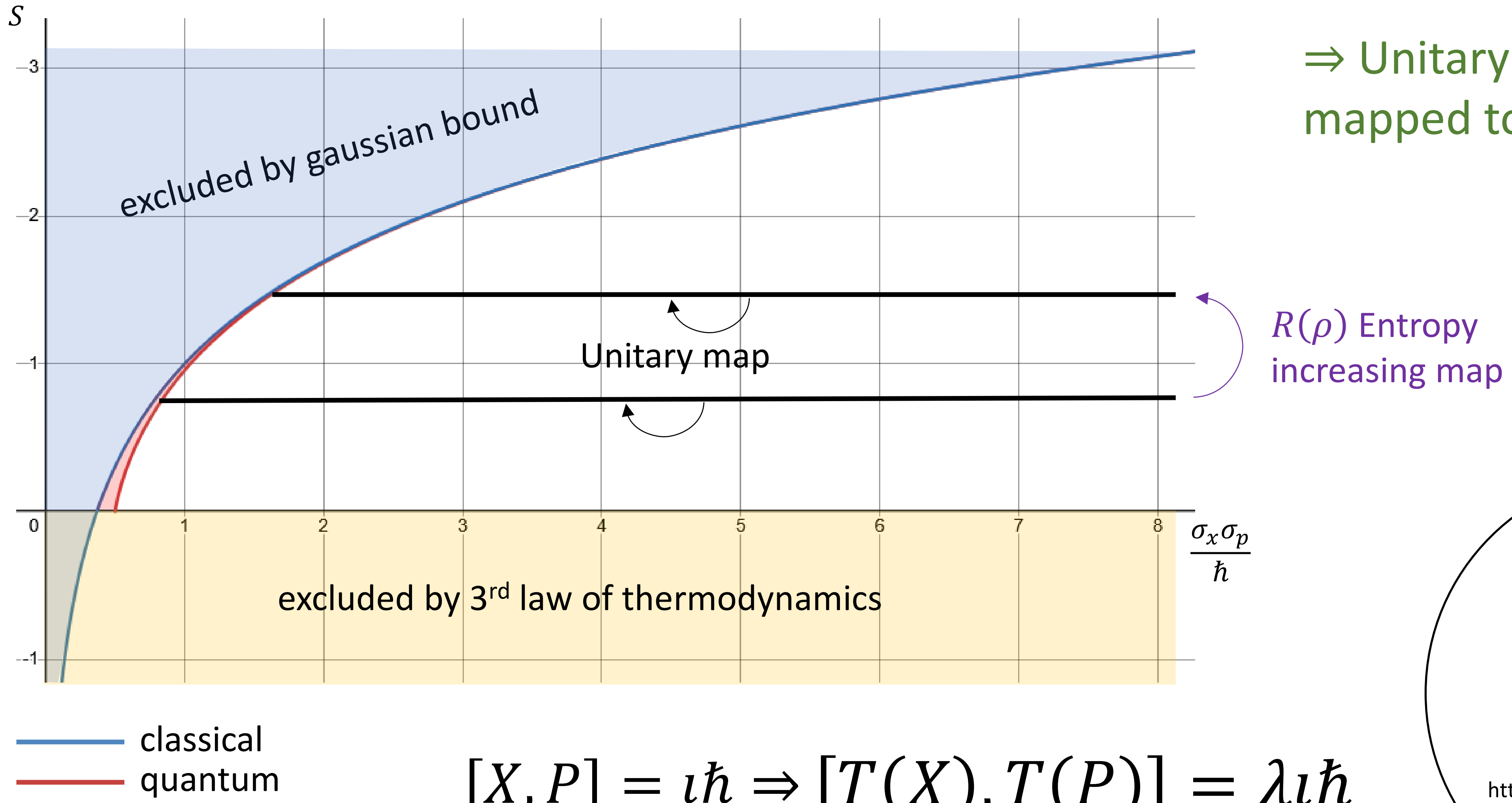
Looking for a map $R(\rho)$ that increases entropy of all mixed states, such that every level set of entropy maps to another level set



⇒ Unitary trans. must be mapped to unitary trans.

$$[X, P] = \imath\hbar \Rightarrow [T(X), T(P)] = \lambda \imath\hbar$$

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Assumptions of Physics

Entropy increasing maps in classical mechanics

We want a map $(x,p) \mapsto (\hat{x},\hat{p})$ that increases the entropy of all distributions by the same value

$$S[\rho(x,p)] = -\int \rho \log \rho \, dxdp = -\int \hat{\rho}|J| \log(\hat{\rho}|J|) \, dxdp$$

$$= -\int \hat{\rho} \log(\hat{\rho}|J|) \, d\hat{x}d\hat{p}$$

$$= -\int \hat{\rho} \log \hat{\rho} \, d\hat{x}d\hat{p} - \int \hat{\rho} \log|J| \, d\hat{x}d\hat{p}$$

$$= S[\hat{\rho}(\hat{x},\hat{p})] - \int \hat{\rho} \log|J| \, d\hat{x}d\hat{p}$$

Poisson bracket!

