Responses to reviewer comments on PHYSSCR-110477, G. Carcassi and C.A. Aidala  
May 25, 2020

Referee: 1  
  
COMMENTS TO THE AUTHOR(S)  
[[Some identifying information has been removed from this report by the Physica Scripta editorial team]]  
  
Dear Authors:  
  
I am skeptical of the articles claim that space-time structure is an extensive aspect of topology (a direct formulation of mathematical analysis).  But most clearly the authors are in contact with the essence of experimental verifiability; to keep in mind that topological properties require causal effect (of measurement) to be of importance to physical theory.   
  
The authors formal explanation, i.e., formal systems, is consistent but incomplete.  But as the physical world follows principles of quantum theory -- at the atomic and cosmological scale, not all measurements are whole and/or complete.  Yet the authors are competent in acknowledging this.  For physical properties are not the central importance of any experimental effect, rather its dynamics of motion within an 'experimental apparatus.'  
  
That be said, the authors demonstrate experimental competence consistent for further research within the confines of the scientific method; especially, the experimental process.  
  
I will "hesitantly" but agree to have the article accepted for publication in the Institute of Physics journal, Physica Scripta.

We thank the first referee for the positive recommendation for publication. We hope that the modifications made to the manuscript in more direct response to the second reviewer would also serve to improve it in the eyes of the first. The modifications seek to clarify certain points and to relate the mathematical structures more explicitly to familiar notions of physical space-time structure.

Board Member: 2  
  
COMMENTS TO THE AUTHOR(S)  
This work presents a foundational view of spacetime structure that I found very interesting. It makes us rethink some of the fundamental concepts of physical reality that we so often take for granted, and I, therefore, think it is of interest to the physics community.  
  
We thank the second reviewer for their interest in the work and their recommendation of further consideration for publication after clarifications. We hope that the modifications described below address the points raised such that the manuscript is deemed suitable for publication in Physica Scripta.

My only concern is that the work delves so much into topological structures that the main subject of the study, spacetime structures, have been given less focus and exposition. Given that the mathematical foundations of the problem have been discussed in greater detail in there previous work (Ref [8]), I would have like to see more concrete examples of the physics of spacetime structures discussed here.

In order to increase the focus on the physics of space-time structures, text has been added in several sections.

We have added the following paragraph near the end of the Introduction:

“We will concentrate on a single real line (i.e.~total ordering) and not address higher dimensional spaces specifically. Since space-time in relativity is four dimensional, it would seem we are ignoring the most interesting cases, but it is not so. The issue is that time itself plays a special role: in every local frame, we must be able to use the time coordinate as the affine parameter for the evolution of a particle by writing $x^i=x^i(x^0)$. As we mentioned before, functions need to be, at the very least, topologically continuous or they would break the notion of experimental verifiability (i.e.~verifiable statements would not be mapped to other verifiable statements). This means that in any theory that includes time evolution, the topology one gives to time will severely constrain the topology of the space within which time evolution takes place. Moreover, in any theory of space-time we have the additional constraint that time and spatial coordinates can be mixed. Therefore the argument will work in reverse as well: the topology one gives to space has to be the same as the one of time because one may use spatial distance as a time parameter. For example, two spaceships drifting at constant velocity may use their spatial separation as a clock.”

The following paragraph has been added to Section 1.1:

“The idea is that the quantities themselves are not a priori objects, but a construction built upon a set of references that form our system of measurement. A reference frame in space-time, then, is also a construction consisting of fixed elements (e.g. the stars, the borders of the experiment table, the elements of a timing system in a particle accelerator, ...) and signals exchanged among them. The topology of space-time is then an idealized characterization of the set of all possible such constructions, that is, all possible reference systems. Our goal is to understand the extent and the limitation of such idealization.”

The following text has been added to the very end of Section 1.1:

“At large scale, geometrical structure will need to emerge to recover the established theories. The point is that these large scale geometric structures cannot emerge from other geometrical structures, as these would necessarily suffer from the same fundamental problems.”

The following text has been added to the end of Section 5.3 on refinable references:

“If we take the traditional manifold structure of space-time literally, this would mean having at our disposal ever shrinking references that can be placed anywhere in space and time. For example, it would require being able to create a timing system with as many well separated pulses as desired distributed to as many places as desired. All of this, without changing the nature of the process we are studying.”

Moreover, I think the claim that "there are clear ﬁxed ordering relationships between the references. For example, every time something is before one reference it will also be after the other" is problematic for it assumes the absolute time and absolute space paradigm of Newtonian physics. We know from relativistic physics that the order of events may not necessarily be respected in all reference frames. I think some clarity is needed here.

In order to address this point, the following two paragraphs have been added to the end of Section 5.2 on aligned references:

“To give a better understanding of how these definitions work in a multidimensional setting, let us see how they apply to a Cartesian frame with coordinates $(x, y, z)$. A reference for the $x$ coordinate would be something like $r\_1=$(``$x < 0$'',``$0 \leq x \leq 2$'', ``$ x > 2$''). Note that it partitions the whole space in three disjoint regions. The on case acts as a divider and extends throughout $y$ and $z$. The reference $r\_2=$(``$y < 0$'',``$0 \leq y \leq 1$'', ``$y > 1$'') is not aligned with $r\_1$ as something can be before $r\_1$ and still be before, on or after $r\_2$. On the other hand, $r\_3=$(``$x < -1$'',``$-1 \leq x \leq 1$'', ``$x > 1$'') is aligned with $r\_1$ even though it is not either before or after $r\_1$ because the on regions overlap. Note that these references would not have such a simple expression in, say, spherical coordinates. Yet the regions themselves are coordinate independent and, therefore, the relationships between them are too.

What happens is that each reference frame would have its own set of references that can be nicely expressed using its coordinates. The before/after relationships expressed using one set of references may not be simply expressible using another, precisely because they are not aligned with each other. Relativistically, the references used for time by one frame will not be aligned with the ones of a boosted frame: the coordinate surfaces do not partition (i.e. do not foliate) space-time in the same way. Reference alignment essentially captures these various requirements with one simple formal definition.”

Another minor observation I noticed is that the abstract talks of a "talk" where the results of the work are being shown. Sure thing, this work might have been given as a talk somewhere, but it's being presented as a "paper" here.

We thank the reviewer for catching this. It has been corrected.