Assumptions of Physics:
a new principled approach to the foundations of physics

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Assumptions of Physics

Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

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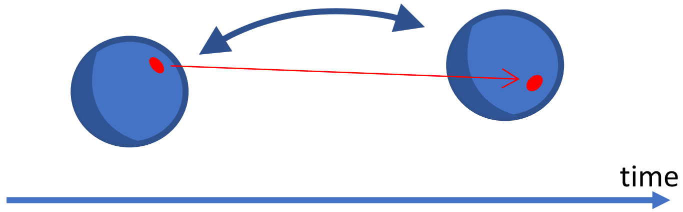
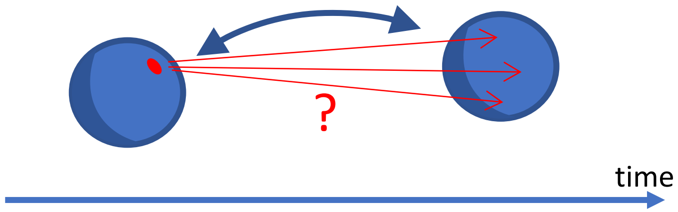
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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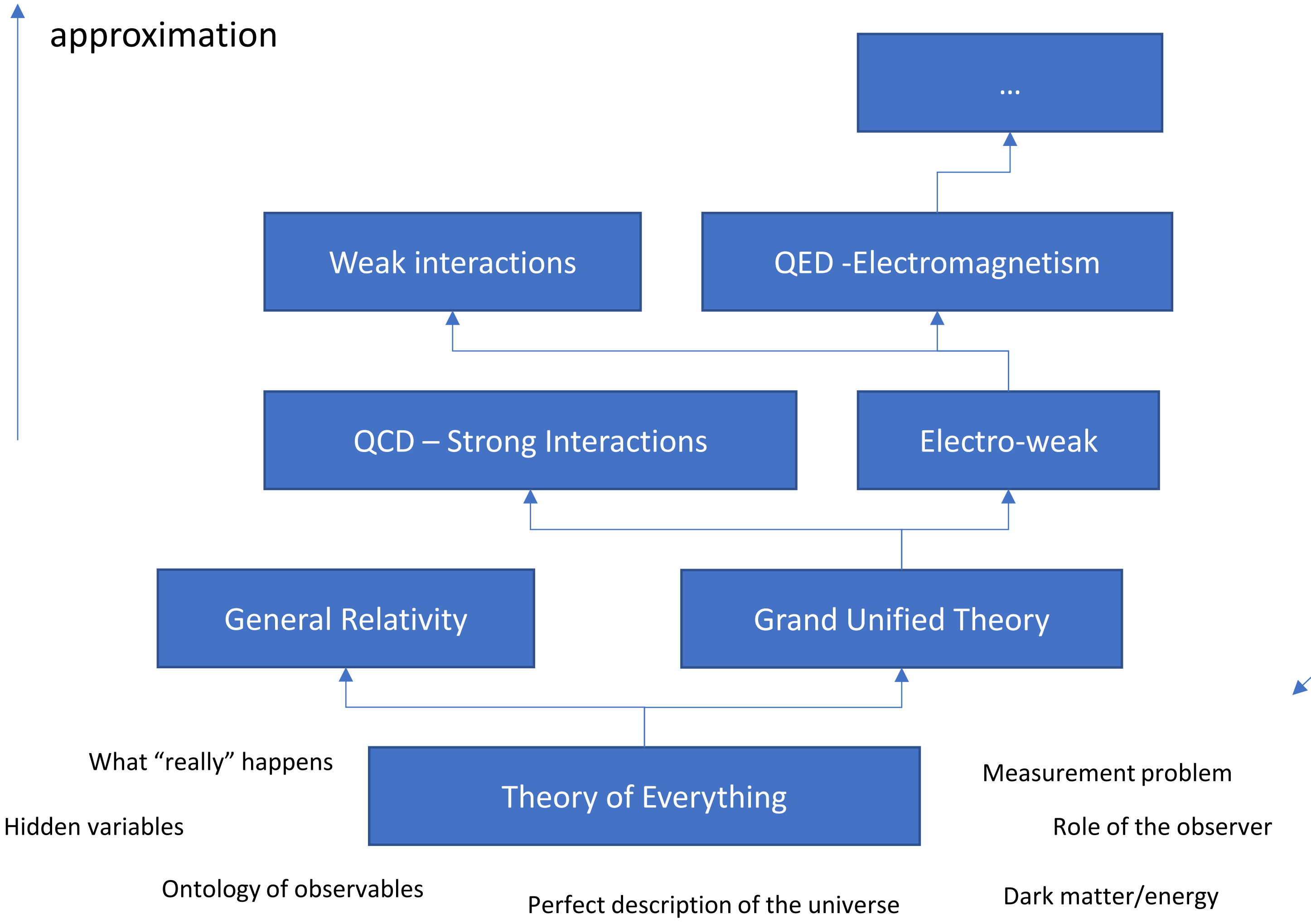
Standard view of the foundations of physics



Goal of physics is to find the true laws of the universe!

The "real" physics!

The foundations of physics!

Everything else is an approximation

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We found:

Experimental verifiability $\Rightarrow$ topologies and σ-algebras

Geometrical structures $\Leftrightarrow$ Entropic structures

Hamiltonian evolution $\Leftrightarrow$ det-rev/isolation + DOF independence

Massive particles and potential forces $\Leftrightarrow$ + Kinematic eq

Physical requirements and assumptions drive most of the theoretical apparatus

~~Goal of physics is to find the true laws of the universe!~~

Less productive point of view

Goal of physics is to find models that can be empirically tested

More productive point of view

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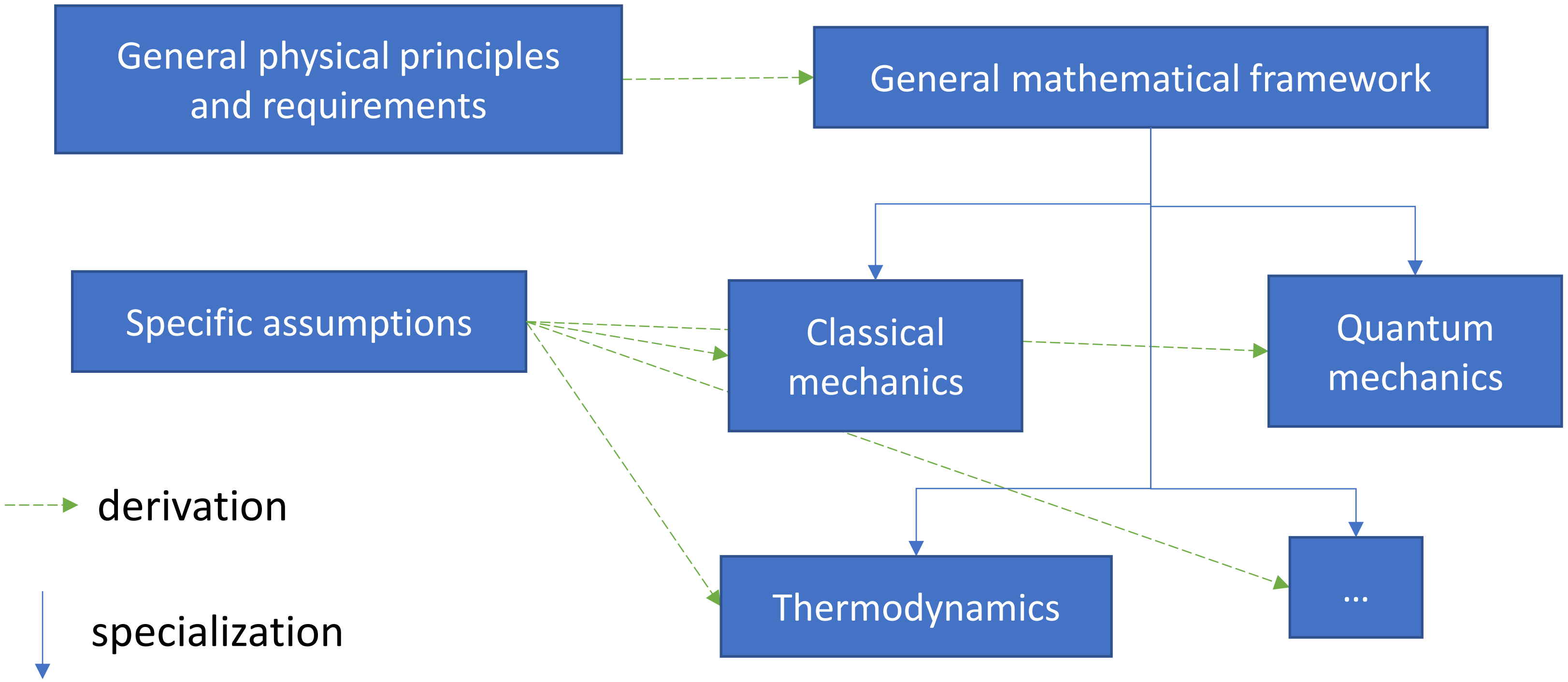
Our view of the foundations of physics



Foundations of physics
⇕
The theory of physical models

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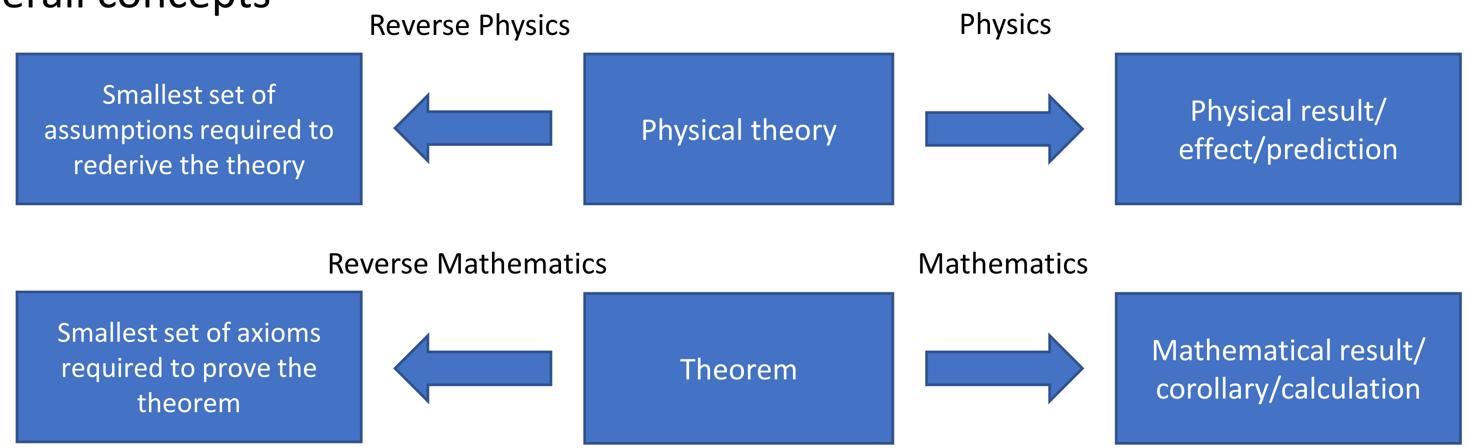
Find the right overall concepts

