Assumptions of Physics
Summer School 2024

Quantum Physics

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Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

Assumptions of Physics

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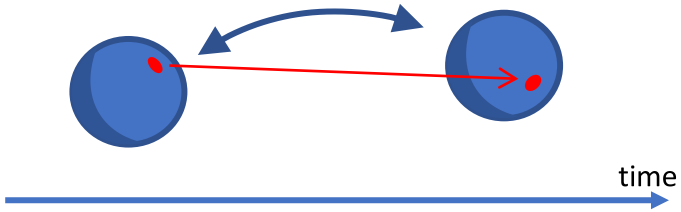
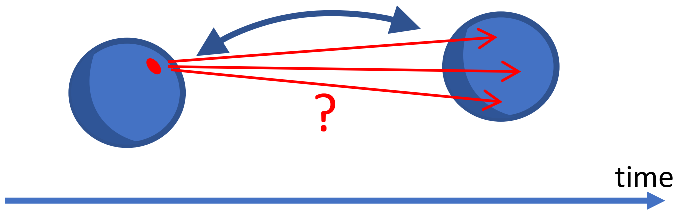
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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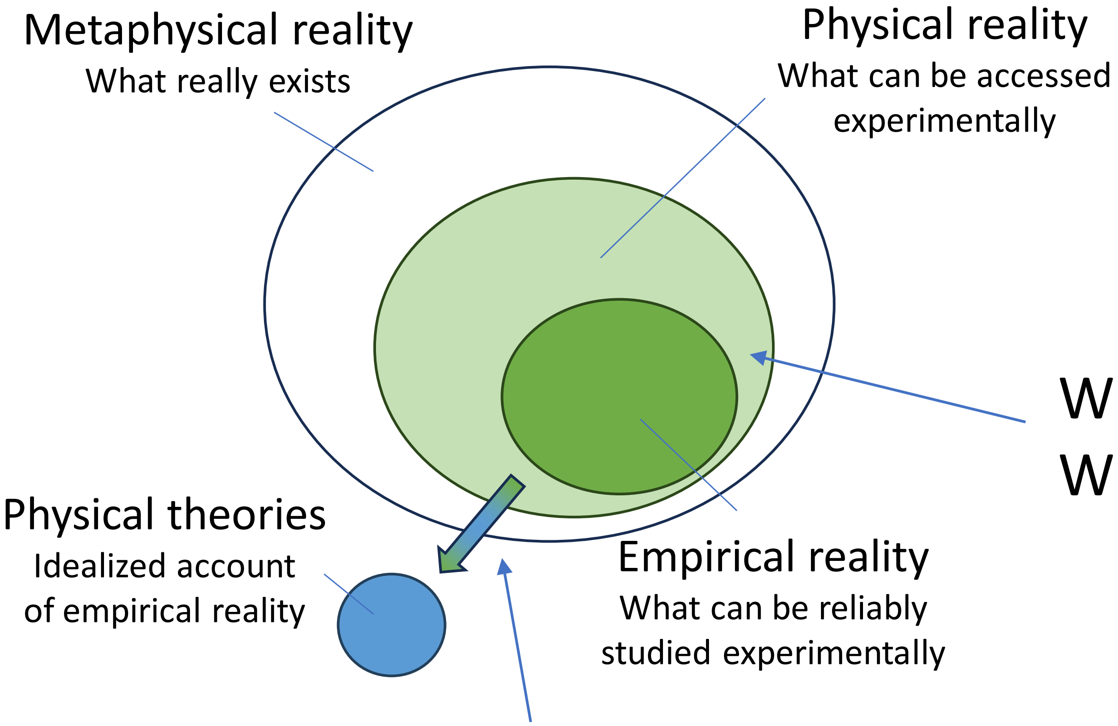
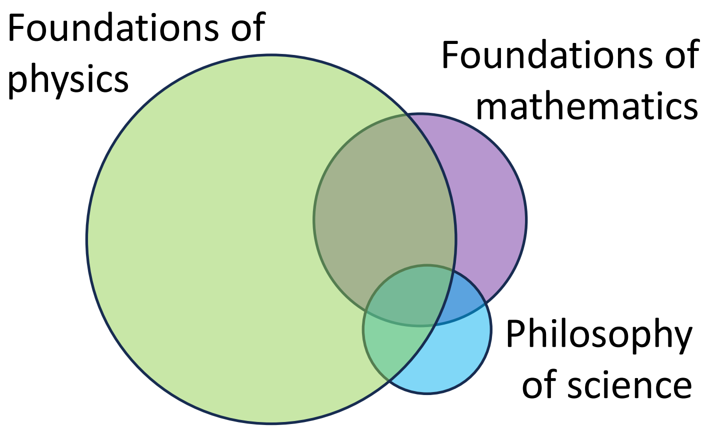
Underlying perspective



What is the boundary?

What are the requirements?

How exactly does the abstraction/idealization process work?



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Assumptions of Physics

If physics is about creating models of empirical reality, the foundations of physics should be a theory of models of empirical reality

Requirements of experimental verification, assumptions of each theory, realm of validity of assumptions, …

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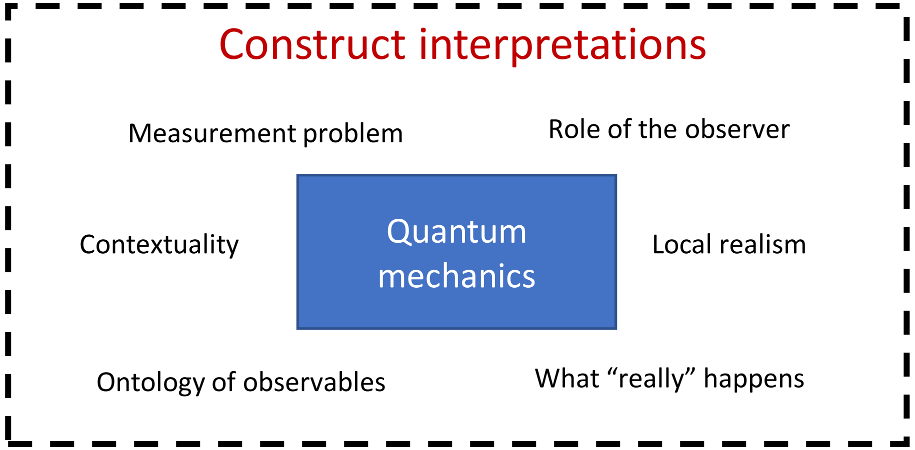
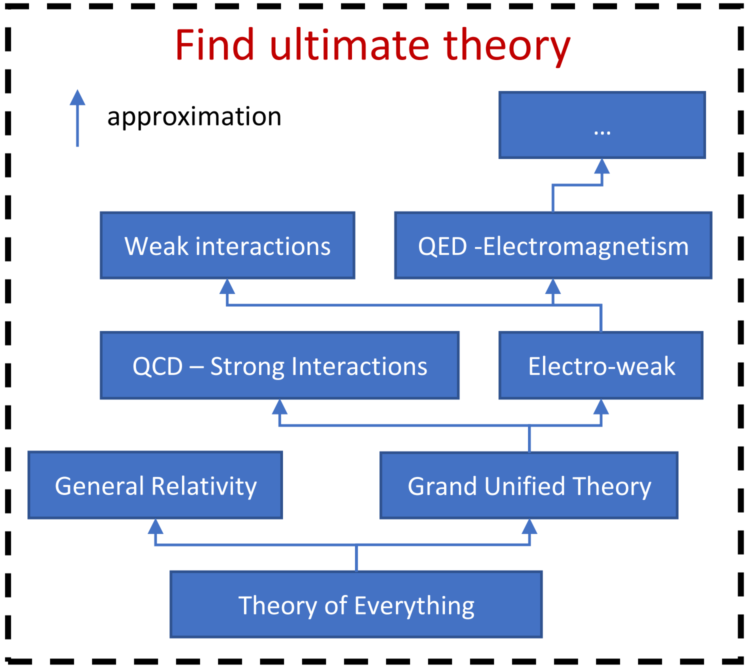
Assumptions of Physics

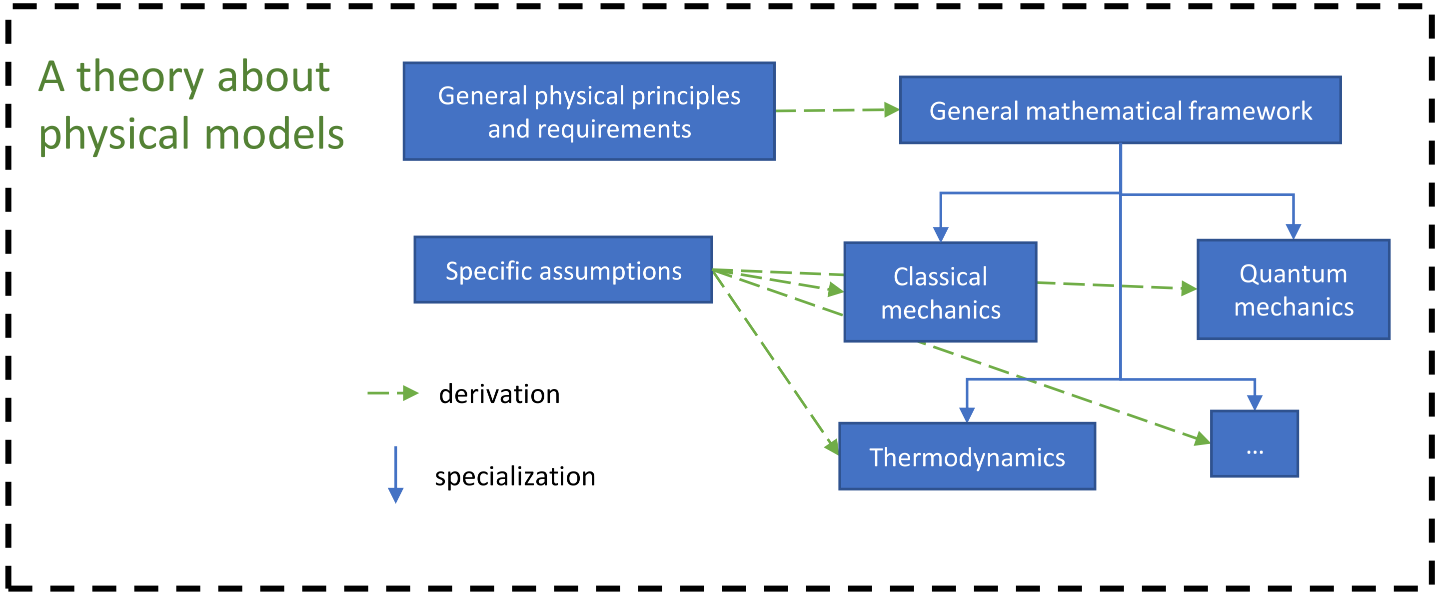
Different approach to the foundations of physics

Typical approaches





Our approach



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Assumptions of Physics

Find the right overall concepts

Reverse physics:

Start with the equations, reverse engineer physical assumptions/principles

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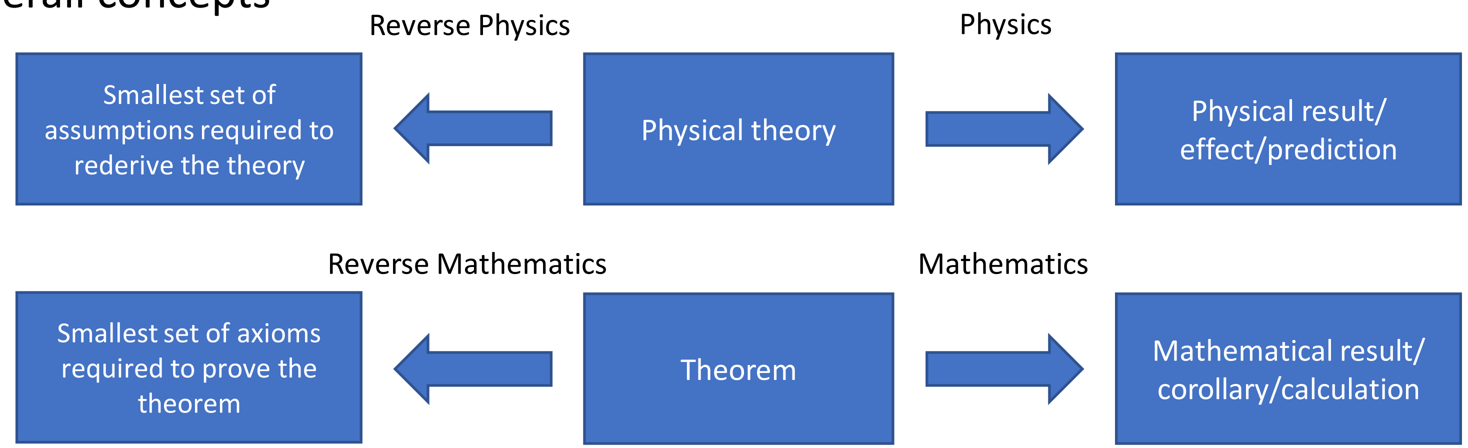
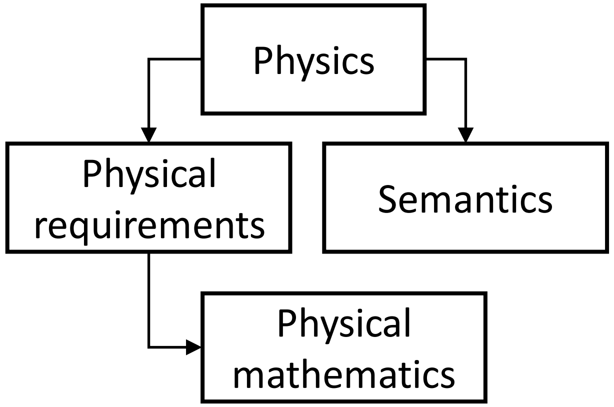