$\Rightarrow$ Jacobian determinant must be a constant!

$$\frac{\partial \hat{x}}{\partial x}\frac{\partial \hat{p}}{\partial p} - \frac{\partial \hat{x}}{\partial p}\frac{\partial \hat{p}}{\partial x} = \lambda \qquad S[R(\rho)] = S[\rho] + \log \lambda$$

Entropy increasing maps in classical mechanics

$$S[R(\rho)] = S[\rho] + \log\lambda \Leftrightarrow \{R(x), R(p)\} = \lambda\{x, p\}$$

Let's call "stretching maps" those R that satisfy this property

Let's call "pure stretching map" the map T such that $(x,p) \mapsto \left(\sqrt{\lambda}\,x, \sqrt{\lambda}\,p\right)$

They also satisfy

$$\langle T(x^n p^m)\rangle = \left(\sqrt{\lambda}\right)^{n+m}\langle x^n p^m\rangle$$

In classical mechanics

$$S\big(R(\rho)\big) = S(\rho) + \log\lambda \Leftrightarrow \{R(x), R(p)\} = \lambda\{x, p\}$$

Jacobian determinant is a constant: all volumes rescaled by the same factor

In quantum mechanics

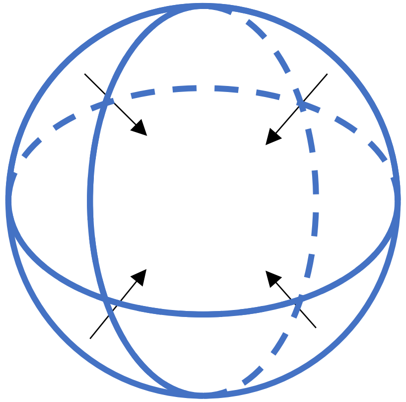
Stretching map $$[R(X), R(P)] = \lambda[X, P]$$

Stretching maps can't be a unitary transformation

They do not conserve entropy nor preserve commutation relationship

They must be quantum channels

They are maps between density matrices, not pure states



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Assumptions of Physics

Linblad master equation for open quantum systems

Infinitesimal stretching map must obey

$$\frac{dX}{dt} = \frac{\imath}{\hbar}[H, X] + \gamma\left(L^\dagger X L - \frac{1}{2}\{L^\dagger L, X\}\right)$$

Unitary/conservative part

Non-unitary/non-conservative part

Quantum pure stretching map

$$T(X) = \sqrt{\lambda}\, X \qquad T(P) = \sqrt{\lambda}\, P$$

What about products?

Only unitary maps preserve products!

1. Suppose $T(\rho_1\rho_2) = T(\rho_1)T(\rho_2)$. Since pure states are exactly the density matrices for which $\rho^2 = \rho$, $T(\rho) = T(\rho^2) = T(\rho)T(\rho) = T(\rho)^2$ and therefore T maps pure states to pure states. Therefore, it is a unitary map!
2. Suppose $T(AB) = T(A)T(B)$. Then $T([A,B]) = T(AB) - T(BA) = [T(A), T(B)]$

Which product ordering recovers

$$\langle T(x^n p^m)\rangle = \left(\sqrt{\lambda}\right)^{n+m} \langle x^n p^m \rangle?$$

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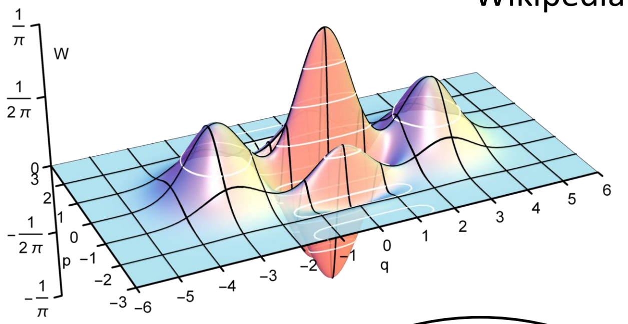
Symmetrized operator averages

$$\langle T_W(\Pi(\underbrace{X,...,X}_{\text{n times}},\underbrace{P,...,P}_{\text{m times}}))\rangle = (\sqrt{\lambda})^{(n+m)}\langle\Pi(\underbrace{X,...,X}_{\text{n times}},\underbrace{P,...,P}_{\text{m times}})\rangle$$

$$\Pi(A_1, A_2, \ldots, A_n) = \frac{1}{n!}\sum_{\pi} A_{\pi(1)}A_{\pi(2)}\cdots A_{\pi(n)}$$

We take the average of all permutations

Wikipedia



$$\langle\Pi(\underbrace{X,...,X}_{\text{n times}},\underbrace{P,...,P}_{\text{m times}})\rangle = \int_M x^n p^m W(x,p)\,dx\,dp$$

Also given by the expectations of the Wigner function

A stretching map that rescales symmetrized operator averages is a stretching map in phase space for the Wigner function…

… and it can't work!!!