Reverse physics:

Start with the equations, reverse engineer physical assumptions/principles

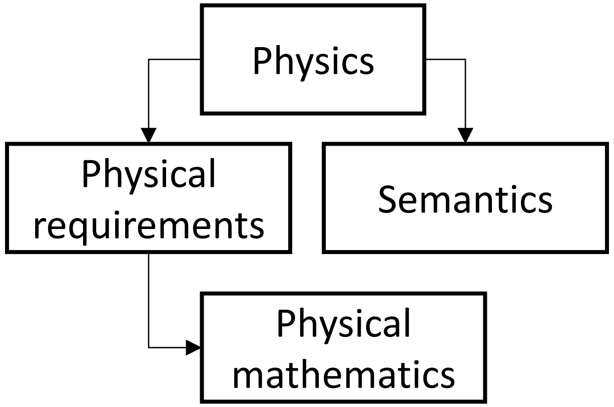
Found Phys **52**, 40 (2022)



Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

Physical mathematics:

Start from scratch and rederive all mathematical structures from physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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Reverse Physics

Assumptions of Physics,
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J. Phys. Commun. **2** 045026 (2018)

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Assumptions of Physics

Assumption DR (Determinism and Reversibility). *The system undergoes deterministic and reversible evolution. That is, specifying the state of the system at a particular time is equivalent to specifying the state at a future (determinism) or past (reversibility) time.*

The displacement field is divergenceless: $\partial_a S^a = 0$ (DR-DIV)
The Jacobian of time evolution is unitary: $\left|\partial_b \hat{\xi}^a\right| = 1$ (DR-JAC)
Densities are conserved through the evolution: $\hat{\rho}(\hat{\xi}^a) = \rho(\xi^b)$ (DR-DEN)
Volumes are conserved through the evolution: $d\hat{\xi}^1 \cdots d\hat{\xi}^n = d\xi^1 \cdots d\xi^n$ (DR-VOL)

The evolution is deterministic and reversible. (DR-EV)
The evolution is deterministic and thermodynamically reversible (DR-THER)
The evolution conserves information entropy (DR-INFO)
The evolution conserves the uncertainty of peaked distributions (DR-UNC)

Assumption IND (Independent DOFs). *The system is decomposable into independent degrees of freedom. That is, the variables that describe the state can be divided into groups that have independent definition, units and count of states.*

The system is decomposable into independent DOFs (IND-DOF)
The system allows statistically independent distributions over each DOF (IND-STAT)
The system allows informationally independent distributions over each DOF (IND-INFO)
The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF (IND-UNC)

$\Updownarrow$

The evolution leaves ω_{ab} invariant: $\hat{\omega}_{ab} = \omega_{ab}$ (DI-SYMP)
The evolution leaves the Poisson brackets invariant (DI-POI)
The rotated displacement field is curl free: $\partial_a S_b - \partial_b S_a = 0$ (DI-CURL)

$\Updownarrow$

$$d_t q^i = \partial_{p_i} H$$
$$d_t p_i = -\partial_{q^i} H$$

$$S_a = S^b \omega_{ba} = \partial_a H$$

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Assumptions of Physics

Reversing the principle of stationary action

DR

$$\nabla \cdot \vec{S} = 0$$

No state is “lost” or “created” as time evolves

$[p, 0, -H(q,p)]$

$$\vec{S} = -\nabla \times \vec{\theta}$$

(Minus sign to recover Ham eq)

KE

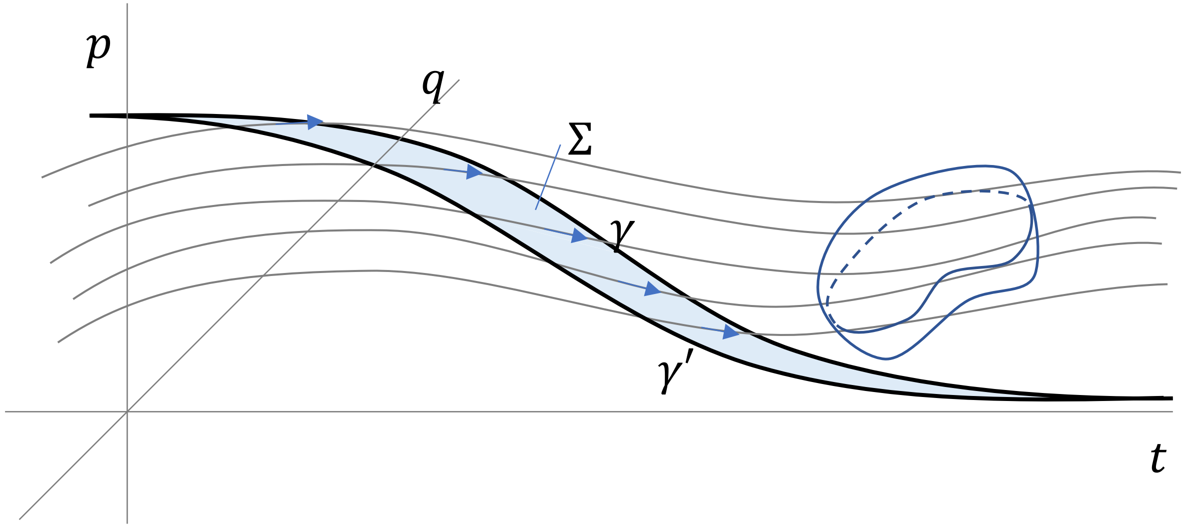
$p\frac{dq}{dt} + 0\frac{dp}{dt} - H\frac{dt}{dt}$

$$\mathcal{A}[\gamma] = \int_\gamma L dt = \int_\gamma \vec{\theta} \cdot d\vec{\gamma}$$

Sci Rep **13**, 12138 (2023)

unphysical

The action is the line integral of the vector potential of the flow of states



Variation of the action

$$\delta\mathcal{A}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$
$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent, physical!

Variation of the action measures the flow of states (physical).
Variation = 0 ⇒ flow of states tangent to the path.

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Is the uncertainty principle really a feature of quantum mechanics alone?

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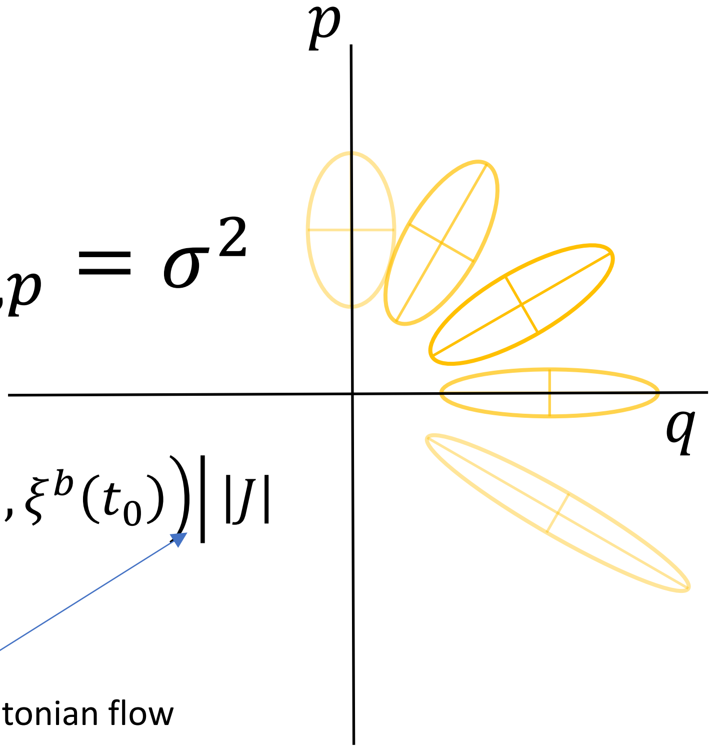
Determinant of covariance matrix:

evolution of covariance matrix



$$\left|cov\left(\xi^a, \xi^b\right)\right| = \begin{vmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{vmatrix} = \sigma_q^2 \sigma_p^2 - cov_{q,p}^2 = \sigma^2$$

Peaked distribution
⇒ flow is almost linear
⇒ covariance matrix transforms linearly

$$\left|cov\left(\xi^a(t), \xi^b(t)\right)\right| = |J| \left|cov\left(\xi^a(t_0), \xi^b(t_0)\right)\right| |J|$$

1 under Hamiltonian flow

$$\sigma_q^2(t)\sigma_p^2(t) - cov_{q,p}^2(t) = \sigma^2(t_0)$$

$$\sigma_q(t)\sigma_p(t) \geq \sigma(t_0)$$

Uncertainty is bounded during classical evolution

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Assumptions of Physics

Let's plot entropy against uncertainty

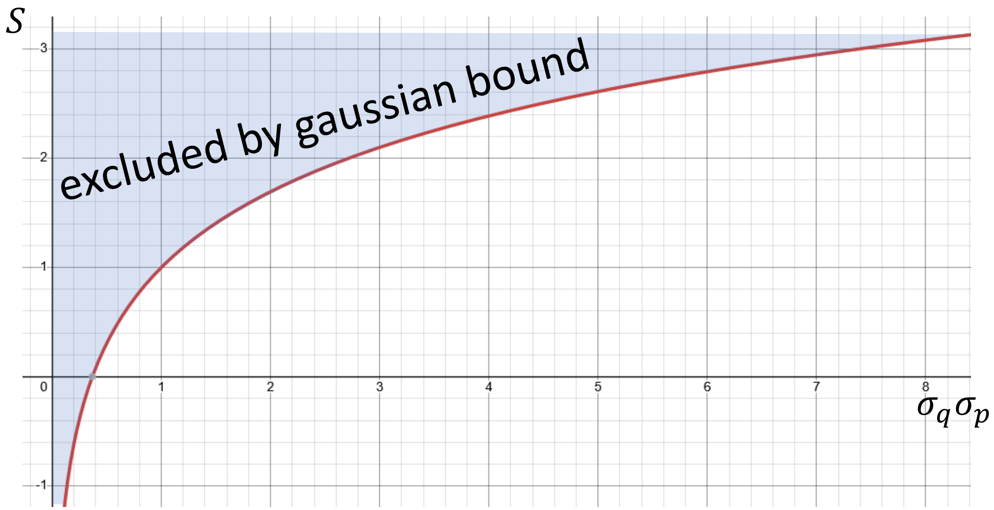
$$S(\rho) = -\int \rho \log h\rho \, dqdp$$