Goal: find the right overall physical concepts, "elevate" the discussion from mathematical constructs to physical principles

Physical mathematics:

Start from scratch and rederive all mathematical structures from physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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Assumptions of Physics

This session

Reverse Physics:
Quantum Physics

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Assumptions of Physics

Classical failure (isolation)

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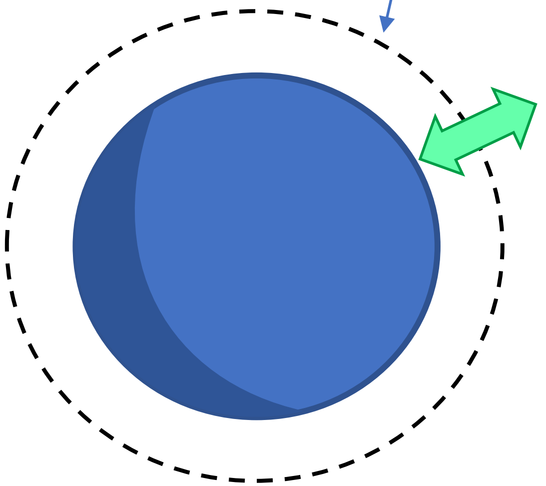
Assumptions of Physics

To define a system, we have to define a boundary

The interaction at the boundary determines what states can be defined for the system



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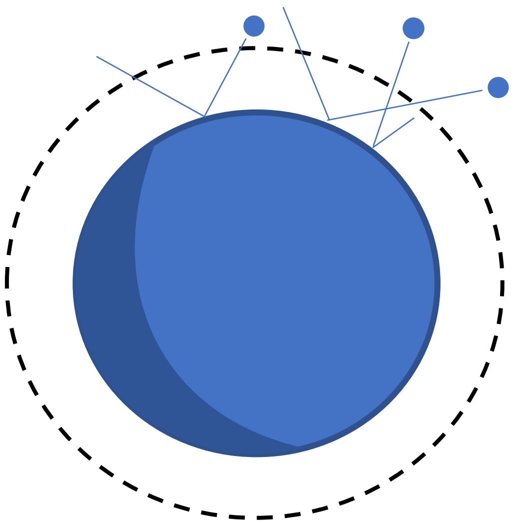
Assumptions of Physics

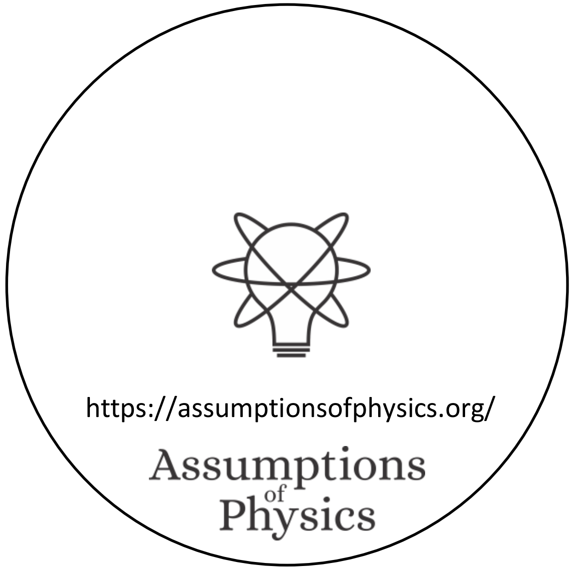
Suppose we want to study the motion of a cannonball

Air will scatter off its surface



However, the effect will be negligible

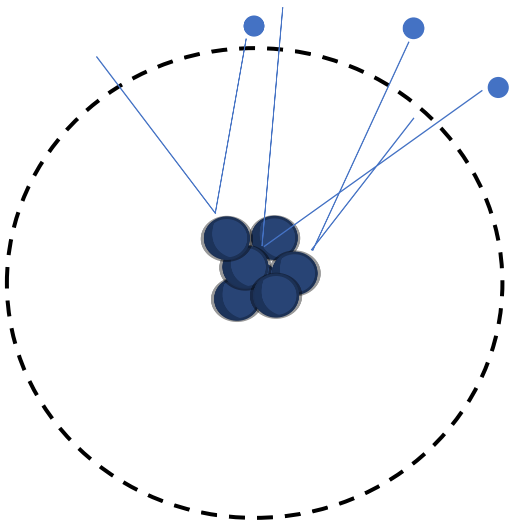
The state of the cannonball can be taken to be a precise value of position and momentum



Suppose we want to study the motion of a speck of dust

Air will scatter off its surface

The effect will not be negligible

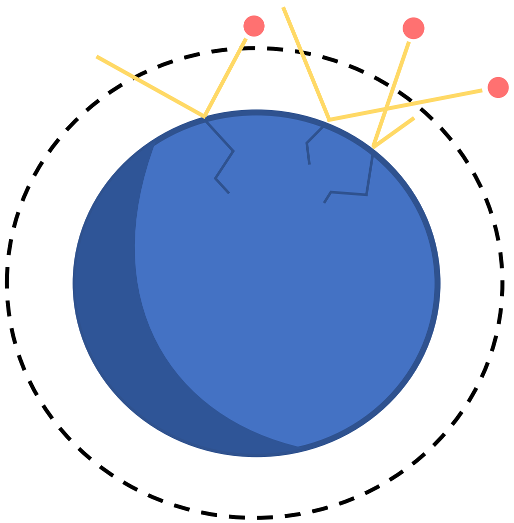


The state of the speck of dust will be a probability distribution over position and momentum

Suppose we want to study the motion of a cannonball on the surface of the sun

Plasma will scatter off its surface

The effect will be catastrophic



The cannonball will melt and cease to exist as a cannonball

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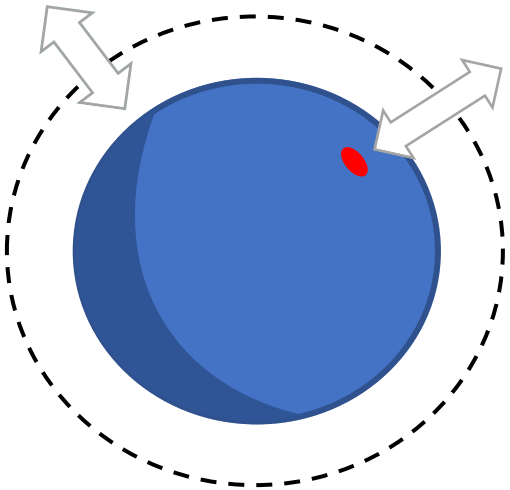
Assumptions of Physics

Interaction at the boundary is important for the very definition of a system

Classical mechanics assumes objects can be adequately isolated

$$x + \eta$$



Classical mechanics assumes we can study parts of objects, as small as we want

These two assumptions are "incompatible": at some point parts are going to be so small that they cannot be assumed to be adequately isolated

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Assumptions of Physics

Classical mechanics fails because we can never completely isolate a system

On practical grounds – we simply cannot do it

On theoretical grounds – we cannot shield gravitational interactions, we cannot eliminate thermal radiation

On logical grounds – complete isolation means no possible interaction with the system, signals would pass through, no possible measurement,
no gravity, the system disappears from our universe

therefore the most accurate description must be statistical/probabilistic in nature

Classical failure (entropy)

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Assumptions of Physics

Logarithm of accessible microstates

$$\log W$$

W is the phase-space volume

volume of a point is zero

$$\log 0 \to -\infty$$

Gibbs/Shannon entropy

$$-\int \rho \log \rho$$

ρ is a δ-function

ρ non-zero only where $\rho \to \infty$

$$-\infty \log \infty \to -\infty$$

The entropy of a "pure" microstate in classical statistical mechanics is $-\infty$

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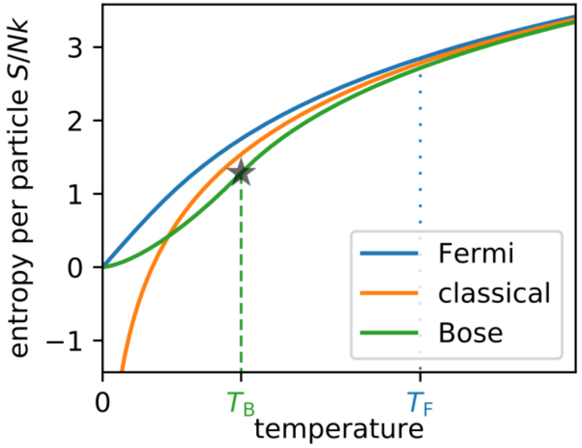
Assumptions of Physics

Recall the third law of thermodynamics

Every system has positive finite entropy. The entropy of a perfect crystal at absolute zero temperature is zero

Classical perfect crystal → single microstate → entropy is $-\infty$

Classical mechanics is inconsistent with the third law of thermodynamics



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Assumptions of Physics

Recall the second law of thermodynamics

We cannot create an engine that converts heat into work without increasing entropy

A system with entropy $-\infty$ provides a loophole: since $-\infty + \Delta S = -\infty$ for all finite ΔS, we can effectively “dump” all the entropy increase into it

We could avoid the effects of the second law of thermodynamics

What is zero entropy?

Entropy is additive for independent systems: $S_{A+B} = S_A + S_B$

The empty system $\emptyset$ acts as a zero under system combination: $A + \emptyset = A$

Therefore it must be that the entropy of the empty system is zero: $S_{\emptyset} = 0$

There is only one possible state for the empty system, and it is a complete description

Entropy lower than zero would correspond to a description that is more refined, more precise, than that of an empty system

From an information theory perspective, no system can have entropy lower than zero

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Assumptions of Physics

Classical mechanics fails because it allows for the possibility of statistical ensembles that can never exist

On practical grounds – they would allow us to bypass the second law

On theoretical grounds – they fail to respect the third law

On logical grounds – they would provide more information about the system than stating that the system does not exist, which is already a complete description of the system

Quantum mechanics solves this: all pure states have zero entropy and mixed states have positive entropy

Takeaways

- Classical mechanics fails at a conceptual level
- It doesn't take into account the relationship between system and environment
- It does not provide a lower bound on entropy

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Assumptions of Physics

Quantum states as equilibrium ensembles

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Assumptions of Physics

Parallels between QM and thermodynamics

$U = e^{\frac{O\Delta t}{\imath\hbar}}$

Eigenstates $\rightarrow$ states unchanged by the process $\rightarrow$ equilibria of the process

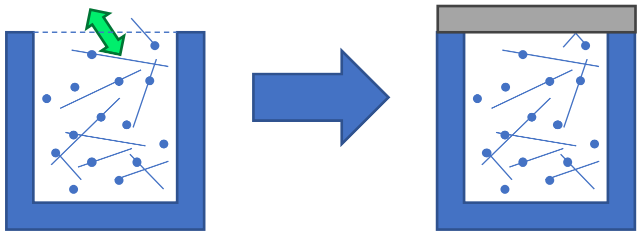
Every state is an eigenstate of some unitary / Hermitian operator $\rightarrow$ all states are equilibria

Every mixed state commutes with some unitary operator (same eigenstates used to calculate entropy)

