Transformations Groups

LA2⋄1. Suppose $T_v, S_H, R_{v,\varphi} \in \mathrm{Isom}(\mathbb{R}^3)$ are a parallel translation on a vector v, a reflection on the hyperplane H and a rotation on an angle φ around the line with a directing vector v. Find out when are true written below equality, and in all cases when they are correct, express the parameters of the motion on the right-hand side through the parameters of the movements from the left
a) $S_{H_1} \circ S_{H_2} = R_{v,\varphi}$, b) $S_{H_1} \circ S_{H_2} = T_v$, c) $S_H \circ R_{v,\varphi} \circ S_H = R_{u,\psi}$,
d) $R_{v_1,\varphi_1} \circ R_{v_2,\varphi_2} = T_v \circ R_{u,\psi}$, e) $R_{u,\varphi} \circ S_{H_1} \circ R_{u,-\varphi} = S_{H_2}$.

LA2⋄2. Prove that $\mathrm{SO}_2(\mathbb{R})$ is an abelian group, but $\mathrm{O}_2(\mathbb{R})$ is not.

LA2⋄3. Find the linear span of the group $\mathrm{SL}_n(\mathbb{R})$ in the vector space $\mathrm{Mat}_n(\mathbb{R})$.

LA2⋄4. Suppose $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ is some motion, i.e. $F \in \mathrm{Isom}(\mathbb{R}^3)$. Find these motions:
a) $F \circ T_v \circ F^{-1}$, b) $F \circ S_H \circ F^{-1}$, c) $F \circ R_{v,\varphi} \circ F^{-1}$.

LA2⋄5. Find the motion $S_{e_1} \circ \ldots \circ S_{e_n}$ in n-dimensional vector space V.

LA2⋄6. Suppose $F\colon \mathbb{R}^4 \to \mathbb{R}^3$ is a factorization of $\mathbb{R}^4$ by the line $\ell = \langle e_1 + e_2 + e_3 + e_4 \rangle$. Find the image of the 4-dimensional unit cube.