Making Accurate O–C Diagrams

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Abstract. In order to study fine period changes in extrinsic variables we developed and tested a special method for zero phase timing exploiting maximum phase information from observations of various kind using phenomenological modelling. Results of this sophisticated method are demonstrated on several recently studied binary systems.

1. O-C diagrams and their accuracy

Most of variable stars show cyclic or more or less periodic light variations. Light changes are repeating with the period $P(t)$ that is the time interval between consecutive moments of the same selected phase (mostly the phases of light curve extrema). As the first approximation we can assume that the period is constant $P(t) = P_0$. Then we can predict moments $C(E)$ when the selected photometric phase will occur or occurred in the past by the simple relation: $C(E) = M_0 + E \times P_0$, where E is an integer called 'epoch' and $M_0 = C(E = 0)$. The differences between the real observed time $O(E)$ of the certain phase of the light curve and the corresponding calculated time in the same epoch $C(E)$ are called (O-C) values. Their dependence on epoch or time is then known as the **O-C diagram**. The diagram belongs to a few of the most significant astrophysical diagrams and serves namely for the fine diagnostics of the period changes and the ephemeris improvements.

$$P(t) = \frac{\mathrm{d}O}{\mathrm{d}E};\ P_0 = \frac{\mathrm{d}C}{\mathrm{d}E};\ \Delta P(t) = P(t) - P_0 = \frac{\mathrm{d(O\text{-}C)}}{\mathrm{d}E} \doteq \frac{\mathrm{d(O\text{-}C)}}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}E} = P_0\frac{\mathrm{d(O\text{-}C)}}{\mathrm{d}t}. \quad (1)$$

The reliability of the models of the variability of individual variable stars critically depends on the accuracy of the time evolution of the observed period $P(t)$ and hence on the preciseness of the basic O-C diagrams. The credibility of O-C diagrams rely primarily on the reliability of the used determinations of extrema times and good knowledge of their uncertainties.

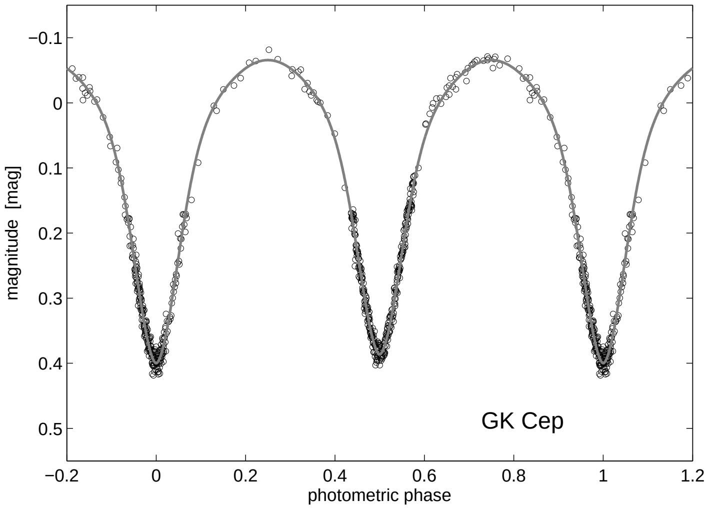

Figure 1. The yet unpublished phase diagram of the close eclipsing binary GK Cephei fitted by the phenomenological model of the light curve. The fit is sufficiently good for our purposes.

Unfortunately, techniques of extrema times determination are as a rule rather obsolete - mostly they were based on the fit of a parabola or other symmetric functions through the observed parts of light curves. Such extrema times determinations give correct results only exceptionally, mostly they are corrupted from several reasons. At first these techniques of extrema derivations malfunction in the case of incomplete or asymmetric light curves. Given uncertainties of the found moments are usually incredible because of the fitted curves are not similar to real light curves. The main insufficiency of these standard methods consists in the fact that they do not exploit the fact that the light curve is periodic!

We developed a new revolutionized method which processes all available measurements simultaneously and uses instead of vague moment of extrema more precise phase shift via the template light curve. Applying the method the real accuracy of O-C determination is enhanced as a rule 2-3 times!! This approach firstly outlined in Mikulášek et al. (2006) has proved to be competent not only in the case when we have at our disposal a set of unbroken measurements in the vicinity of light curve extrema but mainly if we treated individual measurements coming from various surveys of Hipparcos or ASAS observations which were obtained in times not depending on the extrema timing of the particular object.

