***Preamble**: the overall goal of [Assumptions of Physics] is to create a framework where physics, mathematics and philosophy are well integrated. As such, it makes no sense to fully develop one aspect without a proof of concept that it can be integrated successfully with all other aspects. The purpose of these notes is to flesh out some key arguments on the philosophical side, and see their connection with the math and the physics to get early feedback and find potential collaborators.*

The general idea is that a physical theory must support a statistical description, which means it must (at least) provide a notion of ensemble and basic operations on those ensembles. This requirement is enough to derive a mathematical framework that recovers much of the mathematics that is used in physics in a general way, so that we can have an abstract theory for physical theories, of which each particular theory (i.e. classical mechanics, quantum mechanics, …) is a more constrained instance. On the philosophical side, it provides a minimal interpretation that is common to all physical theories, and is enough to understand what all mathematical objects are.

More details on the math and physics can be found in our living draft. Funding for the development of the non-additive measure part of the project is provided by The John Tempelton Foundation.

Assumptions of Physics: find a minimal set of physical assumptions from which the laws can be rederived. Comprehensive work is carried out in out open access and open source book.

Reverse Physics: start from equations, reverse engineer physical assumptions/principles. Some findings:

Classical mechanics as the high-entropy limit of quantum mechanics (2024): shows how classical mechanics can be recovered as the high-entropy limit of quantum mechanics.

How quantum mechanics requires non-additive measures (2023): shows a quantum analogue of the classical state counting Liouville measure must be non-additive. Published in Entropy.

On the reality of the quantum state once again: A no-go theorem for Ψ-ontic models (2024): shows how Ψ-ontic models as defined by Harrigan and Spekkens cannot reproduce quantum information theory and quantum thermodynamics. Published in Foundations of Physics.

Geometric and physical interpretation of the action principle (2022): shows how the variation of the action is related to the flow of evolutions in the extended phase space. Published in Scientific Reports.

Reverse Physics: From Laws to Physical Assumptions (2021): showcases our approach that starts from the physical laws to find physical principles that rederive them. Published in Foundations of Physics.

Physical Mathematics: start from scratch and rederive all mathematical structures from physical requirements. Some findings:

The unphysicality of Hilbert spaces (2023): shows how Hilbert spaces are unsuitable to model quantum state spaces as they do not handle infinity correctly. Published in Quantum Studies: Mathematics and Foundations.

On the Common Logical Structure of Classical and Quantum Mechanics (2022): shows that quantum mechanics follows classical logic, and that classical mechanics can be given a version of 'quantum logic'. Published in Erkenntnis.

The four postulates of quantum mechanics are three (2020): shows that the tensor product postulate for composite system can be recovered from the state and measurement postulates. Published in Physical Review Letters.

Variability as a better characterization of Shannon entropy (2019): provides a physically more meaningful way to characterize information entropy and a more precise account of its relationship to statistical mechanics. Published in European Journal of Physics.

Topology and experimental distinguishability (2017): first attempt to formalize the first part of the derivation up to topological spaces. Published in Topology Proceedings.

Constitutive principle: starting point that establishes the subject of the discourse; **evidential principle**: starting point that is established by experimental evidence (e.g. invariance of the speed of light); derive as much as possible from constitutive principles. The current two constitutive principles we are considering are:

Principle of scientific objectivity (PSO): science is universal, non-contradictory and evidence based

Principle of scientific reproducibility (PSR): physical laws describe relationships that can always be experimentally replicated

PSO → experimental verifiability (verifiable statements) → topologies and σ-algebras: universality and non-contradiction implies the need for statements (i.e. assertions that can only be true or false for everybody); evidence based implies the need for verifiable statements (i.e. statements that are associated with an experimental test that terminates **in finite time** if and only if the statement is true). A physical theory, then, must be a collection of statements, that can be "generated" by a verifiable subset (i.e. all information is contained in the verifiable ones).