Quantum contexts

$\Longleftrightarrow$

Boundary conditions between system and environment



$[\mu, V, T]$

$[N_1, V, T]$

$[N_2, V, T]$

$[\dots, V, T]$

Different equilibria, different variables

$|x^+\rangle$

$|z^+\rangle$

$|z^-\rangle$

Different contexts, different variables

Equilibration

Projections $\Longleftrightarrow$ ~~Measurements~~

Unitary $\Longleftrightarrow$ Quasi-static

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Assumptions of Physics

Eigenstates

Eigenvalues

Observable
Hermitian operator

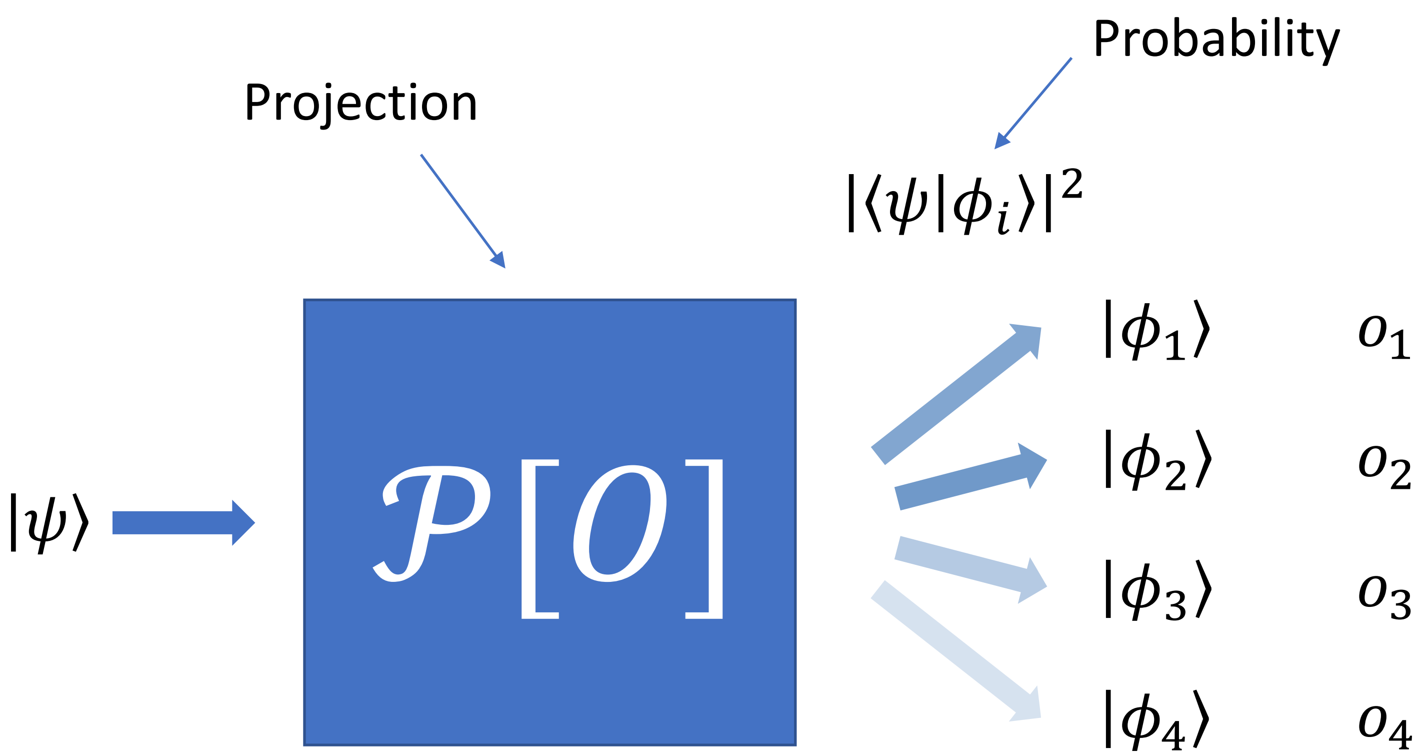
$$O$$

$$|\phi_1\rangle \qquad o_1$$

$$|\phi_2\rangle \qquad o_2$$

$$|\phi_3\rangle \qquad o_3$$

$$|\phi_4\rangle \qquad o_4$$



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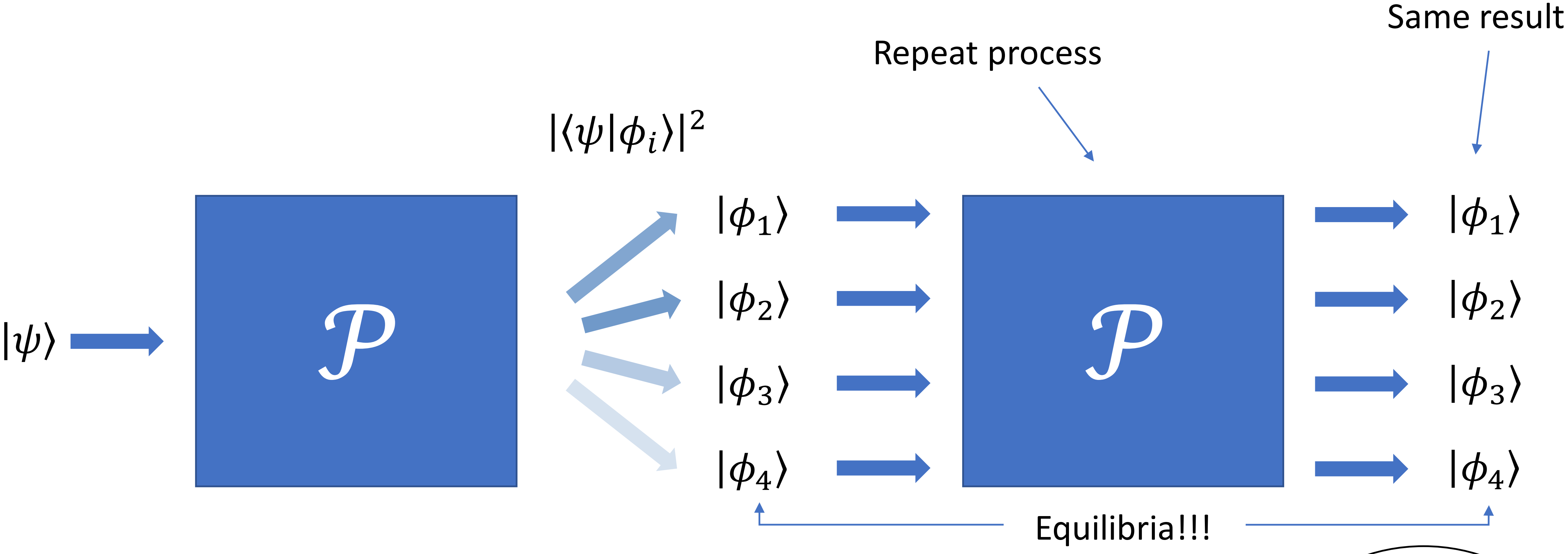
Assumptions of Physics



Eigenstates are equilibria of measurements

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Assumptions of Physics

All quantum states are eigenstates of an observable

$$|\psi\rangle \qquad O = |\psi\rangle\langle\psi|$$

$\|\psi\rangle$	1
all other cases	0

All quantum states are equilibria of measurements

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Assumptions of Physics

Every observable generates a unitary transformation

$$O \to e^{\frac{O d\alpha}{\imath\hbar}}$$

$$O^{\dagger}O = e^{-\frac{O d\alpha}{\imath\hbar}} e^{\frac{O d\alpha}{\imath\hbar}} = I$$

Same eigenstates

$\Rightarrow$ All quantum states are equilibria of unitary processes

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Assumptions of Physics

Same is true for every mixed state

$$\rho \to e^{\frac{\rho d\alpha}{\imath\hbar}}$$

All quantum states (pure and mixed) are equilibria of some time evolution and some measurement processes

Pure states can be always understood as ensembles with lowest entropy

All quantum states (pure and mixed) are equilibrium ensembles for some time evolution and some measurement processes

Not up to interpretation: mathematical fact in QM

Can we argue the converse?

The goal of physics is to establish laws that are valid in all circumstances

$F = ma$ $\quad A = B \quad$ $\vec{\nabla} \cdot \vec{E} = \rho$

Whenever I prepare this… … I find this

Repeatability (i.e. whenever) is implicitly assuming ensembles (i.e. infinite copies)

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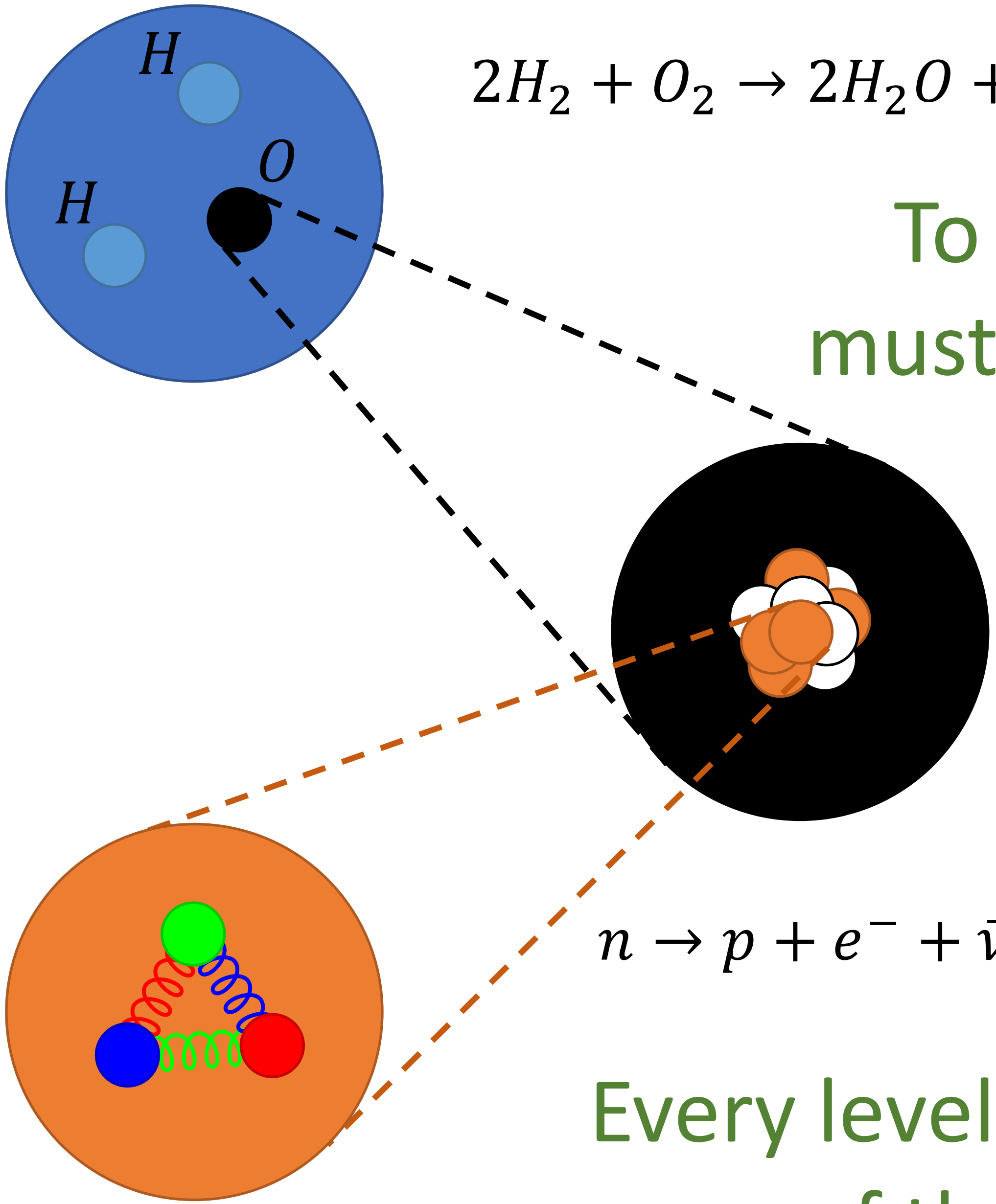
Assumptions of Physics



$$2H_2 + O_2 \rightarrow 2H_2O + Energy$$

To define/manipulate an object it must "stay the same" for long enough

$${}^{12}_{6}C + {}^{4}_{2}He \rightarrow {}^{16}_{8}O + Energy$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Every level is an equilibrium of the lower one

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Assumptions of Physics

$\Rightarrow$ Makes sense to assume that states are ensembles in equilibrium

In classical mechanics, we saw connections between geometry, probability and information theory

$$\mu(U)$$

Symplectic manifold

ρ_U uniform over U

Classical probability

Information theory

$$\rho_U(x) = \frac{1}{\mu(U)}$$

$$H(\rho_U) = \log \mu(U)$$

Classical geometric structure is exactly the structure that allows us to define ensembles (i.e. statistics) and entropy

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Assumptions of Physics

What about quantum mechanics?

