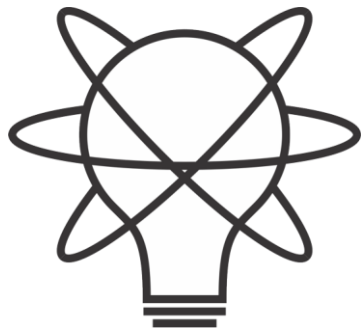


# Assumptions of Physics

## Project overview

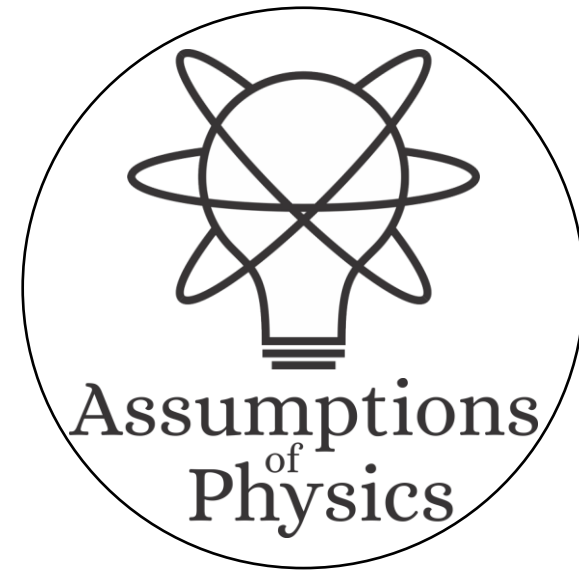
Gabriele Carcassi and Christine A. Aidala

Physics Department  
University of Michigan



Assumptions  
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Physics

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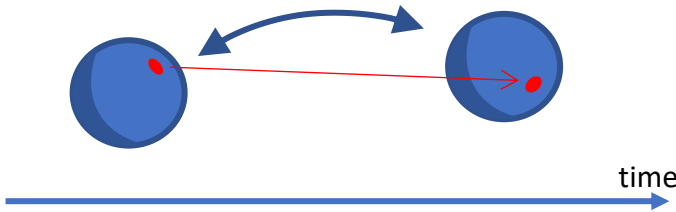
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<sup>of</sup>  
Physics

# Main goal of the project

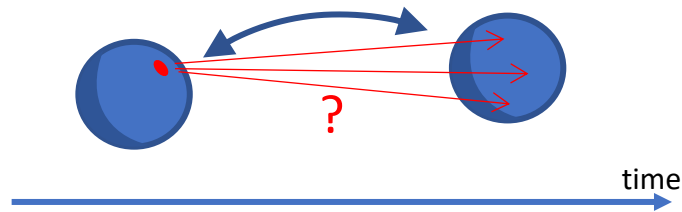
*Identify a handful of physical starting points from which the basic laws can be rigorously derived*

For example:

Infinitesimal reducibility  $\Rightarrow$  Classical state



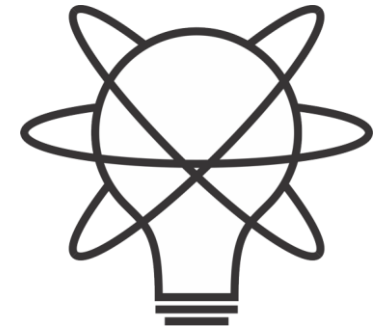
Irreducibility  $\Rightarrow$  Quantum state



This also requires rederiving all mathematical structures from physical requirements

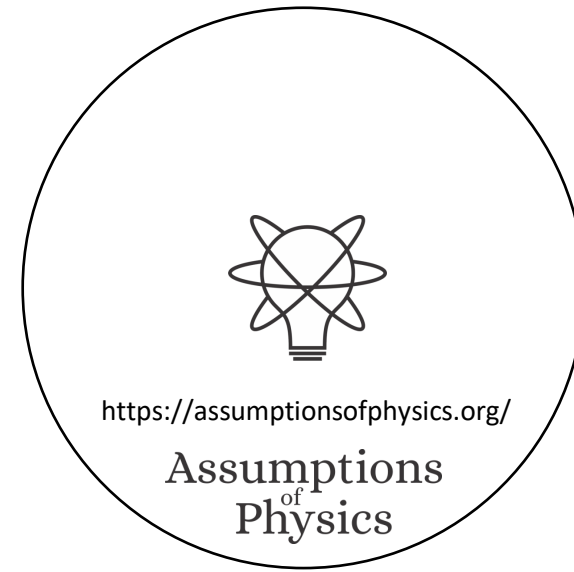
For example:

Science is evidence based  $\Rightarrow$  scientific theory must be characterized by experimentally verifiable statements  $\Rightarrow$  topology and  $\sigma$ -algebras

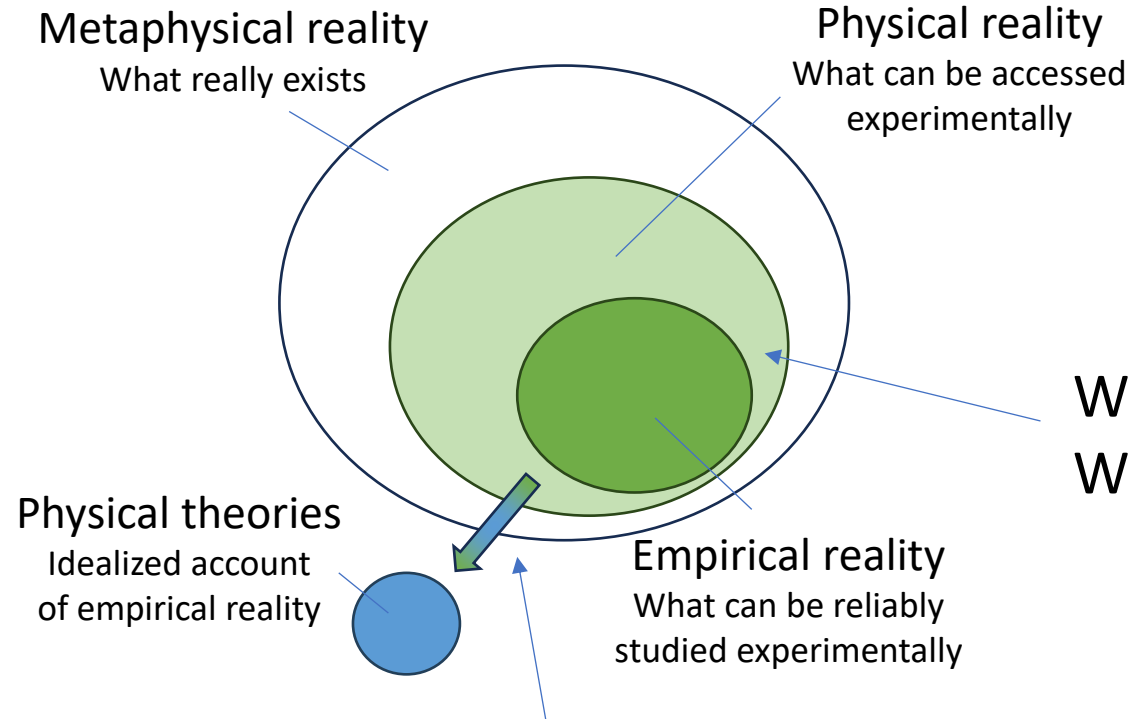


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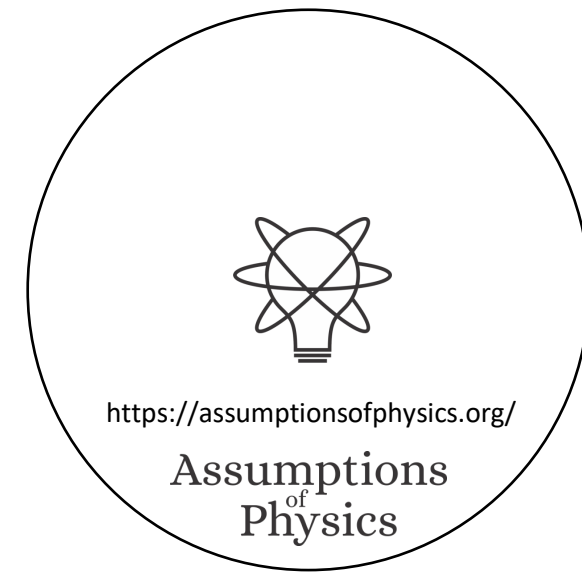
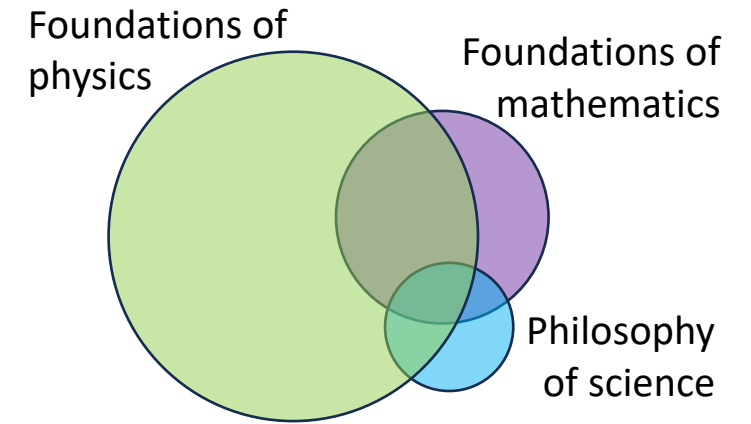


# Underlying perspective



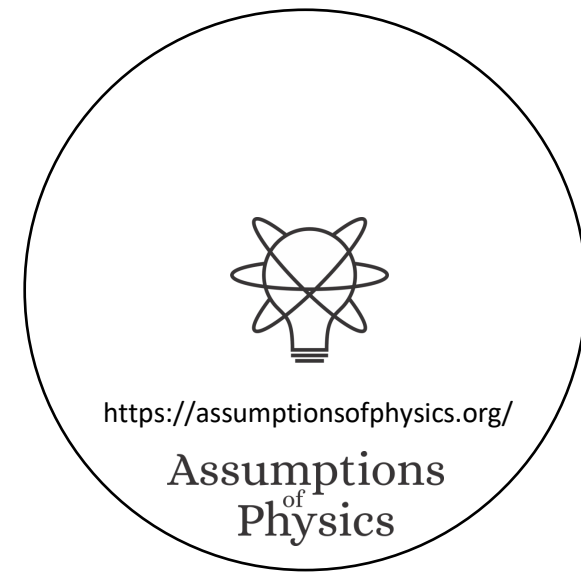
What is the boundary?  
What are the requirements?

How exactly does the abstraction/idealization process work?



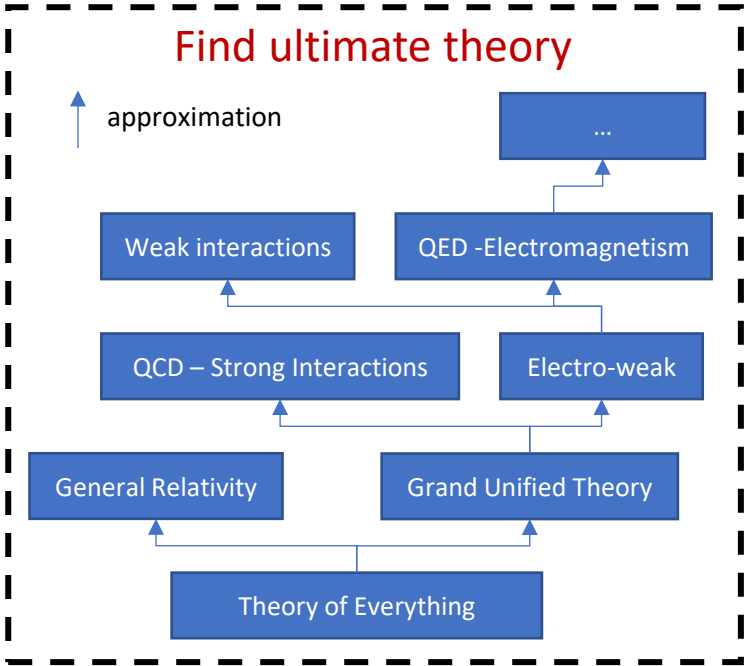
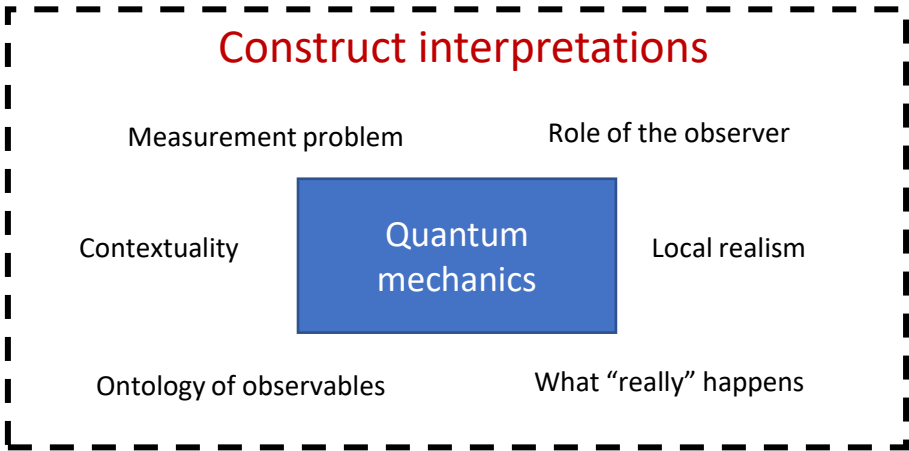
If physics is about creating models of empirical reality, the foundations of physics should be a theory of models of empirical reality

Requirements of experimental verification, assumptions of each theory, realm of validity of assumptions, ...