Negation of a verifiable statement is not a verifiable statement (e.g. "there are at least n quarks" is verifiable, but "there at most n-1 quarks" is not)

A set of experimentally definable cases is a T_0 second-countable topological space where the **open sets correspond to verifiable statements** of the form "x is in U" (e.g. "the mass of the photon is less than 10^{-13} eV). The reals with their standard topology are recovered by assuming a dense set of linearly ordered references (e.g. "the value is before reference A", "the value is after reference B").

The Borel algebra corresponds to statements that are associated with a test regardless of termination. Interior → test success; exterior → test failure; boundary → test does not terminate

Perfect correspondence between math and physical ideas. Explains why the Banach-Tarski paradox does not apply (non-Borel sets are not experimentally meaningful). Explains why sets with cardinality greater than the continuum are not used in physics (can't be given a T_0 second-countable topology). Explains why functions in physics are "well-behaved" (topologically continuous functions preserve experimental verifiability).

Converse: topologies and σ-algebras are most likely meaningless without experimental connections (i.e. no grounds to say that hidden variables or parallel worlds are topological spaces or have a σ-algebra, that maps are well-behaved, …).

PSR → ensembles → convex spaces, entropy, geometry, probability, …: physical laws do not describe relationships at one instant in time, but relationships between reproducible preparations and measurements; physical laws are relationships between ensembles; also, ensembles are what we can prepare in a lab anyway (i.e. no perfect replications of conditions); reproducibility requires being able to always replicate the experiment "one more time:" ensembles represent infinite collections. Physical theories, then, are relationships between idealized objects.

A physical theory must, at least, describe what ensembles are allowed in the theory: **a physical theory must provide an ensemble space**. The ensemble space must at least allow the following:

Experimental verifiability: ensembles must be a T_0 second-countable topological space

Statistical mixing: an ensemble space must allow convex combinations

Entropy: each ensemble must have a well-defined entropy that quantifies the variability of the instances of the corresponding preparation

The three aspects have compatibility conditions (e.g. statistical mixing must be compatible with experimental verification, therefore the convex structure is continuous; the average variability can only increase during mixture and therefore the entropy is strictly concave) and provide a more constraining structure.

Various constructions and consequences that rely on previous assumptions

Uniqueness of the Shannon entropy. The upper-bound increase of the average entropy during mixing is proven to correspond to the Shannon entropy, without reference to the underlying space (e.g. classical, quantum, or any other theory).

Separateness and orthogonality explain classical vs quantum. Two ways to define "different ensembles."

Separateness: they do not have a common component. That is, given a and b, there is no ensemble c such that $a = \lambda c + (1 - \lambda)d$ and $b = \mu c + (1 - \mu)e$. Conceptually, this tells us whether there is a "preparation in common."

Orthogonality: they maximize the entropy during mixture. That is, an equal mixture of the two ensembles makes the entropy rise by one bit. Conceptually, this tells us whether the two ensembles are completely distinguishable (i.e. one instance of preparation of one is enough to tell the ensembles apart).

Separateness and orthogonality coincide only in classical spaces. That is, in non-classical theories we can have ensembles that do not have a common subensemble but are not perfectly distinguishable. That is, the common part cannot be reliably reproduced (it is not another ensemble). This happens if and only if ensembles allow for multiple decompositions. Having multiple decompositions of ensembles in QM is equivalent to the ability to have superpositions (i.e. one pure state is a superposition of other pure states if and only if there is an ensemble for which all elements are components of the ensemble).

Universality of vector spaces. The linearity induced by statistical mixing and constrained by the continuity and strict concavity of the entropy is responsible for all linear structures in physics. Statistical quantities are linear functions of the ensemble space.

Ensemble spaces embed into a compact set of a vector space. The continuity and strict concavity of the entropy force the convex set to embed into a vector space. That is, an "unmixing" operation is well defined. The upper bound of the entropy forces the ensemble space to be a subset of a compact subset (i.e. taken any ensemble and a direction, at some point one finds an "end" to the ensemble space).