2. Mathematical basis of the method

Let we do the period analysis of a variable star on the basis using its n individual photometric observations divided into N arbitrary groups. It is useful to describe belongings of individual measurements to particular groups by a matrix $N \times n$ of zeros and ones, with an matrix element δ_{ik}. If the i-th ($i = 1, 2, ...n$) measurement belongs in the k-th

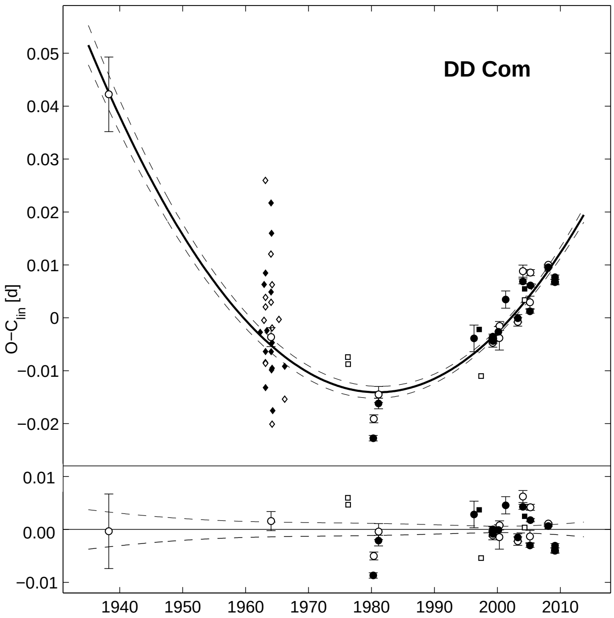

Figure 2. O-C diagram of strongly interacting close eclipsing binary DD Com - taken from Zhu et al. (2010).

($k = 1, 2, ...N$) group then $\delta_{ik} = 1$, otherwise $\delta_{ik} = 0$. Let t_i is $\mathrm{JD_{hel}}$ moment of the i-th observation, y_i is the measured magnitude and w_i is the weight of the i-th measurement.

The observed magnitudes create a light curve expressed by the function F. The predicted (calculated) magnitude y_{pi} for the i-th measurement is then determined by the function value $F(\mathbf{a}, \vartheta_i)$, where $\mathbf{a} = [a_1, a_2, ...a_j, ...a_g]$ is the vector of g parameters describing the shape of the light curve, and ϑ_i is the phase function for time t_i corrected for $(\text{O-C})_k$, mean O-C of the k-th group: $\vartheta_i = \left(t_i - M_0 - \sum_{k=1}^{N} \delta_{ik}\, (\text{O-C})_k\right)/P_0$, where M_0 and P_0 are the starting parameters of the linear ephemeris of the variable star.

Totally $(g + N)$ free parameters $\mathbf{a}$ and $\{(\text{O-C})_k\}$ can be found by the weighted non-linear least square method assuming that the weighted sum of the quadrat of deflections S is minimal:

$$S = \sum_{i=1}^{n} \left[y_i - F(\mathbf{a}, \vartheta_i)\right]^2 w_i; \quad \delta S = \mathbf{0}; \quad \Rightarrow \tag{2}$$

$$\sum_{i=1}^{n} \Delta y_i \frac{\partial F}{\partial a_j} w_i = 0; \tag{3}$$

$$\sum_{i=1}^{n} \Delta y_i \frac{\partial F}{\partial \vartheta_i} \frac{\partial \vartheta_i}{\partial (\text{O-C})_k} w_i = -\sum_{i=1}^{n} \Delta y_i \frac{\partial F}{\partial \vartheta_i} \frac{\delta_{ik}}{P_0} w_i = 0; \tag{4}$$

what led to the g ($j = (1, 2, ...g)$, see Eq. 3) plus N (see Eq. 4) equations. The system of equations has to be solved iteratively by generalised Newton-Raphson or Levenberg-

Marquart methods. The procedure results in determination of g parameters **a** and N $(\text{O-C})_k$ values essential for the setting of the O-C diagram including the estimate of their uncertainties.