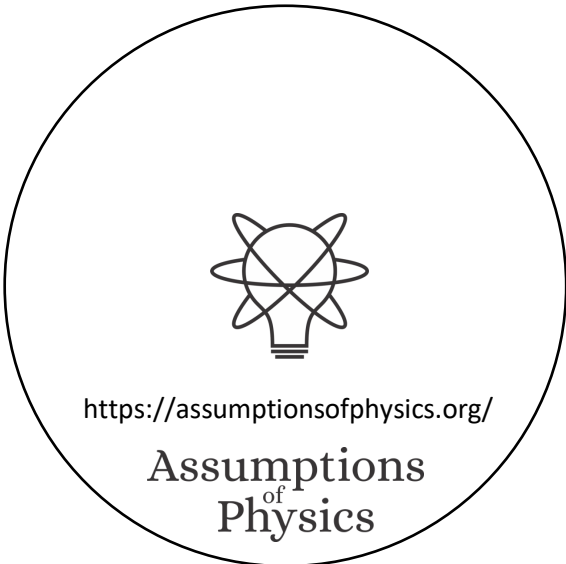
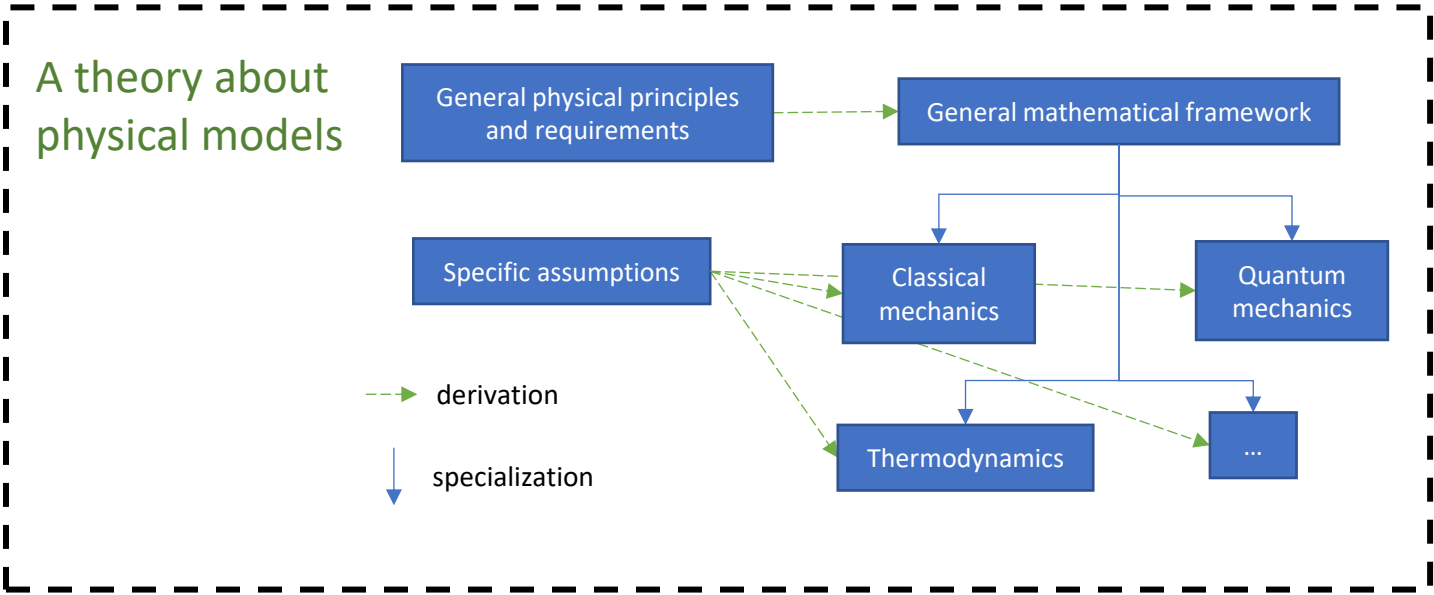


# Different approach to the foundations of physics

## Typical approaches



## Our approach

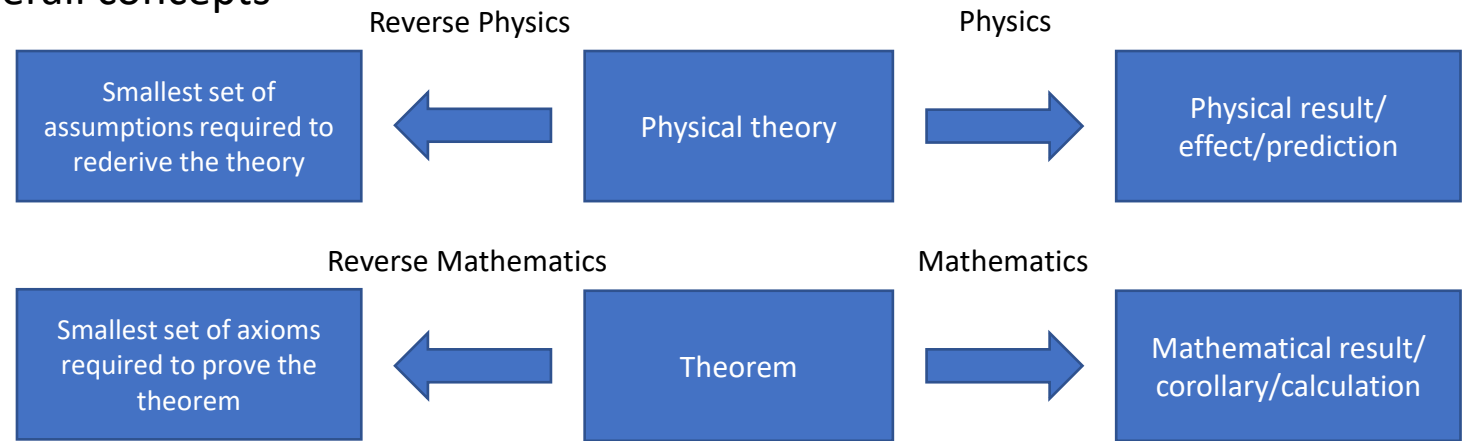


Find the right overall concepts

## *Reverse physics:*

Start with the equations,  
reverse engineer physical  
assumptions/principles

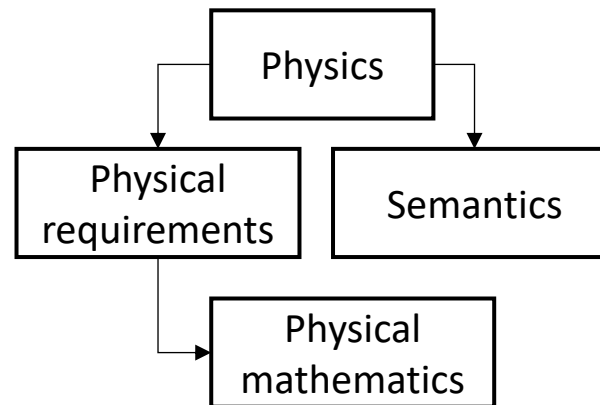
*Found Phys* **52**, 40 (2022)



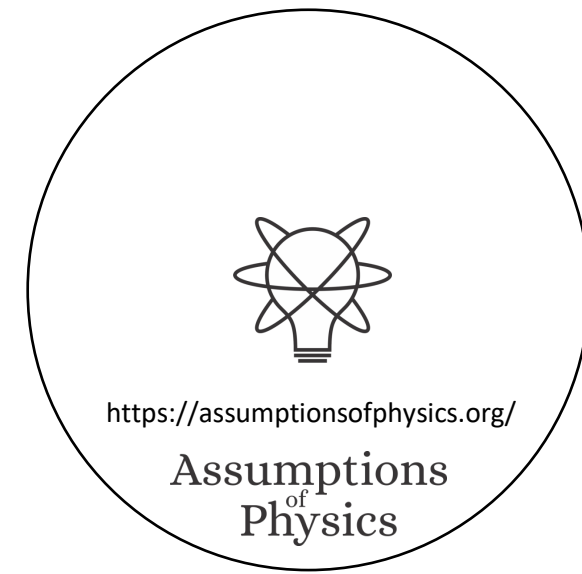
Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

## *Physical mathematics:*

Start from scratch and rederive  
all mathematical structures from  
physical requirements



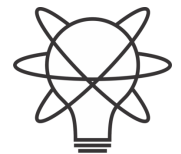
Goal: get the details right, perfect one-to-one map between mathematical and physical objects



# Reverse Physics

**Assumptions of Physics,**  
*Michigan Publishing* (v2 2023)

*J. Phys. Commun.* **2** 045026 (2018)



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**Assumptions  
of  
Physics**

# 7 equivalent characterizations of Hamiltonian mechanics

← 12 in the book

(1) Hamilton's equations

$$S^q = \frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$S^p = \frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

(2) Divergenceless displacement

$$\text{div}(S^a) = \frac{\partial S^q}{\partial q} + \frac{\partial S^p}{\partial p} = 0$$

(3) Area conservation ( $|J| = 1$ )

$$dQdP = |J|dqdp$$

(4) Deterministic and reversible evolution

Area conservation  $\Leftrightarrow$  state count conservation  
 $\Leftrightarrow$  deterministic and reversible evolution

(5) Deterministic and thermodynamically reversible evolution

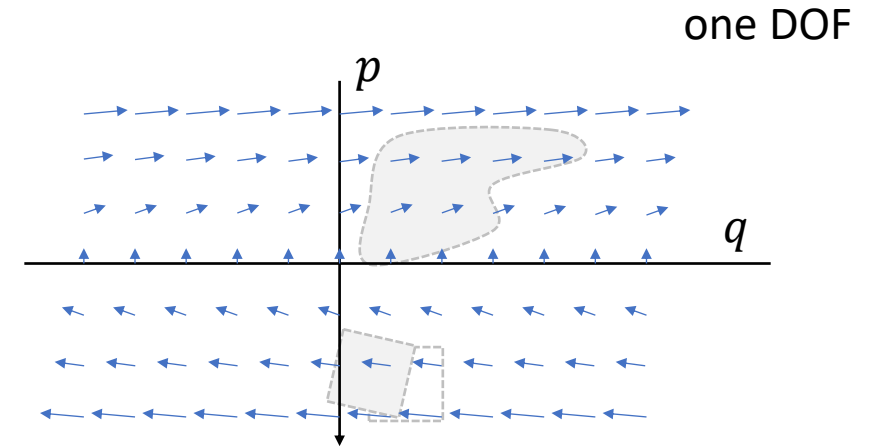
$$S = k_B \log W$$

Area conservation  $\Leftrightarrow$  entropy conservation  
 $\Leftrightarrow$  thermodynamically reversible evolution

(6) Information conservation

$$I[\rho(t + dt)] = I[\rho(t)] - \int \rho \log |J| dqdp$$

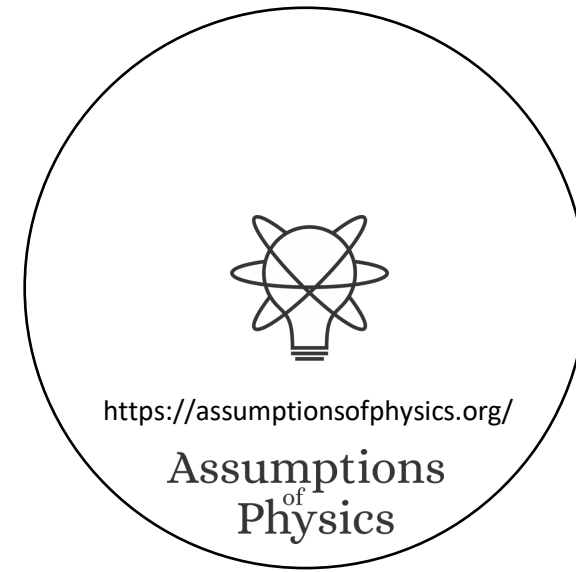
A full understanding of classical mechanics  
means understanding these connections