$\langle \psi | \phi \rangle$

Projective Hilbert space

$$\rho = \frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi$$

Quantum probability

$$p(\psi|\phi) = |\langle \psi | \phi \rangle|^2$$

Quantum information theory

$$H(\rho) = H\left(\frac{1+\sqrt{p}}{2}, \frac{1-\sqrt{p}}{2}\right)$$

Inner product is equivalent to defining entropy of mixtures

Even in quantum mechanics, geometry/probability/information theory are different aspects of the same structure

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Assumptions of Physics

Uncertainty principle makes it look like some states are more determined than others

Property of the ensemble,
not of measurement

$$\sigma_X \sigma_P \geq \frac{\hbar}{2}$$

Recall, same bound in classical mechanics
from imposing lower bound in entropy

But: all pure states have the same entropy

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Assumptions of Physics

For every state $|\psi\rangle$, we can find a pair of observables A and B such that $\sigma_A \sigma_B = \hbar/2$

Let $|\phi\rangle$ be a gaussian wave packet for X and P

Always exists

Let U be a unitary operator such that $U|\psi\rangle = |\phi\rangle$

Consider $A = U^\dagger X U$ and $B = U^\dagger P U$, we have:

$$\langle A\rangle_\psi = \langle\psi|A|\psi\rangle = \langle\psi|U^\dagger X U|\psi\rangle = \langle\phi|X|\phi\rangle = \langle X\rangle_\phi$$

$$\langle A^2\rangle_\psi = \langle\psi|AA|\psi\rangle = \langle\psi|U^\dagger X U U^\dagger X U|\psi\rangle = \langle\phi|XX|\phi\rangle = \langle X^2\rangle_\phi$$

$$\langle B\rangle_\psi = \langle P\rangle_\phi \qquad \langle B^2\rangle_\psi = \langle P^2\rangle_\phi$$

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Assumptions of Physics

For every state $|\psi\rangle$, we can find a pair of observables A and B such that $\sigma_A \sigma_B = \hbar/2$

$$\langle A \rangle_\psi = \langle X \rangle_\phi \qquad \langle A^2 \rangle_\psi = \langle X^2 \rangle_\phi$$

$$\langle B \rangle_\psi = \langle P \rangle_\phi \qquad \langle B^2 \rangle_\psi = \langle P^2 \rangle_\phi$$

$$\sigma_{A,\psi} = \sqrt{\langle A^2 \rangle_\psi - \langle A \rangle_\psi^2} = \sqrt{\langle X^2 \rangle_\phi - \langle X \rangle_\phi^2} = \sigma_{X,\phi}$$

$$\sigma_{B,\psi} = \sigma_{P,\phi}$$

$$\sigma_{A,\psi}\sigma_{B,\psi} = \sigma_{X,\phi}\sigma_{P,\phi} = \frac{\hbar}{2}$$

For every state $|\psi\rangle$, we can find a pair of observables A and B such that $\sigma_A \sigma_B = \hbar/2$

$$[A, B] = AB - BA = U^\dagger X U U^\dagger P U - U^\dagger P U U^\dagger X U$$
$$= U^\dagger X P U - U^\dagger P X U = U^\dagger [X, P] U = \imath\hbar U^\dagger U = \imath\hbar$$

$$[A, B] = \imath\hbar$$

Every state is a Gaussian state for some pair of operators!

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Assumptions of Physics

Takeaways

- Quantum states are (at least) ensembles in equilibrium
- It doesn’t take into account relationship between system and environment
- TODOs
  - Clean up and organize the ideas
  - Connect to other literature (theoretical and experimental)
    - Typicality, Eigenstate Thermalization Hypothesis, …

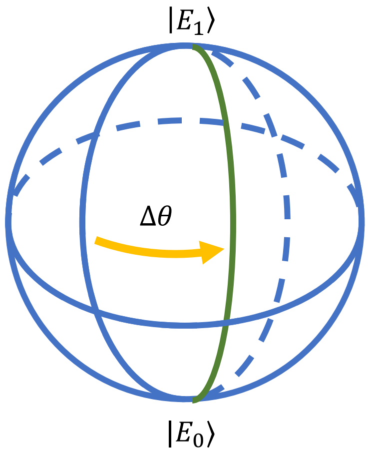
Quantum processes

Time evolution and measurements

$$\rho_I \longrightarrow \boxed{P} \longrightarrow \rho_O = P(\rho_I)$$

Any process (deterministic or stochastic) will take an ensemble as input and return an ensemble as output

$$P(p_1\rho_1 + p_2\rho_2) = p_1 P(\rho_1) + p_2 P(\rho_2)$$



Deterministic and reversible

$$\rho_I \longrightarrow \boxed{P} \xrightarrow{\rho_O} \boxed{P^{-1}} \longrightarrow \rho_I$$

Conserves probability and allows an "inverse"

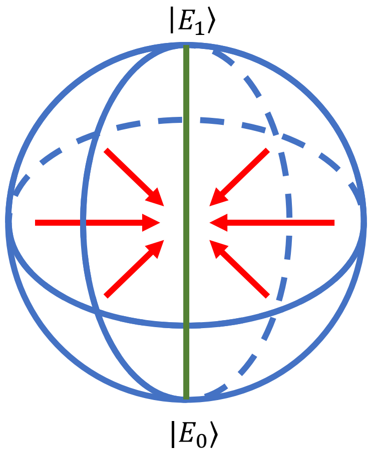
⇒ Unitary operation

Measurement

$$\rho_I \longrightarrow \boxed{P} \xrightarrow{\rho_O} \boxed{P} \longrightarrow \rho_O$$

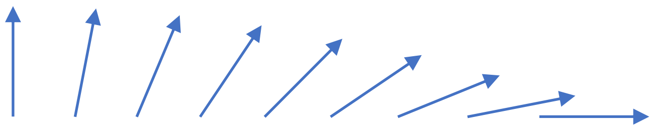
Must be repeatable

⇒ Projection



Measurement problem: unitary ⇏ projections … projections ⇒ unitary

Unitary evolution ≡ sequence of infinitesimal projections



Unitary evolution is for det/rev, isolated processes
System being measured can't be isolated

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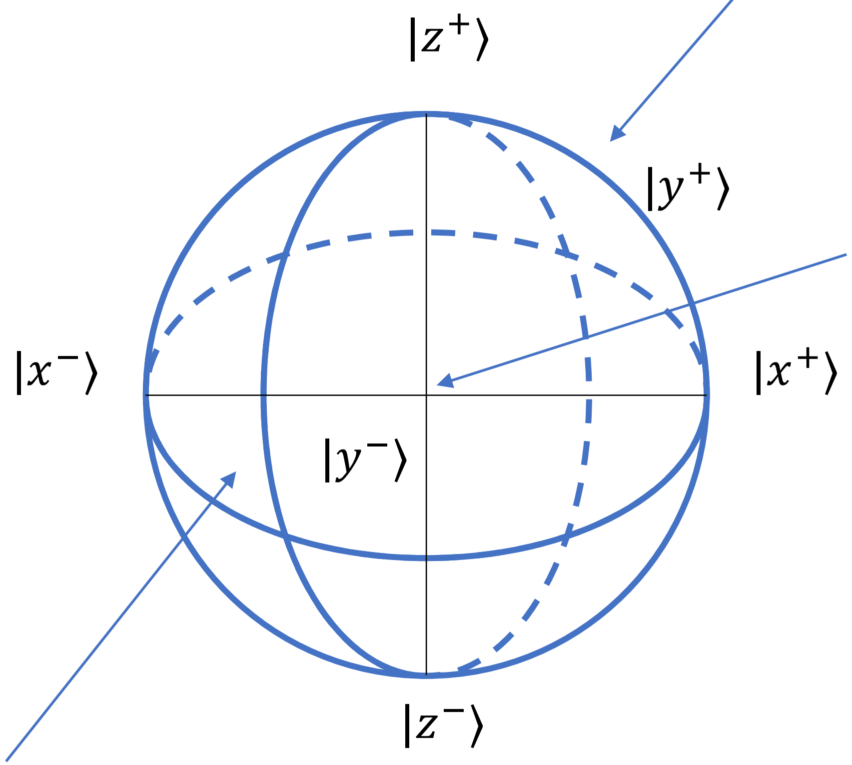
Assumptions of Physics

Geometry of mixed states

Pure states: Bloch ball surface

$S = 0$



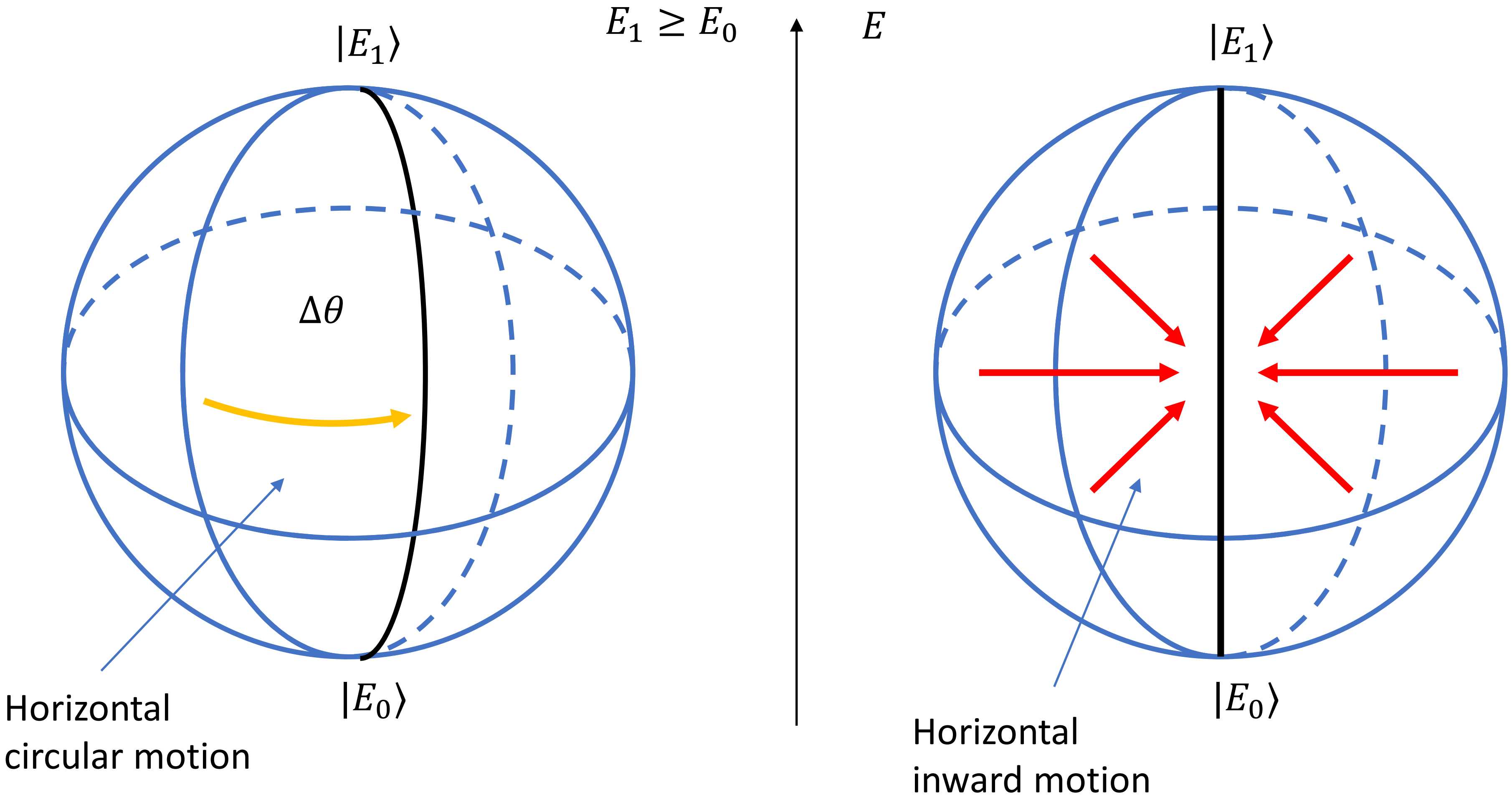
$$1/2|x^+\rangle\langle x^+| + 1/2|x^-\rangle\langle x^-|$$
$$= 1/2|z^+\rangle\langle z^+| + 1/2|z^-\rangle\langle z^-|$$

$S = 1$

Mixed states: Bloch ball interior

Time evolution

Measurement



Change at constant energy
and constant entropy

Change at constant energy
that maximizes entropy

Two steps:

1) Prepare a mixture of possible outcomes
entropy-increasing irreversible process