The situation simplifies substantially if we know the shape of the light curve(s) before, for example from the analysis of a set of very good photometric observations of the variable done in the past, or from the good solution of the physical model of variability of the star. In that case we can omit the g equations of light curves parameters (see Eq. 3) and to solve N equations (Eq. 4) directly. Then we obtain $\{(\text{O-C})_k\}$ and a well defined estimate of their uncertainties $\delta(\text{O-C})_k$:

$$(\text{O-C})_k = -P_0 \frac{\sum_{i=1}^{n} \delta_{ik} \Delta y_i \frac{\partial F}{\partial \vartheta_i} w_i}{\sum_{i=1}^{n} \delta_{ik} \left(\frac{\partial F}{\partial \vartheta_i}\right)^2 w_i}; \quad \delta(\text{O-C})_k = s \sqrt{\frac{\sum_{i=1}^{n} w_i}{n \sum_{i=1}^{n} \delta_{ik} \left(\frac{\partial F}{\partial \vartheta_i}\right)^2 w_i}}, \tag{5}$$

where s is the weighted standard deviation of one measurement in magnitudes. Weights are normalised so that $\overline{w}=1$.

3. Some applications of the method to eclipsing binaries

3.1. Phenomenological model of the light curves of eclipsing binaries

The best template light curves $F(\theta)$ can be obtained using physical models of eclipsing binaries in PHOEBE or Wilson-Deviney codes. We used this approximation in the case of very short period eclipsing binaries DD Com and AR Aur. We could use also the phenomenological model of light curve. We developed a bundle of eclipsing binaries light curves models which are able to describe majority of real cases more than satisfactorily using minimum of free parameters. Example of model for close eclipsing binary with zero eccentricity (minima at phases 0, 0.5), proximity effects, O'Connell effect and eclipses (even U shape minima) applicable for most of close binaries has only 7 (!) free parameters.

$$F(\vartheta) = m_0 + a_1 \left\{1 - \left[1 - \exp\left(-\frac{\varphi_{\text{I}}^2}{d^2}\right)\right]^C\right\} + a_2 \left\{1 - \left[1 - \exp\left(-\frac{\varphi_{\text{II}}^2}{d^2}\right)\right]^C\right\} + \tag{6}$$

$$+a_3\, \cos(4\pi\vartheta) + a_4 \left[\tfrac{5}{16}\sin(2\pi\vartheta) - \tfrac{5}{32}\sin(6\pi\vartheta) + \tfrac{1}{32}\sin(10\pi\vartheta)\right], \tag{7}$$

$$\text{where} \quad \varphi_{\text{I}} = \vartheta - \text{round}(\vartheta); \quad \varphi_{\text{II}} = \vartheta - \text{floor}(\vartheta) - \tfrac{1}{2}. \tag{8}$$

m_0 is the mean magnitude outside of eclipses, a_1 and a_2 are the depths of the primary and the secondary eclipses, d is the parameter describing the width of eclipses, C determines the sharpness of eclipses, a_3 is the semi-amplitude of proximity changes outside eclipses and a_4 is the amplitude of the O'Connell effect.

The model is appropriate for prevailed majority of eclipsing binaries. Very simply we can adopt the model for the case of eclipsing binaries with eccentric orbit and the apsidal motion.

3.2. β Lyrae star GK Cephei

We used the model for the β Lyrae type GK Cephei with initial ephemeris $M_0 = 2\,438\,694.706$, $P_0 = 0\overset{\text{d}}{.}936\,164$. We analysed the Hipparcos survey observations in

Table 1. GK Cep - new calculation of times of light curve minima.

E	O [HJD]	O-C [d]	filter	n	author
10412	2 448 442.0342	-0.01130(46)	*Hp*	106	Hipparcos
16010	2 453 682.7038	0.01220(41)	*R*	137	
16241.5	2 453 899.4269	0.01325(17)	*V*	291	
17486	2 455 064.4733	0.00360(35)	neutral	87	

Hp colour and three observational sets - two from the vicinity od the primary minimum and one mapping the secondary eclipse.

At first we realised that the O'Connell effect is very small (if any) so we could neglect it. Then we found the following parameters of the light curve: a_1 = 0.334(10), a_2 = 0.321(10), and a_3 = 0.0656(55) mag, parameter d = 0.0693(19), and C = 0.882(25). Fig. 1 displays that the phenomenological model fit of the observed light curve optimally and can serve as the template function for the reliable determination of times of minima. We found at the same time the following times of minima and their uncertainties which couldserve for further period analysis.