(7) Uncertainty conservation

$$|\Sigma(t + dt)| = |J||\Sigma(t)||J|$$

for peaked  
distributions



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Assumptions  
of  
Physics



**Assumption DR** (Determinism and Reversibility). *The system undergoes deterministic and reversible evolution. That is, specifying the state of the system at a particular time is equivalent to specifying the state at a future (determinism) or past (reversibility) time.*

- The displacement field is divergenceless:  $\partial_a S^a = 0$  (DR-DIV)
- The Jacobian of time evolution is unitary:  $|\partial_b \hat{\xi}^a| = 1$  (DR-JAC)
- Densities are conserved through the evolution:  $\hat{\rho}(\hat{\xi}^a) = \rho(\xi^b)$  (DR-DEN)
- Volumes are conserved through the evolution:  $d\hat{\xi}^1 \dots d\hat{\xi}^n = d\xi^1 \dots d\xi^n$  (DR-VOL)
- The evolution is deterministic and reversible. (DR-EV)
- The evolution is deterministic and thermodynamically reversible (DR-THER)
- The evolution conserves information entropy (DR-INFO)
- The evolution conserves the uncertainty of peaked distributions (DR-UNC)



- The evolution leaves  $\omega_{ab}$  invariant:  $\hat{\omega}_{ab} = \omega_{ab}$  (DI-SYMP)
- The evolution leaves the Poisson brackets invariant (DI-POI)
- The rotated displacement field is curl free:  $\partial_a S_b - \partial_b S_a = 0$  (DI-CURL)



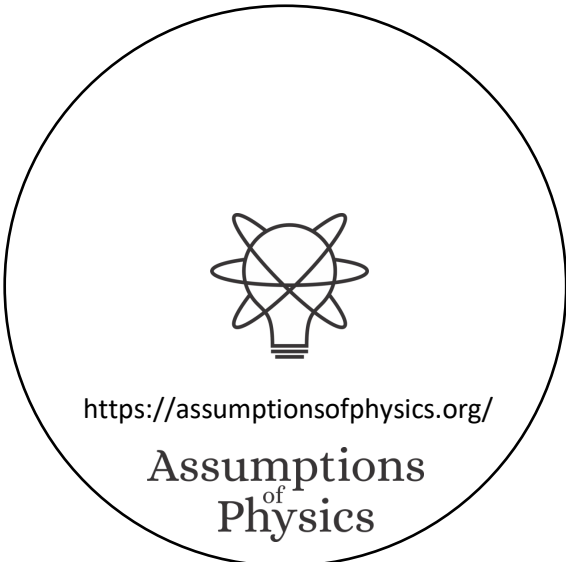
$$\begin{aligned} d_t q^i &= \partial_{p_i} H \\ d_t p_i &= -\partial_{q^i} H \end{aligned}$$

$$S_a = S^b \omega_{ba} = \partial_a H$$

**Assumption IND** (Independent DOFs). *The system is decomposable into independent degrees of freedom. That is, the variables that describe the state can be divided into groups that have independent definition, units and count of states.*

- The system is decomposable into independent DOFs (IND-DOF)
- The system allows statistically independent distributions over each DOF (IND-STAT)
- The system allows informationally independent distributions over each DOF (IND-INFO)
- The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF (IND-UNC)

Mathematical conditions and physical assumptions are not necessarily one-to-one



# Reversing the principle of least action

DR

$$\nabla \cdot \vec{S} = 0$$

No state is “lost” or  
“created” as time evolves

$[p, 0, -H]$

$$\vec{S} = -\nabla \times \vec{\theta}$$

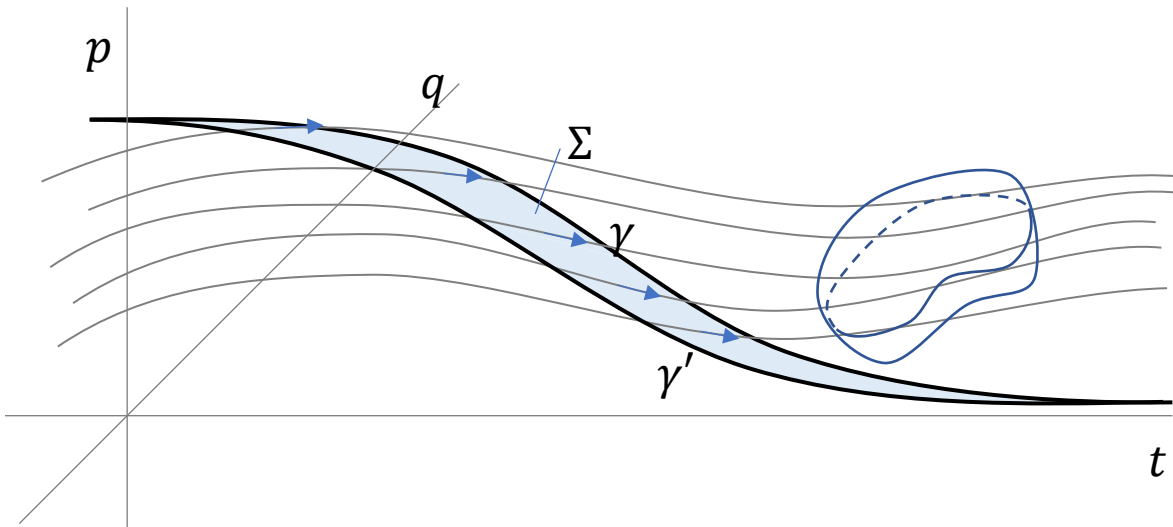
(Minus sign to match convention)

KE

$$\mathcal{S}[\gamma] = \int_{\gamma} L dt = \int_{\gamma} \vec{\theta} \cdot d\vec{\gamma}$$

*Sci Rep* **13**, 12138 (2023)

The action is the line integral of the vector potential (unphysical)



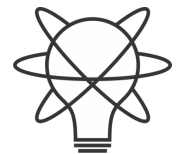
Variation of the action

$$\begin{aligned} \delta \mathcal{S}[\gamma] &= \oint_{\partial \Sigma} \vec{\theta} \cdot d\vec{\gamma} \\ &= - \iint_{\Sigma} \vec{S} \cdot d\vec{\Sigma} \end{aligned}$$

Gauge independent,  
physical!

Variation of the action measures the flow of states (physical).

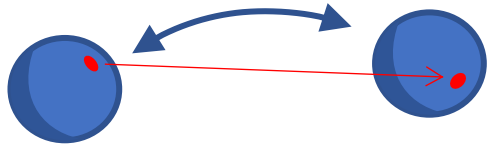
Variation = 0  $\Rightarrow$  flow of states tangent to the path.



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Assumptions  
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Physics

# Assumptions of classical mechanics



(IR) Infinitesimal reducibility

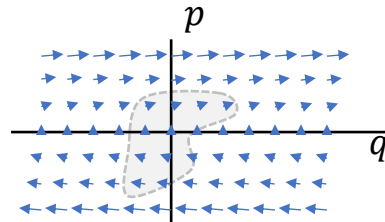
(IND) Degree of freedom independence

$$\#S = \#c_1 \#c_2$$



$[q^i, p_i]$   
Classical Phase Space

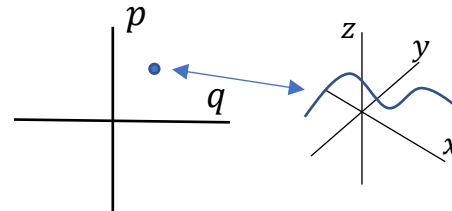
(DR) Determinism /Reversibility



$$\frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial q^i}$$

Hamiltonian Mechanics

(KE) Kinematic Equivalence



$$\delta \int_{\gamma} L(q^i, \dot{q}^i, t) dt = 0$$

Lagrangian Mechanics

Massive particles under potential forces

$$H = \frac{1}{2m} (p_i - qA_i) g^{ij} (p_j - qA_j) + qV$$

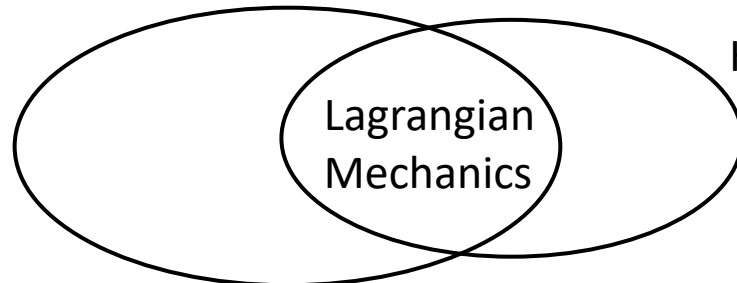


weak

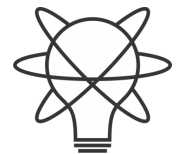
full



Newtonian Mechanics



Hamiltonian Mechanics



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Assumptions  
of  
Physics

# Reverse physics gives us links between theories

Deterministic and reversible evolution

⇒ existence and conservation of energy (Hamiltonian)

Why?

Stronger version of the first law of thermodynamics

Deterministic and reversible evolution

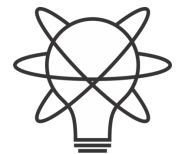
⇒ past and future depend only on the state of the system

⇒ the evolution does not depend on anything else

⇒ the system is isolated

First law of thermodynamics!

⇒ the system conserves energy



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Assumptions  
of  
Physics

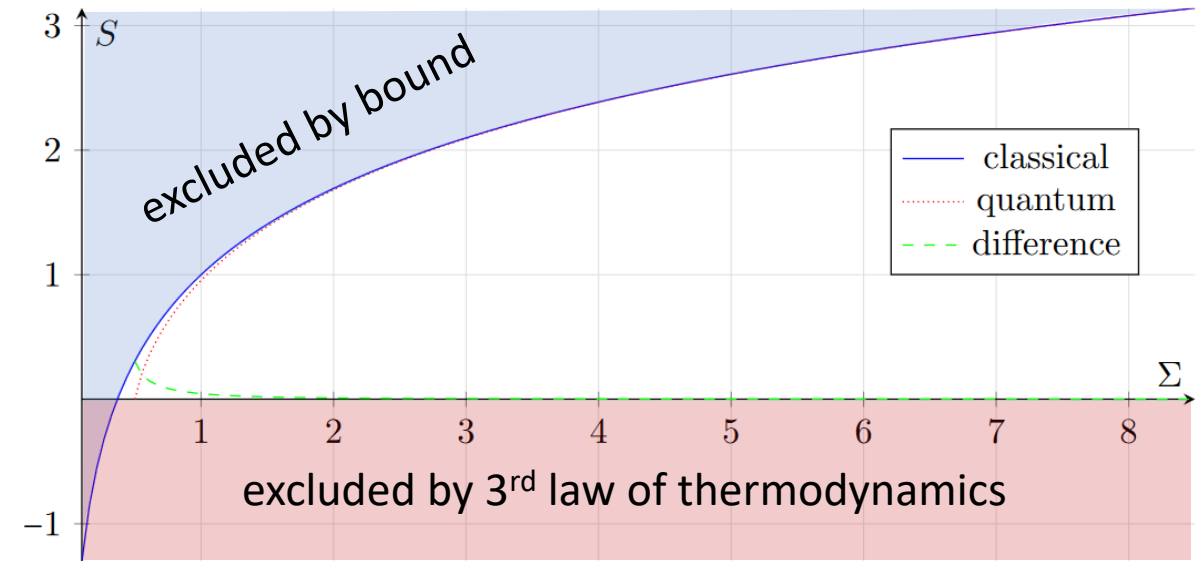
# Classical uncertainty principle

Gaussian states minimize uncertainty at a given entropy

Let  $h$  be the volume of phase space over which a uniform distribution has zero entropy.