Fixes units

Uniform distribution over volume h has zero entropy

$$S(\rho) \leq \log 2\pi e \frac{\sigma_q \sigma_p}{h}$$

Gaussian maximizes entropy for a given uncertainty



$$\sigma_q \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$

Entropy puts a lower bound on the uncertainty

Hamiltonian evolution conserves entropy

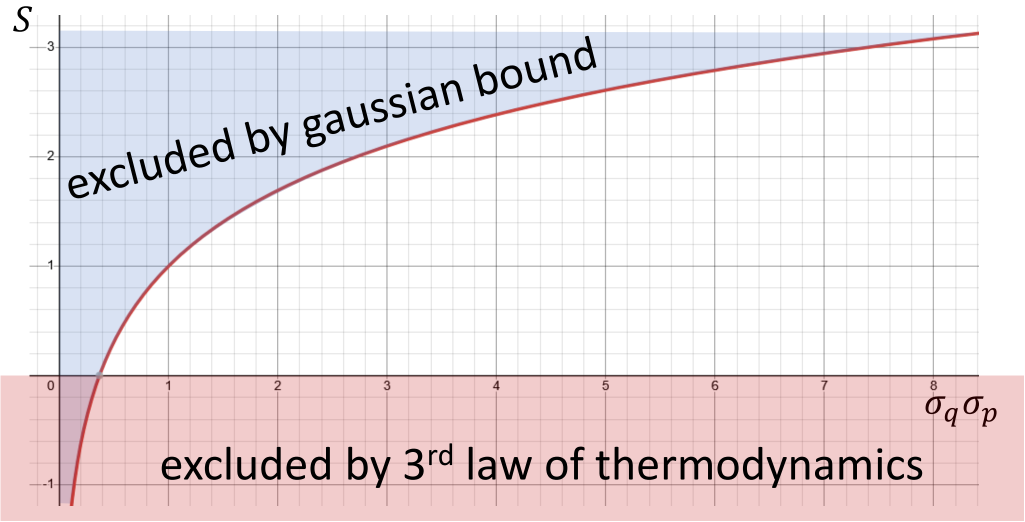
Is there anything that puts a lower bound on the entropy?

Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.

G. N. Lewis and M. Randall, Thermodynamics and the free energy of chemical substances (McGraw-Hill, 1923)

The third law of thermodynamics!

Third law puts a lower bound on the entropy which puts a lower bound on the uncertainty



$$\sigma_q \sigma_p \geq \frac{\hbar}{e} e^0 = \frac{\hbar}{e}$$

Classical uncertainty principle!

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Comparing theories

classical

$$\sigma_q \sigma_p \geq \frac{\hbar}{e}$$

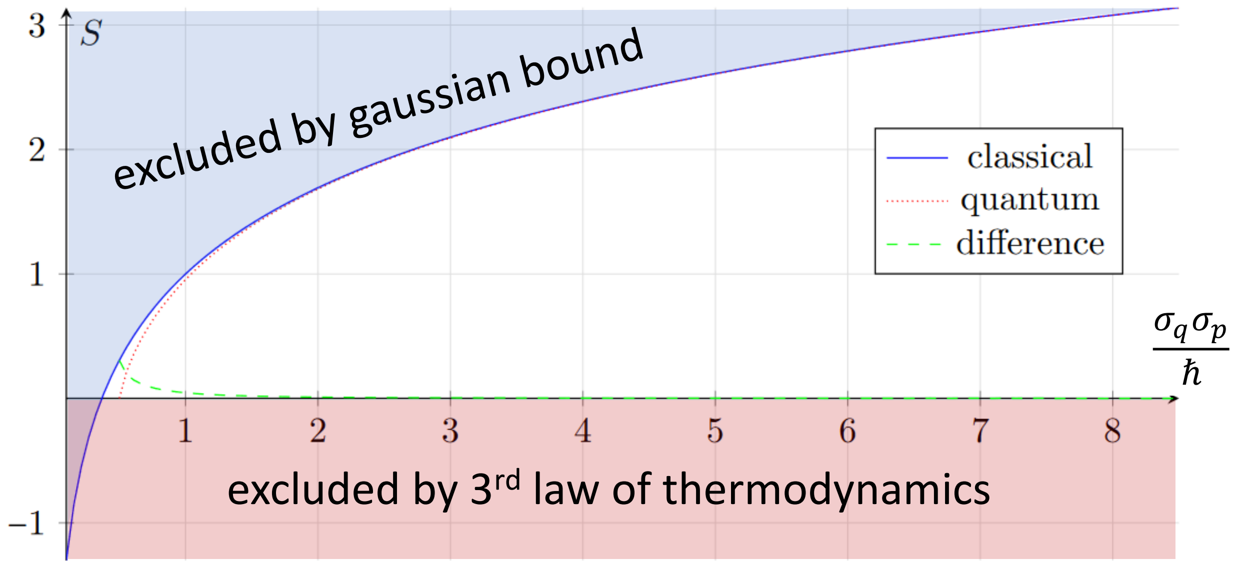
2.71828…

quantum

$$\sigma_q \sigma_p \geq \frac{\hbar}{2}$$

Entropy of quantum states is already non-negative

The gaussian bound quickly becomes very similar across theories



$$S_C = \ln e\sigma$$

$$S_Q = \left(\sigma + \tfrac{1}{2}\right) \ln \left(\sigma + \tfrac{1}{2}\right) - \left(\sigma - \tfrac{1}{2}\right) \ln \left(\sigma - \tfrac{1}{2}\right)$$

Quantum mechanics incorporates the third law

Classical mechanics does not

Is this the only difference?

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Suppose the lower bound on the entropy is the only difference, then in the limit of high entropy of quantum mechanics we should recover classical mechanics

Can we?

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Classical mechanics as high entropy limit?

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greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

Reply | Reply All | Forward | ...

Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

[Submitted on 1 Nov 2024 (v1), last revised 3 Dec 2024 (this version, v2)]

Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

Comments: 14 pages, 3 figures
Subjects: **Quantum Physics (quant-ph)**
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(or arXiv:2411.00972v2 **[quant-ph]** for this version)
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References & Citations

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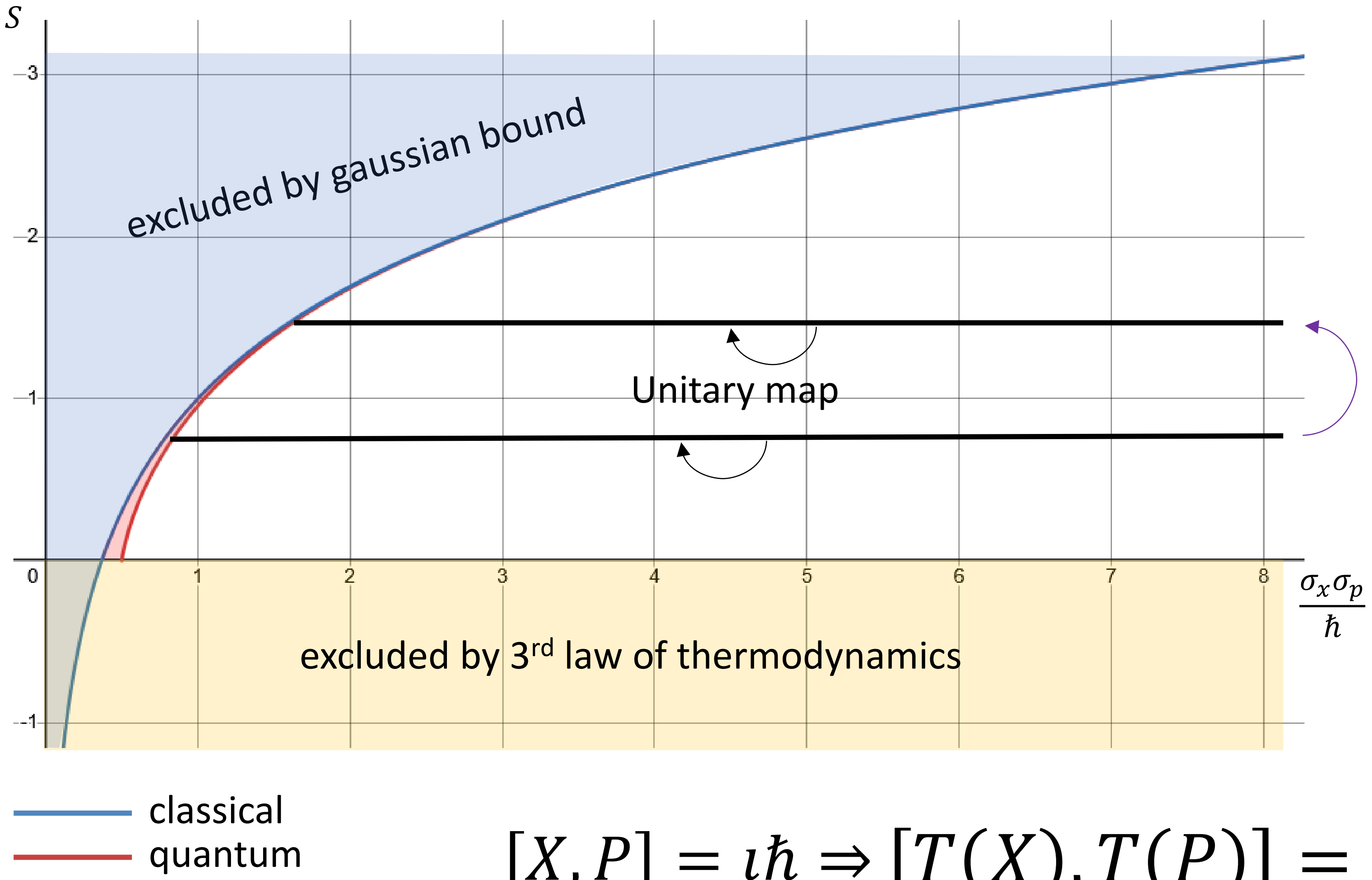
Assumptions of Physics

Looking for a map $R(\rho)$ that increases entropy of all mixed states, such that every level set of entropy maps to another level set



$\Rightarrow$ Unitary trans. must be mapped to unitary trans.