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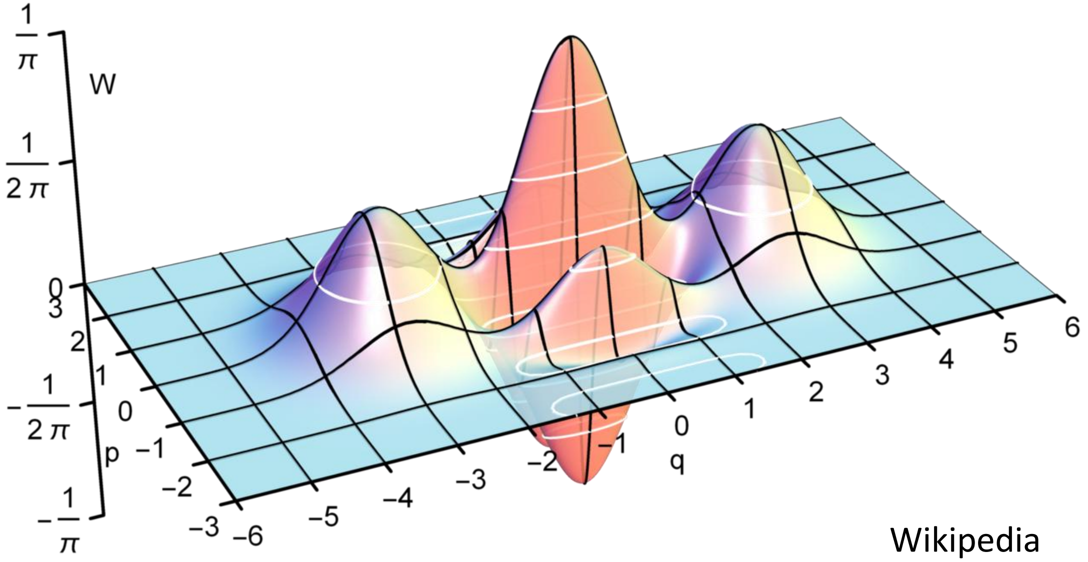
Symmetrized operator averages



Wikipedia

Wigner functions can have negative values, but they must be restricted to “small” regions

A stretching map would make those regions arbitrarily large, so it can’t map valid quantum states to valid quantum states

Symmetrized operator averages are ruled out!

Normal ordering

$$\langle T_P((a^\dagger)^n a^m)\rangle = (\sqrt{\lambda})^{(n+m)}\langle (a^\dagger)^n a^m\rangle$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\imath}{m\omega}P\right) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\imath}{m\omega}P\right)$$

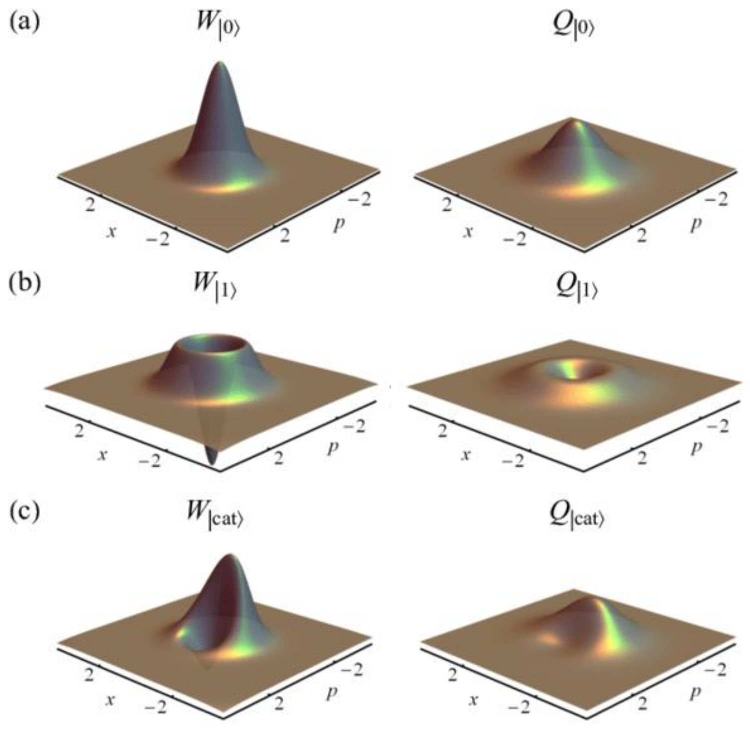
For the vacuum state we have $a|0\rangle = 0$. The mean for all expectations for the vacuum state is zero. Therefore, after the map, all expectations are zero. Which means the vacuum is mapped to the vacuum. Entropy is not increased for all states!