$$\sigma_q \sigma_p \geq \frac{h}{2\pi e} = \frac{\hbar}{e}$$

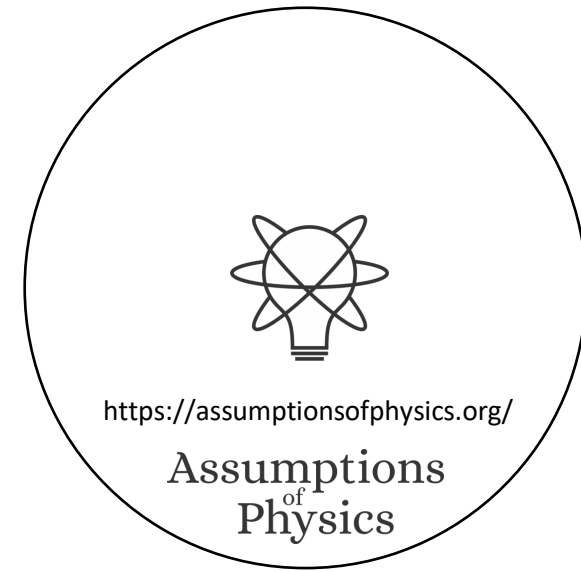
Lower bound on entropy  
 $\Rightarrow$  lower bound on uncertainty



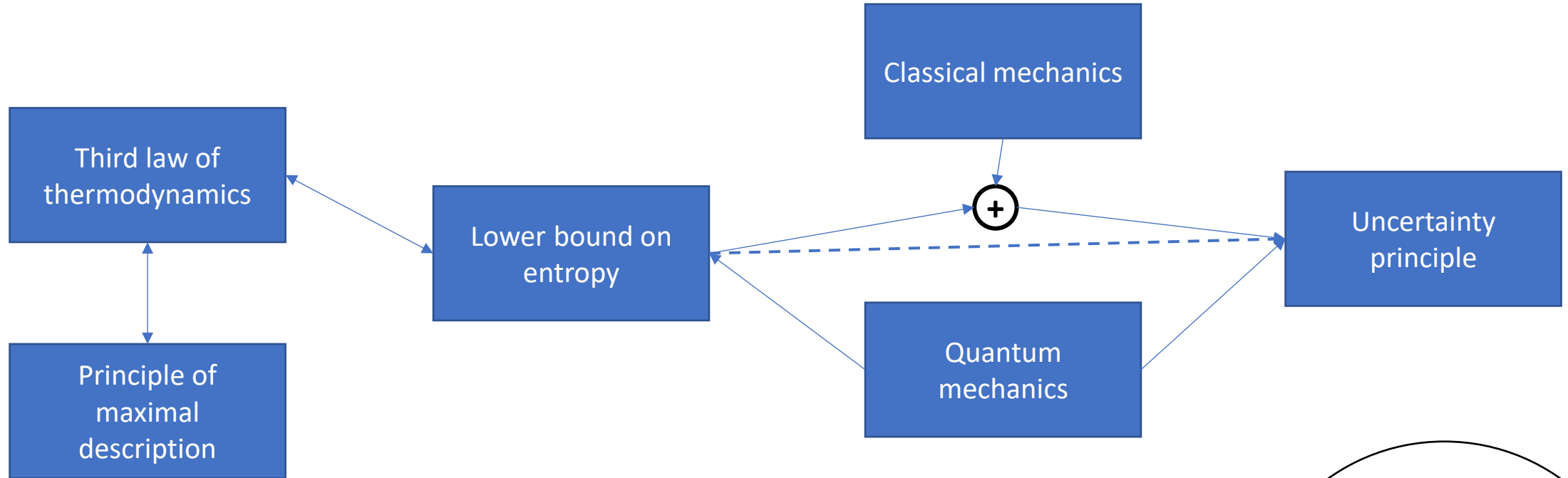
Entropy  $S$  in nats for a Gaussian state as a function of uncertainty  $\Sigma$  (in units of  $\hbar$ )

$$S_C = \ln \Sigma + 1$$

$$S_Q = \left( \Sigma + \frac{1}{2} \right) \ln \left( \Sigma + \frac{1}{2} \right) - \left( \Sigma - \frac{1}{2} \right) \ln \left( \Sigma - \frac{1}{2} \right)$$



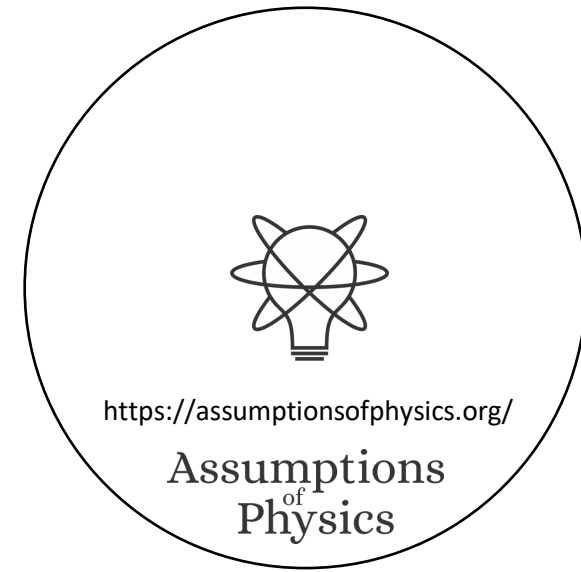
# 3<sup>rd</sup> law of thermodynamics and uncertainty principle



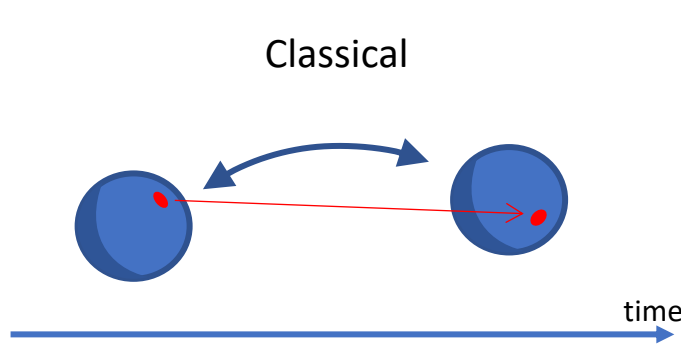
No state can describe a system more accurately than stating the system is not there in the first place

The uncertainty principle is a consequence of the principle of maximal description

Can we understand the rest of quantum mechanics in the same way?

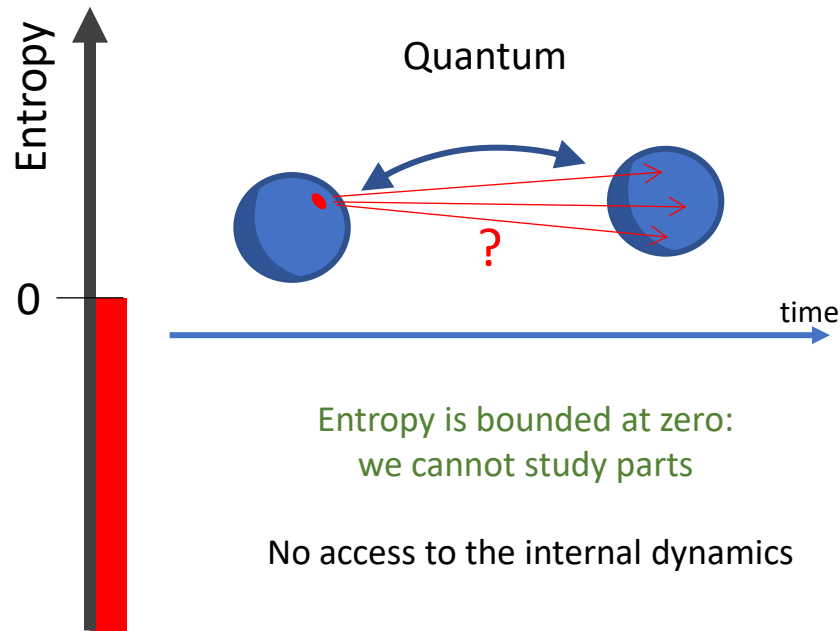


# Quantum mechanics as irreducibility



Can prepare ensembles at arbitrarily low entropy: we can study arbitrarily small parts

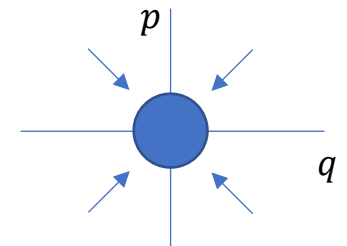
We always have access to the internal dynamics



Entropy is bounded at zero: we cannot study parts

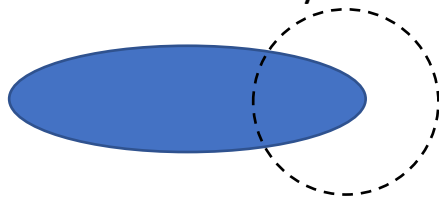
No access to the internal dynamics

Minimum uncertainty



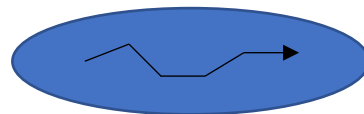
Can't squeeze ensemble arbitrarily

Non-locality



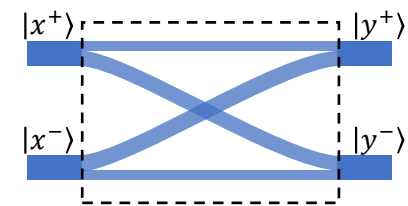
Can't refine ensembles  $\Rightarrow$   
Can't interact with parts

Superluminal effects  
that can't carry information



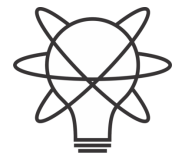
Can't refine ensembles  $\Rightarrow$   
Can't extract information

Probability of transition



$$p(x^+|y^-) = p(y^-|x^+)$$

Symmetry of the inner product



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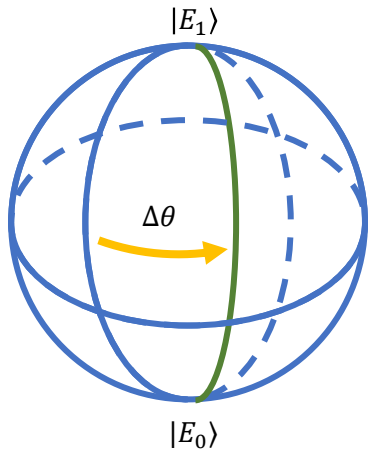
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# Time evolution and measurements

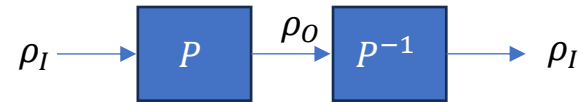
Any process (deterministic or stochastic) will take an ensemble as input and return an ensemble as output

$$\rho_I \longrightarrow \boxed{P} \longrightarrow \rho_O = P(\rho_I)$$

$$P(p_1\rho_1 + p_2\rho_2) = p_1P(\rho_1) + p_2P(\rho_2)$$

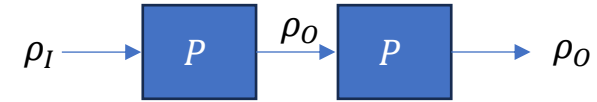


Deterministic and reversible

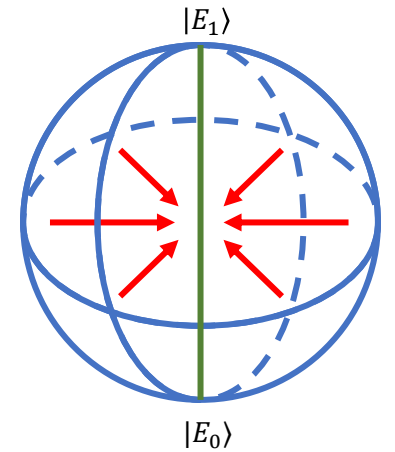


Conserves probability and allows an “inverse”  
 $\Rightarrow$  Unitary operation

Measurement

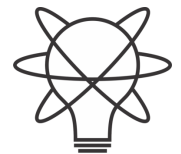
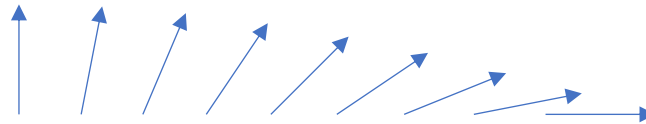


Must be repeatable  
 $\Rightarrow$  Projection



Measurement problem: unitary  $\nRightarrow$  projections ... projections  $\Rightarrow$  unitary

Unitary evolution  $\equiv$  sequence of infinitesimal projections



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Assumptions  
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Physics