$R(\rho)$ Entropy increasing map

$$[X, P] = \imath\hbar \Rightarrow [T(X), T(P)] = \lambda\imath\hbar$$

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Another perspective: move the pure states to minus infinite entropy

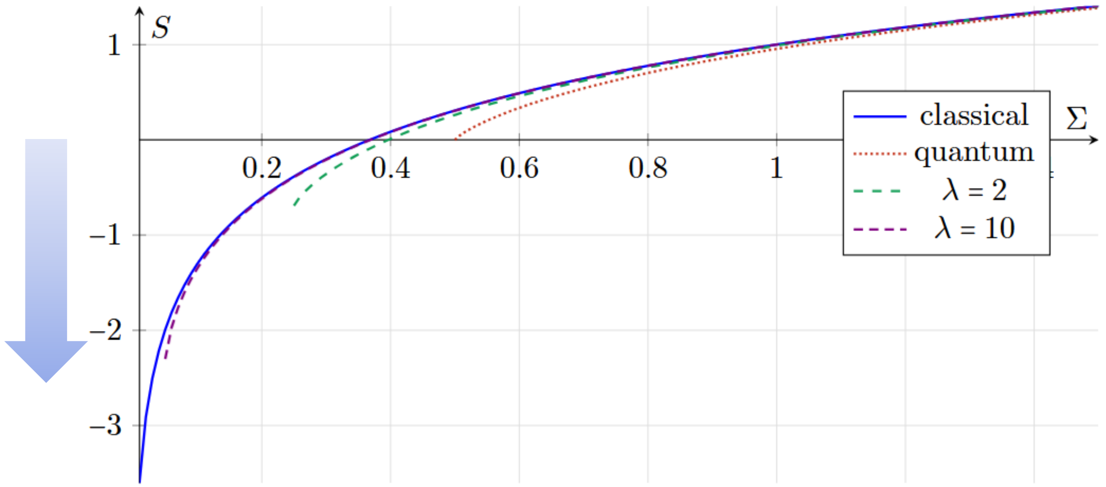
Instead of

$$[X, P] = \imath\hbar \qquad [T(X), T(P)] = \lambda\imath\hbar$$

Redefine original space such that

$$[\hat{X}, \hat{P}] = \frac{\imath\hbar}{\lambda} \qquad [T(\hat{X}), T(\hat{P})] = \imath\hbar$$

$$\lambda \to \infty \Rightarrow \frac{\hbar}{\lambda} \to 0$$



Mathematically equivalent to lowering the entropy of a pure state to $-\infty$, or $\hbar \to 0$ (group contraction)

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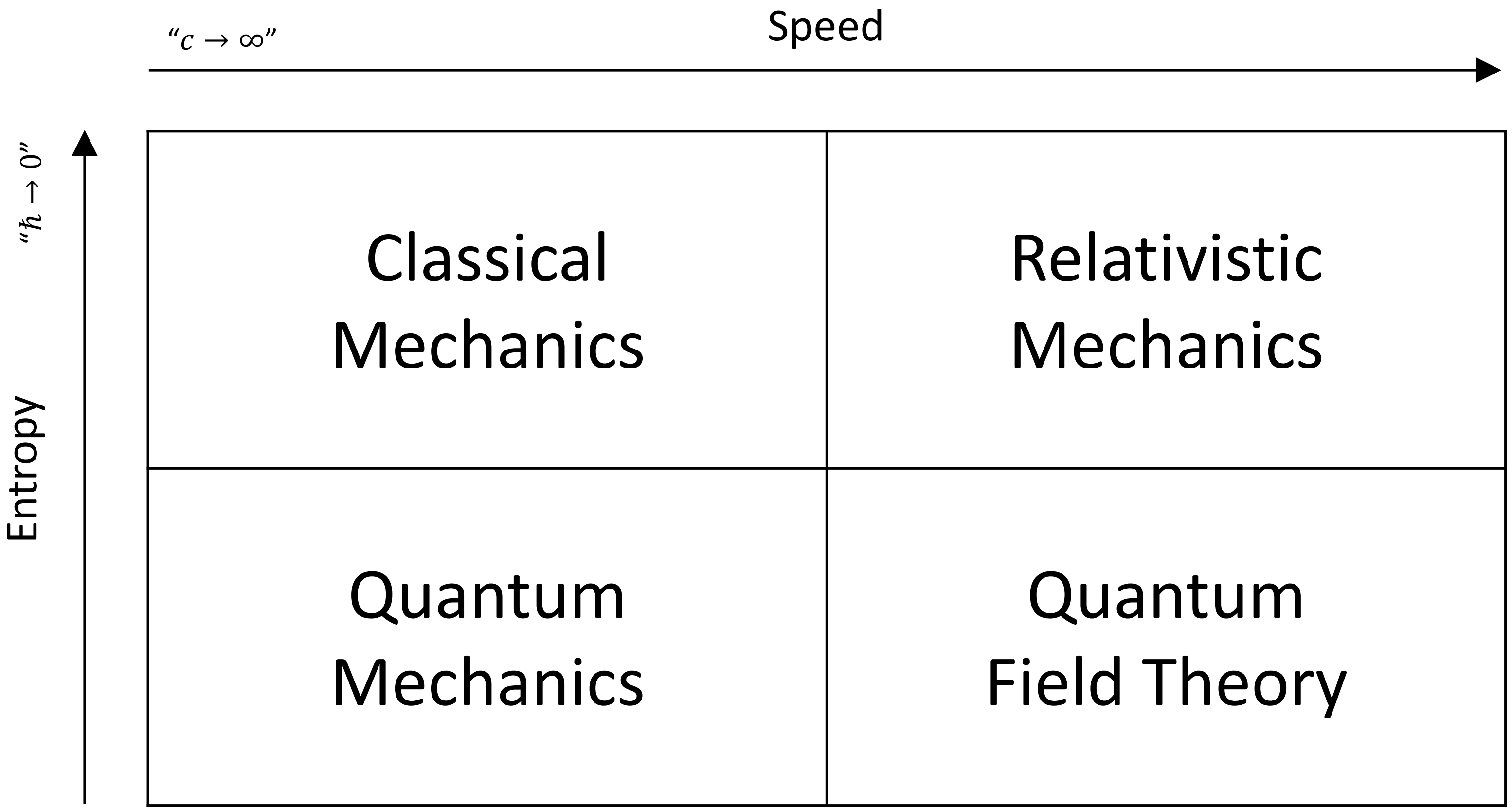

No-mechanism limit
(same as non-relativistic limit)

$$\{A, B\} \longrightarrow \frac{[A, B]}{\imath\hbar}$$

Dirac's correspondence principle: give me a theory with an entropic lower bound that recovers the classical one at high entropy

Only one way to do it

Moyal bracket is the unique one-parameter Lie-algebraic deformation of the Poisson bracket

Quantizing a classical theory means putting a lower bound on the entropy

Geometry is entropy!

The geometric structures of both classical and quantum mechanics are equivalent to the entropic structure

$\mu(U) = \int_U \omega^n$

Symplectic manifold

ρ uniform over U

Standard probability

$\rho(x) = \frac{1}{\mu(U)}$

Information theory

$S(\rho) = \log \mu(U)$

Thermodynamics/Statistical mechanics are not built on top of mechanics

Mechanics is the ideal case of thermodynamics/statistical mechanics

$\langle \psi | \phi \rangle$

Projective Hilbert space

$\rho = \frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi$

Quantum probability

$p(\psi|\phi) = |\langle \psi | \phi \rangle|^2$

Quantum information theory

$S(\rho) = S\left(\frac{1+\sqrt{p}}{2}, \frac{1-\sqrt{p}}{2}\right)$

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Assumptions of Physics

Reverse Physics tells us there are not many "independent concepts" in a physical theory

Extracting principles/assumptions behind the laws gives us solid intuition that cuts across fields and leads to new insights/results

But: the only way to be sure we got all the concepts is to derive all the math from scratch

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Physical mathematics

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Assumptions of Physics

Examples of unphysical mathematics

In differential geometry, tangent vectors are derivations

$$v : C^\infty(X) \to \mathbb{R}$$

$$v(f) = v^i \, \partial_i(f)\Big|_P$$

component

basis

In polar coordinates

$$\partial_r + \partial_\theta = ???$$

$[\mathrm{m}]$ $[\mathrm{rad}]$

Doesn't work with units

In phase space

$$\partial_q + \partial_p = ???$$

$[\mathrm{m}]$ $[\mathrm{Kg\ m\ s^{-1}}]$

Mathematically precise $\not\Rightarrow$ physically precise

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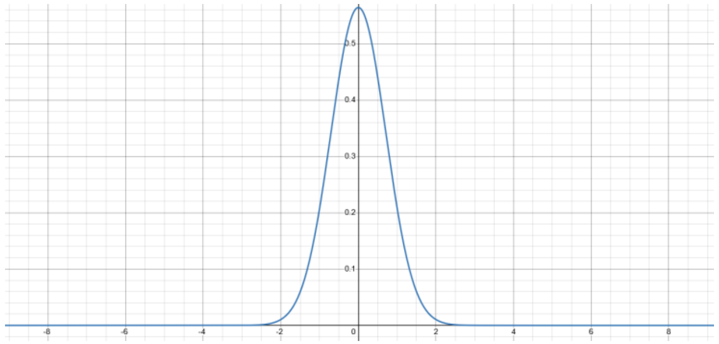
Assumptions of Physics

Quantum states represented by L^2 Hilbert space

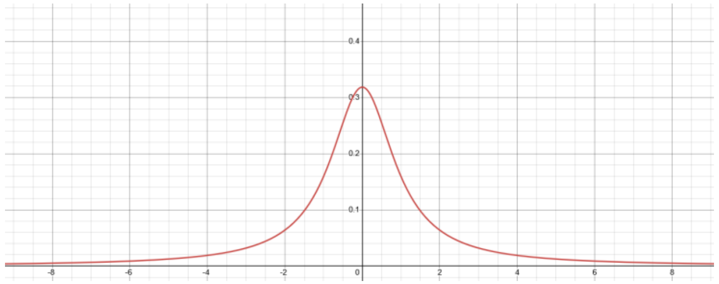
$$\psi(x) = \sqrt{\frac{e^{-x^2}}{\sqrt{\pi}}} \qquad \int |\psi|^2 dx = 1$$

$$\rho_\psi(x) = \frac{e^{-x^2}}{\sqrt{\pi}} \qquad \langle X^2 \rangle_\psi = \frac{1}{2}$$



$$\phi(y) = \sqrt{\frac{1}{\pi(y^2+1)}} \qquad \int |\phi|^2 dx = 1$$

$$\rho_\phi(y) = \frac{1}{\pi(y^2+1)} \qquad \langle Y^2 \rangle_\phi \to \infty$$



Different observers see finite/infinite expectation

$$y = \tan\left(\frac{\pi}{2}\mathrm{erf}(x)\right)$$

$$\psi(y) = \psi(x)\sqrt{\frac{dx}{dy}}$$

Expectation can have finite/infinite oscillations

$$x(x_0, t) = x_0 \cos^2\frac{\pi t}{2} + \tan\left(\frac{\pi}{2}\mathrm{erf}(x_0)\right)\sin^2\frac{\pi t}{2}$$