Normal ordering is ruled out!

Anti-normal ordering

$$\langle T_Q(a^n(a^\dagger)^m)\rangle = (\sqrt{\lambda})^{(n+m)}\langle a^n(a^\dagger)^m\rangle$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\iota}{m\omega}P\right) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\iota}{m\omega}P\right)$$



Maurice de Gosson

$$\langle a^n(a^\dagger)^m\rangle = \int \alpha^n(\alpha^*)^m Q_1(\alpha)d^2\alpha$$

Also given by the expectations of the Husimi Q function

T_Q stretches the Husimi Q…

… which is non-negative!!!

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Assumptions of Physics

So, we want: $\langle T_Q(a^n(a^\dagger)^m)\rangle = (\sqrt{\lambda})^{(n+m)}\langle a^n(a^\dagger)^m\rangle$

Lindblad master equation

$$\frac{dX}{dt} = \frac{\imath}{\hbar}[H,X] + \gamma\left(L^\dagger X L - \frac{1}{2}\{L^\dagger L, X\}\right)$$

We set $H = 0 \quad L = a^\dagger$

Recall $[a, a^\dagger] = 1$

$$\left[a,(a^\dagger)^m\right] = m(a^\dagger)^{m-1}$$

$$\left[a^n, a^\dagger\right] = n a^{n-1}$$

$$\frac{d}{dt}a^n(a^\dagger)^m = \gamma\left(a a^n (a^\dagger)^m a^\dagger - \frac{1}{2} a a^\dagger a^n (a^\dagger)^m - \frac{1}{2} a^n (a^\dagger)^m a a^\dagger\right)$$

$$= \gamma\left(a a^n (a^\dagger)^m a^\dagger - \frac{1}{2} a a^n (a^\dagger)^m a^\dagger - \frac{1}{2} a[a^\dagger, a^n](a^\dagger)^m - \frac{1}{2} a a^n (a^\dagger)^m a^\dagger - \frac{1}{2} a^n \left[(a^\dagger)^m, a\right] a^\dagger\right)$$

$$= \gamma\left(-\frac{1}{2} a(-n a^{n-1})(a^\dagger)^m - \frac{1}{2} a^n \left(-m(a^\dagger)^{m-1}\right) a^\dagger\right)$$

$$= \frac{\gamma}{2}(n+m) a^n (a^\dagger)^m$$

first-order linear ODE

$(\sqrt{\lambda})^{n+m}$

$$\Rightarrow \left(a^n(a^\dagger)^m\right)(t) = e^{\frac{\gamma}{2}t(n+m)} a^n (a^\dagger)^m$$

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Effects of stretching map on phase space representations

Husimi Q is simply stretched

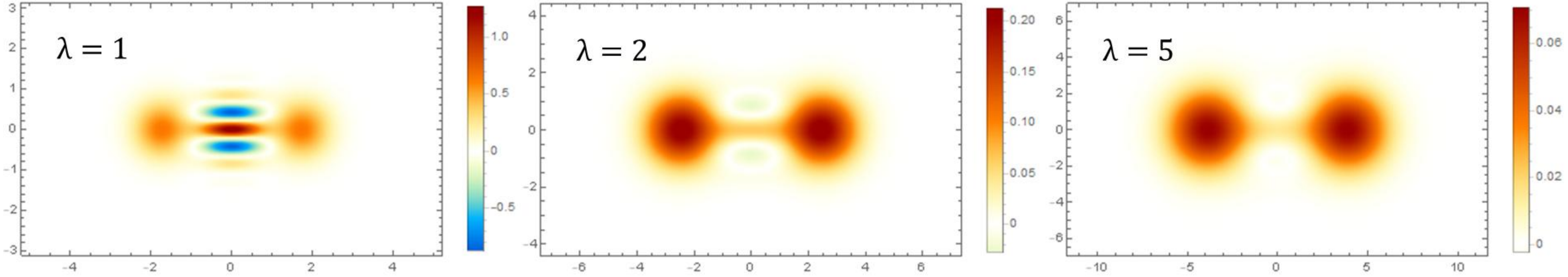
Wigner function is stretched and convolved with a gaussian, and coincides with the Husimi Q in the limit $\lambda \to \infty$