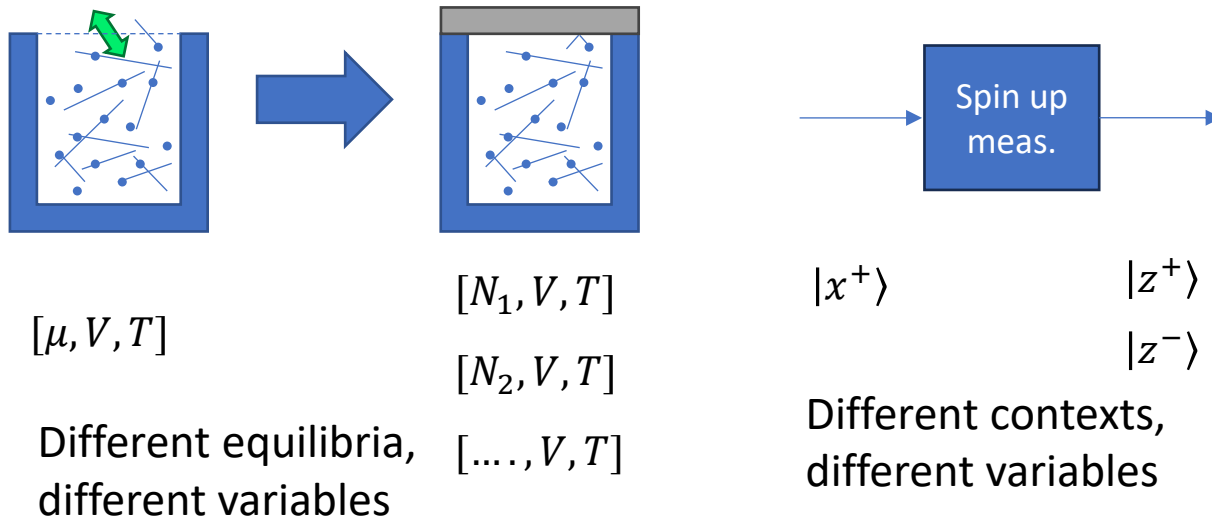
# Parallels between QM and thermodynamics

$$U = e^{\frac{i\hbar \Delta t}{\hbar}}$$

Eigenstates → states unchanged by the process → equilibria of the process

Every state is an eigenstate of some unitary /  
Hermitian operator → all states are equilibria

Every mixed state commutes with some unitary operator  
(same eigenstates used calculate entropy)



Quantum contexts

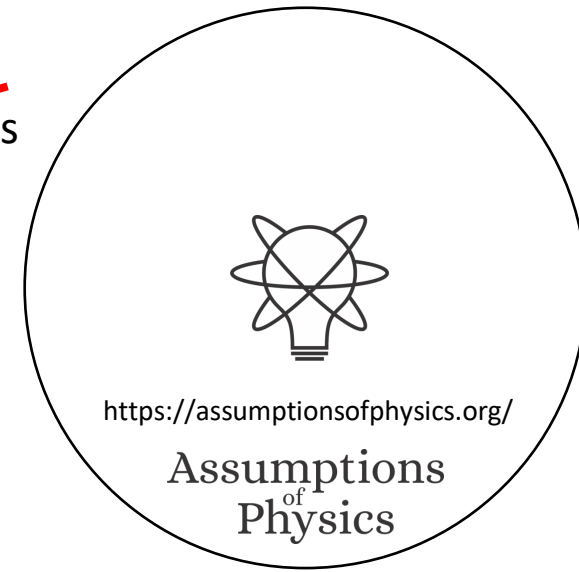


Boundary conditions  
between system and  
environment

Equilibration

Projections  $\Leftrightarrow$  ~~Measurements~~

Unitary  $\Leftrightarrow$  Quasi-static



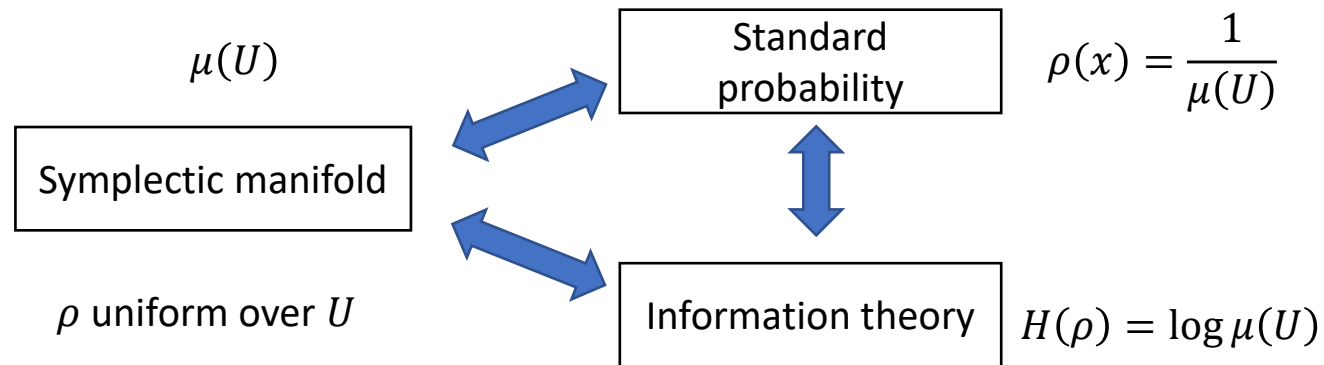
# Entropic nature of physical theories

Thermodynamics/Statistical mechanics are not built on top of mechanics

Mechanics is the ideal case of thermodynamics/statistical mechanics

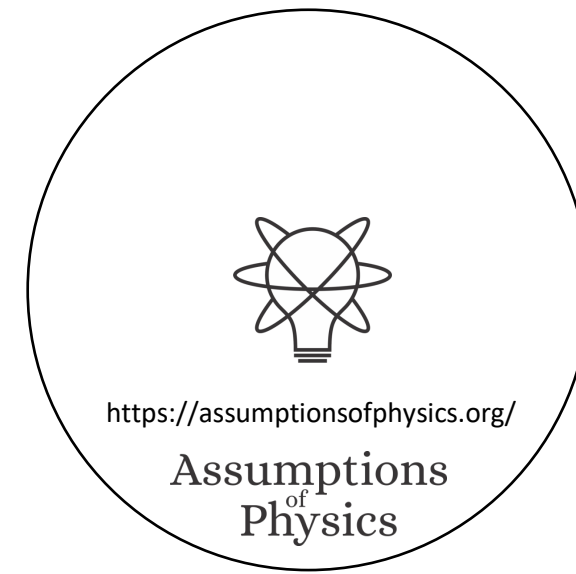
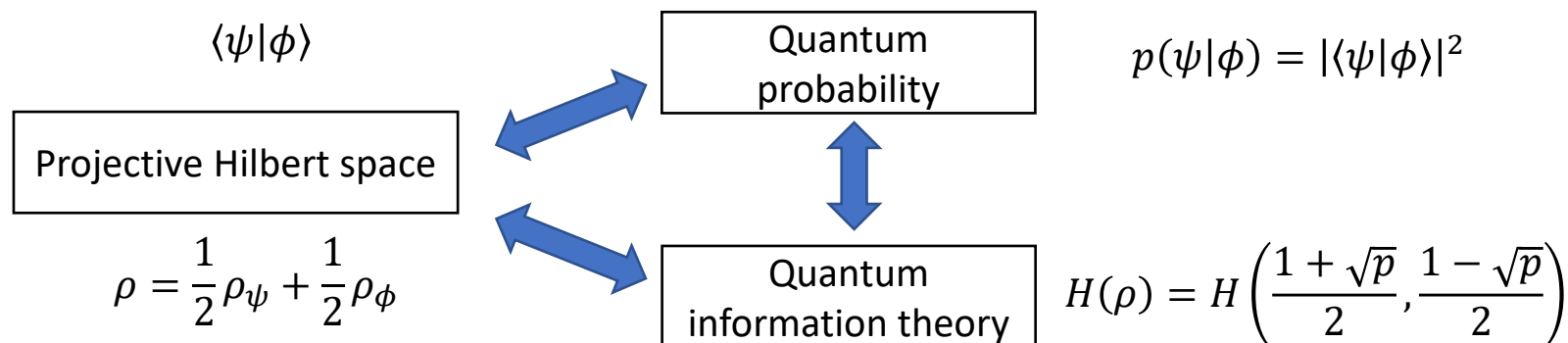
Best preparation  $\Rightarrow$  pure state

Best process  $\Rightarrow$  map between pure states



The geometric structure of both classical and quantum mechanics is ultimately an entropic structure

We can only prepare/measure ensembles. Ensembles can offer a unified way of thinking about states.



# Unphysicality of Hilbert spaces

Hilbert space: 

complete	inner product vector space
----------	----------------------------



Redundant on finite-dimensional spaces. For infinite-dimensional spaces, it allows us to construct states with infinite expectation values from states with finite expectation values

Exactly captures measurement probability/entropy of mixtures and superposition/statistical mixing

Physically required

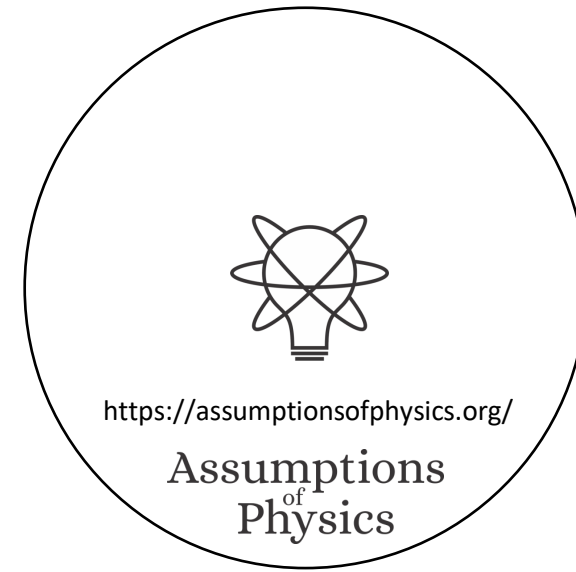
Extremely physically suspect!!!

⇒ Thus requires us to include unitary transformations (e.g. change of representations and finite time evolution) that change finite expectation values into infinite ones

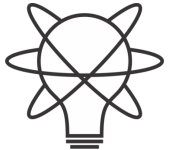
Suppose we require all polynomials of position and momentum to have finite expectation

⇒ Schwartz space

Maybe more physically appropriate?



# Physical mathematics

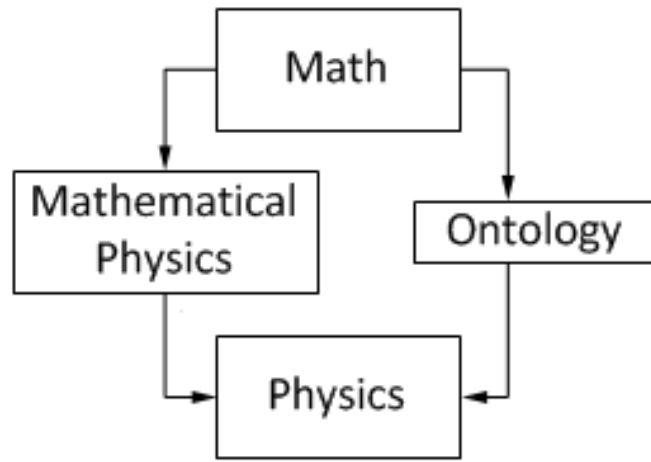


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Assumptions  
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Physics

In modern physics, mathematics is used as the foundation of our physical theories

From Hossenfelder's *Lost in Math*: "[...] finding a neat set of assumptions from which the whole theory can be derived, is often left to our colleagues in mathematical physics [...]"



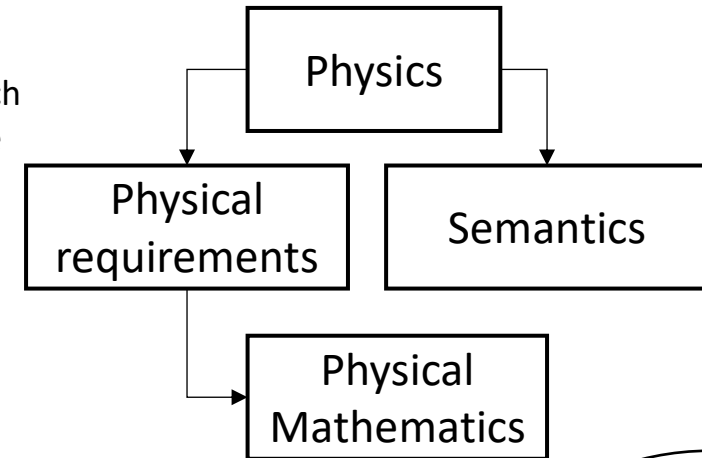
From Wikipedia "Mathematical Physics"

Mathematical content of a theory can never tell us the full physical content

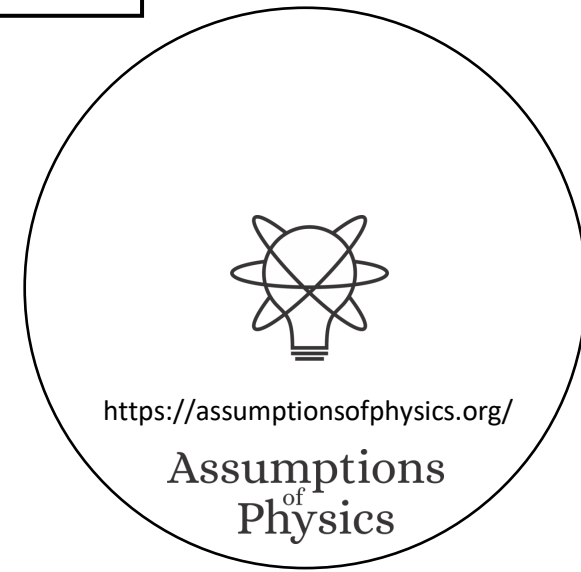
David Hilbert: "Mathematics is a game played according to certain simple rules with meaningless marks on paper."