Statistical quantities. Quantities that represent expectation values are linear functions of ensembles (i.e. the value on a mixture is the average of the values). In QM, these correspond to Hermitian operators. Note, however, that one can define quantities on ensembles that are not linear (i.e. they are not an expectation). The entropy is one such quantity. Therefore, not all quantities in QM are necessarily Hermitian operators (e.g. there is no angle operator for spin ½).

Geometry is entropy. Geometric structures are equivalent to entropic structures: the geometry allows you to calculate the entropy and the entropy allows you to reconstruct the geometry. The Liouville volume corresponds to the entropy of uniform distributions. The inner product in QM can be recovered from the entropy increase during mixing.

Average entropy increase leads to pseudo-distance. Given two ensembles, we can calculate the entropy increase during an equal mixture (i.e. $MS(a,b) = S\left(\frac{1}{2}a + \frac{1}{2}b\right) - \left(\frac{1}{2}S(a) + \frac{1}{2}S(b)\right)$). This a number from zero to one, where zero means $a = b$ and one means $a \perp b$. This recovers the Jensen-Shannon divergence in both classical and quantum mechanics. In quantum mechanics, the inner product between two pure states is bijective with respect to their pseudo-distance.

Riemannian metric from entropy. Since the entropy is strictly concave, its Hessian is negative definite. The negation of the Hessian $g = -\frac{1}{8}\frac{\partial^2 S}{\partial S^2}$ is a positive definite function of two variations. It recovers the Fisher-Rao metric in both classical and quantum mechanics.

Points as limits of ensembles. Instead of defining ensembles as probability distributions over idealized points (e.g. pure states in quantum mechanics, microstates in classical mechanics), points are defined as the limit of ensembles. This allows the generalization, as ensembles work the same but in general points do not. In fact, it highlights key differences and false parallels.

Pure states vs microstates. The parallel between quantum pure states and classical microstates does not work. Classical microstates are the limit for distributions over phase-space with decreasing support. The entropy would be minus infinity, and therefore they are not proper ensembles. Quantum pure states are ensembles that can no longer be decomposed as the mixture of other states (i.e. extreme points in the convex space). Classical mechanics has no such ensembles. Eigenstates of position in quantum mechanics are the limit of wavefunctions with decreasing support. It is a limit of a sequence of pure states (not a limit of ensembles with decreasing entropy as in the classical case) that does not converge in the ensemble space.

Classical contexts. A classical context is a subset of an ensemble space where we recover single decomposition in terms of separate ensembles. We can define subspaces where we can properly say that an ensemble is either in one subspace or another. In these subsets we can define sequences of subspaces that become smaller and smaller: these are the points of the spectra of the context. These correspond, in both classical and quantum mechanics, to the possible outcomes (i.e. the sample space) of measurements. While the whole ensemble space in classical mechanics is a classical context (i.e. we always have single decomposition), a quantum ensemble space (and beyond) can provide single decomposition only on a part.

Classical/quantum transition. Starting from ensembles and recovering points maps better to empirical practice: the points are never prepared or measured. It provides a better way to understand the quantum/classical transition. Classical mechanics has to fail as ensembles cannot be made arbitrarily more precise (i.e. there is a lower bound on entropy given by the third law of thermodynamics). Conversely, classical mechanics is recovered as the high-entropy limit of quantum mechanics (in the same way that non-relativistic mechanics is the low-speed limit of relativistic mechanics). It is the inability to consistently prepare a system with lower uncertainty that drives the problem, as laws at a more refined level cannot be empirically established. The mechanism as to why this happens has no bearing (in the same way that why speeds are low in the non-relativistic limit has no bearing).

Quantization of space. A similar treatment should be given to points of physical space, as they should represent how ensembles over fields can be recursively broken down into the product of ensemble spaces of different regions (still to be done). As regions become smaller, however, ensemble spaces on them can no longer be seen as the product of ensemble spaces on smaller regions (i.e. factorization of ensemble spaces must break). This gives a principled way to explore the quantization of space, which is likely to require new mathematical techniques, which can be developed first as a better way to reorganize field theories in the ensemble space framework.