2) Determine the outcome
same as classical

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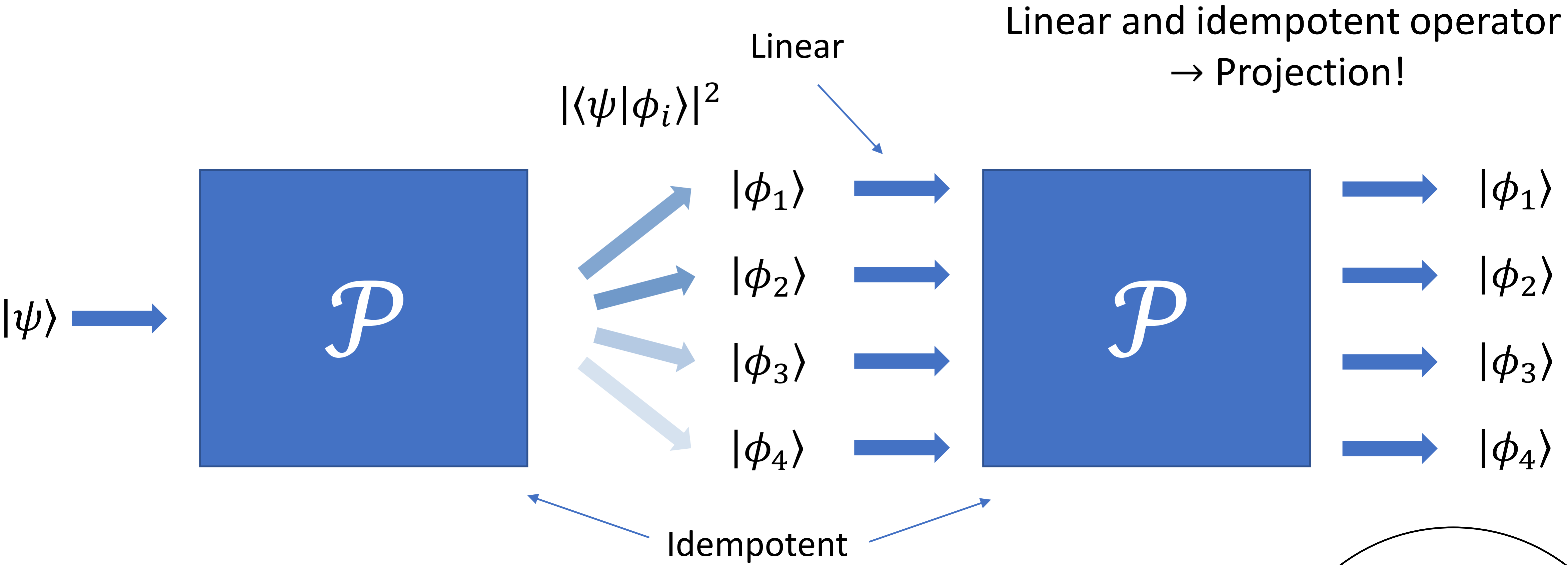
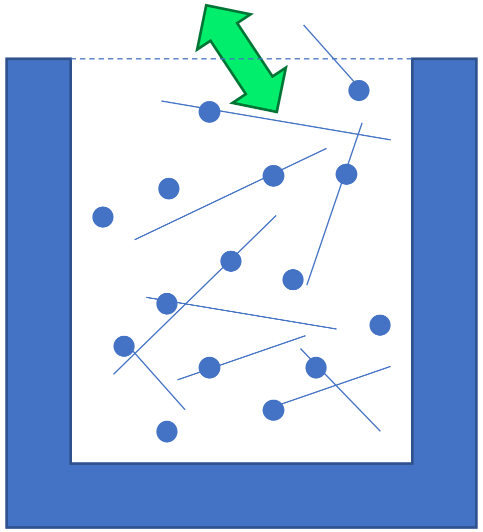
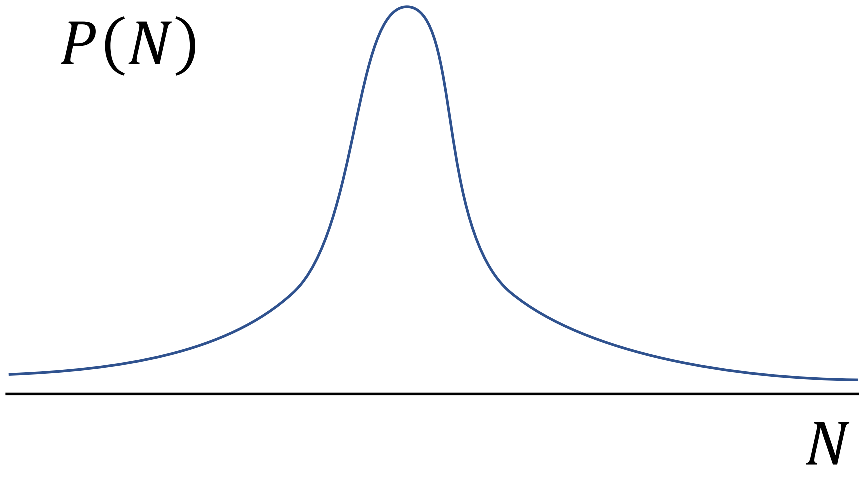
Assumptions of Physics



Projections $\Longleftrightarrow$ black-box equilibration processes





Equilibrium of an open system
does not define a unique number of particles

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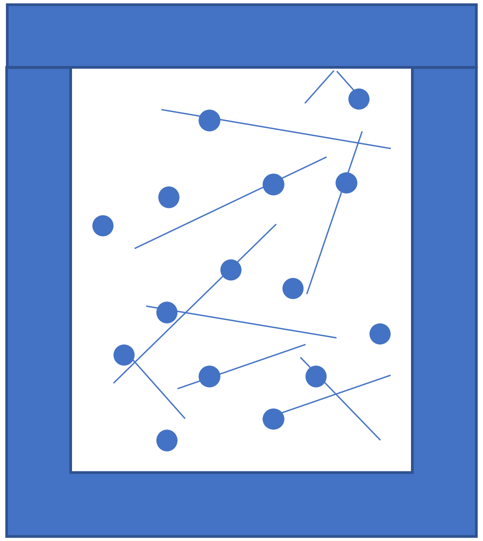
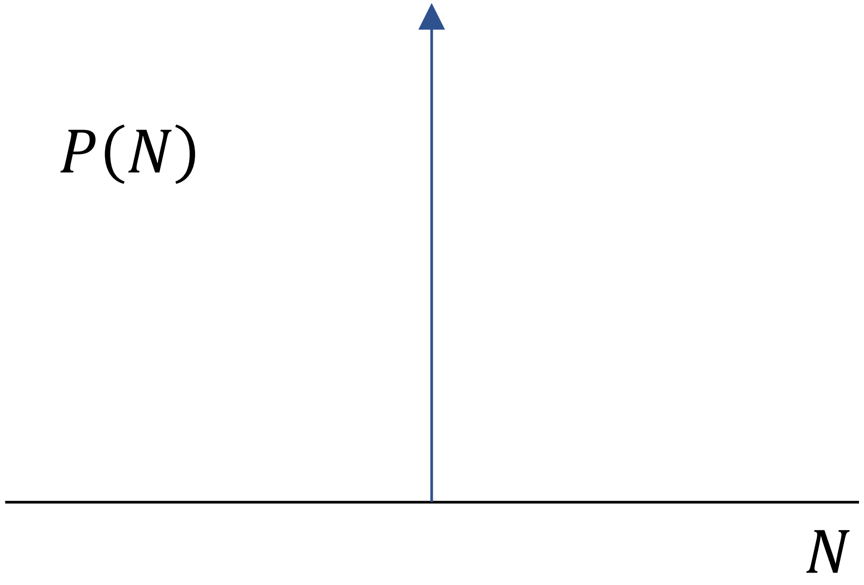
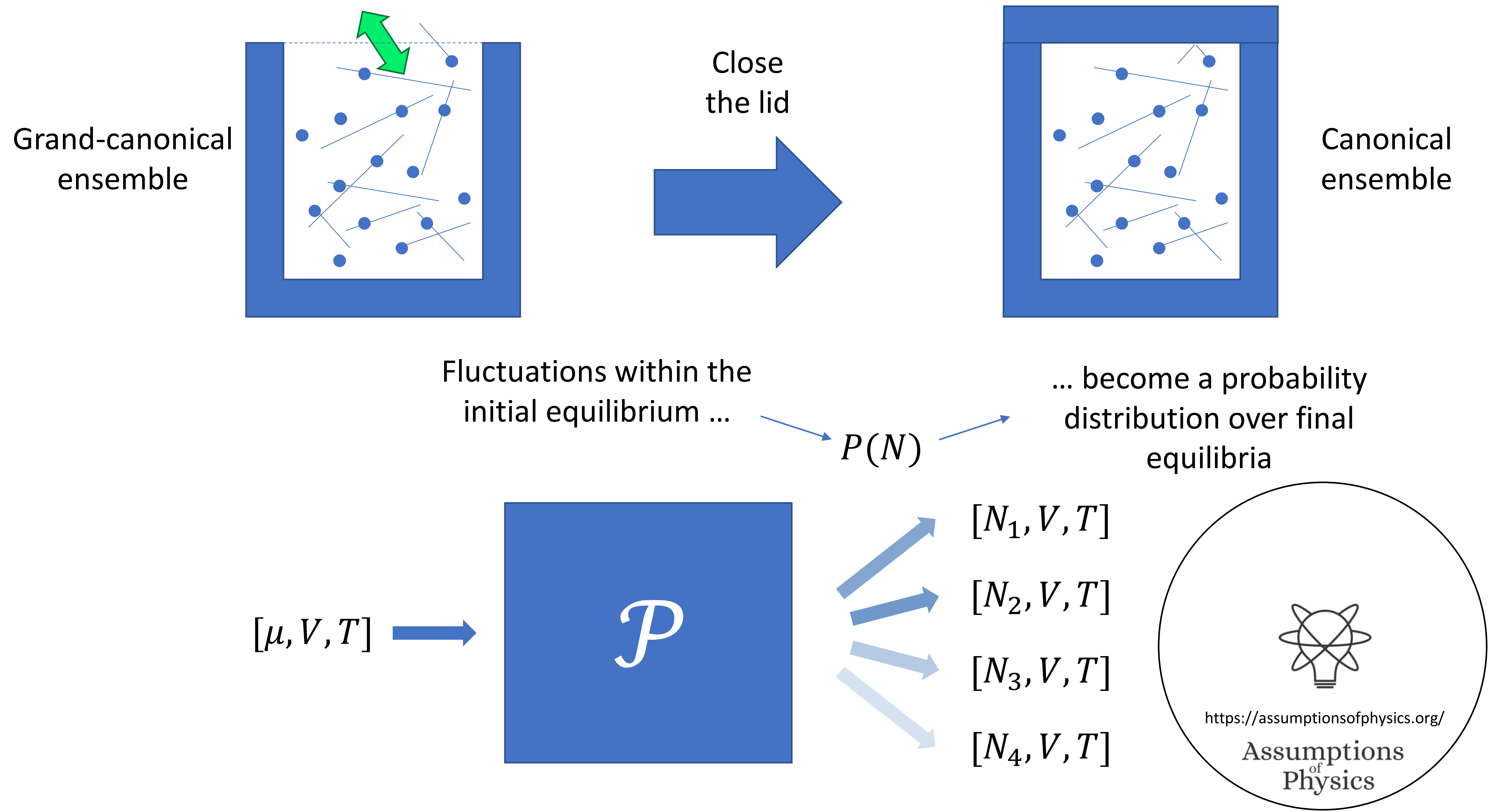




Equilibrium of a closed system
defines a unique number of particles


Grand-canonical ensemble
Close the lid
Canonical ensemble
Fluctuations within the initial equilibrium …
$P(N)$
… become a probability distribution over final equilibria
$[\mu, V, T]$
$\mathcal{P}$
$[N_1, V, T]$
$[N_2, V, T]$
$[N_3, V, T]$
$[N_4, V, T]$
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Assumptions of Physics

Think of quantum states as different ensembles identified by different quantities

In both cases, we cannot describe the equilibration process: it is not in terms of equilibrium states!

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Assumptions of Physics

Schrödinger equation – (unitary) time evolution

$$H|\psi\rangle = \imath\hbar\partial_t|\psi\rangle$$

Hamiltonian

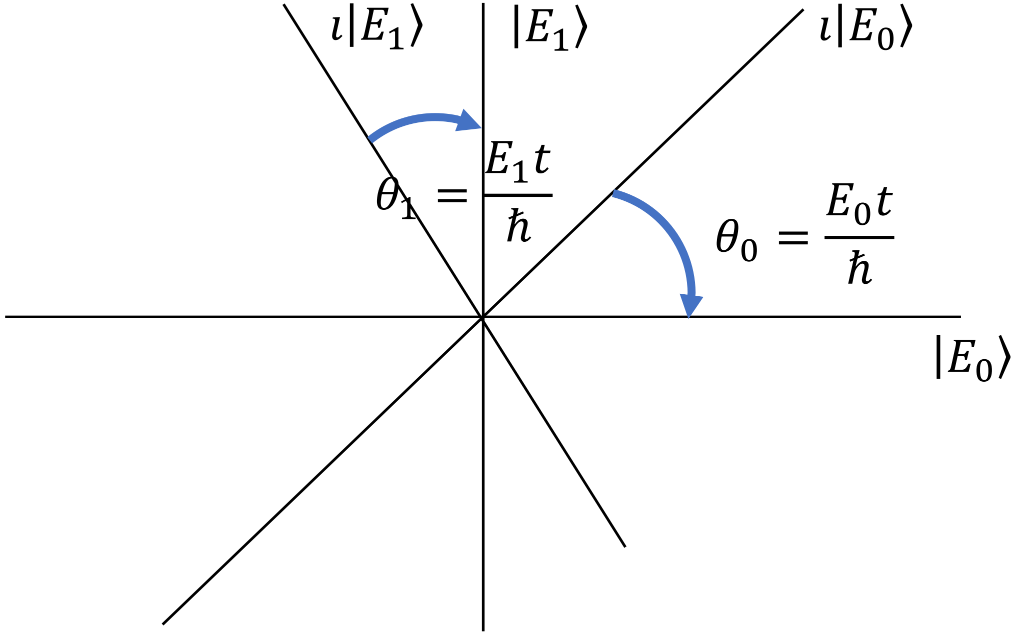
$$H = \begin{bmatrix} E_1 & 0 \\ 0 & E_0 \end{bmatrix}$$

diagonalized

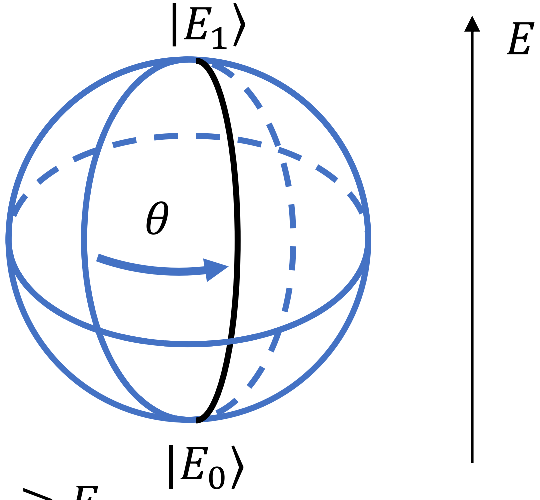
$$|\psi(t)\rangle = U(t)|\psi_0\rangle = e^{\frac{Ht}{\imath\hbar}}|\psi_0\rangle$$