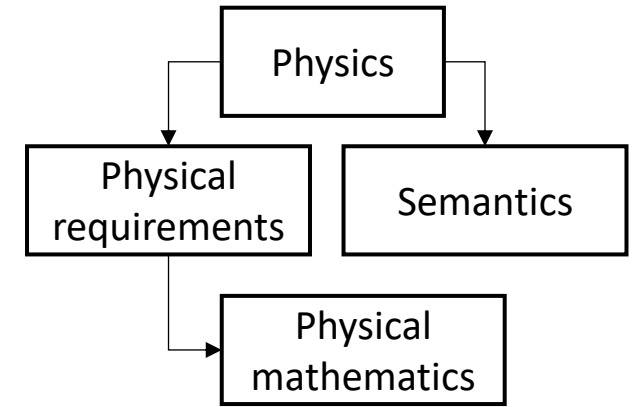
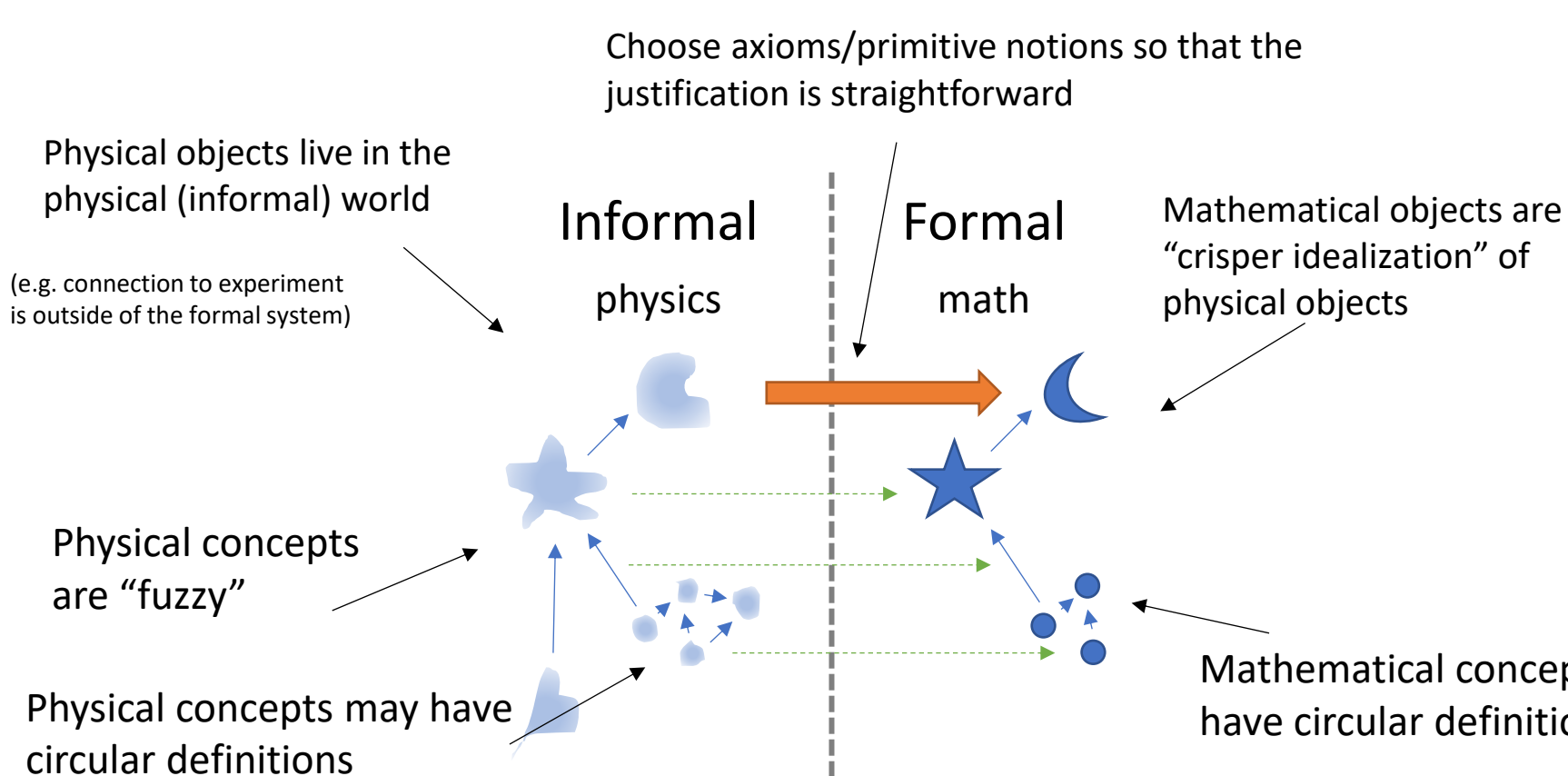
Bertrand Russell: "It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is, of which it is supposed to be true."

We need to identify which parts of mathematics are "correct" to capture physical properties in a specific realm of applicability



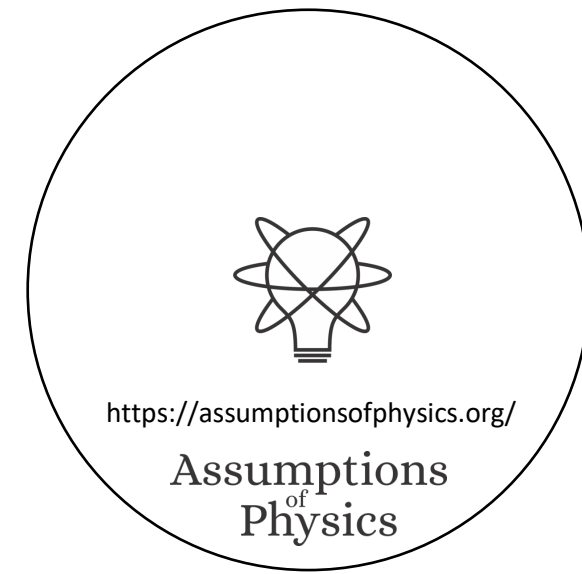
Mathematical structures must be justified by physical requirements



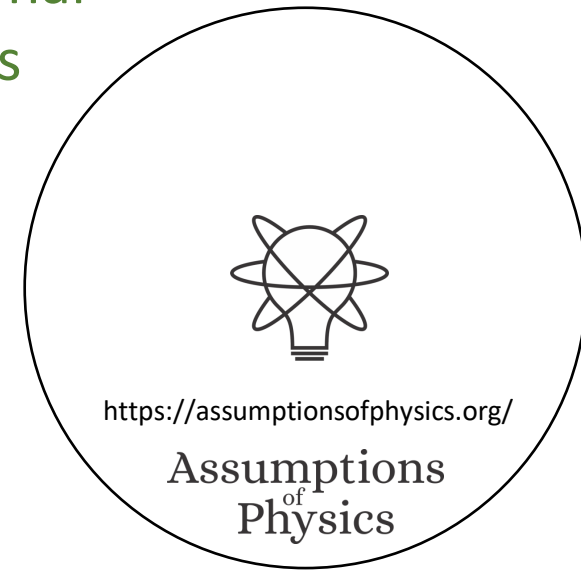
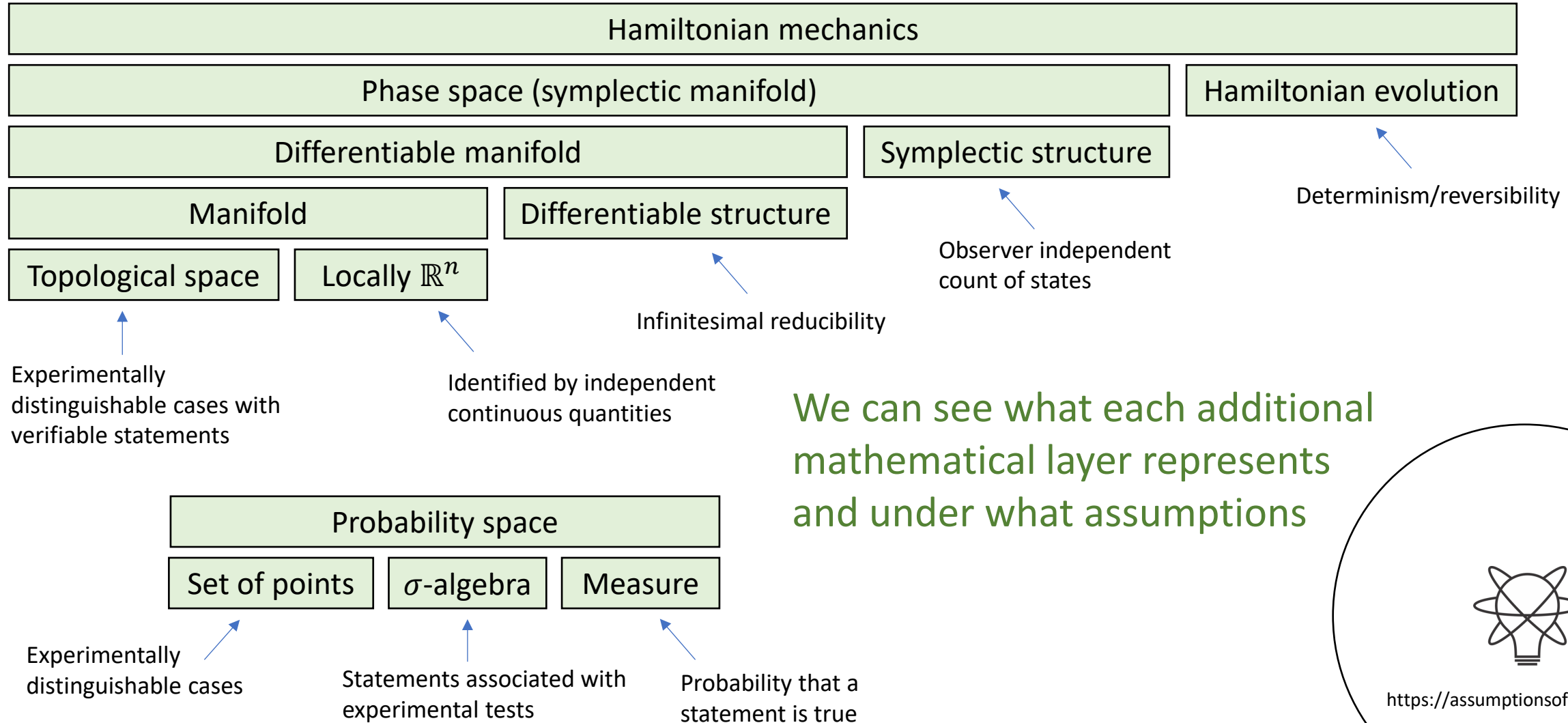


All physical content is captured by the definitions and axioms

The map between informal and formal is the most delicate and important step, and it is also the least studied!!!