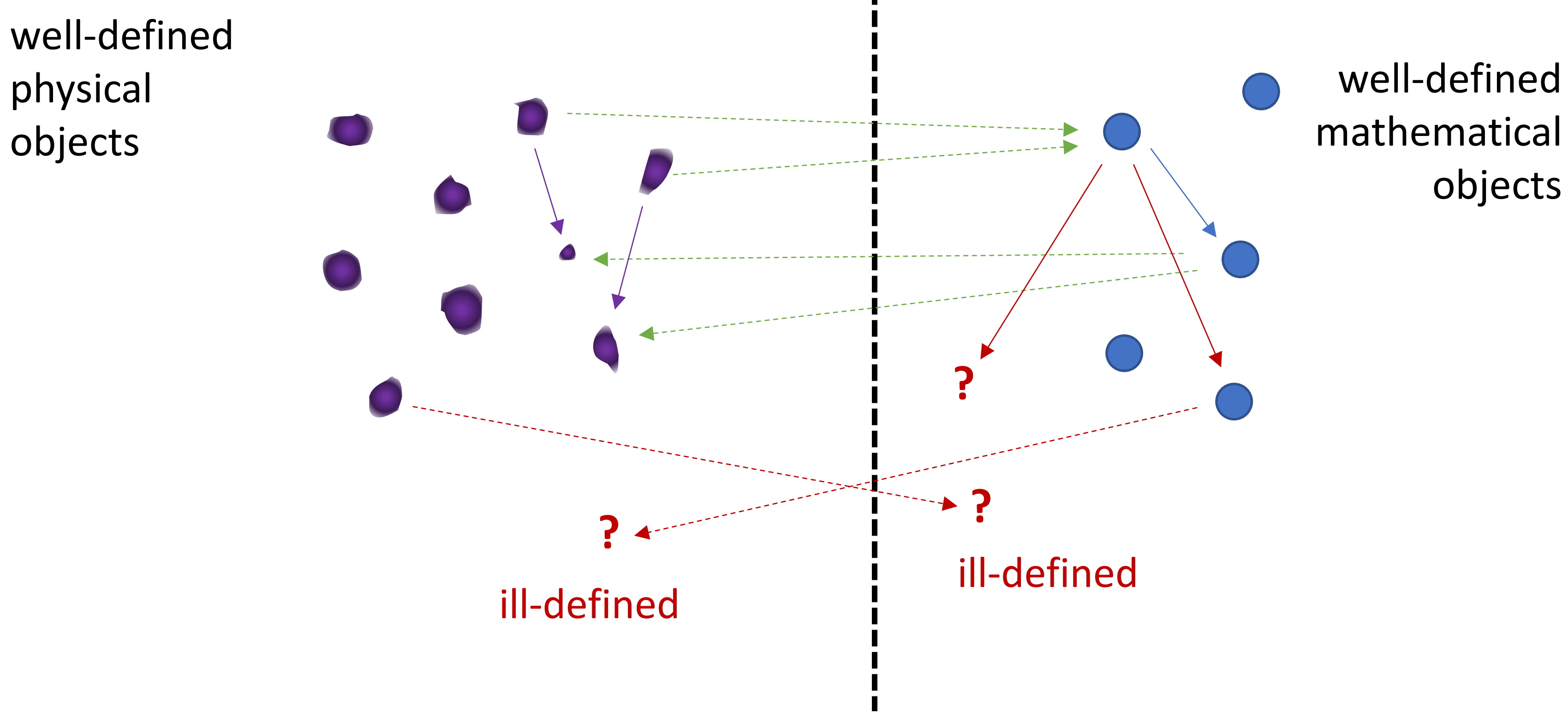
Every continuous linear operator defined on the whole Hilbert space is bounded ⇒ position/momentum/energy/number of particles are not defined on the whole Hilbert space!!!

Physical world

(informal system)

Mathematical representation

(formal system)



Current state of the art in theoretical physics

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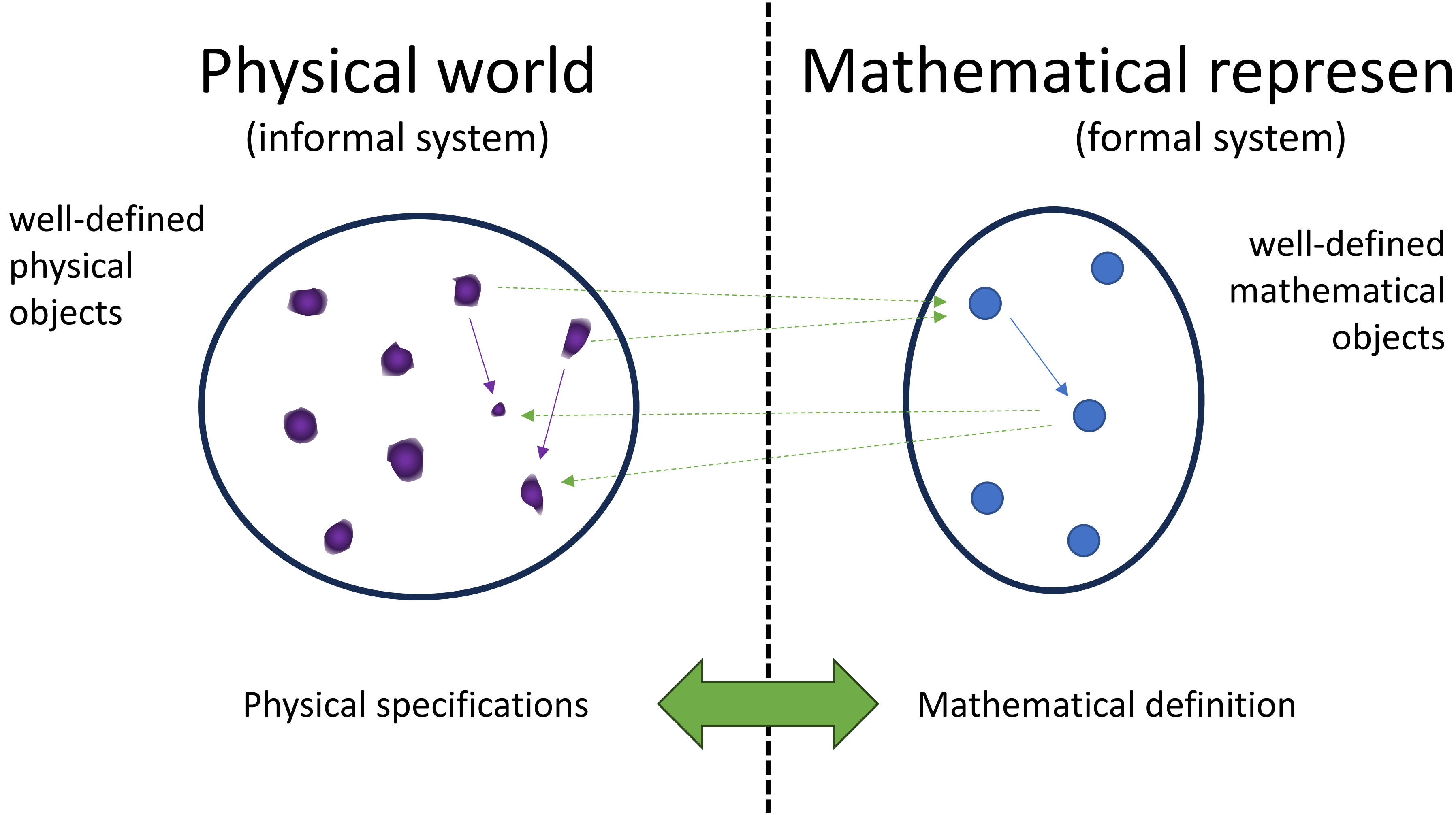
Assumptions of Physics

Physical world
(informal system)

Mathematical representation
(formal system)



A mathematical definition is **physical** if it captures and only captures an aspect of the physical system

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble.*

Informal intuitive statement

(something that makes sense to a physicist or an engineer)

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Assumptions of Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+ : [0,1] \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ called* ***mixing****, noted with the infix notation $p\mathsf{a} + \bar{p}\mathsf{b}$, with the following properties:*

- ***Continuity:*** *the map $+(p, \mathsf{a}, \mathsf{b}) \to p\mathsf{a} + \bar{p}\mathsf{b}$ is continuous (with respect to the product topology of $[0,1] \times \mathcal{E} \times \mathcal{E}$)*
- ***Identity:*** $1\mathsf{a} + 0\mathsf{b} = \mathsf{a}$
- ***Idempotence:*** $p\mathsf{a} + \bar{p}\mathsf{a} = \mathsf{a}$ *for all* $p \in [0,1]$
- ***Commutativity:*** $p\mathsf{a} + \bar{p}\mathsf{b} = \bar{p}\mathsf{b} + p\mathsf{a}$ *for all* $p \in [0,1]$
- ***Associativity:*** $p_1\mathsf{e}_1 + \bar{p}_1\left(\overline{\left(\frac{p_3}{\bar{p}_1}\right)}\mathsf{e}_2 + \frac{p_3}{\bar{p}_1}\mathsf{e}_3\right) = \bar{p}_3\left(\frac{p_1}{\bar{p}_3}\mathsf{e}_1 + \overline{\left(\frac{p_1}{\bar{p}_3}\right)}\mathsf{e}_2\right) + p_3\mathsf{e}_3$ *where* $p_1 + p_3 \leq 1$

Informal intuitive statement

(something that makes sense to a physicist or an engineer)

Formal requirement

(something a mathematician will find precise)

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Assumptions of Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+ : [0,1] \times \mathcal{E} \times \mathcal{E} \to \mathcal{E}$ called* ***mixing****, noted with the infix notation $p\mathsf{a} + \bar{p}\mathsf{b}$, with the following properties:*

- ***Continuity****: the map $+(p, \mathsf{a}, \mathsf{b}) \to p\mathsf{a} + \bar{p}\mathsf{b}$ is continuous (with respect to the product topology of $[0,1] \times \mathcal{E} \times \mathcal{E}$)*
- ***Identity****: $1\mathsf{a} + 0\mathsf{b} = \mathsf{a}$*
- ***Idempotence****: $p\mathsf{a} + \bar{p}\mathsf{a} = \mathsf{a}$ for all $p \in [0,1]$*
- ***Commutativity****: $p\mathsf{a} + \bar{p}\mathsf{b} = \bar{p}\mathsf{b} + p\mathsf{a}$ for all $p \in [0,1]$*
- ***Associativity****: $p_1\mathsf{e}_1 + \bar{p}_1\left(\left(\overline{\frac{p_3}{\bar{p}_1}}\right)\mathsf{e}_2 + \frac{p_3}{\bar{p}_1}\mathsf{e}_3\right) = \bar{p}_3\left(\frac{p_1}{\bar{p}_3}\mathsf{e}_1 + \left(\overline{\frac{p_1}{\bar{p}_3}}\right)\mathsf{e}_2\right) + p_3\mathsf{e}_3$ where $p_1 + p_3 \leq 1$*

Justification. This axiom captures the ability to create a mixture merely by selecting between the output of different processes. Let e_1 and e_2 be two ensembles that represent the output of two different processes P_1 and P_2. Let a selector S_p be a process that outputs two symbols, the first with probability p and the second with probability $\bar{p}$. Then we can create another process P that, depending on the selector, outputs either the output of P_1 or P_2. All possible preparations of such a procedure will form an ensemble. Therefore we are justified in equipping an ensemble space with a mixing operation that takes a real number from zero to one, and two ensembles.