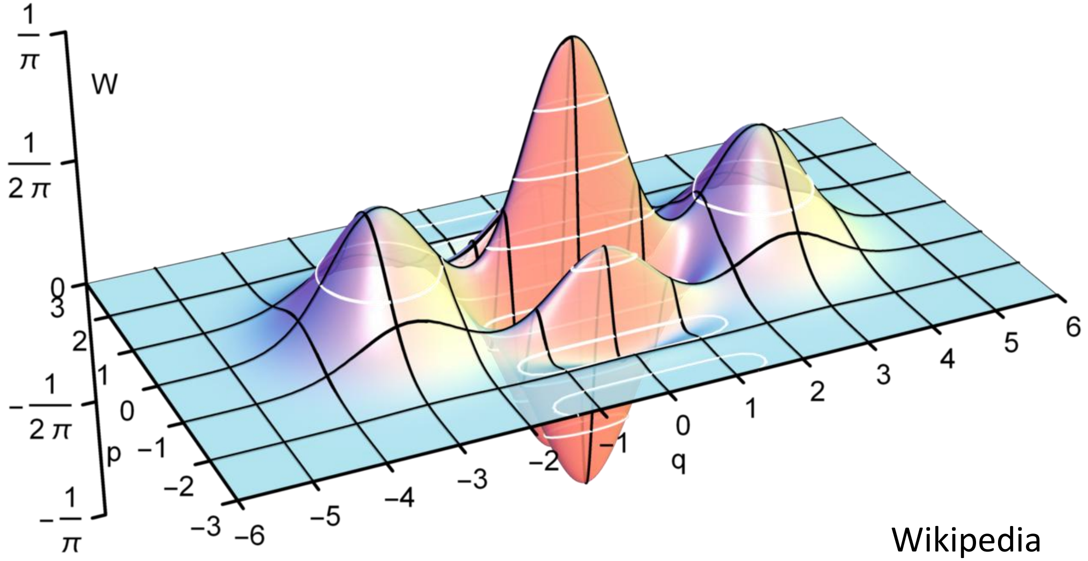
It is interesting to consider what happens to the negative regions of W under the stretching map. We know that W can have negative regions, but their size is limited by the uncertainty principle. In fact, convolving W with a 2D Gaussian with unitary spread, as in the definition of Q, returns a function that is never negative. In the limit $\lambda \gg 1$, the formula for W_λ reduces to

$$W_\lambda(\beta) \to_{\lambda \gg 1} \frac{2}{\pi \lambda^2} \int W_1\left(\frac{\alpha}{\sqrt{\lambda}}\right) e^{-\frac{2}{\lambda}|\alpha-\beta|^2} d^2\alpha = Q_\lambda(\beta). \tag{53}$$

Therefore, while negative regions can be in principle found at any finite λ, W tends to a positive function in the limit. As usual for the W distribution, the phase space size of negative regions is limited to $\hbar$ by the uncertainty principle. The weight of the function in such regions is limited between $\pm 2/\hbar$ for pure states. The effect of the stretching map is to reduce this bound to $\pm 2/(\lambda\hbar)$ for large values of λ. A clear interpretation can be made by working directly on the Fourier transform of W; see Eq. (51). The function $F(W_\lambda)$ is a scaled version of $F(W_1)$ with a Gaussian filter applied to it. Quantum information in W_1 is carried by spectral weights with k-vectors larger than 1. The bandwidth of the Gaussian filter is given by $\lambda/(\lambda - 1)$. This cutoff approaches 1 in the limit, filtering away the interference terms.