Time evolution operator

$$U(t) = e^{\frac{Ht}{\imath\hbar}} = \begin{bmatrix} e^{\frac{E_1 t}{\imath\hbar}} & 0 \\ 0 & e^{\frac{E_0 t}{\imath\hbar}} \end{bmatrix}$$



$$\theta = \theta_1 - \theta_0 = \frac{(E_1 - E_0)t}{\hbar}$$



$E_1 \geq E_0$

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Assumptions of Physics

Unitary evolution ⟺ det/rev evolution

$$|\psi(t+dt)\rangle - |\psi(t)\rangle = \mathcal{T}(t)dt|\psi(t)\rangle$$

Change of states depends only on previous state (determinism)

$$\langle\psi(t+dt)|\psi(t+dt)\rangle = 1$$

Map to only one state (reversibility)

$$= \langle(1+\mathcal{T}(t)dt)\psi(t)|(1+\mathcal{T}(t)dt)\psi(t)\rangle$$

$$= \langle\psi(t)\big|(1+\mathcal{T}(t)dt)^{\dagger}(1+\mathcal{T}(t)dt)\big|\psi(t)\rangle$$

$$= \langle\psi(t)\big|1+\mathcal{T}(t)^{\dagger}dt+\mathcal{T}(t)dt+\mathcal{T}(t)^{\dagger}\mathcal{T}(t)dt^{2}\big|\psi(t)\rangle$$

$$= 1 + dt\langle\psi(t)\big|\mathcal{T}(t)^{\dagger}+\mathcal{T}(t)\big|\psi(t)\rangle + O(dt^{2})$$

$$\Rightarrow \mathcal{T}(t)^{\dagger} = -\mathcal{T}(t)$$

Self-adjoint

$$\mathcal{T}(t)dt = -\frac{H(t)dt}{\imath\hbar}$$

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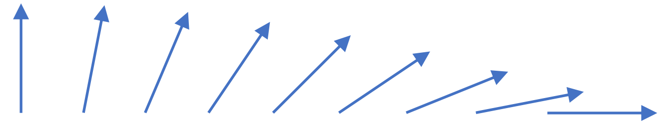
Assumptions of Physics

Unitary evolution $\Longleftrightarrow$ quasi-static evolution

Unitary evolution
$\Longleftrightarrow$
sequence of infinitesimal projections



$$|\langle\psi(t+dt)|\psi(t)\rangle|^2 = 1$$

$$= \langle\psi(t+dt)|\psi(t)\rangle\langle\psi(t)|\psi(t+dt)\rangle$$

$$= (1 + \langle d\psi(t)|\psi(t)\rangle)(1 + \langle\psi(t)|d\psi(t)\rangle)$$

$$= 1 + (\langle d\psi(t)|\psi(t)\rangle + \langle\psi(t)|d\psi(t)\rangle) + O(dt^2)$$

$$= 1 + dt(\langle\mathcal{T}(t)\psi(t)|\psi(t)\rangle + \langle\psi(t)|\mathcal{T}(t)\psi(t)\rangle) + O(dt^2)$$

$$\Rightarrow \mathcal{T}(t)^\dagger = -\mathcal{T}(t)$$

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Assumptions of Physics

Deterministic and reversible evolution

Black-box process with equilibria

Quasi-static evolution

Unitary evolution

Projection

Every preparation is a measurement

Time evolution prepares the system at each time

⇒ Time evolution is a series of measurements

Takeaways

- Projections are processes with equilibria
  - Measurements are processes with equilibria
- Unitary evolution is deterministic and reversible evolution
- Solution to the inverse measurement problem: unitary evolution is a series of measurements
- TODOs
  - Clean up and organize the ideas

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Assumptions of Physics

Quantum irreducibility

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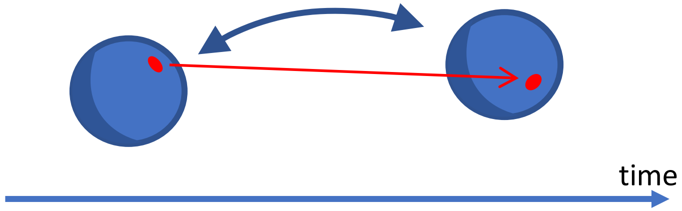
Assumptions of Physics

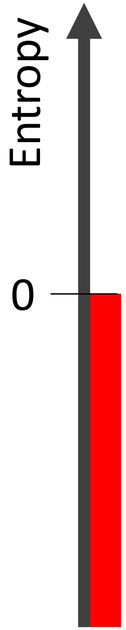
Quantum mechanics as irreducibility

Classical

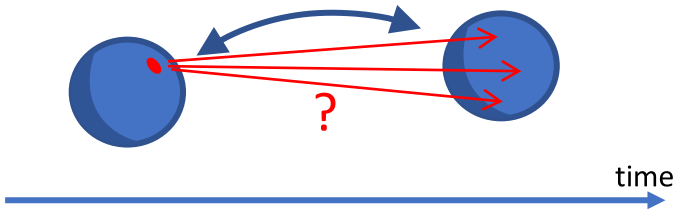


Can prepare ensembles at arbitrarily low entropy: we can study arbitrarily small parts

We always have access to the internal dynamics



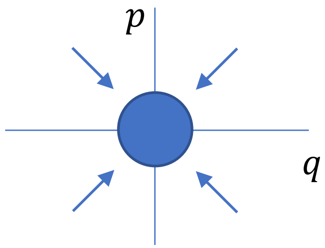
Quantum



Entropy is bounded at zero: we cannot study parts

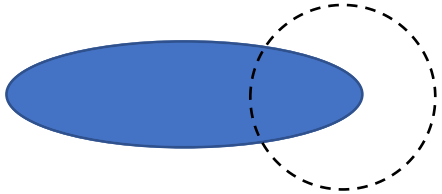
No access to the internal dynamics

Minimum uncertainty



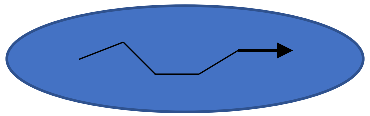
Can't squeeze ensemble arbitrarily

Non-locality



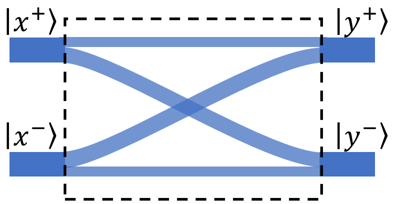
Can't refine ensembles ⇒
Can't interact with parts

Superluminal effects that can't carry information



Can't refine ensembles ⇒
Can't extract information

Probability of transition



$$p(x^+|y^-) = p(y^-|x^+)$$

Symmetry of the inner product

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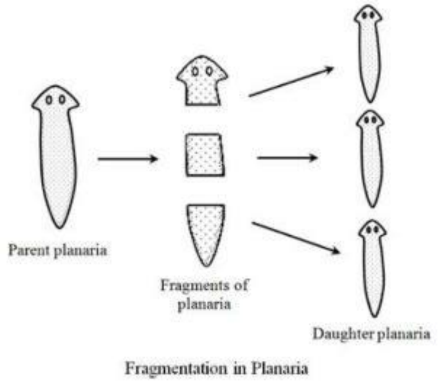
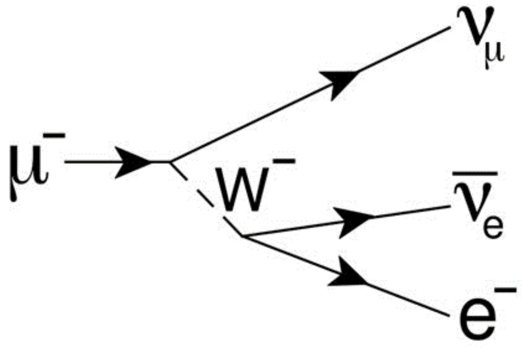
Assumptions of Physics

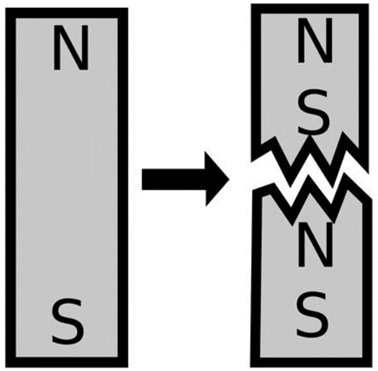
Divisible vs Reducible

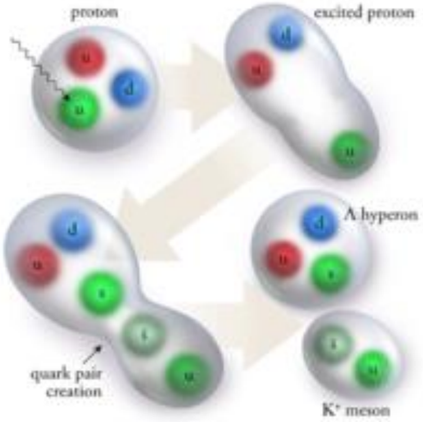




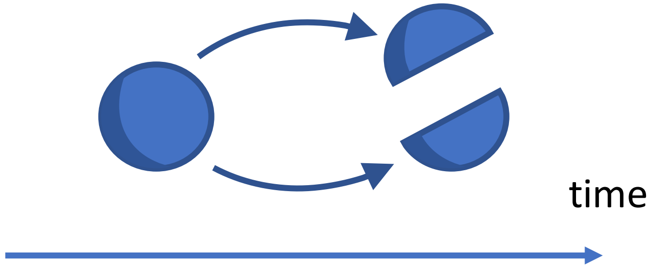
divisible but not reducible

reducible but not divisible

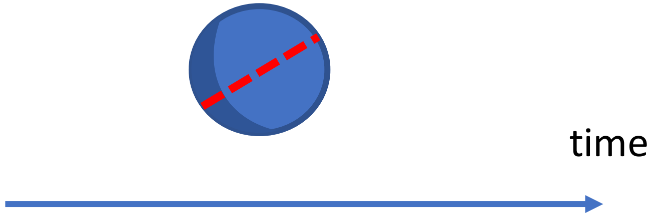




$$\mathcal{P}_t : \mathcal{S} \to \mathcal{S}_1 \times \mathcal{S}_2$$



$$\mathcal{S} \equiv \mathcal{S}_1 \times \mathcal{S}_2$$



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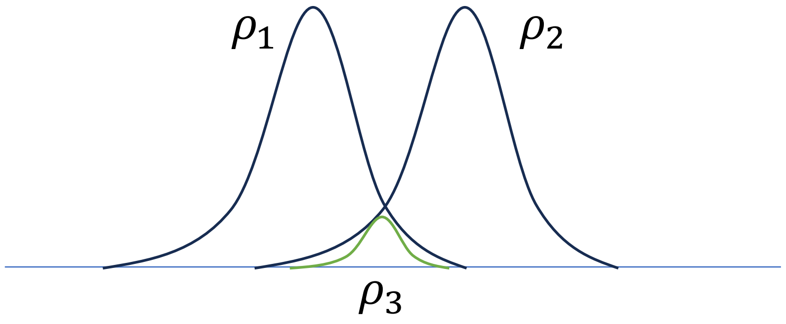
Assumptions of Physics

Reducibility in terms of ensembles



Detect overlap

Common component

$$\exists \rho_3 \quad \begin{aligned} \rho_1 &= p\rho_3 + (1-p)\rho_4 \\ \rho_2 &= \lambda\rho_3 + (1-\lambda)\rho_5 \end{aligned}$$

$$\int_X \rho_1 \rho_2 dx \neq 0$$

Not orthogonal

Classical physics: common component $\Leftrightarrow$ not orthogonal

If two ensembles have something in common, there exists an ensemble for the common part

Two ensembles can have something in common,
but the common part cannot be reliably prepared and studied