Given that mixing represents an experimental relationship, and all experimental relationships must be continuous in the natural topology, mixing must be a continuous function. Note that p is a continuously ordered quantity, where no value is perfectly experimentally verifiable, and therefore the natural topology is the one of the reals. This justifies continuity.

If $p = 1$, the output of P will always be the output of P_1. This justifies the identity property. If P_1 and P_2 are the same process, then the output of P will always be the output of P_1. This justifies the idempotence property. The order in which the processes are given does not matter as long as the same probability is matched to the same process. The process P is identical under permutation of P_1 and P_2. This justifies commutativity. If we are mixing three processes P_1, P_2 and P_3, as long as the final probabilities are the same, it does not matter if we mix P_1 and P_2 first or P_2 and P_3. This justifies associativity. ☐

Informal intuitive statement

(something that makes sense to a physicist or an engineer)

Formal requirement

(something a mathematician will find precise)

Show that the formal requirement follows from the intuitive statement

Principle of scientific objectivity. Science is universal, non-contradictory and evidence based.

⇒ Science is about statements that are associated to experimental tests

Statements must be either true or false for everybody

Tests may or may not terminate (i.e. be conclusive)

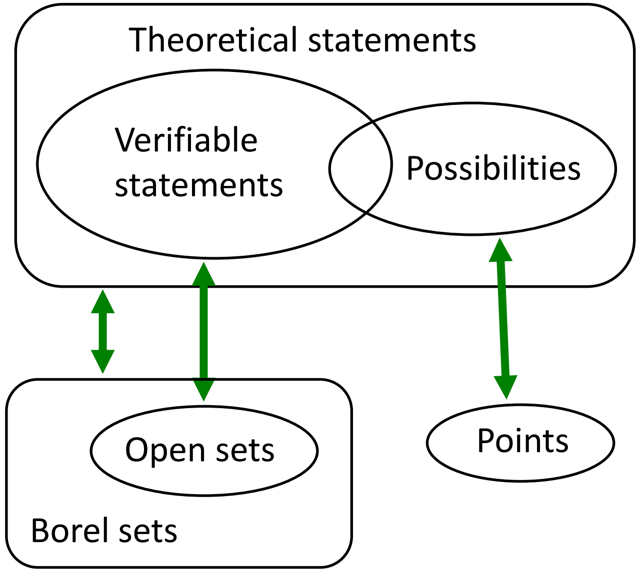
Statement	Test Result
T	SUCCESS (in finite time)
	UNDEFINED
F	UNDEFINED
	FAILURE (in finite time)

Verifiable statement	Test Result
T	SUCCESS (in finite time)
F	UNDEFINED
	FAILURE (in finite time)

Topology and σ-algebra

Experimental verifiability $\Rightarrow$ topology and σ-algebras (foundation of geometry, probability, …)

Perfect map between math and physics



s_1	Test Result
T	SUCCESS (in finite time)
T	UNDEFINED
F	UNDEFINED
F	FAILURE (in finite time)

$int(A)$ corresponds to the verifiable part of a statement

∂A corresponds to the undecidable part of a statement

$ext(A)$ corresponds to the falsifiable part of a statement

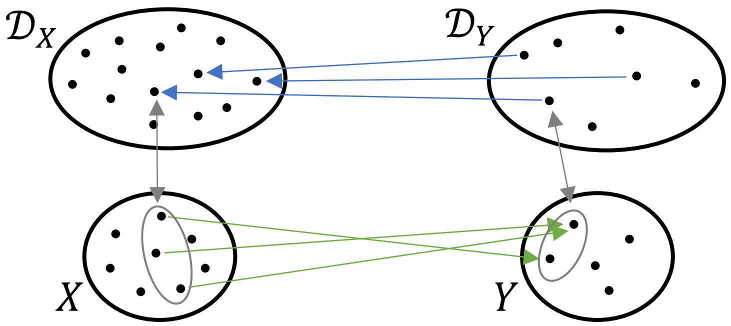
NB: in physics, topology and σ-algebra are parts of the **same** logic structure

Open set $(509.5, 510.5) \Leftrightarrow$ Verifiable "the mass of the electron is 510 ± 0.5 KeV"

Closed set $[510] \Leftrightarrow$ Falsifiable "the mass of the electron is exactly 510 KeV"

Borel set $\mathbb{Q}$ $(int(\mathbb{Q}) \cup ext(\mathbb{Q}) = \emptyset) \Leftrightarrow$ Theoretical "the mass of the electron in KeV is a rational number" (undecidable)

Inference relationship $r: \mathcal{D}_Y \to \mathcal{D}_X$ such that $r(s) \equiv s$



Causal relationship $f: X \to Y$ such that $x \preccurlyeq f(x)$

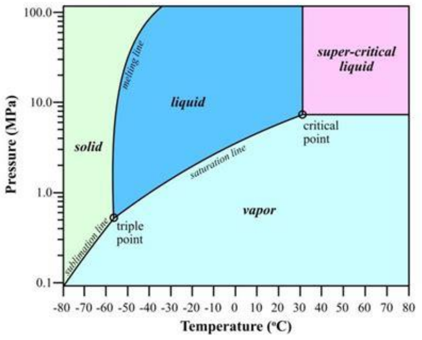
Inference relationship

$\Leftrightarrow$

Causal relationship

Relationships must be topologically continuous

Topologically continuous consistent with analytic discontinuity on isolated points



Phase transition $\Leftrightarrow$ Topologically isolated regions

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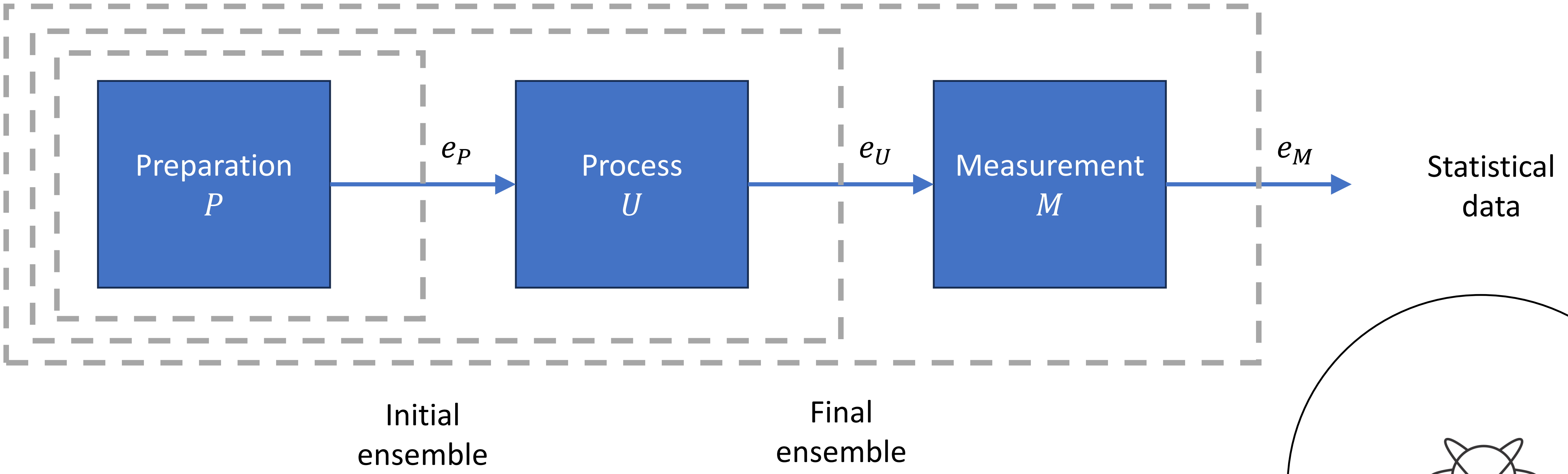
Assumptions of Physics

Principle of scientific reproducibility. Scientific laws describe relationships that can always be experimentally reproduced.

⇒ Scientific laws are relationships between ensembles



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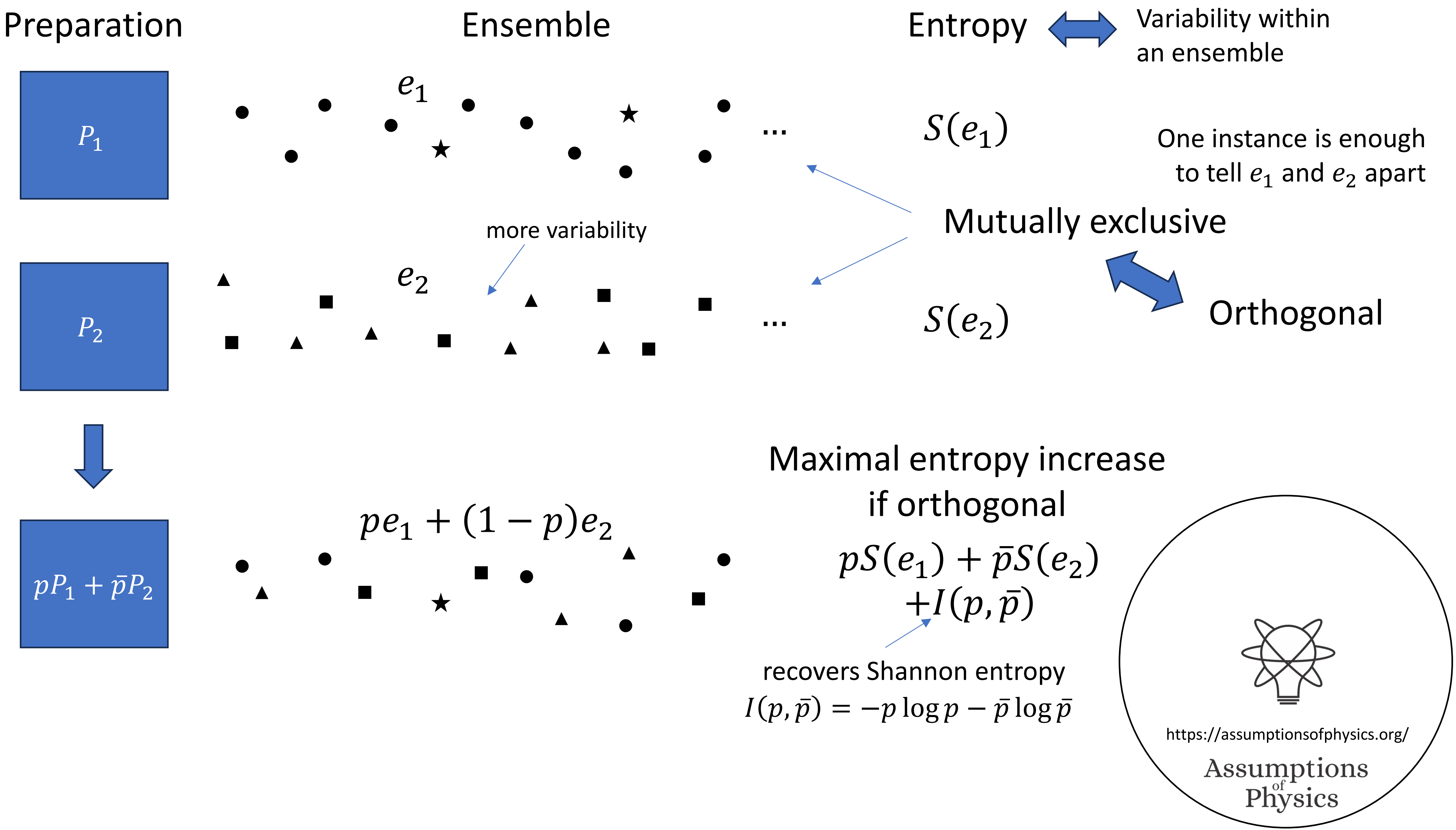
Assumptions of Physics