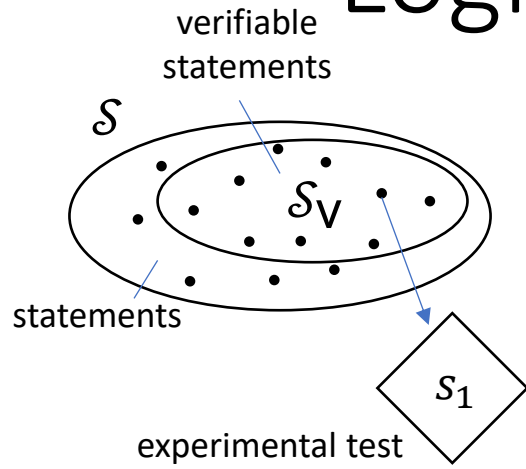


# Examples: symplectic space and probability spaces



# Logic of experimental verifiability

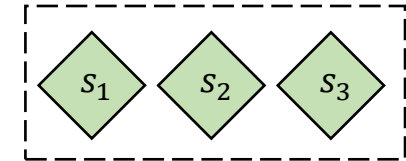
Top. Proc. **54**  
pp. 271-282 (2019)



$s_1$	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED

Finite conjunction  
(logical AND)

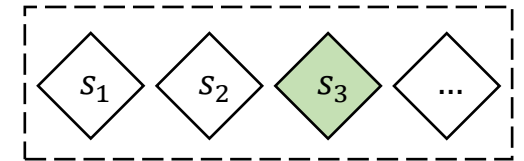
$$\bigwedge_{i=1}^n s_i$$



All tests must succeed

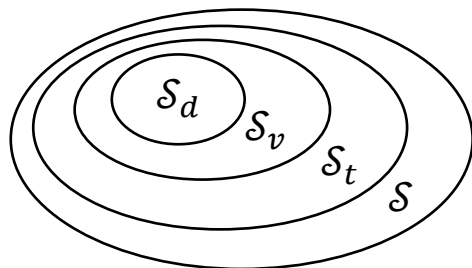
Countable disjunction  
(logical OR)

$$\bigvee_{i=1}^{\infty} s_i$$



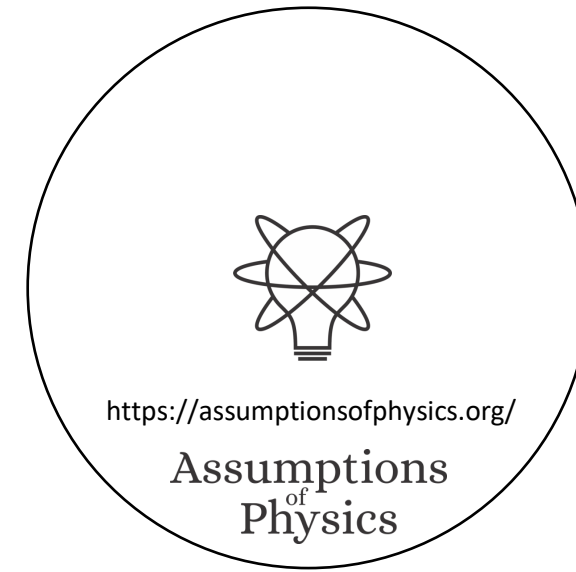
One successful test is sufficient

Physical theories (evidence based)  
⇒ all theoretical statements associated with tests



Operator	Gate	Statement	Theoretical Statement	Verifiable Statement	Decidable Statement
Negation	NOT	allowed	allowed	disallowed	allowed
Conjunction	AND	arbitrary	countable	finite	finite
Disjunction	OR	arbitrary	countable	countable	finite

Some mathematical theories (formally well-posed)  
have “too many statements” to be physically meaningful





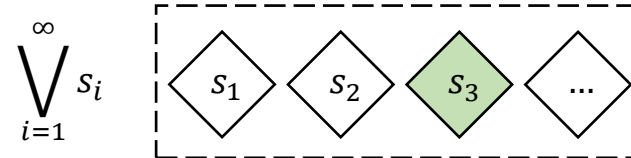
**Axiom 1.32** (Axiom of countable disjunction verifiability). *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let  $\{s_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_v$  be a countable collection of verifiable statements. Then the disjunction  $\bigvee_{i=1}^{\infty} s_i \in \mathcal{S}_v$  is a verifiable statement.*

Disjunction (OR) of verifiable statements:  
check that ONE test terminates successfully

$\vee (e_i)$ :

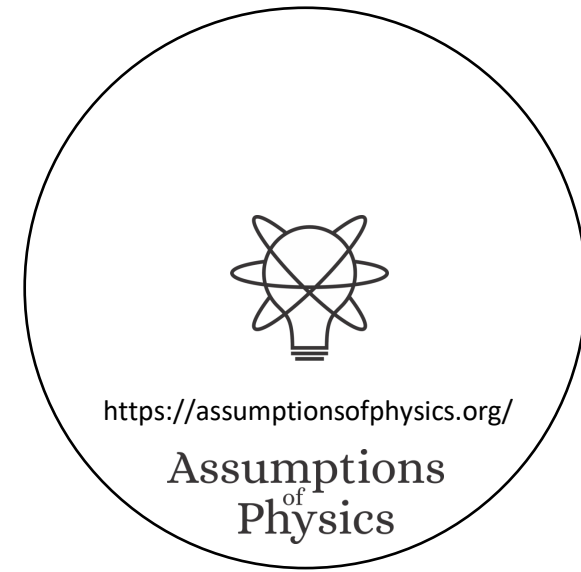
1. Initialize  $n$  to 1
2. For each  $i = 1 \dots n$ 
  - a) Run  $e_i$  for  $n$  seconds
  - b) If  $e_i$  succeeds, return SUCCESS
3. Increment  $n$  and go to 2

watch out for non-termination!



$s_1$	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED

$\Rightarrow$  Only countable disjunction can reach all tests

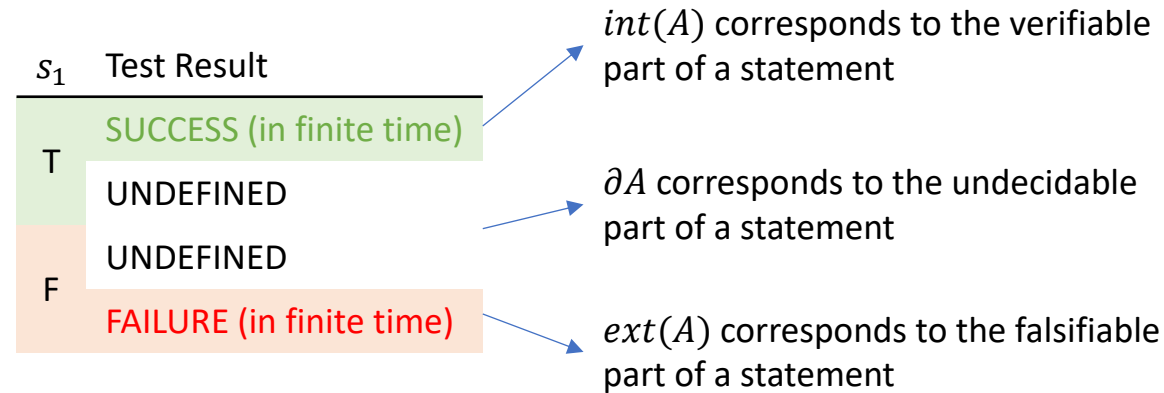
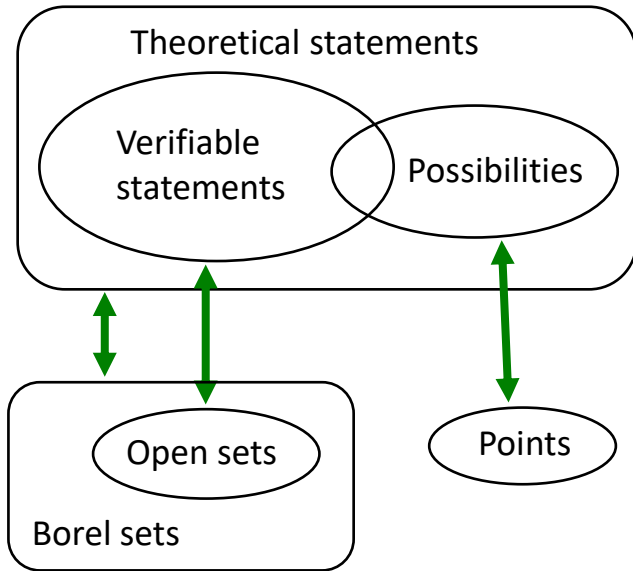


# Topology and $\sigma$ -algebra

Experimental verifiability  $\Rightarrow$   
topology and  $\sigma$ -algebras  
(foundation of geometry,  
probability, ...)

Perfect map  
between math and  
physics

NB: in physics, topology and  
 $\sigma$ -algebra are parts of the  
**same** logic structure

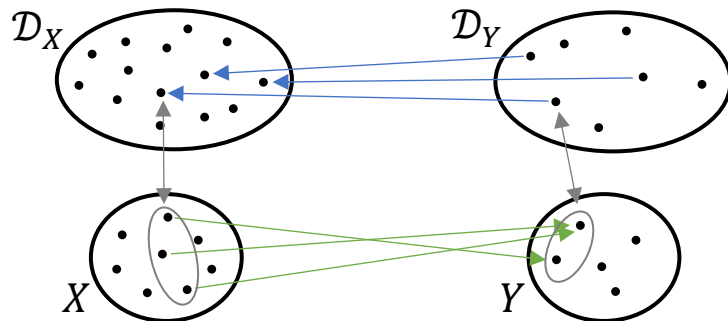


Open set  $(509.5, 510.5) \Leftrightarrow$  Verifiable "the mass of the electron is  $510 \pm 0.5$  KeV"

Closed set  $[510] \Leftrightarrow$  Falsifiable "the mass of the electron is exactly 510 KeV"

Borel set  $\mathbb{Q}$  ( $int(\mathbb{Q}) \cup ext(\mathbb{Q}) = \emptyset$ )  $\Leftrightarrow$  Theoretical "the mass of the electron in KeV is a rational number" (undecidable)

Inference relationship  $r: \mathcal{D}_Y \rightarrow \mathcal{D}_X$  such that  $r(s) \equiv s$



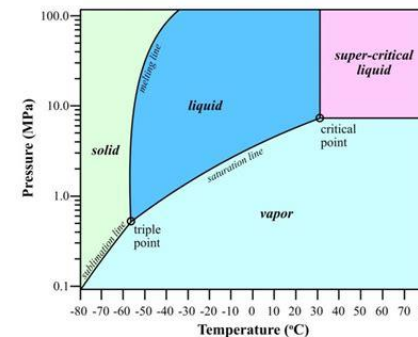
Inference relationship

Causal relationship

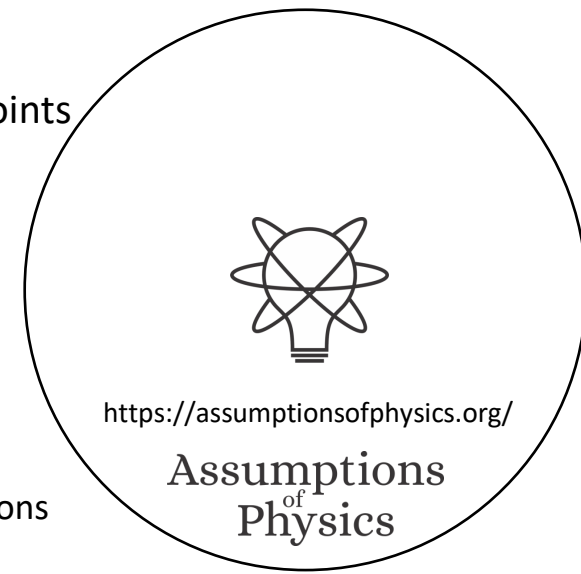
Relationships must be  
topologically continuous

Causal relationship  $f: X \rightarrow Y$  such that  $x \leq f(x)$

Topologically continuous consistent  
with analytic discontinuity on isolated points



Phase transition  $\Leftrightarrow$  Topologically isolated regions

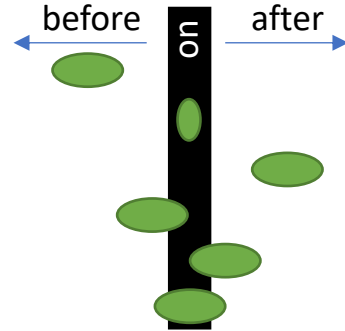


# Quantities and ordering

Phys. Scr. **95** 084003 (2020)

Goal: deriving the notion of quantities and numbers (i.e. integers, reals, ...) from an operational (metrological) model

A **reference** (i.e. a tick of a clock, notch on a ruler, sample weight with a scale) is something that allows us to distinguish between a before and an after

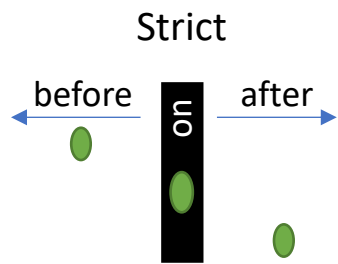


Mathematically, it is a triple  $(b, o, a)$  such that:

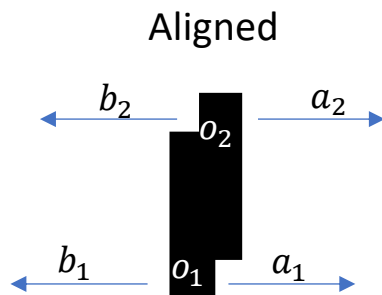
- $b$  and  $a$  are verifiable
- The reference has an extent ( $o \not\equiv \perp$ )
- If it's not before or after, it is on ( $\neg b \wedge \neg a \leq o$ )
- If it's before and after, it is on ( $b \wedge a \leq o$ )

Numbers defined by  
metrological assumptions,  
NOT by ontological assumptions

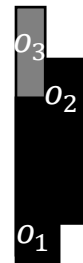
To define an **ordered** sequence of possibilities, the references must be (nec/suff conditions):



$$\Rightarrow (X, \leq)$$

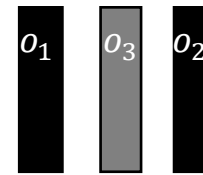


Refinable



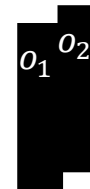
+

Dense



$$\Rightarrow (X, \leq) \cong (\mathbb{R}, \leq)$$

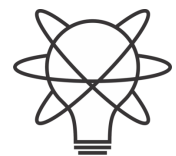
Sparse



$$\Rightarrow (X, \leq) \cong (\mathbb{Z}, \leq)$$

The hard part is to  
recover ordering. After  
that, recovering reals  
and integers is simple.

