Trigonometric & Hyperbolic Substitutions in Integrands

In each case it is supposed that a is a positive number. The forms involving $\sqrt{x^2-a^2}$ can be done using different secant substitutions depending upon the definition of $\sec^{-1}$ that is used.

Trigonometric Substitutions

Form	Substitution	New Expression
$\sqrt{a^2-x^2}$	$x=a\cdot\sin\theta$ $\theta=\sin^{-1}\left(\frac{x}{a}\right)$	$dx=a\cdot\cos\theta\,d\theta$ $\sqrt{a^2-x^2}=a\cdot\cos\theta$
$\sqrt{a^2+x^2}$	$x=a\cdot\tan\theta$ $\theta=\tan^{-1}\left(\frac{x}{a}\right)$	$dx=a\cdot\sec^2\theta\,d\theta$ $\sqrt{a^2+x^2}=a\cdot\sec\theta$
$\sqrt{x^2-a^2}$	$x=a\cdot\sec\theta$ $\theta=\sec^{-1}\left(\frac{x}{a}\right)$ If $\sec^{-1}\left(\frac{x}{a}\right)=\cos^{-1}\left(\frac{a}{x}\right)$	$dx=a\cdot\sec\theta\tan\theta\,d\theta$ $\sqrt{x^2-a^2}=a\cdot\operatorname{sgn}(x)\cdot\tan\theta$

Hyperbolic Substitutions

Form	Substitution	New Expression
$\sqrt{a^2-x^2}$	$x=a\cdot\tanh\theta$ $\theta=\tanh^{-1}\left(\frac{x}{a}\right)$	$dx=a\cdot\operatorname{sech}^2\theta\,d\theta$ $\sqrt{a^2-x^2}=a\cdot\operatorname{sech}\theta$
$\sqrt{a^2+x^2}$	$x=a\cdot\sinh\theta$ $\theta=\sinh^{-1}\left(\frac{x}{a}\right)$	$dx=a\cdot\cosh\theta\,d\theta$ $\sqrt{a^2+x^2}=a\cdot\cosh\theta$
$\sqrt{x^2-a^2}$	$x=a\cdot\coth\theta$ $\theta=\coth^{-1}\left(\frac{x}{a}\right)$	$dx=-a\cdot\operatorname{csch}^2\theta\,d\theta$ $\sqrt{x^2-a^2}=a\cdot\operatorname{sgn}(x)\cdot\operatorname{csch}\theta$