Section 13: Nonhomogeneous Second Order DE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2nd Order ODEs

$$ay'' + by' + cy = f(t)$$

Homogenous
$f(t) = 0$

Nonhomogenous
$f(t) = not\ 0$

2 (Distinct) Real	2 (Repeated) Real	2 (Conjugate) Complex
$y_1(t) = e^{r_1 t}$ $y_2(t) = e^{r_2 t}$	$y_1(t) = e^{rt}$ $y_2(t) = te^{rt}$	$r = \alpha \pm i\beta$ $y_1(t) = e^{\alpha t}\cos \beta t$ $y_2(t) = e^{\alpha t}\sin \beta t$
$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$	$y(t) = c_1 e^{rt} + c_2 te^{rt}$	$y(t) = c_1 e^{\alpha t}\cos \beta t + c_2 e^{\alpha t}\sin \beta t$

Use homogenous solutions combined with Good Guessing (formally called Method of Undetermined Coefficients)

Trial Solutions for the Method of Undetermined Coefficients

	Form of $g(x)$	Guess for particular solution
1.	1 (any constant)	A
2.	$5x + 7$	$Ax + B$
3.	$3x^2 - 2$	$Ax^2 + Bx + C$
4.	$\sin 4x$	$A\cos 4x + B\sin 4x$
5.	$\cos 4x$	$A\cos 4x + B\sin 4x$
6.	e^{5x}	Ae^{5x}
7.	$(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
8.	$x^2 e^{5x}$	$(Ax^2 + Bx + C)e^{5x}$ increase Polynomial if not applicable
9.	$e^{3x}\sin 4x$	$Ae^{3x}\cos 4x + Be^{3x}\sin 4x$
10.	$5x^2 \sin 4x$	$(Ax^2 + Bx + C)\cos 4x + (E^2 + Fx + G)\sin 4x$
11.	$xe^{3x}\cos 4x$	$(Ax + B)e^{3x}\cos 4x + (Cx + E)e^{3x}\sin 4x$
12.	$(5x + 7) + \sin 4x$	$(Ax + B) + (C\cos 4x + D\sin 4x)$
13.	$e^x + x^3$	$Ae^x + Bx^3 + Cx^2 + Dx + E$