Stretching a cat state





Wikipedia

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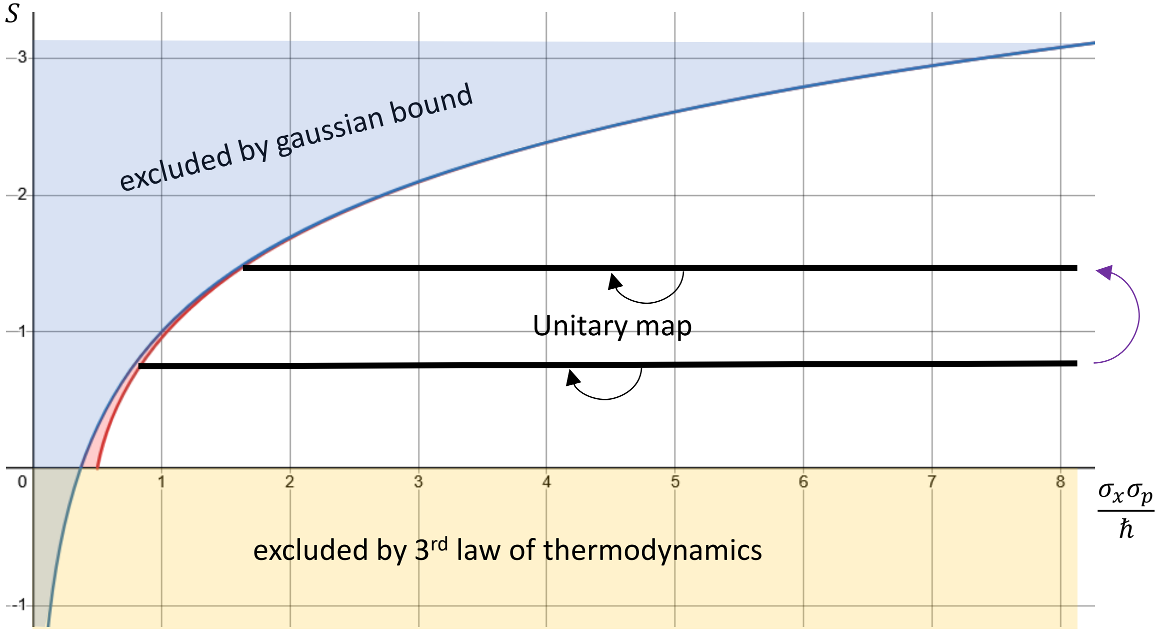
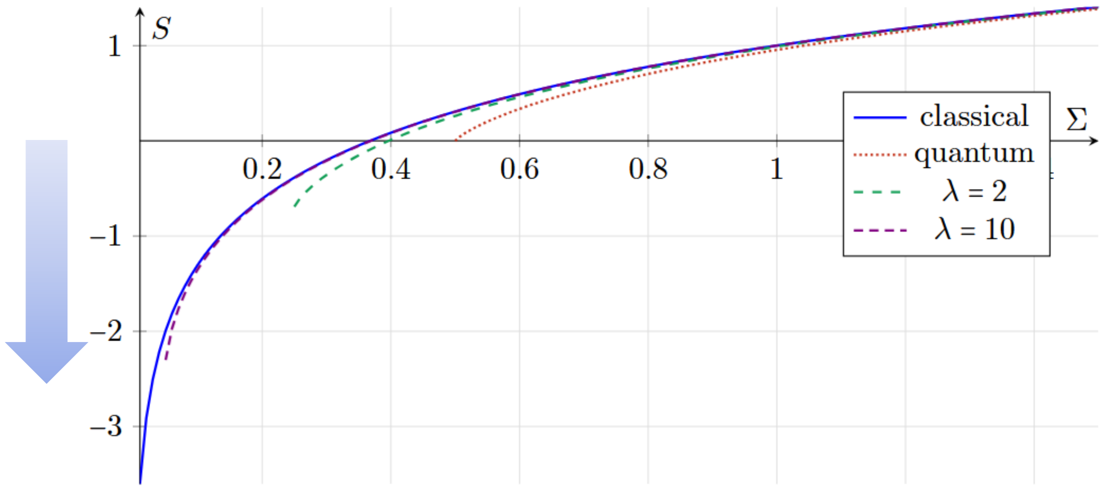
Recover $\hbar \to 0$