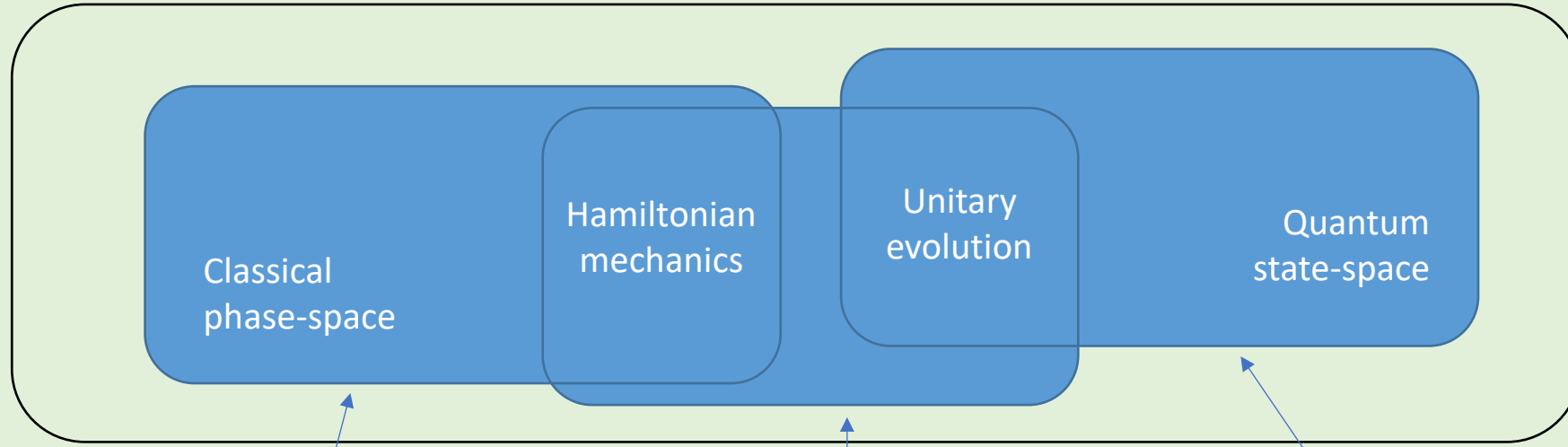
Assumptions untenable at Planck scale:  
no consistent **ordering**: no “objective” “before” and “after”



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Assumptions  
of  
Physics

Space of the well-posed scientific theories



## Physical theories

Specializations of the general theory under the different assumptions

## Assumptions

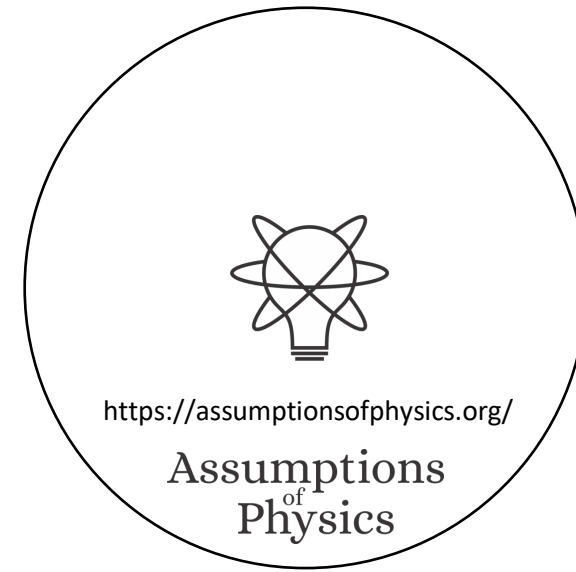
States and processes

Information granularity

Experimental verifiability

## General theory

Basic requirements and definitions valid in all theories



If physical theories are to be repeatably experimentally testable, then they must (at least) be able to describe statistical ensembles (i.e. outputs of repeatable procedures)

If physical laws describe relationship that are always applicable (i.e. whenever this is prepared, this is measured), then they are statements about statistical ensembles

⇒ Want a general theory of ensemble that is applicable to all physical theories (i.e. minimum requirements for a space of ensemble)



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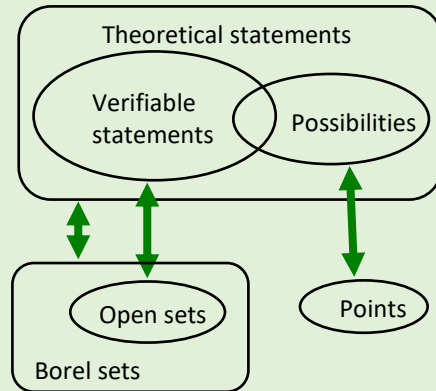
# Axiom of ensemble

Since a physical theory needs to provide repeatedly testable results, it must be able to describe statistical ensembles that are distinguishable experimentally.

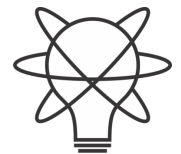
## ⇒ Topological structure

Every ensemble space must be a  $T_0$  second countable topological space. The open sets represent statements that are experimentally verifiable: there is a test and the test succeeds in finite time if and only if the statement is true.

The Borel sets represent statements that are associated to a test, regardless of termination.



Covered by previous work



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Physics

# Fraction capacity

## Generalized non-additive probability

Given an ensemble  $e$  and a set of ensembles  $A$ , what is the biggest component of  $e$  that can be achieved with a mixture of  $A$ ?

$$\text{fcap}_e(a) = \sup(\{p \in [0,1] \mid e = pa + \bar{p}e_1\})$$

$$\text{fcap}_e(A) = \sup(\text{fcap}_e(\text{hull}(A)))$$

$\Rightarrow$  non-negative, unit bounded, monotonic,  
sub-additive set function  $\Rightarrow$  fuzzy measure

Recovers probability (additive) in classical  
mechanics and quantum measurements

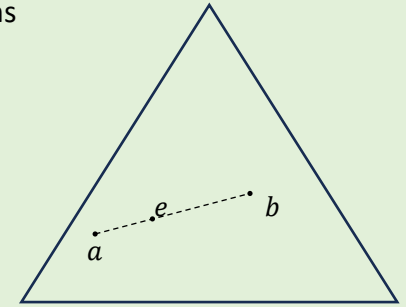
# Axioms of mixture

Given two ensembles, we can always obtain new ones using statistical mixtures (e.g. selecting one 40% of the times and the other 60%).

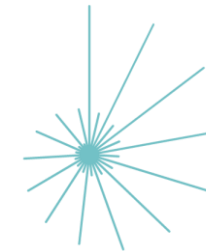
## $\Rightarrow$ Convex structure

Ensemble spaces allow convex combinations

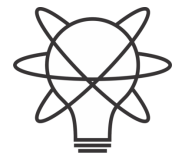
$$e = pa + \bar{p}b$$
$$\bar{p} = 1 - p$$



Ultimately responsible for all linear  
and probabilistic structures



JOHN  
TEMPLETON  
FOUNDATION  
*Inspiring Awe & Wonder*



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# Axioms of entropy

Every ensemble must have a well defined entropy that represents the variability of the elements within the ensemble.

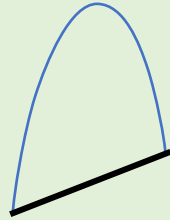
## $\Rightarrow$ Entropic structure

Entropy is strictly concave

$$S(p_1 e_1 + p_2 e_2) \geq \sum p_i S(e_i)$$

Upper bound on entropy increase

$$S(p_1 e_1 + p_2 e_2) \leq \sum p_i S(e_i) - p_i \log p_i$$



# Entropic geometry

Pseudo-distance (recovers Jensen-Shannon Divergence)

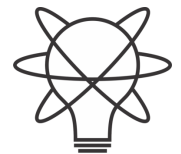
$$0 \leq S\left(\frac{1}{2}e_1 + \frac{1}{2}e_2\right) - \frac{1}{2}(S(e_1) + S(e_2)) \leq 1$$

Strict concavity of entropy  $\Rightarrow$  Hessian negative definite (recovers Fisher-Rao metric and Bures metric)

$$g(\delta e_1, \delta e_2) = -\frac{\partial^2 S}{\partial e^2}(\delta e_1, \delta e_2)$$

Orthogonality!

Ultimately responsible for  
all geometric structures  
(i.e. metrics, symplectic forms and inner products)



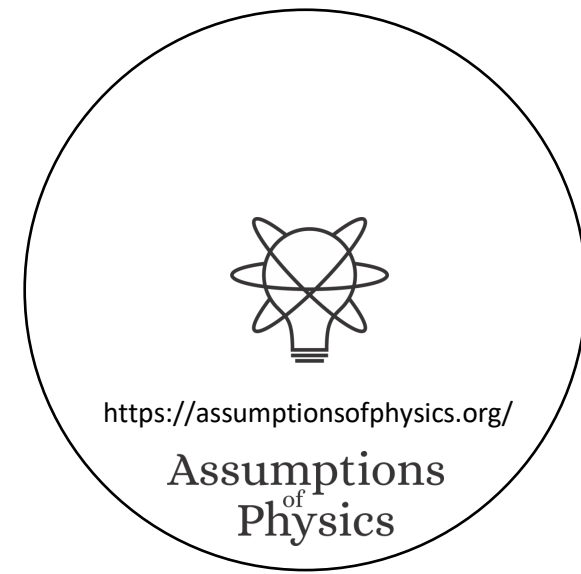
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# Wrapping it up

- Different approach to the foundations of physics
  - No interpretations, no theories of everything: physically meaningful starting points from which we can rederive the laws and the mathematical frameworks they need
- Reverse physics (reverse engineer principles from the known laws)
  - Classical mechanics is “completed”; very good ideas for both thermodynamics and quantum mechanics; still do not know how to generalize to field theories
- Physical mathematics (rederive the mathematical structures from scratch)
  - Topology and  $\sigma$ -algebras are derived from experimental verifiability; started to formalize states/processes
- The goal is ambitious and requires a wide collaboration
  - Always looking for people to collaborate with in physics, math, philosophy, ...



# To learn more

- Project website

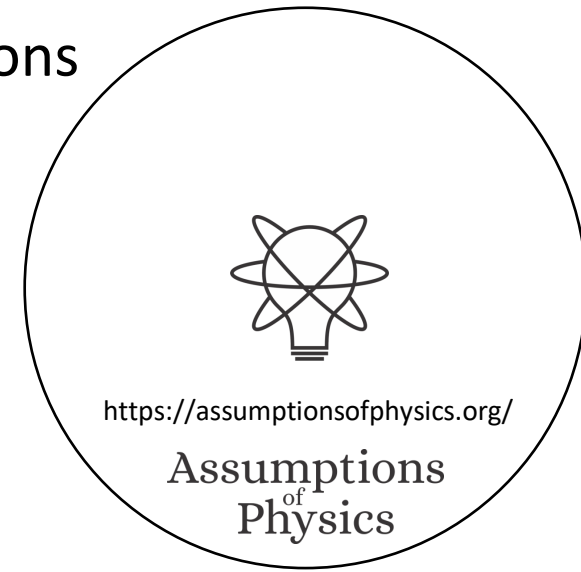
- <https://assumptionsofphysics.org> for papers, presentations, ...
- <https://assumptionsofphysics.org/book> for our open access book (updated every few years with new results)

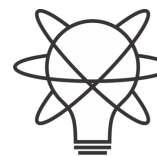
- YouTube channels

- <https://www.youtube.com/@gcarcassi>  
Videos with results and insights from the research
- <https://www.youtube.com/@AssumptionsofPhysicsResearch>  
Research channel, with open questions and livestreamed work sessions

- GitHub

- <https://github.com/assumptionsofphysics>  
Book, research papers, slides for videos...





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