E.g. spin up and spin left

Quantum physics:
common component $\Rightarrow$ not orthogonal

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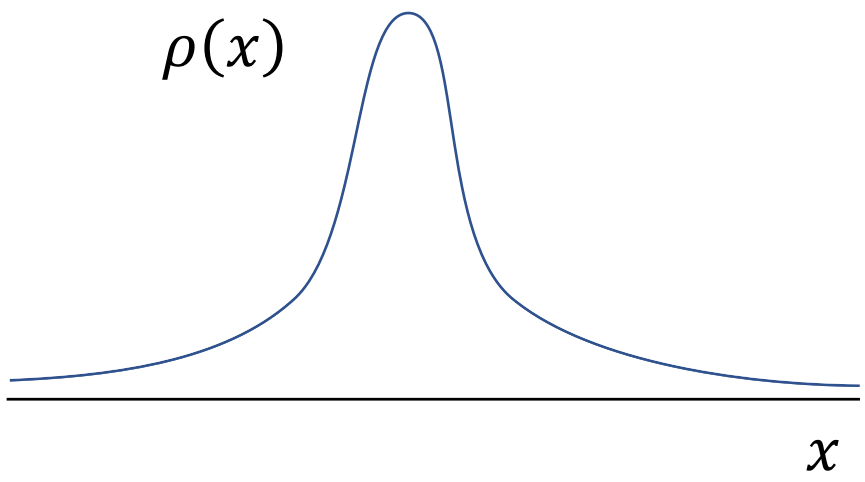
Assumptions of Physics

Statistical distribution: the matter is spread across space
i.e. 50% of the mass is in a particular region

Wave nature of the quantum system



These cases merge in quantum mechanics

The ability to tell statistical from probability distributions requires having access to the ensembles at lower entropy

Probability distribution: the matter is concentrated but "jumps around"
i.e. the whole mass is in a particular region 50% of the time

Particle nature of the quantum system

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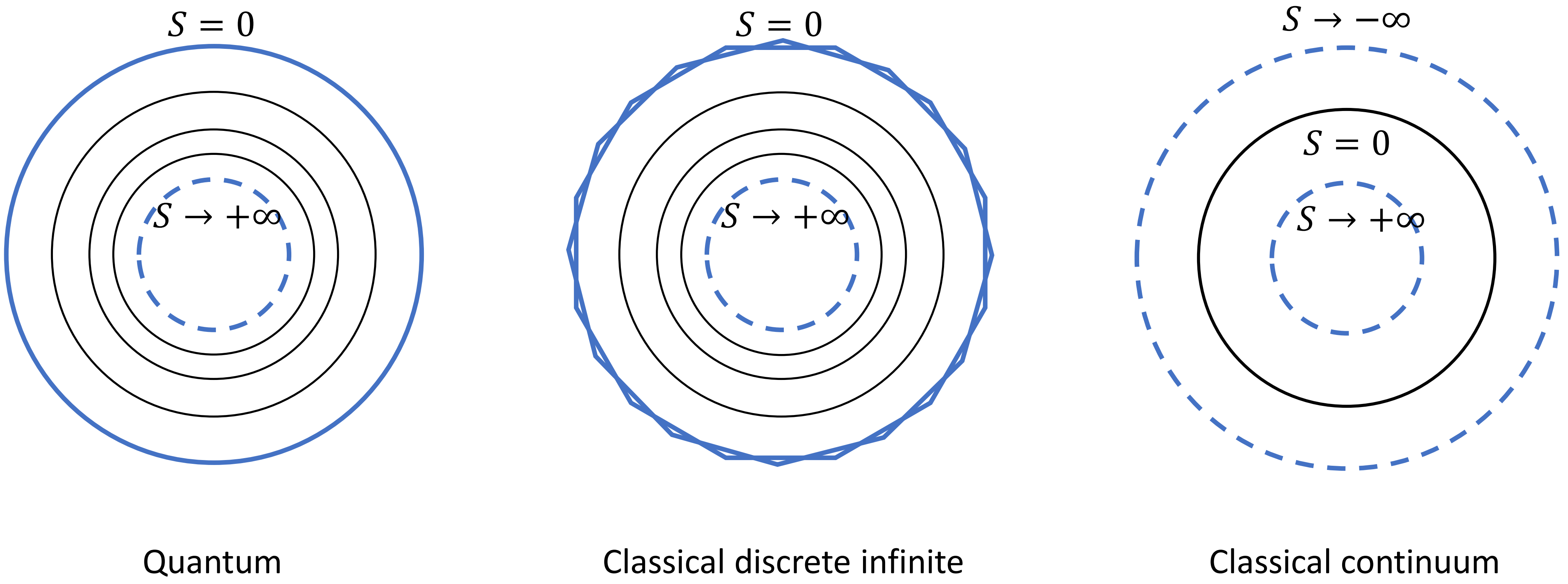
Assumptions of Physics



Quantum mechanics is a hybrid between discrete and continuum

Quantum pure states form a manifold (like classical continuum) where each state has zero entropy (like classical discrete)

Quantum mixed states have no single decomposition in terms of pure states, classical continuum mixed states have no single decomposition in terms of zero entropy states

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Assumptions of Physics

Takeaways

- Irreducibility is the key difference for quantum systems
- All quantum properties can be qualitatively understood in terms of irreducibility
- TODOs
  - Prove mathematically that it is the only difference
    (i.e. QM can be fully recovered)

Non-additive measures

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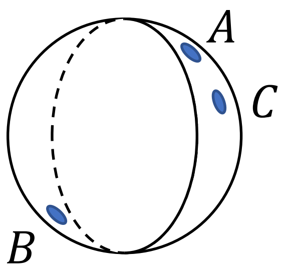
Assumptions of Physics

Need for non-additive measure

Want to generalize $S = \log \mu$



$\mu(\{A\}) = 2^0 = 1$

$\mu(\{A, B\}) = 2^1 = 2$

not additive

$\mu(\{A, C\}) < 2 = \mu(\{A\}) + \mu(\{C\})$

In quantum mechanics, literally $1 + 1 \leq 2$

	Single point		Finite continuous range	
	$\mu(U)$	$\log \mu(U)$	$\mu(U)$	$\log \mu(U)$
Counting measure $\mu(U) = \#U$ Number of points	1	0	$+\infty$	$+\infty$
Lebesgue measure $\mu([a, b]) = b - a$ Interval size	0	$-\infty$	$< \infty$	$< \infty$
"Quantized" measure $\mu(U) = \sup(2^{S(\mathrm{hull}(U))})$ Entropy over uniform distribution	1	0	$< \infty$	$< \infty$

Pick two!

1. Single point is a single case (i.e. $\mu(\{\psi\}) = 1$)
2. Finite range carries finite information (i.e. $\mu(U) < \infty$)
3. Measure is additive for disjoint sets (i.e. $\mu(\cup\, U_i) = \sum \mu(U_i)$)

Physically, we count states all else equal

Contextuality $\Longleftrightarrow$ non-additive measure

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Assumptions of Physics

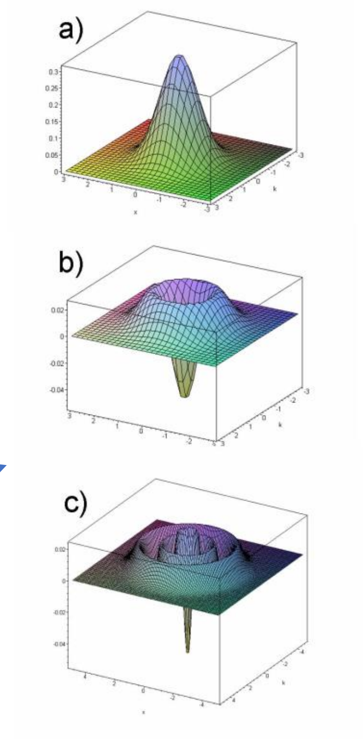
Failure of classical probability in quantum mechanics

CHSH inequality

Bell type theorems

$$|E(a,b) - E(a,b') + E(a',b') + E(a',b)| \leq 2$$

In quantum mechanics, $2 < |\cdot| \leq 2\sqrt{2}$



Wigner quasiprobability distribution

$$W(x,p) = \frac{1}{\pi\hbar}\int_X \psi^*(x+y)\psi(x-p)e^{2\imath py/\hbar}dy$$

$$|\psi(x)|^2 = \int W(x,p)dp \quad |\psi(p)|^2 = \int W(x,p)dx$$

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Assumptions of Physics

Classical probability

$\sum p(x) = 1$

$\int \rho(q,p)\,dq\,dp = 1$

Sample space (i.e. classical states)

Wigner function

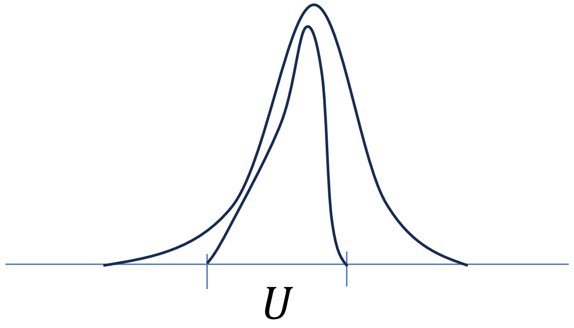
$\int W(q,p)\,dq\,dp = 1$

Not the sample space (i.e. quantum states)

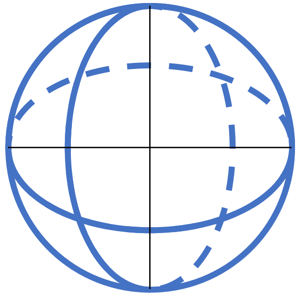
Generalized probability

Probability of a subset: weight for the biggest part that has support in that subset

$$p(x) = p(x|U)p(U) + p(x|U^C)p(U^C)$$



Maximally mixed state: probability for each pure state equals 1/2



Non-additive

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Assumptions of Physics

Takeaways

- Classical (Kolmogorov) probability does not work in QM
- Successful use of signed probability (e.g. Wigner function)
  - No physical interpretation for negative probability
- Potential use of non-additive measures
- TODOs
  - Construct a full theory of non-additive probability

Classical limit

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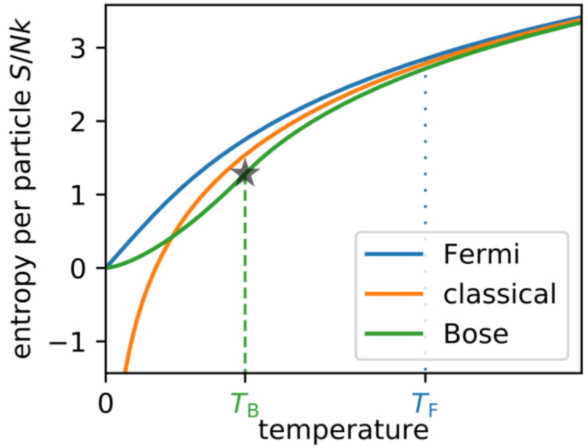
Assumptions of Physics

	slow $\left(\frac{v}{c} \ll 1\right)$	fast	
$(\hbar \to 0)$ big	Classical Mechanics	Relativity	high entropy $(S \to +\infty)$
small	Quantum Mechanics	Quantum Field Theory	low entropy



Quantum effects at large scale

Constants of nature are the same for all systems

Classical statistical mechanics
fails at low entropy

Classical system has high entropy;
$\hbar$ quantifies uncertainty at zero entropy