3.3. W UMa star DD Comae Berenices

DD Comae Berenices ($P = 0\overset{\mathrm{d}}{.}26920811$) is the very short period close (VSP) interacting binary with apparent changes in its orbital period. Zhu et al. (2010) collected all available photometric data about the system with the emphasis on the individual observational data. We treated them simultaneously using the model light curves computed by the 2003 version of Wilson–Van Hamme code as templates $F(c, \vartheta)$. The computation of the model light curves in all needed colors were based on two precise complete $(IR)_{\mathrm{C}}$ light curves obtained recently. We recalculated published times of light minimum and added new ones from our own to construct O-C diagram which spans over 70 years.

Using LSM orthogonal quadratic model function, we found that the orbital period of DD Com is continuously increasing with $\dot{P} = 0.00401(22)\,\mathrm{s\,y^{-1}}$, which corresponds to the conservative mass transfer from the secondary component to the primary one at a rate of $\dot{M} = 7.2 \times 10^{-8}\,\mathrm{M_{\odot}\,yr^{-1}}$.

With the period increase, the binary is evolving from the present shallow contact phase to the broken stage predicted by the thermal relaxation oscillation (TRO) theory. Compared with other VSP systems, DD Com is a rare system which lies on the expanding phase of the TRO cycle. Till now, only four such systems including DD Com are found in this stage. Thus, this target is another good observational proof of the TRO theory in very short period region.

3.4. Eclipsing binary with the third body AR Aurigae

Very young eclipsing binary AR Aur consisting of two HgMn chemically peculiar stars shows well defined light time effect caused by a third body in the system. Using template light curves derived from the model of Johansen (1970) we determined timings of 61 light minima (both primary and secondary ones) and extended them by 36 times of minima adopted from O-C databases. See the O-C diagram plotted on the Fig. 3.4The efficiency of this approach is substantiated by the fact that the scatter of the published

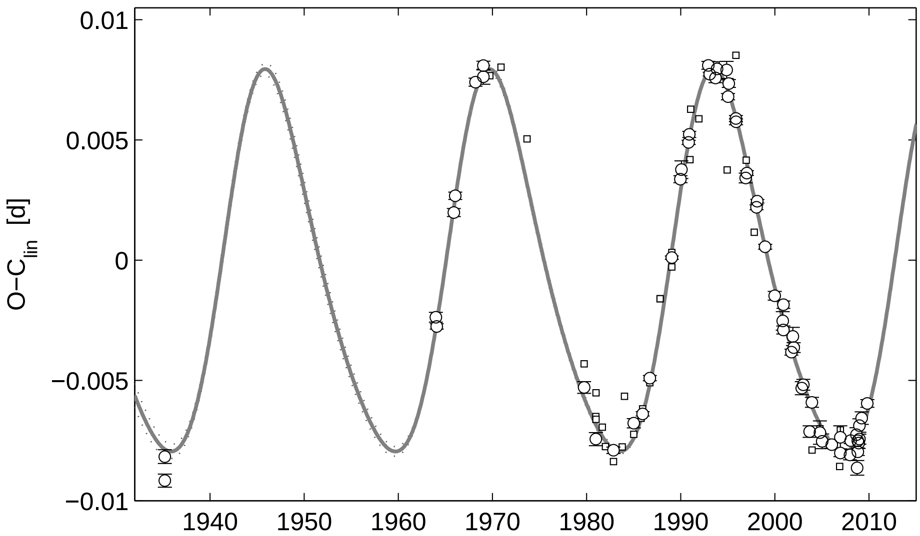

Figure 3. The textbook O-C diagram of the eclipsing binary AR Aurigae with the third body in the system. Taken from Mikulášek et al. (2010)

times of minima is more than 2.5 times larger than the scatter of the values obtained by our method.

We found new values of LiTE period and compared them with parameters obtained earlier. We improved the orbital of the third body to $P_3 = 23.79 \pm 0.09$ yr and found another eccentricity of its orbit $\varepsilon = 0.320 \pm 0.016$.

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