## Affine and Vector Spaces

**LA1⋄1.** Suppose $\ell_1$ and $\ell_2$ are skew lines in the space $\mathbb{R}^3$. Is it true that lines $PQ$, where $P \in \ell_1$, $Q \in \ell_2$, sweep the whole space?

**LA1⋄2.** Find a basis of the vector space $V = \{p(x) \in \mathbb{R}_4[x] : p'(5) = 0\}$.

**LA1⋄3.** Find a dimension and a basis of the vector space
(a) of all symmetric matrices $A \in \mathrm{Mat}_n(\mathbb{R})$
(b) of all skew-symmetric matrices $A \in \mathrm{Mat}_n(\mathbb{R})$.
(c) $\mathfrak{sl}_n(\mathbb{R}) = \{A \in \mathrm{Mat}_n(\mathbb{R}) : \mathrm{tr}\, A = 0\}$.

**LA1⋄4.** Give an example of a finite dimensional space $V$ and three its pairwise transversal subspaces $U, W, T$ (that is, intersecting only at the origin) such that $\dim U + \dim W + \dim T = \dim V$, but $U + W + T \neq V$.

**LA1⋄5.** Suppose $\dim(U + V) = \dim(U \cap V) + 1$ for some two vector subspaces $U, V \subset \mathbb{R}^n$. Is it true that $U + V$ equals one of the subspaces $U, V$ and $U \cap V$ is equal to another?

**LA1⋄6.** Is it possible that the intersection of the positive orthant $\{(x_1, x_2, x_3, x_4) : x_1, x_2, x_3, x_4 \geq 0\} \subset \mathbb{R}^4$ with some 2-dimensional plane is a square?

**LA1⋄7.** Let $V$ be a vector space of dimension $n$ over the finite field $\mathbb{F}_q$ of $q$ elements. How many
(a) vectors (b) bases (c) $k$-dimensional subspaces
are there in $V$?

**LA1⋄8.** Let $V$ be an affine space of dimension $n$ over the finite field $\mathbb{F}_q$ of $q$ elements. How many $k$-dimensional affine subspaces are there in $V$?

**LA1⋄9.** For which $c \in \mathbb{R}$ the hyperplane $\sum x_j = c$ intersects with
(a) the three dimensional cube $I^3 = \{(x_1, x_2, x_3) \in R^3 : |x_j| \leq 1\ \forall j\}$
(b) the four dimensional cube $I^4 = \{(x_1, x_2, x_3, x_4) \in R^4 : |x_j| \leq 1\ \forall j\}$ ?
Please draw all the polygons and polyhedra respectively that are cut from the cube by such hyperplanes.

**LA1⋄10.** Prove that the vector space of all continuous functions on $\mathbb{R}$ is infinite dimensional.