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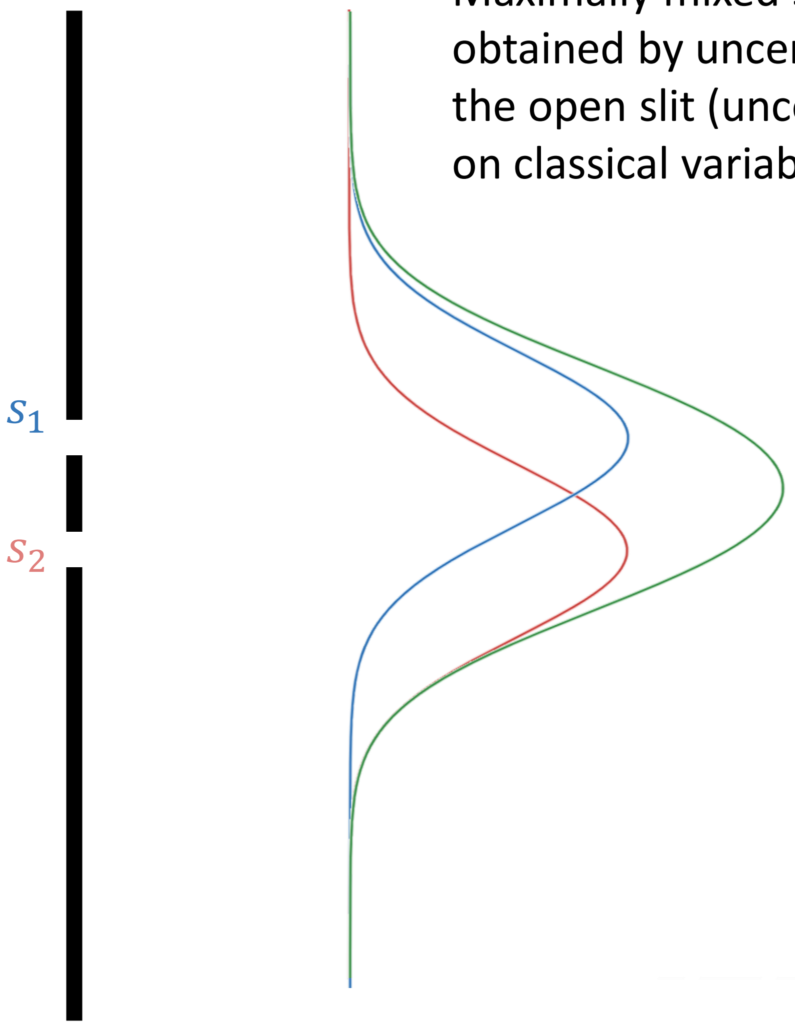
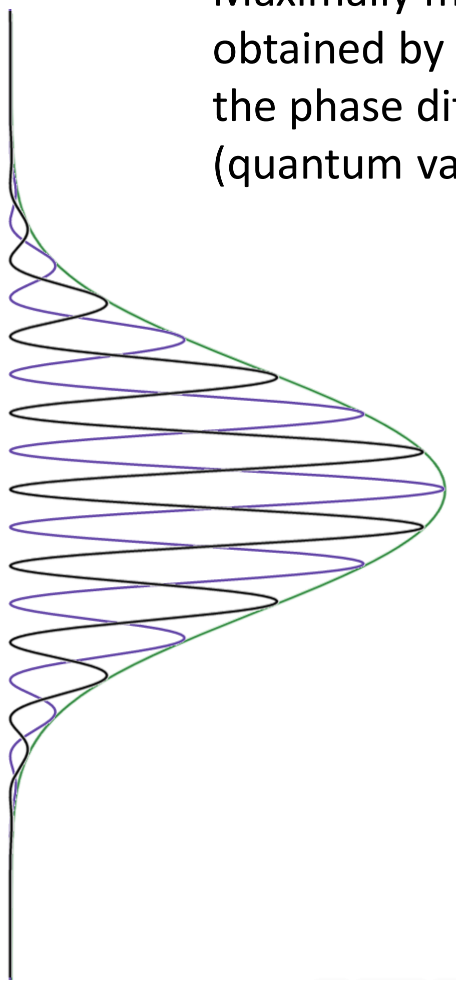
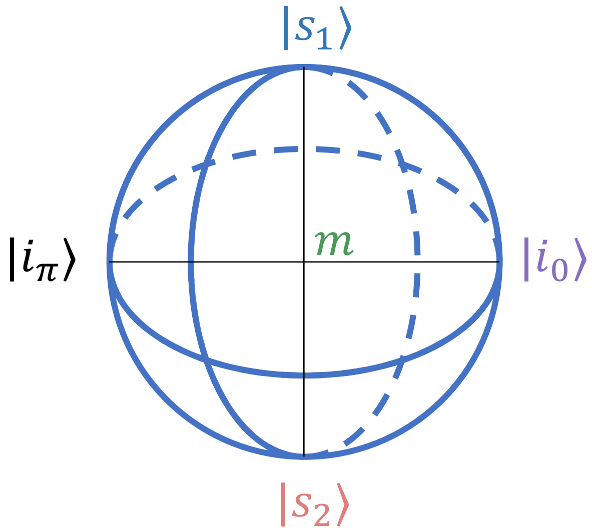
Assumptions of Physics



Maximally mixed state obtained by uncertainty on the open slit (uncertainty on classical variable)

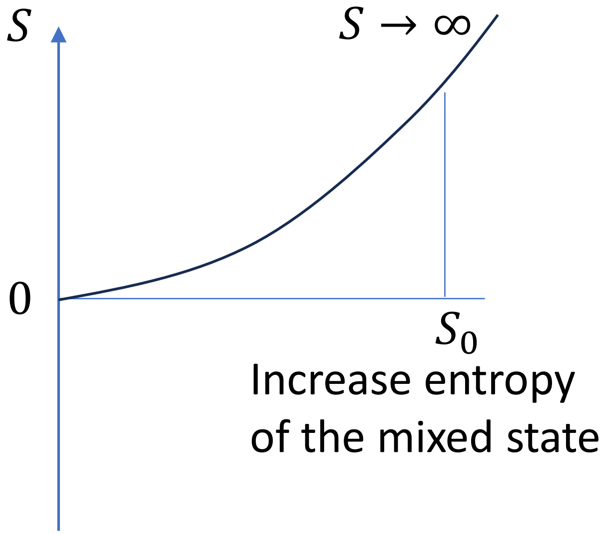


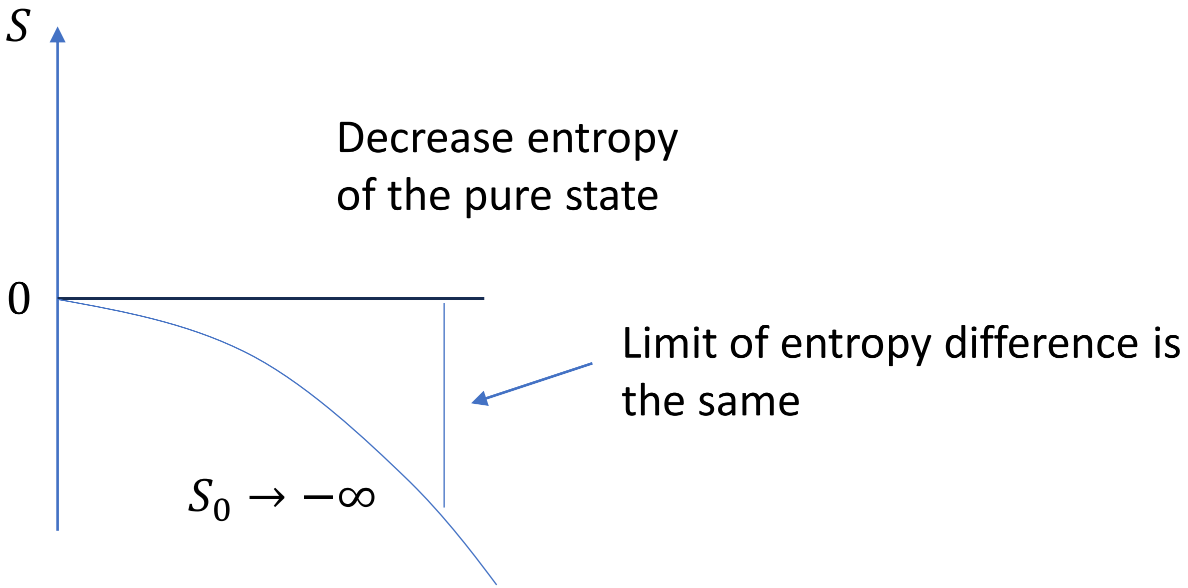
Maximally mixed state obtained by uncertainty on the phase difference (quantum variable)



Uncertainty on quantum variables can be represented by uncertainty on classical variables

May be able to recycle formal proofs $\hbar \to 0$





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Assumptions of Physics

Takeaways

- Classical mechanics may be recovered for high entropy states
- No mechanism: high entropy “hides” quantum effects
- TODOs
  - Actually prove the conjecture

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Assumptions of Physics

Wrapping it up

- Quantum mechanics can be seen as a combination of classical mechanics and thermodynamics
- Minimal interpretation: using concepts and only concepts that are strictly in the equations (e.g. ensembles in equilibrium is supported by the math)
- Main goal is to clean up all these ideas and make it a consistent theory (conceptual/mathematical) with experimental support

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Assumptions of Physics

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Assumptions of Physics

MATERIAL

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Assumptions of Physics

Hilbert spaces are precisely the Banach spaces where we have a projection for each subspace

Mathematically, it makes sense that quantum states are equilibria

A system to be assigned a state must exist in a stable way for some finite time (even if it may be very short)

Physically, it makes sense that quantum states are equilibria

Reverse Physics: Quantum mechanics

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Assumptions of Physics

Entropic nature of physical theories

Thermodynamics/Statistical mechanics are not built on top of mechanics

Mechanics is the ideal case of thermodynamics/statistical mechanics

Best preparation ⇒ pure state

Best process ⇒ map between pure states

$\mu(U)$

Symplectic manifold

ρ uniform over U

Standard probability

Information theory

$$\rho(x) = \frac{1}{\mu(U)}$$

$$H(\rho) = \log \mu(U)$$

The geometric structure of both classical and quantum mechanics is ultimately an entropic structure

We can only prepare/measure ensembles. Ensembles can offer a unified way of thinking about states.

$\langle\psi|\phi\rangle$

Projective Hilbert space

$$\rho = \frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi$$

Quantum probability

Quantum information theory

$$p(\psi|\phi) = |\langle\psi|\phi\rangle|^2$$

$$H(\rho) = H\left(\frac{1+\sqrt{p}}{2}, \frac{1-\sqrt{p}}{2}\right)$$

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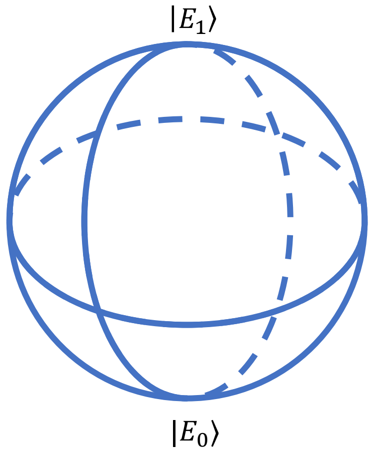
Assumptions of Physics

Recovering QM from assumptions on ensembles



Ensembles can mix ⇒ Form a convex space

Irreducibility ⇒ Extreme points in the convex space

Continuous time ⇒ Extreme points form a manifold (not discrete)

Frame-invariance ⇒ Manifold is symplectic

Homogeneity ⇒ All two dimensional subspaces are spheres

2-sphere only symplectic sphere

Is this enough to recover complex projective spaces?

Unphysicality of Hilbert spaces

Hilbert space: $\boxed{\text{complete}}\boxed{\text{inner product vector space}}$

Redundant on finite-dimensional spaces. For infinite-dimensional spaces, it allows us to construct states with infinite expectation values from states with finite expectation values

Exactly captures measurement probability/entropy of mixtures and superposition/statistical mixing

Physically required

Extremely physically suspect!!!

⇒ Thus requires us to include unitary transformations (e.g. change of representations and finite time evolution) that change finite expectation values into infinite ones

Suppose we require all polynomials of position and momentum to have finite expectation

⇒ Schwartz space

Maybe more physically appropriate?

Closed under Fourier transforms
Used as starting point for theories of distributions

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Assumptions of Physics

QM postulates revisited

⇒ Recover mathematical structure of quantum mechanics from properties of ensembles

State postulate:
states are rays of a complex vector space

→ Recovered from properties of ensembles and rules of ensemble mixing

Measurement postulate:
projection measurement and Born rule

→ Projections as processes with equilibria

Born rule recoverable from entropy of mixing

Composite system postulate:
tensor product for composite system

→ Derived from other postulates

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Evolution postulate:
unitary evolution (Schrödinger equation)

→ Deterministic/reversible evolution

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Assumptions of Physics

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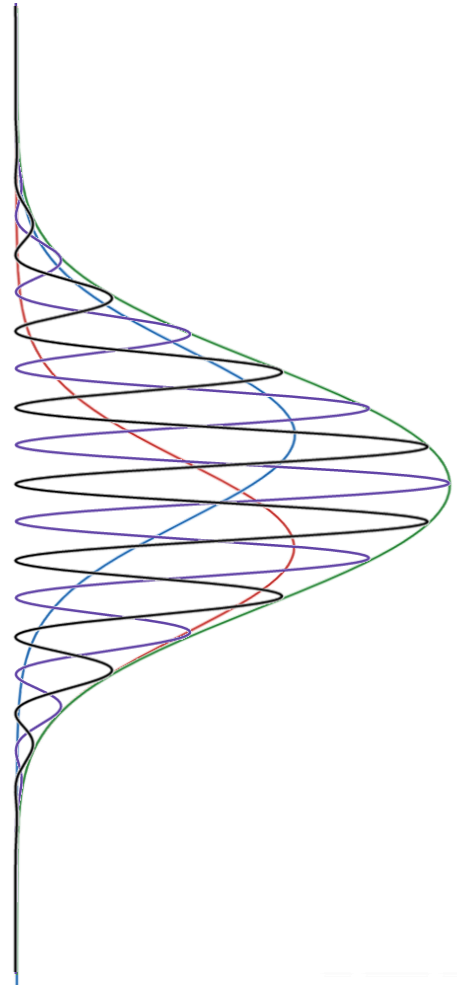
Assumptions of Physics



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Assumptions of Physics