Physical limit:
entropy of mixed state is high
$S \gg 0$



Mathematical limit:
entropy of pure states is minus infinity
$\hbar \to 0$



In relativity

Physical limit: velocities are small $v \ll c$

Mathematical limit: speed of light is infinite $c \to +\infty$

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We started with

$$X \qquad P$$

$$[X, P] = \imath\hbar$$

After the map, we found

$$T(X) \qquad T(P)$$

$$[T(X), T(P)] = \lambda\imath\hbar$$

We want to start with

$$\hat{X} \qquad \hat{P}$$

Therefore, we must have

$$[\hat{X}, \hat{P}] = \frac{\imath\hbar}{\lambda}$$

So that after the map we find

$$T(\hat{X}) \qquad T(\hat{P})$$

$$[T(\hat{X}), T(\hat{P})] = \imath\hbar$$

But $\lambda \to \infty \;\Rightarrow\; \frac{\hbar}{\lambda} \to 0$

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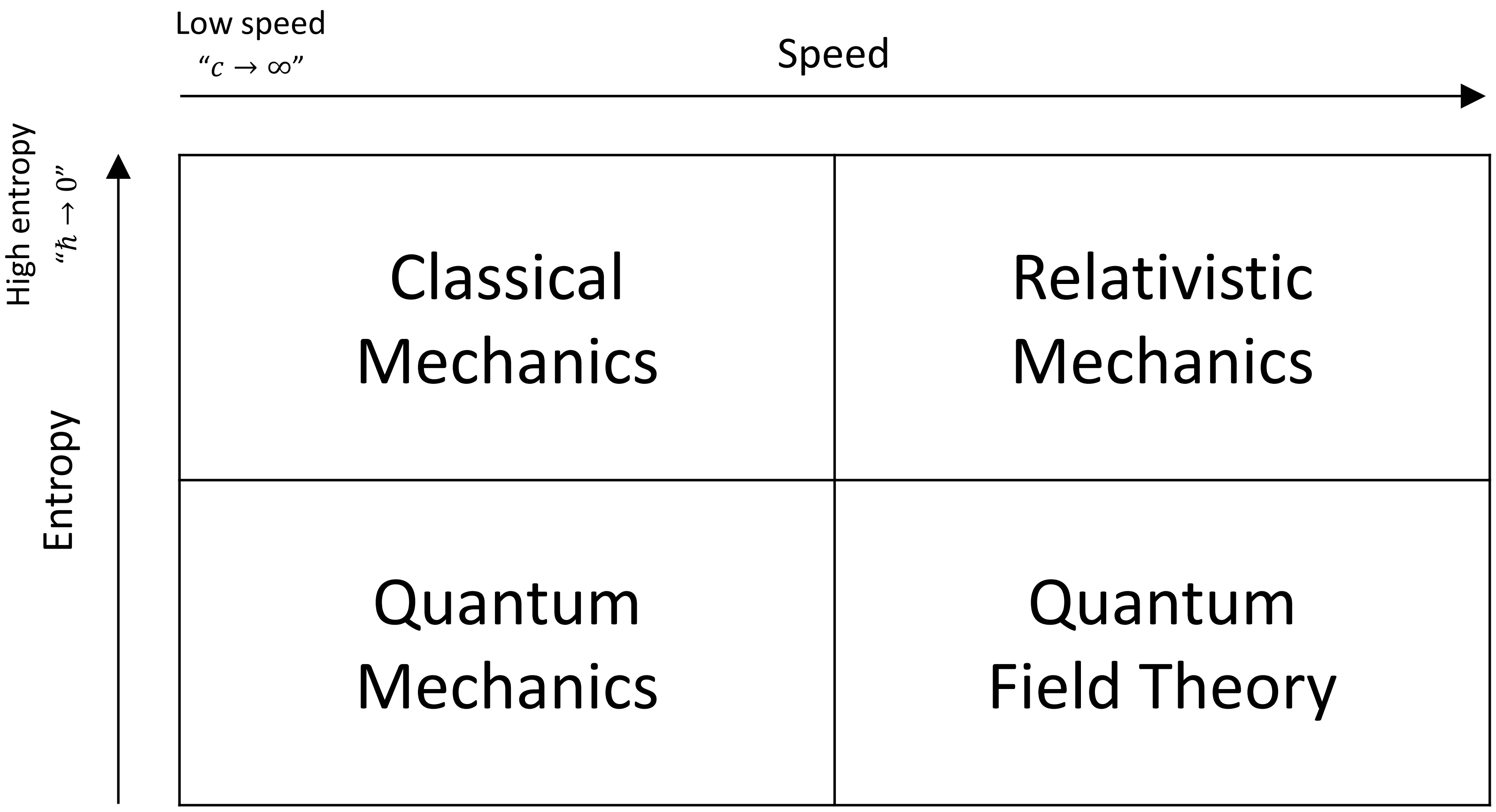

No-mechanism limit
(same as non-relativistic limit)

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Meatballs

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Low speed
"$c \to \infty$"

Speed

High entropy
"$\hbar \to 0$"

Entropy

Classical Mechanics	Relativistic Mechanics
Quantum Mechanics	Quantum Field Theory

No-mechanism limit
(same as non-relativistic limit)

Can we say the converse?

That quantization is putting a low entropy bound on a classical theory?

Suppose we ask for a theory with an entropic lower bound that recovers the classical one at high entropy

Moyal bracket is the unique one-parameter Lie-algebraic deformation of the Poisson bracket

Only one way to do it

Dirac's correspondence principle

$$\{A, B\} \longrightarrow \frac{[A, B]}{\imath\hbar}$$

Quantizing a classical theory means putting a lower bound on the entropy

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Assumptions of Physics

Assumptions of Physics: find a minimal set of physical assumptions from which the physical laws can be rigorously rederived

- Logic of experimental verifiability ⇒ topologies and σ-algebras
- Real valued quantities recovered from a metrological model
- Variability of elements within an ensemble ⇒ entropy
- Geometric structures ⇔ entropic structures
- Invariance of entropy across observers ⇒ symplectic structure
- Determinism/reversibility ⇔ conservation of entropy ⇔ Hamiltonian evolution
- Geometric and physical interpretation of the action principle
    - Action is the line integral of the vector potential of the flow of states
- Statistical mixing ⇒ all linear structure in physics
- …