State capacity. Given an ensemble, we want to quantify its spread over experimentally distinct states/configurations. This can be recovered from the exponential of the entropy. This is the inverse of the definition of Boltzmann entropy in classical mechanics, and extends to quantum mechanics recovering vector space dimensionality over subspaces.

Exponential of the entropy. A count of distinct states that support an ensemble should be monotonic with the entropy (i.e. greater entropy means the ensemble spreads over more configurations) and at most additive (i.e. a mixture of ensembles can at most be spread over the sum of configurations). The exponential of the entropy has this property (likely the only function).

Counting states. Given a set of ensembles, the exponential of the highest entropy reachable through mixing represents the count of distinct states/configurations. This is a sub-additive measure that is additive over orthogonal sets, such as subspaces. On classical subspaces it recovers the Liouville measure; on quantum subspaces it recovers the dimensionality of the subspace.

Fraction capacity. Given an ensemble e we can ask what fraction can be expressed as a mixture taken from a set of ensembles A. We call this the fraction capacity of A towards e, and it is a number from zero to one. This provides a generalization of probability theory that works the same in classical and quantum mechanics. The focus is not on the probability of outcome, but on the mixing coefficients of preparation. While in classical mechanics the two are the same, they are not the same in general.

Classical probability over classical contexts. Each ensemble in a classical context can be represented as a probability distribution over the spectrum (i.e. the points) of the context. Since classical ensemble spaces are classical contexts, we recover that every classical space is a probability distribution (i.e. preparation and measurements are the same). In quantum mechanics, a classical context is given by the mixed states that commute with a maximal set of observables. For position, this recovers the space of functions of position that integrate to one.

Measurements. Classical and quantum measurements can now be treated on equal footing. The output of a measurement will be an ensemble of a classical ensemble space. First, we need to project the starting ensemble space into a classical context. If the ensemble is already part of the context, nothing happens. This is always the case in classical mechanics, and it is the case in quantum mechanics when the preparation corresponds to the right context. If not, an entropy-increasing process changes the state (i.e. a change of boundary conditions like changing from a canonical to a grand canonical ensemble). This is the part of the measurement process that acts on the system (e.g. responsible for quantum Zeno effect). Then, once the measurement is done, the state can be updated with the new information. This is an entropy-decreasing step and is completely "classical" (e.g. we find what card was face down on the table).

Quasi-probability. Negative quasi-probability measures can be understood simply as affine combinations in the ensemble space: since the space embeds into a vector space, any point can be seen as an affine combination (i.e. a linear combination where the coefficients sum to one, but can be negative) of a suitable number of other points. Mathematically, this can always be done and can be useful, but expands the space to unphysical objects (i.e. it is not possible to give a simple general characterization of which affine combinations are valid).

Non-additive probability theory. The general non-additivity of the fraction capacity and the state capacity may provide a way to extend measure theory and probability theory to the non-classical case. We need a suitable generalization of the Radon-Nikodym derivative to the non-additive case. This requires developing new mathematical tools.

Poisson structure. How to impose a Poisson structure on the space of ensembles is still an open problem. Ideally, given two statistical quantities, we should be able to define a Poisson algebra, which reduces to the expectation of the Poisson brackets and commutators in classical and quantum mechanics respectively. There are examples in the literature, but there are many details that need to be clarified, including whether a new axiom needs to be introduced.

Entropy kills the Poisson bracket. The transformation generated by an observable will correspond to a deterministic and reversible process. Since entropy is constant over such processes, the Poisson bracket between the entropy and any observable will be zero. Foliations of entropy should give us a “symplectic space,” which contains all the orbits of ensembles under det/rev processes.

Single Hamiltonian equation. The goal would be to be able to write Hamilton’s equation on the general ensemble space, without reference to classical or quantum mechanics. It would also allow us to write the equation of motion for quantities that are not statistical, and therefore do not correspond to Hermitian operators in quantum mechanics (e.g. the angle for spin).