$-5-$ $\frac{\int F(x)}{G'(x)}$ ate

# Summary Solving ODEs

$\frac{dy}{dx} = \frac{-F_x}{F_y}$

## Linear

1. Rewrite in the form

$$\frac{dy}{dx} + (terms\ of\ x)y = (terms\ of\ x)$$

2. Determine the integrating factor, $\mu(x) = e^{\int P(x)dx}$, where $P(x)$ is the factor multiplied by $y$ above.

3. Multiply the equation by the integrating factor.

4. Integrate left side using implicit differentiation: You should have:

$$\frac{d}{dx}[\mu(x) * y] = \mu(x) * right\ side$$

5. Integrate the equation with respect to $x$. Do this by removing the $\frac{d}{dx}$ from the left side and adding the $\int dx$ notation to the right. There will be a constant of integration in the result.

6. Solve for $y(x)$.

$\frac{dy}{dx} + \frac{1}{x}y = x$, $P(x) = \frac{1}{x}$

$\mu(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$x\frac{dy}{dx} + \frac{x}{x}y = x \cdot x \rightarrow xy' + 1 \cdot y = x^2$

$\underbrace{\int xy' + 1 \cdot y}_{\text{undo Productrule}} \overset{dx}{=} \int x^2\,dx$

$\frac{\cancel{x} \cdot y}{\cancel{x}} = \frac{x^3}{3}\frac{1}{x}$, $y = \frac{x^2}{3} + C$

## Separable

1. Rewrite in the form

$$\frac{dy}{dx} = f(x) \cdot g(y)$$

2. Then rewrite as

$$(terms\ of\ y)dy = (terms\ of\ x)dx$$

3. Integrate both sides:

$$\int (terms\ of\ y)dy = \int (terms\ of\ x)dx$$

4. Write constants $(+C)$ to independent variable side

5. Solve for $y(x)$, the **explicit** solution. If this is not possible, then your result is an **implicit** solution.

exact: $\frac{da}{dt} = \frac{-a^2 + 2t}{2ta} \rightarrow \overset{N}{(2ta)}da + \overset{M}{(a^2 + 2t)}dt = 0$

$\frac{\partial M}{\partial a} = 2a = \frac{\partial N}{\partial t}$

$\int N = a + C_a$

$\int M = a^2t + t^2 + C_t = t(a^2 + t) + C_t$

$a + C_a = t(a^2 + t) + C_t$

$C = t(a^2 + t) - a$

$\int M(x,y)dx + \int \left(N(x,y) - \left(\frac{\partial}{\partial y}\int M(x,y)dx\right)\right)dy$

## Exact

$F_x + F_y\frac{dy}{dx} = 0$

1. Rewrite in the form

$$Mdx + Ndy = 0, \quad Ndy = -Mdx$$

$F_y dy + F_x dx = 0$

indep vars, dep var

$$(terms\ of\ x,y)dx + (terms\ of\ x,y)dy = 0$$

2. The equation is **exact** if the partial derivative of $M$ with respect to $y$ equals the partial derivative of $N$ with respect to $x$:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

3. Integrate $M$ with respect to $x$ and use $g(y)$ as the constant.

4. Integrate $N$ with respect to $y$ and use $g(x)$ as the constant.

> drop X terms in N

5. Set step 3 and 4 equal to each other

6. Anything that is the same on left and right side of equal sign are in the original equation.

7. Anything "extra" represents the constants of $g(x)$ and $g(y)$.

8. The general solution is formed by adding everything together and setting equal to $C$.

...ccurred, determine the amount of salt in the tank

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get $y^{(n)}$ of degrees wanted
Plug in each $y^{(n)}$ and solve

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(\_)}{n!}(x-\_)$$

$$\frac{d}{dx}[2xy] = 2y + 2xy'$$

ra

# 2nd order ODEs

homogeneous

$F(t) = 0$

$$ay'' + by' + cy = 0$$
$$ar^2 + br + c = 0 \quad \leftarrow y(t) = e^{rt}$$

| distinct roots | repeated roots | complex roots $r = \alpha \pm \beta i$ |
|---|---|---|
| $y_1 = e^{r_1 t}$, $y_2 = e^{r_2 t}$ | $y_1 = e^{rt}$, $y_2 = te^{rt}$ | $y_1 = e^{\alpha t}\cos(\beta t)$ |
| $y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$ | $y = C_1 e^{rt} + C_2 t e^{rt}$ | $y_2 = e^{\alpha t}\sin(\beta t)$ |
| | | $y = C_1 e^{\alpha t}\cos(\beta t) + C_2 e^{\alpha t}\sin(\beta t)$ |

$y'' + 6y' + 9y = 0$, $y(0) = -1$, $y'(0) = 2$

$r^2 + 6r + 9 = 0$

$r = -3$

$-1 = C_1 e^{-3(0)} + \cancel{C_2(0)(e^{-3(0)})}$, $C_1 = -1$

$y' = 3e^{-3t} + C_2[e^{-3t} - t3e^{-3t}]$

$2 = 3 + C_2$, $C_2 = -1$

$y = -e^{-3t} - te^{-3t}$

For IV's solve ~~___~~ y for either constant then get y', solve, find other constant.

Method of undetermined coeffs

$g(x)$

$Ax^n + Bx^{n-1} + C\ldots \rightarrow$ Polynomials

$\sin x/\cos x \rightarrow A\cos x + B\sin x$

$e^x \rightarrow Ae^x$

$e^x(\sin x/\cos x) \rightarrow Ae^x\cos x + Be^x\sin x$

$e^x + x^2 \rightarrow Ae^x + Bx^2 + Cx + D$ mix of $e^x$ & Polynomials

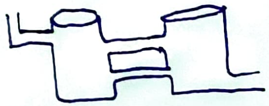
$x^n(\sin x/\cos x) \rightarrow$ (Polynomial)$\cos x$ + (Polynomial)$\sin x$

$\int \cot x\,dx = \ln|\sin x|$

$\int \sec^2 x\,dx = \tan x$

$\int \csc x\,dx = \ln\left|\tan\left(\frac{x}{2}\right)\right|$

$$\frac{d}{dx}(\ln x) = \frac{x'}{x}$$



$\frac{dx}{dt} = \ldots$, $\frac{dy}{dt} = \ldots$

eigenval: $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda I\right) = \det\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = (a-\lambda)(\lambda-d) - cb = 0$

eigenvector: Plug in $\lambda$'s and rref, $\lambda \rightarrow t$ for final function for vectors

$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} a & b & 0 \\ c & d & 0 \end{array}\right] \rightarrow Z(t) = C_1 e^{\lambda_1 t}\begin{bmatrix} x \\ y \end{bmatrix} + C_2 e^{\lambda_2 t}\begin{bmatrix} x \\ y \end{bmatrix}$$

Mixing 'formula'

$$\frac{dx}{dt} = \frac{q^{\text{weight}}}{\text{time}} \cdot \frac{B\ \text{measure}}{\text{weight}} - \frac{x(t)}{\text{Size} + (\text{in-out difference})t} \cdot \text{outflow}$$

$$\frac{dx}{dt} = \text{flowin} - \frac{x(t) \cdot \text{outflow}}{\text{tank size} + (\text{flow difference})t}$$