Preparation
Ensemble
Entropy
Variability within an ensemble
P_1
e_1
...
$S(e_1)$
One instance is enough to tell e_1 and e_2 apart
more variability
Mutually exclusive
P_2
e_2
...
$S(e_2)$
Orthogonal
Maximal entropy increase if orthogonal
$pP_1 + \bar{p}P_2$
$pe_1 + (1-p)e_2$
$pS(e_1) + \bar{p}S(e_2)$
$+I(p,\bar{p})$
recovers Shannon entropy
$I(p,\bar{p}) = -p\log p - \bar{p}\log\bar{p}$
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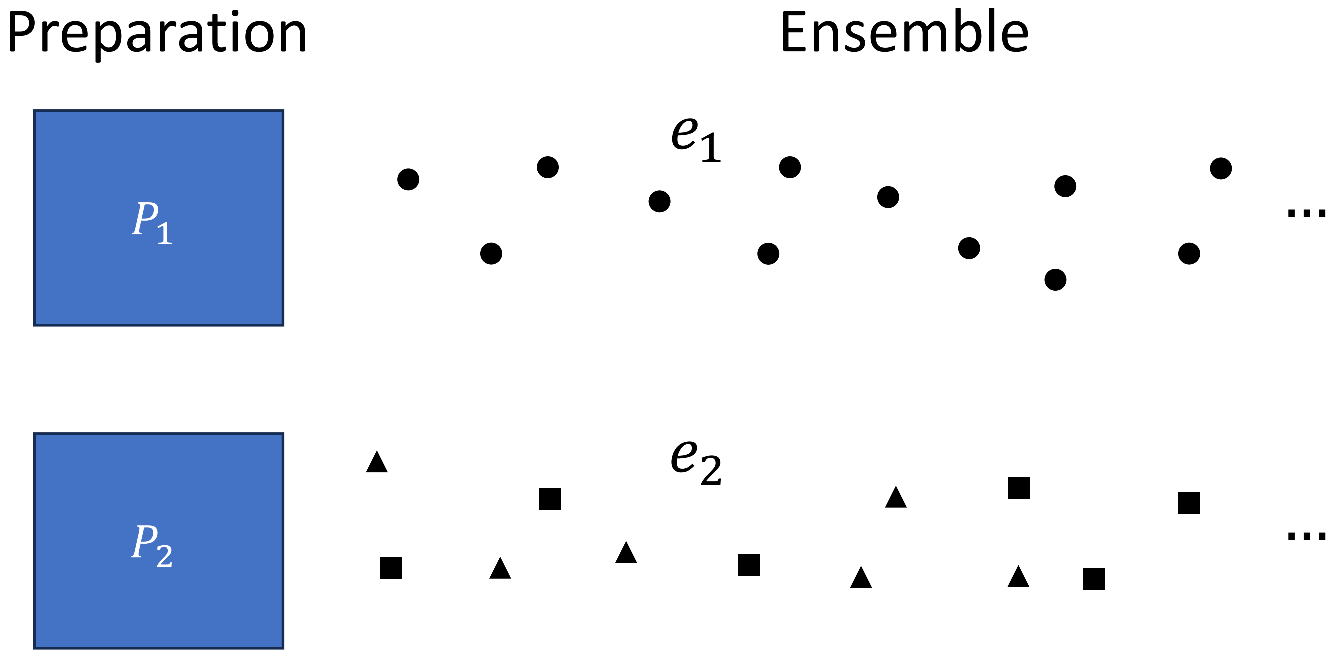
Assumptions of Physics



Identically prepared ensemble

There is no variability in the ensemble

Pure ensemble

No process can reduce the variability of the ensemble

There is no a such that $e_2 = pa + \bar{p}b$ for some $p \in (0,1)$ and $b \neq a$

In classical theories, all pure ensembles are identically prepared

No real distinction between ensembles and instances

In non-classical theories, no identically prepared ensembles

Zero entropy does NOT correspond to identically prepared ensembles

Minimal requirements for an ensemble space

Axiom 1.4 (Axiom of ensemble). *The state of a system is represented by an **ensemble**, which represents all possible preparations of equivalent systems prepared according to the same procedure. The set of all possible ensembles for a particular system is an **ensemble***

Experimental verifiability ⇒ Topology

Responsible for all topological structures

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble.*

Ensembles can be mixed ⇒ Convex structure

Responsible for all linear structures

Axiom 1.21 (Axiom of entropy). *Every element of the ensemble is associated with an **entropy** which quantifies the variability of the preparations of the ensemble. Formally, an*

Ensemble variability ⇒ Entropy

Responsible for all geometric structures

Still trying to find the right starting point for Poisson/commutators structure

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Assumptions of Physics

Some general results/constructions

Theorem 1.25 (Uniqueness of entropy). *The entropy of the coefficients $I(p,\bar{p})$ is the Shannon entropy. That is, $I(p,\bar{p}) = -\kappa\,(p\log p + \bar{p}\log\bar{p})$ where $\kappa > 0$ is the arbitrary multiplicative constant for the entropy. For a mixture of arbitrarily many elements, $I(\{p_i\}) = -\kappa\sum_i p_i \log p_i$.*

The maximal entropy increase $I(p,\bar{p})$ is uniquely determined, independently of physical theory

Ensembles embed into a vector space
(if continuously, we have foundation for calculus)

Theorem 1.65 (Differences from a vector space). *Let $\mathsf{a} \in \mathcal{E}$ be an interior point and let $V = \{[r(\mathsf{b}-\mathsf{a})]\}$ be the set of equivalence classes of ensemble differences from a. Then V is a vector space under the scalar multiplication and addition.*

$$MS(\mathsf{a},\mathsf{b}) = S\left(\frac{1}{2}\mathsf{a} + \frac{1}{2}\mathsf{b}\right) - \left(\frac{1}{2}S(\mathsf{a}) + \frac{1}{2}S(\mathsf{b})\right)$$

Entropy increase during mixing generalizes Jensen-Shannon Divergence (pseudo-distance)

Hessian of the entropy generalizes Fisher-Rao metric from information geometry

$$g_{\mathsf{e}}(\delta\mathsf{e}_1, \delta\mathsf{e}_2) = -\frac{\partial^2 S}{\partial \mathsf{e}^2}(\delta\mathsf{e}_1, \delta\mathsf{e}_2).$$

Definition 1.85. *Let $\mathsf{e} \in \mathcal{E}$ be an ensemble and $A \subseteq \mathcal{E}$ a Borel set. The* ***fraction capacity*** *of A for e is the biggest fraction achievable with convex combinations of A. That is,* $\mathrm{fcap}_{\mathsf{e}}(A) = \sup(\mathrm{frac}_{\mathsf{e}}(\mathrm{hull}(A)) \cup \{0\})$.

Non-additive generalization of probability

Generalization of
$S(\rho_U) = \log \mu(U)$

Definition 1.156. *Let $U \subseteq \mathcal{E}$ be the subset of an ensemble space. The* ***state capacity*** *of U is defined as* $\mathrm{scap}(U) = \sup(2^{S(\mathrm{hull}(U))})$ *if $U \neq \varnothing$ and* $\mathrm{scap}(U) = 0$ *otherwise.*

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Assumptions of Physics

The problem with counting on the continuum

We'd like:

1. Every state is a single case (i.e. $\mu(\{\psi\}) = 1$)
2. Finite continuous range carries finite information (i.e. $\mu(U) < \infty$)
3. Count is additive for disjoint sets (i.e. $\mu(\cup U_i) = \sum \mu(U_i)$)

Incompatible!

Pick two!

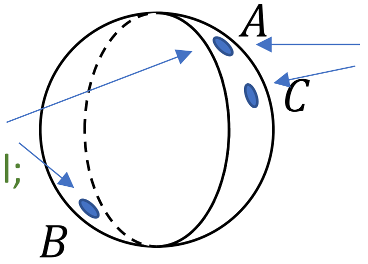
Discard 1 $\Rightarrow$ Lebesgue measure   Discard 2 $\Rightarrow$ counting measure

Discard 3 $\Rightarrow$ "Quantum measure"

$$\mu(U) = 2^{\sup\left(S(\mathrm{hull}(U))\right)}$$

Orthogonal states: different states all else equal; mutually exclusive

additive



sub-additive

Non-orthogonal states: different states but in different contexts; not mutually exclusive

$\mu(\{A\}) = 2^0 = 1$

$\mu(\{A, B\}) = 2^1 = 2$

$\mu(\{A, C\}) < 2 = \mu(\{A\}) + \mu(\{C\})$

Exponential of the maximum entropy reachable with convex combinations (statistical mixtures) of U (reduces to counting/Liouville measure)

Quantum mechanics $\Rightarrow$ lower bound on #conf (entropy) on continuous DOF

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Assumptions of Physics

JOHN TEMPLETON FOUNDATION
Inspiring Awe & Wonder

Conjecture: quantum gravity ⇒ lower bound on DOF count

$$\#\text{conf} = \#\text{DOFs}\ \frac{\#\text{conf}}{\#\text{DOFs}}$$

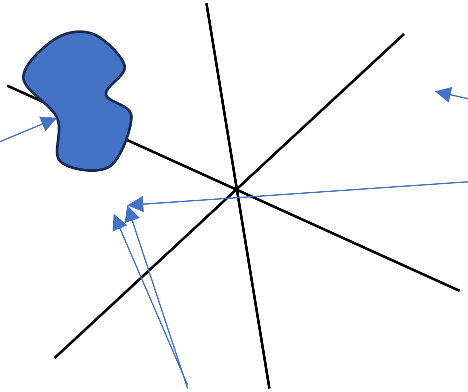
Lower bound on this…

…requires a lower bound on this

Same problem!

