Let $p = \frac{a}{b}, q = \frac{b}{c}, r = \frac{c}{a}$ be rational numbers.

Notice that we don't make the problem stronger with this substitution since $\forall p, q, r \in \mathbb{Q} \exists a, b, c \in \mathbb{Z} : (p, q, r) = \left(\frac{a}{b}, \frac{b}{c}, \frac{c}{a}\right)$

Now, observe that $\frac{b}{a} = \frac{1}{\frac{a}{b}} = \frac{1}{p} = \frac{pqr}{p} = qr$, thus $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = pq + qr + rp$. Furthermore, the sum $p+q+r$ is by definition an integer and the product pqr is one, also an integer. Thus, by Vietta, p, q, r are the roots of a unary polynomial with integer coefficients, and therefore, integers. Thus, since $pqr = 1$, we have $p, q, r = \pm 1$.