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- Program website
  - https://assumptionsofphysics.org for papers, presentations, …
  - https://assumptionsofphysics.org/book for our open access book (updated every few years with new results)
- YouTube channels
  - https://www.youtube.com/@gcarcassi
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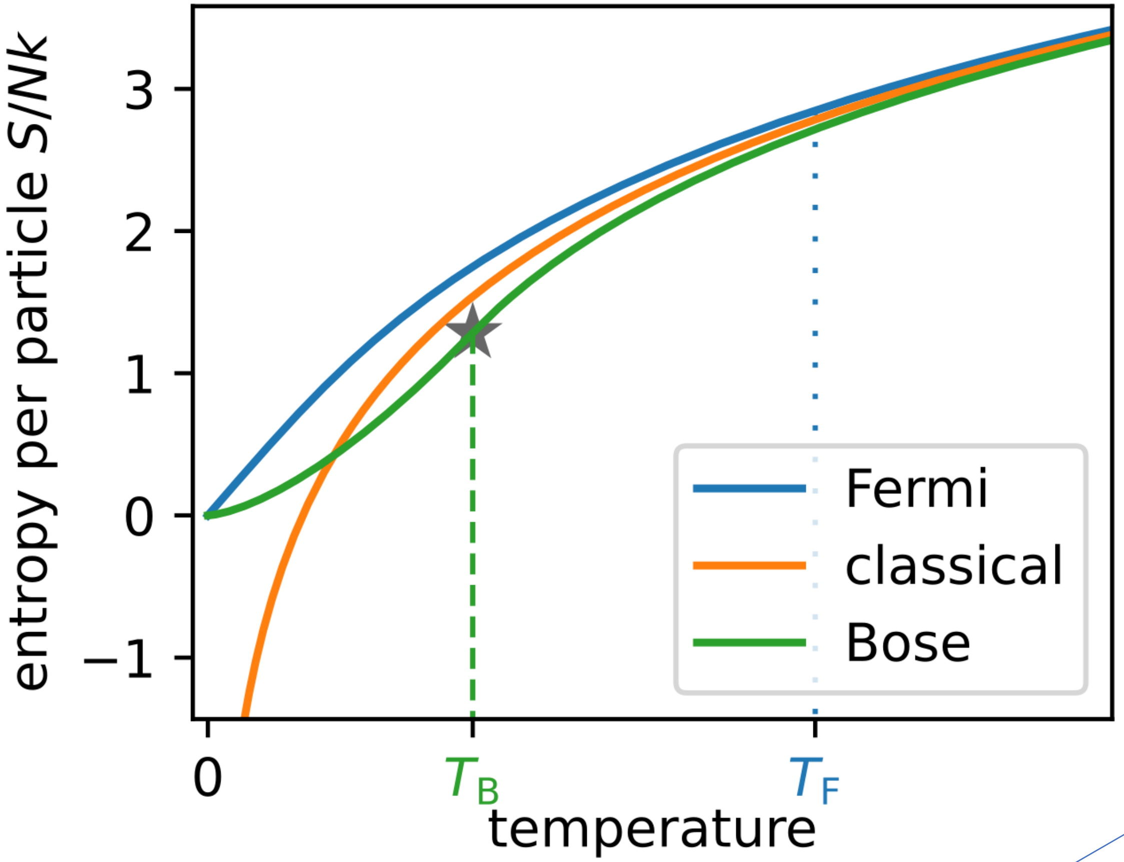
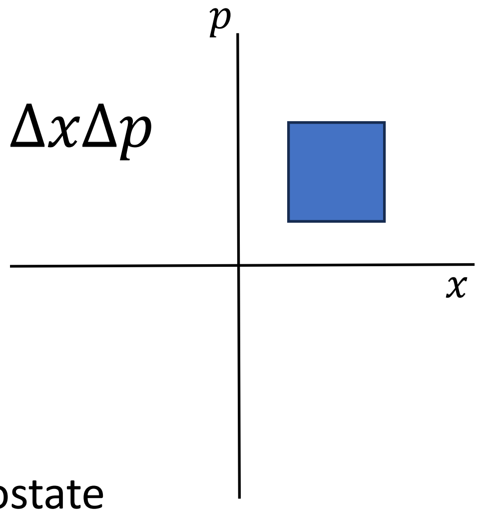
Temperature/entropy curve for ideal gases



Entropy in phase space

$$S = \log \Delta x \Delta p$$



For a single microstate

$$S = \log 0 = -\infty$$

Classical mechanics allows arbitrarily large negative entropy!!!

Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

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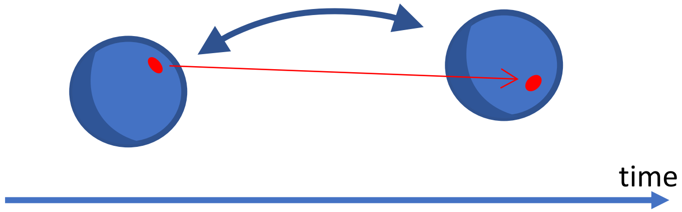
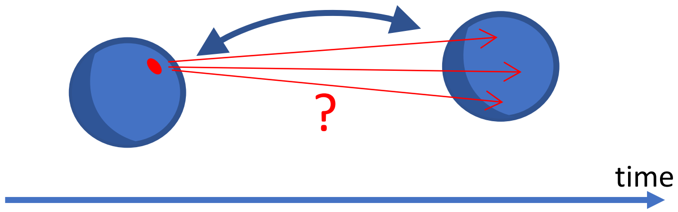
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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