1. Every point is a single DOF (i.e. $\mu(\{x\}) = 1$)
2. Finite volume carries finitely many DOFs (i.e. $\mu(U) < \infty$)
3. Count is additive for disjoint regions (i.e. $\mu(\cup\, U_i) = \sum \mu(U_i)$)

#DOF ~ Spatial volume $\int_U \sqrt{-g}\, d^3x$



Distant points: independent DOFs

additive

sub-additive

Close points: DOFs not independent

From QM: Lower bound on state count requires a severe revisitation of particle state space

Does lower bound on DOF count require an equally severe revisitation of space-time?

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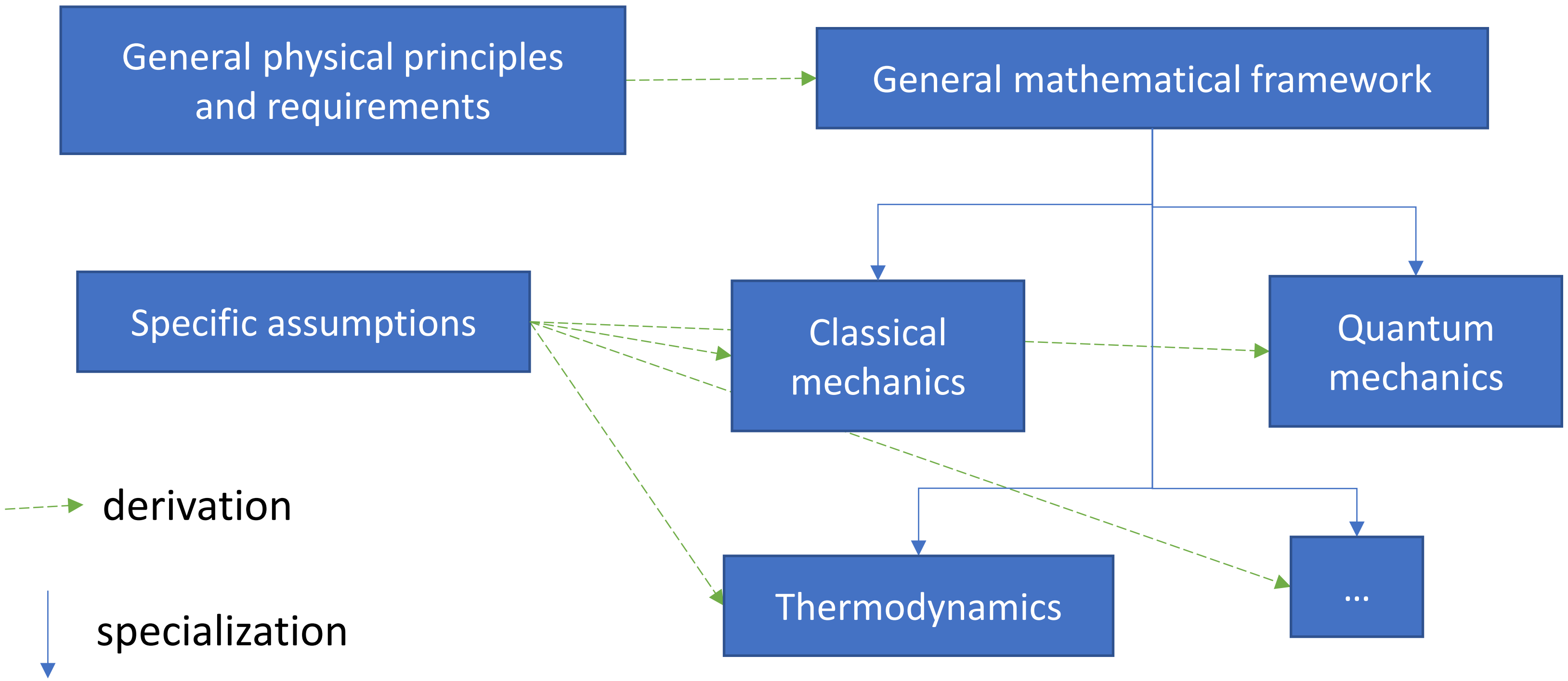
Assumptions of Physics

Experimental verifiability: topologies/σ-algebras
Ensembles: convex space, entropy

Connections to: measure theory, vector spaces, functional analysis, differential geometry, …



Foundations of physics
⇕
The theory of physical models

It must be a concerted effort across physics, math, information theory, philosophy, …

… and I can't know everything!

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Assumptions of Physics

Assumption of Physics is an open project

Our main output is an open access book: https://assumptionsofphysics.org/book/

All our material is developed on GitHub: https://github.com/assumptionsofphysics

One YouTube channel dedicated to publicize results: https://www.youtube.com/user/gcarcassi

Livestream discussions

Another YouTube channel dedicated to research: https://www.youtube.com/@AssumptionsofPhysicsResearch

Activities coordinated through a Discord server (contact me for an invite)

Always looking for experts to gain insights and/or help

Always looking for collaborations

Always looking for editors/journals/conferences that are sympathetic to the mission

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Assumptions of Physics

Wrapping it up

- Different approach to the foundations of physics
  - No interpretations, no theories of everything: physically meaningful starting points from which we can rederive the laws and the mathematical frameworks they need
- Reverse physics (reverse engineer principles from the known laws)
  - Classical mechanics is “completed”; very good ideas for both thermodynamics and quantum mechanics; still do not know how to generalize to field theories
- Physical mathematics (rederive the mathematical structures from scratch)
  - Topology and σ-algebras are derived from experimental verifiability; Good progress on a generic theory of states
- The goal is ambitious and requires a wide collaboration
  - Always looking for people to collaborate with in physics, math, philosophy, …

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Assumptions of Physics

To learn more

- Project website
  - https://assumptionsofphysics.org for papers, presentations, …
  - https://assumptionsofphysics.org/book for our open access book (updated every few years with new results)
- YouTube channels
  - https://www.youtube.com/@gcarcassi
    Videos with results and insights from the research
  - https://www.youtube.com/@AssumptionsofPhysicsResearch
    Research channel, with open questions and livestreamed work sessions
- GitHub
  - https://github.com/assumptionsofphysics
    Book, research papers, slides for videos...

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Assumptions of Physics

Principle of scientific objectivity. Science is universal, non-contradictory and evidence based.

⇒ Science is about statements that are associated to experimental tests

Statements must be either true or false for everybody

Tests may or may not terminate (i.e. be conclusive)

Statement	Test Result
T	SUCCESS (in finite time)
	UNDEFINED
F	UNDEFINED
	FAILURE (in finite time)

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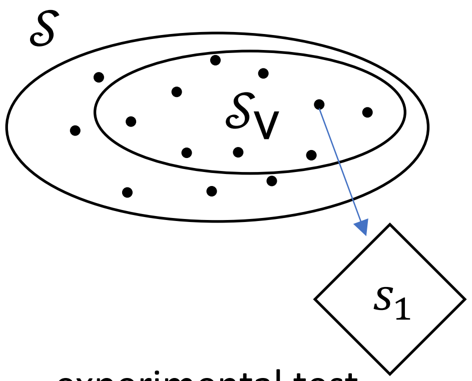
Assumptions of Physics

Axiom 1.27 (Axiom of verifiability). *A **verifiable statement** is a statement that, if true, can be shown to be so experimentally. Formally, each logical context $\mathcal{S}$ contains a set of statements $\mathcal{S}_v \subseteq \mathcal{S}$ whose elements are said to be verifiable. Moreover, we have the following properties:*

- *every certainty $\top \in \mathcal{S}$ is verifiable*
- *every impossibility $\bot \in \mathcal{S}$ is verifiable*
- *a statement equivalent to a verifiable statement is verifiable*

Remark. The **negation or logical NOT** of a verifiable statement is not necessarily a verifiable statement.

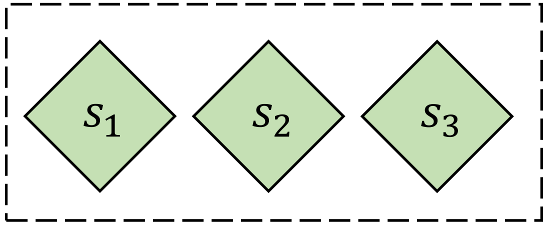


experimental test

outside the formal system

s_1	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED

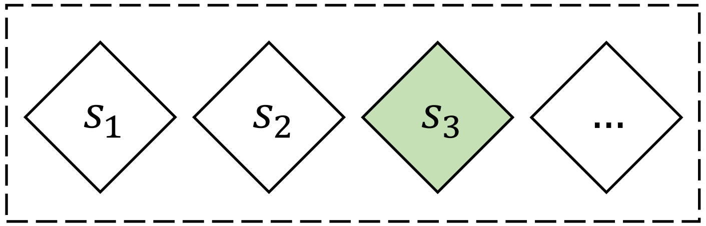
$$\bigwedge_{i=i}^{n} s_i$$



All tests must succeed

Axiom 1.31 (Axiom of finite conjunction verifiability). *The conjunction of a finite collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{n} \subseteq \mathcal{S}_v$ be a finite collection of verifiable statements. Then the conjunction $\bigwedge_{i=1}^{n} s_i \in \mathcal{S}_v$ is a verifiable statement.*

$$\bigvee_{i=1}^{\infty} s_i$$



One successful test is sufficient

Axiom 1.32 (Axiom of countable disjunction verifiability). *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_v$ be a countable collection of verifiable statements. Then the disjunction $\bigvee_{i=1}^{\infty} s_i \in \mathcal{S}_v$ is a verifiable statement.*

$\Rightarrow$ Verifiable statements form a frame/Heyting algebra

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Assumptions of Physics

Axiom 1.32 (Axiom of countable disjunction verifiability). *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let* $\{\mathsf{s}_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_{\mathsf{v}}$ *be a countable collection of verifiable statements. Then the disjunction* $\bigvee_{i=1}^{\infty} \mathsf{s}_i \in \mathcal{S}_{\mathsf{v}}$ *is a verifiable statement.*

Disjunction (OR) of verifiable statements: check that ONE test terminates successfully

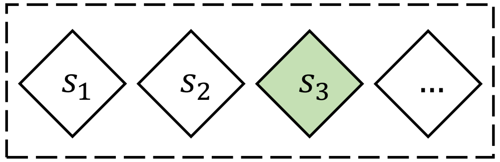
s_1	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED

watch out for non-termination!

$\vee\,(e_i)$:

1. Initialize n to 1
2. For each $i = 1 \dots n$
   a) Run e_i for n seconds
   b) If e_i succeeds, return SUCCESS
3. Increment n and go to 2

$\bigvee_{i=1}^{\infty} s_i$



$\Rightarrow$ Only countable disjunction can reach all tests

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Assumptions of Physics